MATH - 6205 Assignment - 2

1. SDE Discretization for GBM and European Option Pricing

Your assignment number below is: M = [(Your student ID) mod 25].

Call Option

Use parameters: $r = 0.02 + 0.002 * M$, $\sigma = 0.25 + 0.005 * M$, $T = 0.5$, $\Delta t = 1/12$, $S(0) = 100$.

1.1 Generate N paths of a stock price $S(t)$ in risk neutral measure with constant volatility σ and risk-free rate r of the length T with time step Δt for Geometric Brownian Motion (GBM) SDE $dS = rSdt + \sigma SdW$ using:

a) exact discretization [G, pp. 81, 94] for the Brownian Motion $X(t) = \ln(S(t)/S(0)) = (r - \sigma^2/2)t + \sigma W(t)$

b) Euler scheme for $S(t)$ in the GBM SDE above [G, pp. 339-340].

Use the same seeds for a) and b). Calculate mean squared discretization error between a) and b) for $S(T)$ and plot a histogram for the lognormal distribution of $S(T)$ for case a) based on $N = 10{,}000$ paths. Plot first 10 paths for both methods a) and b) (with the same seeds). Discuss results.

1.2 Calculate the price for the European at-the-money stock option with maturity T based on Monte Carlo scenarios from 1.1 a) and compare with Black-Scholes price.