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E-M algorithm for t distribution (parameter ν)

As before, we have to compute

$$f_{W|X}(w|x) = \frac{f_{X|W} \cdot h}{f_X}$$

For the t distribution, this is

$$f_{W|X}(w|x) = \frac{1}{(2\pi)^{d/2}\sqrt{|\Sigma|}\, w^{d/2}} \exp\left[-\frac{(x-\mu)^T\Sigma^{-1}(x-\mu)}{2w}\right] \frac{(\nu/2)^{\nu/2} e^{-\nu/(2w)} w^{-\nu/2-1}}{\Gamma(\frac{1}{2}\nu)}$$

$$\times \frac{(\pi\nu)^{d/2}\sqrt{|\Sigma|}\,\Gamma(\nu/2)}{\Gamma(\frac{\nu}{2}+\frac{d}{2})}\left[1+\frac{(x-\mu)^T\Sigma^{-1}(x-\mu)}{\nu}\right]^{\nu/2+d/2}$$

$$= \left[\frac{\nu+(x-\mu)^T\Sigma^{-1}(x-\mu)}{2}\right]^{\nu/2+d/2} \exp\left[-\frac{\nu+(x-\mu)^T\Sigma^{-1}(x-\mu)}{2w}\right] w^{-\nu/2-d/2-1}$$

so $W|X \sim IG(\alpha, \beta)$ with

$$\alpha = \frac{\nu}{2}+\frac{d}{2}, \quad \beta = \frac{\nu+(x-\mu)^T\Sigma^{-1}(x-\mu)}{2}$$

Now $E(W^{-1}|X) = \dfrac{\alpha}{\beta}$

$$E(\ln W|X) \approx \varepsilon^{-1}\left(\frac{\Gamma(\alpha-\varepsilon)\beta^{\varepsilon}}{\Gamma(\alpha)} - 1\right) \quad \varepsilon \text{ small}$$

So Step 2 now becomes

$$\alpha^{[k]} = \frac{\nu^{[k]}}{2} + \frac{d}{2}$$

$$\beta_i^{[k]} = \frac{\nu^{[k]}}{2} + \frac{(X_i - \mu^{[k]})^T (\Sigma^{[k]})^{-1} (X_i - \mu^{[k]})}{2}$$

$$\delta_i^{[k]} = E(W_i^{-1} \mid X_i ; \theta^{[k]}) = \frac{\alpha^{[k]}}{\beta_i^{[k]}}$$

$$\bar{\delta}^{[k]} = \frac{1}{n} \sum_{i=1}^{n} \delta_i^{[k]}$$

In Step 4, we want

$$\beta_i^{[k,2]} = \frac{\nu^{[k]}}{2} + \frac{(X_i - \mu^{[k+1]})^T (\Sigma^{[k+1]})^{-1} (X_i - \mu^{[k+1]})}{2}$$

$$\delta_i^{[k,2]} = \frac{\alpha^{[k]}}{\beta_i^{[k,2]}}, \quad \xi^{[k,2]} = \epsilon\left(\frac{\Gamma(\alpha^{[k]} - \epsilon)(\beta^{[k,2]})^{\epsilon}}{\Gamma(\alpha^{[k]})} - 1 \right)$$

ϵ small

In Step 5,

$$\sum_{i=1}^{n} \log h(W_i ; \nu) = \sum_{i=1}^{n} \left[\frac{\nu}{2} \log\left(\frac{\nu}{2}\right) - \frac{\nu}{2} W_i^{-1} - \left(\frac{\nu}{2} + 1\right) \log W_i - \log \Gamma\left(\frac{\nu}{2}\right) \right]$$

Substitute $\delta_i^{[k,2]}$ for W_i^{-1}
$\xi_i^{[k,2]}$ for $\log W_i$

Maximize over all ν

$$\theta^{[k+1]} = (\nu^{[k+1]}, \mu^{[k+1]}, \Sigma^{[k+1]})^T$$