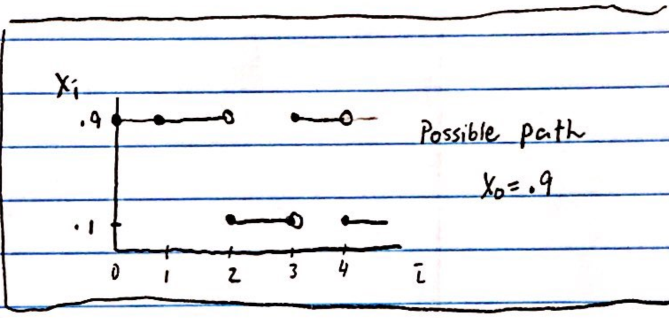
HW 7

3) $R_i = \begin{cases} 1 & p=\frac{1}{2} \\ 0 & p=0 \\ -1 & p=\frac{1}{2} \end{cases}$ $\qquad X_{i+1} = \begin{cases} X_i & R_i=1, & p=\frac{1}{2} \\ \sim U(0,1) & R_i=0, & p=0 \\ 1-X_i & R_i=-1, & p=\frac{1}{2} \end{cases}$ $\qquad X_0 \sim U(0,1)$

$P(X_{i+1} \le u) = \frac{1}{2}P(X_i \le u) + \frac{1}{2}P(X_i \ge 1-u) = u$ $\qquad X_{i+1}$ is uniform.

$$E[X_{i+1}X_i] = \frac{1}{2}\int_0^1 x^2\,dx + \frac{1}{2}\int_0^1 (x-x^2)\,dx$$
$$= \frac{1}{2}\left(\frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{2}-\frac{1}{3}\right)$$
$$= \frac{1}{6} + \frac{1}{12}$$
$$= \frac{1}{4}$$




$E(X_{i+1})E(X_i) = \frac{1}{4}$

$\mathrm{Corr}(X_{i+1}, X_i) = 0$ $\qquad$ *Like before, cov. stationary*

This really reduces to flipping a coin. I'm assuming $X_0$ is set to some value, say $X_0 \sim U(0,1)$ and then the sequence goes from there with $X_i$ either being $X_0$ or $1-X_0$ (heads/tails)

ex) Let $X_0 = .75$ $\qquad X_1 = \begin{cases} .75 & p=\frac{1}{2} \\ .25 & p=\frac{1}{2} \end{cases}$ $\qquad X_2 = \begin{cases} .75 & p=\frac{1}{2} \\ .25 & p=\frac{1}{2} \end{cases}$ ...

This comes from:

$X_2 = \begin{cases} X_1 & p=\frac{1}{2} \\ 1-X_1 & p=\frac{1}{2} \end{cases} \Rightarrow X_2 = \begin{cases} X_0 & p=\frac{1}{4} & .75 \\ 1-X_0 & p=\frac{1}{4} & .25 \\ 1-X_0 & p=\frac{1}{4} & .25 \\ 1-(1-X_0) & p=\frac{1}{4} & .75 \end{cases}$

So the $X$'s are really iid here!

$X_{i+1} = \begin{cases} X_0 & p=\frac{1}{2} \\ 1-X_0 & p=\frac{1}{2} \end{cases}$ $\qquad X_0 \sim U(0,1)$

It is SWN.