(A Structural Approach to Measuring Default Risk)

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Outline

- Option pricing framework for the valuation of equity and debt
- Merton (1974) model
- Estimating the market value of a bank's assets
- Estimating the expected loss of a bank
- Computing credit spreads on the market value of a bank's debt
- Limitations of a diffusion-based approach for the evolution of asset values
- Introducing a jump diffusion process

The Structural Approach to Default

- Structural credit risk models view a firm's liabilities as "contingent claims" on the firm's underlying assets.
- If assets are not enough to repay the debt in full the bank defaults.
- Banks have assets that change in value over time, and a fixed amount of debt that is due at some point of time in the future.
- Assets of a bank are uncertain and change due to factors such as profit flows and risk exposures.
- Default risk over a given horizon is driven by uncertain changes in future asset values relative to promised payments on debt.

The value of the firm

- The value of the firm is split into two - that which goes to the equity holders and that which goes to the debt holders.
- Firms have assets (V) that change in value over time, and a fixed amount of debt (D) that is due at some point in the future (T)
- If, at the time (T) when the debt fall due, the assets have more than enough value to repay the liabilities (D), then the excess value ($V - D$) goes to the equity holders.
- If they don't, the equity holders receive nothing. Equity is a residual claim on assets after debt has been repaid.

Equity as a contingent claim

- This means the equity holder has a European call option on the value of the firm's assets at maturity (T),
- where the payoff is either zero or the value of assets (V) less liabilities (D) whichever is greater.
- The strike price is the nominal value of outstanding debt (D).
- $E = max(V - D, 0)$

Debt as a contingent claim

- If the assets (V) are more than enough to pay off the liabilities (D), then the debt holders receive the full value of debt (D).
- If the assets (V) are not enough to repay the debt in full, then the firm defaults, and the debt holders receive the recovery value of the assets (V).
- Debt holders offer an implicit guarantee by absorbing losses if there is a default.
- They receive a put option premium in the form of a credit spread above the risk-free rate in return for holding risky debt.
- So the value of debt is a equivalent to a risk free debt holding plus a short put option on the value of the assets.

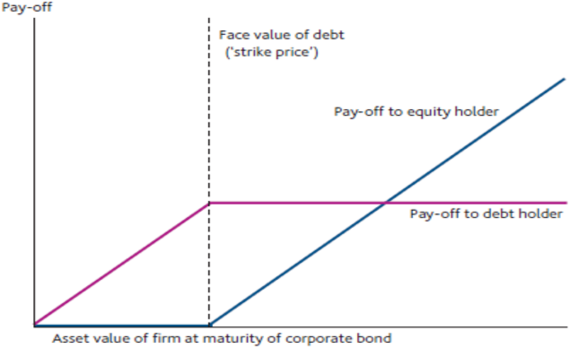
Payoff to debt holders

- The payoff to debt holders at maturity, B, can be described as:
- The nominal value of outstanding debt, D, minus their put option of selling the firm's assets at a strike price D,
- $B = min\left[V, D\right] = D - max\left[D - V, 0\right]$
- So the value of debt is a equivalent to a risk free debt holding plus a short put option on the value of the assets.

Payoff to debt and equity holders

- Merton (1974) developed a model for the valuation of corporate debt which we apply here:

Figure 1 Option-like pay-off to corporate bond and equity investors in the Merton model



Asymmetry in market values of debt and equity

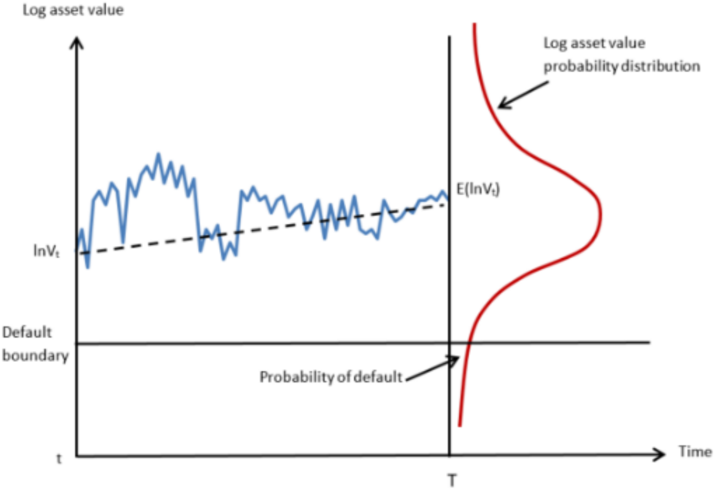
- A bank's creditors participate in the downside risk but receive a maximum payoff equal to the face value of debt.
- Equity holders benefit from upside outcomes where default does not occur but have limited liability on the downside
- So uncertainty in a bank's asset value has an asymmetric on the market values of debt and equity.

Asset value evolution according to a diffusion process

- In the Merton model each bank's assets V evolve according to a Geometric Brownian Motion (GBM) with ex-ante fixed coefficients $[\mu, \sigma]$:
- $\frac{dV}{V} = \mu dt + \sigma dW_t$
- where μ is the drift, which measures the average return.
- The stochastic component , σdW_t, models the random change in asset value in response to external effects.
- σ is equal to the standard deviation of the asset return.
- W_t is a standard Brownian motion that is characterised by independent identically distributed increments that are normally distributed with mean 0 and variance t.

Log-normal distribution of asset returns

- For a GBM the asset value at time t can be calculated from the asset value at time 0 using the following relationship:
- $V_t = V_0 exp\left[\left(\mu_i - \frac{\sigma^2}{2}\right) t + \sigma dW_t\right]$
- The asset value dynamics imply a log-normal distribution of asset returns.
- This means the logarithm of the asset value is normally distributed.
- $d\left(lnV_t\right) = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dW$



Log asset value
Log asset value
probability distribution
E(lnV$_t$)
lnV$_t$
Default
boundary
Probability of default
t
T
Time

The value of equity

- Equity (E) is a call option on the asset value of the firm (V) with a strike price equal to the face value of the firm's debt (D)
- $E = VN(d_1) - De^{-rT}N(d_2)$
- $d_1 = \frac{ln\left(\left(\frac{V}{D}\right)+\left(r+\frac{\sigma^2}{2}\right)T\right)}{\sigma\sqrt{T}}$
- $d_2 = d_1 - \sigma\sqrt{T}$

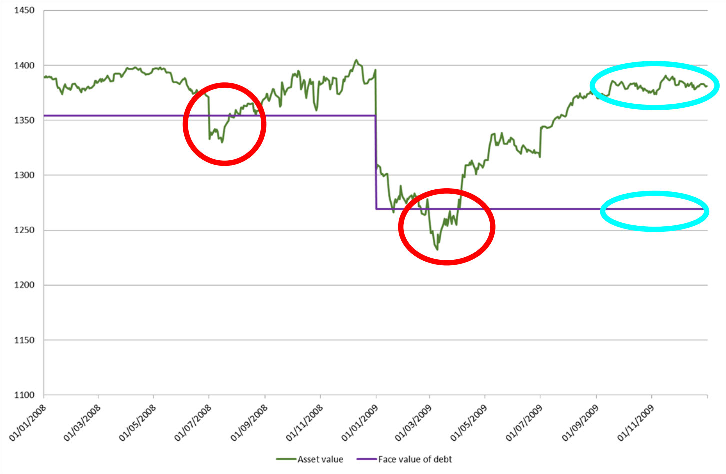
Merton model formula

- E = market value of equity
- V = market value of assets.
- D = nominal value of debt
- σ = implied volatility of V
- T = time to maturity of debt D

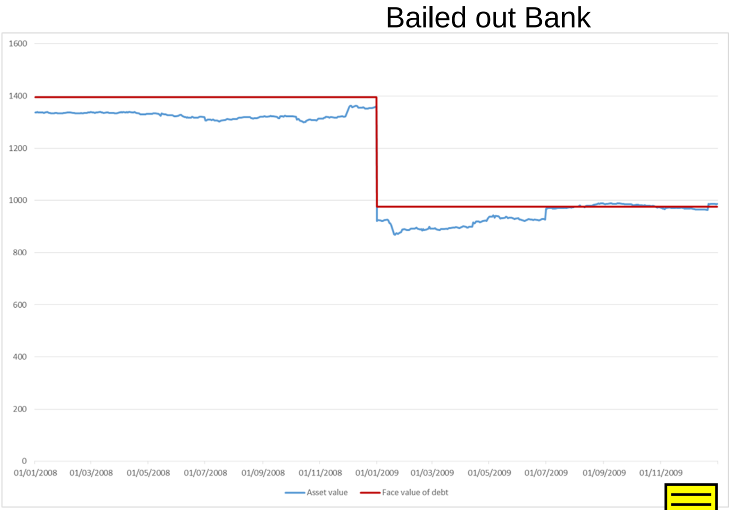
Data

- Time series of market capitalisation for a bank
- Nominal liabilities of a bank
- Risk-free rate of interest

Asset values and debt for a non-crisis bank in 2008-09



Asset values and debt for a crisis bank in 2008-09



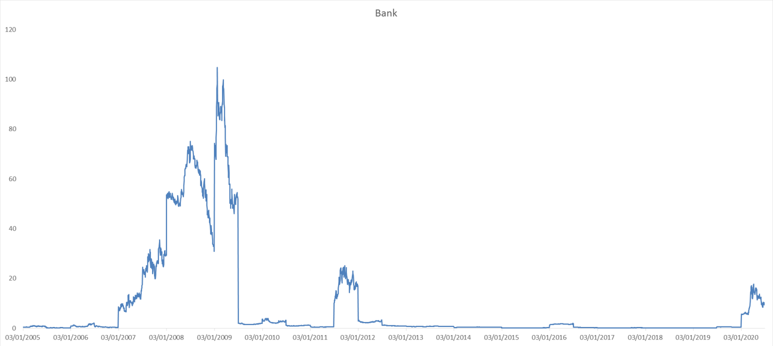
The value of debt

- The payoff to debt holders B_t is the sum of the safe claim payoff and a short position in a put option written on the firm's assets.
- $B_t = De^{-rT} - \left(De^{-rT} N(-d_2) - V_t N\left(-d_1\right)\right)$
- The value of the implicit put option is given by:
- $P_t = De^{-rT} N(-d_2) - V_t N\left(-d_2\right)$
- The market value of debt, B_t, may be described as the default-free value minus the expected loss, as given by the implicit put option:
- $B_t = De^{-rT} - P_t$

LGD determined endogenously

- Expected Loss (EL) is given:
- $P_t = N(-d_2)\left[1 - \frac{N(-d_1)}{N(-d_2)} \frac{V_t}{De^{-rT}}\right] De^{-rT}$
- Probability of default (PD) $= N(-d_2)\left[1 - \frac{N(-d_1)}{N(-d_2)} \frac{V_t}{De^{-rT}}\right]$
- The loss given default (LGD) $= De^{-rT}$
- Banks default when they are not able to pay their debt in full.
- The recovery rate is, in general, larger than zero and endogenously determined.

Expected losses of a bank(Put Option values)in GBP billions



Credit spread on the market value of debt

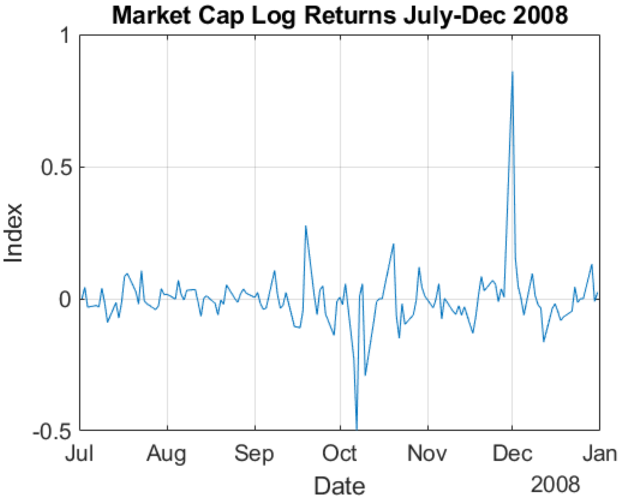
- In order to measure a bank's default risk we calculate the credit spread s on the market value of debt B_t
- To calculate the credit spread we first compute the yield-to-maturity y on market debt.
- $B_t = De^{-(Y-t)}$
- $y_t = \frac{ln\left(\frac{D}{B_t}\right)}{T-t}$
- $s = y_t - r_t$

Empirical deficiencies of a diffusion process

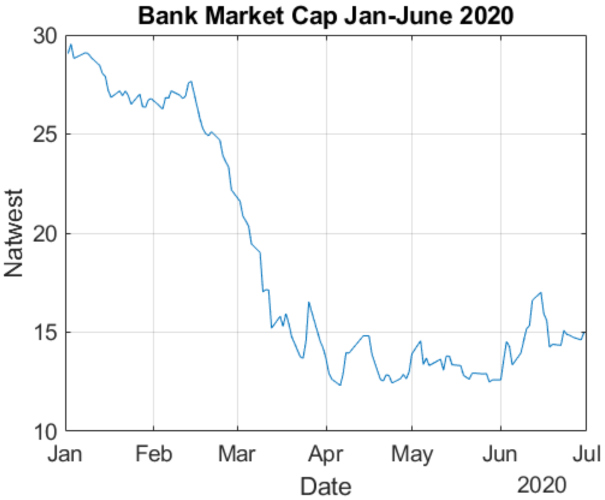
- The empirical distribution of returns on financial assets differ in many ways from the diffusion process assumed in the Black-Scholes (1973) and Merton (1974) models.
- If a firm cannot default unexpectedly the probability of default on its short-term debt will be zero.
- This is not borne out empircally as credit spreads on even investment-grade short-term corporate debt are not zero.

Market Capitalisation for a Bank during GFC

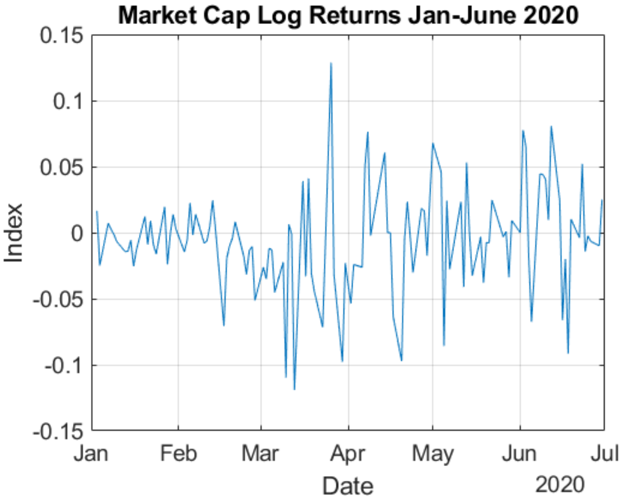
Log Returns of Market Capitalisation during GFC



Market Cap for a bank during COVID crisis



Log Returns of Market Capitalisation during COVID crisis



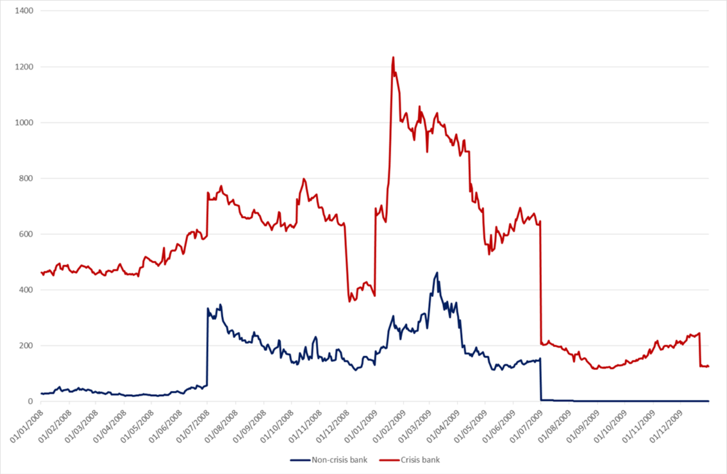
Advantages of a jump-diffusion process

- The reduced-form approach treats default as a random Poisson event involving a sudden loss in market value which cannot be predicted.
- A jump-diffusion model adds this Poisson jump process to the diffusion process in the Black-Scholes framework.
- Jump-diffusions can incorporate, rare large fluctuations in asset prices as witnessed in the GFC in 2008-09 and the ongoing COVID crisis.
- Jumps allow higher moment features such as skewness and leptokurtic behaviour in the distribution of asset price returns.

Jump diffusion model specification

- The continuous-time stochastic process for the asset value V_t is given by the stochastic differential equation
- $\frac{dV_t}{V_t} = \mu dt + \sigma dW_t + d\left(\sum_{q=0}^{N_t}(J_q - 1)\right).$
- where the last term models the jumps.
- The continuous component is given by a Brownian motion, W_t distributed as $dW_t \sim N(0, dt)$
- The discontinuities of the asset value process are described by a Poisson process N_t, and jump size J_q.
- The jump size J_q is a sequence of independent identically distributed nonnegative random variables.
- The Brownian motion and the Poisson process are independent.

Credit Spreads in 2008-09 for a crisis and non-crisis bank



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