

HW 6 #1

1) Bernoulli RV (X_1, X_2)

$X_1 \backslash X_2$	0	1	X_2 marginal
0	.33	.11	.44
1	.26	.30	.56
X_1 marginal	.59	.41	

$$P(X_1 = x_1, X_2 = x_2) = P_{11}^{x_1 x_2} \left[\frac{x_1(1-x_2)}{P_{10}} \right] P_{01}^{(1-x_1)x_2} \left[\frac{(1-x_1)(1-x_2)}{P_{00}} \right]$$

$$\sum_{i=0}^1 \sum_{j=0}^1 P_{ij} = 1$$

$$P(X_1 = x_1) = [P_{10} + P_{11}]^{x_1} [P_{01} + P_{00}]^{1-x_1}$$

$$P(X_2 = x_2) = [P_{11} + P_{01}]^{x_2} [P_{10} + P_{00}]^{1-x_2}$$

$$\text{Cov}(X_1, X_2) = E\{[X_1 - E(X_1)][X_2 - E(X_2)]\} = E[X_1 X_2] - E[X_1]E[X_2]$$

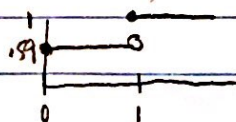
$$= E\{[X_1 - (P_{10} + P_{11})][X_2 - (P_{11} + P_{01})]\}$$

$$= E[X_1 X_2] + (P_{10} + P_{11})(P_{11} + P_{01}) - E[X_1](P_{11} + P_{01}) - E[X_2](P_{10} + P_{11})$$

$$= E[X_1 X_2] - E[X_1]E[X_2]$$

$$= .3 - .41(.56) = .0704$$

$$F_1(x) = \begin{cases} .59 & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases} \quad F_2(x) = \begin{cases} .44 & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$



$F(x_1, x_2) =$

$x_1 < 0, x_2 < 0$
 $x_1 < 0, 0 \leq x_2 < 1$
 $0 \leq x_1 < 1, x_2 < 0$
 $0 \leq x_1 < 1, 0 \leq x_2 < 1$
 $x_1 \geq 1, x_2 < 0$
 $x_1 \geq 1, 0 \leq x_2 < 1$
 $0 \leq x_1 < 1, x_2 \geq 1$
 $x_1 \geq 1, x_2 \geq 1$

	0	1
0	$F(x_1, x_2) = .33$	$F(x_1, x_2) = .44$
1	$F(x_1, x_2) = .59$	$F(x_1, x_2) = 1$

$$F(0,0) = C(F_{x_1}(0), F_{x_2}(0)) \quad F(1,0) = C(F_{x_1}(1), F_{x_2}(0))$$

$$F(0,1) = C(F_{x_1}(0), F_{x_2}(1)) \quad F(1,1) = C(F_{x_1}(1), F_{x_2}(1))$$

$$.33 = C(.59, .44)$$

$$.59 = C(.59, 1)$$

$$.44 = C(1, .44)$$

$$1 = C(1, 1)$$

$$U_1 = F_{x_1}(x_1)$$

$$U_2 = F_{x_2}(x_2)$$

$$C(u_1, u_2) = u_1 u_2 + .0704(1-u_1)(1-u_2)$$

$$C(u_1, u_2) = \min(u_1, u_2) - .4790940767(1-u_1)(1-u_2)$$