1.1. 163029

a) Sean $p_0, \dots, p_l$ vectores no nulos.

Sean $\alpha_i \in \mathbb{R}$ escalares $\ni$ $\sum_{i=1}^{l} \alpha_i p_i = 0$

Fijemos p_j. Entonces

$$0 = \left(\sum_{i=1}^{l} (\alpha_i p_i^T) \right) A p_j = \sum_{i=1}^{l} \alpha_i (p_i^T A p_j)$$

$$= \sum_{\substack{i=1 \\ i \neq j}}^{l} \alpha_i \cancel{p_i^T A p_j}^{0} + \alpha_j p_j^T A p_i$$

$$= \alpha_j (p_j^T A p_j) \qquad \forall j = 1, \dots, l$$

Como A es positiva definida, entonces $p_j^T A p_j > 0 \quad \forall j = 1, \dots, l$

$\Rightarrow \alpha_j = 0 \quad \forall j = 1, \dots, l$

$\therefore \{p_i\}_{i=1}^{l}$ es l.i.

b)
Tomamos n vectores p_i

Como $\{p_i\}_{i=1}^{n}$ es l.i., entonces $x^* - x_0 = \beta_1 p_1 + \dots + \beta_n p_n$, β_i únicos.

Fijemos p_j, entonces

$$p_j^T A (x^* - x_0) = p_j^T A \sum_{i=1}^{n} \beta_i p_i = \sum_{i=1}^{n} \beta_i p_j^T A p_i = \beta_j p_j^T A p_j$$

$$\Rightarrow \beta_j = \frac{p_j^T A (x^* - x_0)}{p_j^T A p_j}$$

que coincide con los α_j del método del gradiente conjugado.

En cada paso se calculan aproximaciones β_i

$\therefore$ Convergemos en a lo más n pasos.

1.2. 163029

$$B_{k+1} = (I - \rho_k y_k s_k^T) B_k (I - \rho_k s_k y_k^T) + \rho_k y_k y_k^T$$

$$H_{k+1} = (I - \rho_k s_k y_k^T) H_k (I - \rho_k y_k s_k^T) + \rho_k s_k s_k^T$$

Entonces

ecuación secante ecuación secante

$$B_{k+1} H_{k+1} = (B_{k+1} - \rho_k B_{k+1} s_k y_k^T) H_k (I - \rho_k y_k s_k^T) + \rho_k B_{k+1} s_k s_k^T$$

$$= (B_{k+1} - \rho_k y_k y_k^T) H_k (I - \rho_k y_k s_k^T) + \rho_k y_k s_k^T$$

$$B_{k+1} = B_k - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k} + \rho_k y_k y_k^T$$

$$= \left(B_k - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k}\right) H_k (I - \rho_k y_k s_k^T) + \rho_k y_k s_k^T$$

$$= \left(I - \frac{B_k s_k s_k^T}{s_k^T B_k s_k}\right)(I - \rho_k y_k s_k^T) + \rho_k y_k s_k^T$$

dado que $\rho_k \dfrac{B_k s_k s_k^T}{s_k^T B_k s_k} y_k s_k^T = \dfrac{1}{s_k^T y_k} \dfrac{B_k s_k (s_k^T y_k) s_k^T}{s_k^T B_k s_k} = \dfrac{B_k s_k s_k^T}{s_k^T B_k s_k}$

$$= I - \cancel{\frac{B_k s_k s_k^T}{s_k^T B_k s_k}} - \cancel{\rho_k y_k s_k^T} + \cancel{\frac{B_k s_k s_k^T}{s_k^T B_k s_k}} + \cancel{\rho_k y_k s_k^T}$$

$$= I$$

$$\therefore B_{k+1} H_{k+1} = I$$