Simulating Generalized Inverse Gaussian

Consider the density of W:

$$h(w) = \frac{\chi^{-\lambda/2}\,\psi^{\lambda/2}}{2K_\lambda(\sqrt{\chi\psi})}\, w^{\lambda-1}\, e^{-\frac{1}{2}(\chi w^{-1}+\psi w)}$$

Now let $U = \sqrt{\psi/\chi}\; W$

Then the density function of U is

$$g(u) = h\left(u\sqrt{\chi/\psi}\right)\sqrt{\chi/\psi}$$

$$= \frac{u^{\lambda-1}}{2K_\lambda(\sqrt{\chi\psi})}\, e^{-\frac{1}{2}\sqrt{\psi\chi}\,(u^{-1}+u)}$$

This function now only has 2 parameters

$$\omega = \sqrt{\chi\psi}\,,\ \lambda.$$

$$g(u) = \frac{u^{\lambda-1}}{2K_\lambda(\omega)}\, e^{-\frac{1}{2}\omega(u^{-1}+u)}$$

To simplify it even further, $Y = \log U$. Then the density becomes

$$\phi(y) = \frac{e^{\lambda y - \cosh y\cdot\omega}}{2K_\lambda(\omega)}\,,\ y \in \mathbb{R}$$

Note that $y \to -y$ or $U \to U^{-1}$ means

$$\lambda \to -\lambda$$

The proof of the uniform speed of the rejection algorithm with this choice of (s, t) (Theorem 2) is given in the Appendix.

THEOREM 2. *[UNIFORM BOUNDEDNESS] With the above choice for (t, s), the expected number of iterations in the rejection method is not more than*

$$\max\left(\frac{1}{1-\exp(-0.34114777\ldots)}, \frac{e}{1-e^{-e}}\right) = 3.459655\ldots.$$

For the sake of completeness, we summarize the random variate generator in its entirety.

[Generator for the GIG distribution with parameters $\omega > 0, \lambda \geq 0$]
[Functions needed]
  Define $\psi(x) = -\alpha(\cosh x - 1) - \lambda(e^x - x - 1)$
  Define $\psi'(x) = -\alpha \sinh x - \lambda(e^x - 1)$
[Set-up]
Set $\alpha = \sqrt{\omega^2 + \lambda^2} - \lambda$
If $-\psi(1) \in [1/2, 2]$, then set $t = 1$
  else if $-\psi(1) > 2$, then set $t = \sqrt{2/(\alpha+\lambda)}$
  else [if $-\psi(1) < 1/2$], then set $t = \log(4/(\alpha + 2\lambda))$
If $-\psi(-1) \in [1/2, 2]$, then set $s = 1$
  else if $-\psi(-1) > 2$, then set $s = \sqrt{4/(\alpha \cosh 1 + \lambda)}$
  else [if $-\psi(-1) < 1/2$], then set $s = \min\left(\frac{1}{\lambda}, \log\left(1 + \frac{1}{\alpha} + \sqrt{\frac{1}{\alpha^2} + \frac{2}{\alpha}}\right)\right)$
Compute $(\eta, \zeta, \theta, \xi) = (-\psi(t), -\psi'(t), -\psi(-s), \psi'(-s))$
Set $(p, r) = (1/\xi, 1/\zeta)$
Compute $t' = t - r\eta$
Compute $s' = s - p\theta$
Compute $q = t' + s'$
[Generation]
Repeat Generate U, V, W uniformly on $[0, 1]$.
  If $U < q/(p+q+r)$ then set $X = -s' + qV$
  else if $U < (q+r)/(p+q+r)$ then set $X = t' + r\log(1/V)$
  else set $X = -s' - p\log(1/V)$
Until $W\chi(X) \leq \exp(\psi(X))$, where
  $\chi(x) = \mathbf{1}_{[[-s',t']]}(x) + \mathbf{1}_{[(t',\infty)]}(x)e^{-\eta-\zeta(x-t)} + \mathbf{1}_{[(-\infty,-s')]}(x)e^{-\theta+\xi(x+s)}$
Return $\left(\frac{\lambda}{\omega} + \sqrt{1 + \frac{\lambda^2}{\omega^2}}\right)e^X$

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