165860 Luis Carlos Bernal M.

1.-

a) Dem.:

Sea $A \in \mathbb{R}^{n \times n}$ simétrica. Sean $p_0, p_1, \dots p_l \in \mathbb{R}^n$ tales que:

$p_i^T A p_j = 0 \quad \forall \; i \neq j$

Sup. que $\exists \; \alpha_j \neq 0$ tal que

$0 = \sum_{i=1}^{l} \alpha_i p_i \; \therefore \; A\left(\sum_{i=1}^{l} \alpha_i p_i\right) = A(0) = 0$. Por otro lado

$p_h^T \alpha p_k = p_h^T A\left(\sum_{i=0}^{j} \alpha_i p_i\right) = \sum_{i=0}^{j} \alpha_i p_h^T A p_i = p_h^T A p_h = 0$ ▽

Como A es positiva definida $\Rightarrow \alpha_j p_j^T A p_j = 0 \iff$

$\alpha_j = 0$! $\therefore \{p_1, \dots, p_l\}$ son L.I.

b) Sea el sistema lineal $\sum_{i=1}^{l} \alpha_i p_i$, considerando este sistema y la sucesión $x_{k+1} = x_k + \alpha_k p_k$. Por Teo. el algoritmo de gradiente conjugado converge en a lo más n pasos. Esto es porque para algunas $\sigma_1, \dots, \sigma_{n-1}$

$x^* - x_0 = \sum_{i=0}^{n-1} \sigma_i p_i \neq 0$ //

1.2

Dem.:

$B_{k+1} s_k = y_k \Rightarrow$

$$B_{k+1} H_{k+1} = B_{k+1}\left[(I - \rho_k s_k y_k^T) H_k (I - \rho_k s_k y_k^T) + \rho_k s_k s_k^T\right] =$$

$$[B_{k+1} - \rho_k y_k y_k^T] H_k [I - \rho_k y_k s_k^T] + \rho_k y_k s_k^T =$$

$$\left[B_k - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k}\right] H_k [I - \rho_k y_k s_k^T] + \rho_k y_k s_k^T =$$

$$\left[I - \frac{B_k s_k s_k^T}{s_k^T B_k s_k}\right](I - \rho_k y_k s_k^T) + \rho_k y_k s_k^T =$$

$$I - \rho_k y_k s_k^T - \frac{B_k s_k s_k^T}{s_k^T B_k s_k}(I - \rho_k y_k s_k^T) + \rho_k y_k s_k^T =$$

$$I - \frac{B_k s_k s_k^T}{s_k^T B_k s_k}(I - \rho_k y_k s_k^T)$$

Como:

$$\frac{B_k s_k s_k^T}{s_k^T B_k s_k} \rho_k y_k s_k^T = \frac{B_k s_k s_k^T}{s_k^T B_k s_k} \Rightarrow$$

$$\frac{B_k s_k s_k^T}{s_k^T B_k s_k}(I - \rho_k y_k s_k^T) = 0 \quad \therefore \quad B_{k+1} H_{k+1} = I \;//$$