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Examen final

Teoría: G.C.

- P.D. Si los vectores no nulos $p_1, p_2, \ldots, p_l$ satisfacen que:

$p_i^T A p_j = 0 \quad \forall i \neq j.$

Si A es simétrica y positiva definida, entonces los vectores son linealmente independientes

Contradicción:

Supongamos $\exists j, k \; \{a_i\}_{i=0}^{j}$ p_k ⋺

$$p_k = \sum_{i=0}^{j} a_i p_i$$

para a_i fijos. Entonces sea $h \neq j$

$$p_h^T A p_k = p_h^T A \left(\sum_{i=0}^{j} a_i p_i\right)$$

$$\Rightarrow \sum_{i=0}^{j} a_i p_h^T A p_i \overset{h,p}{\Rightarrow} p_h^T A p_h > 0$$

∇
O ∴ los p_i's son linealmente independientes. //

- Solución: Como $\mathbb{R}^n = \text{span}\{p_0, \ldots, p_{n-1}\}$ y $p_i^T A p_j = 0$ $\forall i \neq j$, sea x^* el óptimo ⋺ $Ax^* = b$

$\Rightarrow \exists \{\sigma_i\}_{i=0}^{n-1} \in \mathbb{R}$ ⋺ $x^* - x_0 = \sum_{i=0}^{n-1} \sigma_i p_i$

por ser conjugados se tiene que

$$\sigma_k = \frac{p_k^T A(x^* - x_0)}{p_k^T A p_k} = \frac{p_k^T (b - A x_0)}{p_k^T A p_k}$$

dado el método $x_{k+1} = x_k + a_k p_k$

$$\Rightarrow x_{k+1} = x_0 + a_0 p_0 + \dots + a_k p_k$$

se tiene que

$$p_k^T A(x_k - x_0 + x^* - x^*) = p_k^T A(x^* - x_k) + p_k^T \overset{b}{A}(x_k - x_0)$$

$$\therefore p_k^T A(x^* - x_0) = p_k^T A(x^* - x_k) = -p_k^T r_k$$

$$\therefore \sigma_k = \frac{p_k^T A(x^* - x_0)}{p_k^T A p_k} = -\frac{p_k^T r_k}{p_k^T A p_k} = a_k$$

converge en a lo más n pasos //

BFGS:

- Verificar que B_{k+1} y H_{k+1} son inversas

p.d $B_{k+1} H_{k+1} = I$

se tiene que $H_{k+1} = (I - \rho_k s_k y_k^T) H_k (I - \rho_k s_k y_k^T) + \rho_k s_k s_k^T$

$$B_{k+1} = B_k - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k} + \frac{y_k y_k^T}{y_k^T s_k}$$

y $B_{k+1} s_k = y_k$

$\Rightarrow$

$$B_{k+1} H_{k+1} = B_{k+1}(I - \rho_k s_k y_k^T) H_k (I - \rho_k y_k s_k^T) + \rho_k s_k s_k^T$$

$$= (B_{k+1} - \rho_k y_k y_k^T) H_k (I - \rho_k y_k s_k^T) + \rho_k y_k s_k^T$$

$$= \left(B_k - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k}\right) H_k (I - \rho_k y_k s_k^T) + \rho_k y_k s_k^T$$

$$= \left(I - \frac{B_k s_k s_k^T}{s_k^T B_k s_k}\right)(I - \rho_k y_k s_k^T) + \rho_k y_k s_k^T$$

$$= I - \rho_k y_k s_k^T - \frac{B_k s_k s_k^T}{s_k^T B_k s_k}(I - \rho_k y_k s_k^T) + \rho_k y_k s_k^T$$

$$= I - \underbrace{\frac{B_k s_k s_k^T}{s_k^T B_k s_k}(I - \rho_k y_k s_k^T)}$$

$$\frac{B_k s_k s_k^T}{s_k^T B_k s_k} \rho y_k s_k^T = \frac{1}{s_k^T y_k} \frac{B_k s_k (s_k^T y_k) s_k^T}{s_k^T B_k s_k} = \frac{B_k s_k s_k^T}{s_k^T B_k s_k}$$

$$\Rightarrow \frac{B_k s_k s_k^T}{s_k^T B_k s_k}(I - \rho_k y_k s_k^T) = \frac{B_k s_k s_k^T}{s_k^T B_k s_k} - \frac{B_k s_k s_k^T}{s_k^T B_k s_k} = 0$$

$\therefore$ $B_{k+1} H_{k+1} = I$ //