

# Factor Models

$$F = \frac{\sqrt{p}}{1+p(d-1)} \sum_{i=1}^d X_i + \sqrt{\frac{1-p}{1+p(d-1)}} Y$$

$$\text{Var}(X_i) = 1 \quad E(X_i) = 0 \quad Y, X_i \text{ independent} \quad \text{Var}(Y) = 1$$

$$B = \begin{pmatrix} \sqrt{p} \\ \vdots \\ \sqrt{p} \end{pmatrix} \quad X = BF + \varepsilon \quad \text{Cov}(X_i, X_j) = p$$

$$\varepsilon_j = X_j - \sqrt{p} F$$

$$\begin{aligned} \text{Cov}(F, \varepsilon_j) &= \text{Cov}(F, X_j) - \sqrt{p} \text{Var}(F) \\ &= (d-1) \cdot \frac{\sqrt{p}}{1+p(d-1)} \cdot p + \frac{\sqrt{p}}{1+p(d-1)} \sqrt{p} \text{Var}(F) \end{aligned}$$

$$= \sqrt{p} - \sqrt{p} \text{Var}(F)$$

$$\text{Var Var}(F) = \frac{dp}{(1+p(d-1))^2} + d(d-1) \frac{p^2}{(1+p(d-1))^2} + \frac{(1-p)}{1+p(d-1)}$$

$$= \frac{dp}{(1+p(d-1))} + \frac{1-p}{1+p(d-1)} = 1$$

$$\Rightarrow \text{Cov}(F, \varepsilon_j) = 0.$$

$$\begin{aligned} \text{Cov}(\varepsilon_i, \varepsilon_j) &= \text{Cov}(X_i, X_j) - \sqrt{p} \text{Cov}(X_j, F) - \sqrt{p} \text{Cov}(X_i, F) \\ &\quad + p \text{Var}(F) \end{aligned}$$

$$= p - p = 0.$$