CHAPTER EIGHT

Transmission-Line Networks

The previous chapters of this text have considered the analysis of uniform transmission lines that have one important restriction: all $(n + 1)$ conductors are parallel to each other. Numerous practical configurations consist of interconnections of these types of lines as illustrated in Fig. 8.1(a). These practical configurations will be referred to as *transmission-line networks.* Lines may end in *termination networks* or may be interconnected by *interconnection networks.* Each transmission line of the network will be referred to as a *tube* after [1–3]. A convenient way of describing the overall network is with a *graph* as illustrated in Fig. 8.1(b) [1, 2, 4]. The transmission lines are represented with single lines or *branches* of the graph. The termination networks are defined as a node having only one tube incident on it and are represented by rectangles. The interconnection networks are defined as a node having more than one tube incident on it and are represented by circles. The excitation for the network may be in the form of lumped sources in the termination or interconnection networks or it may be due to either distributed excitation from an incident electromagnetic field or a point excitation along the line as with the direct attachment of a lightning stroke. Point excitation of a tube as in the case of a direct attachment of a lightning stroke can be handled by characterizing the segments of the tube to the left and right of the excitation point with any of the following models and treating the point excitation as an interconnection network between these tube subsegments. Lumped sources in this interconnection network then represent this point excitation at the junction. Distributed excitation must be included in the overall characterization of the tube as described in Chapter 7, whereas lumped sources within the termination/interconnection networks are included in their description. The purpose of this chapter is to examine methods for characterizing these types of interconnected lines.

Evidently, any method for characterizing this network seeks to characterize each tube in some fashion, as outlined in the previous chapters, and to interconnect these tubes by enforcing the constraints on the line voltages and

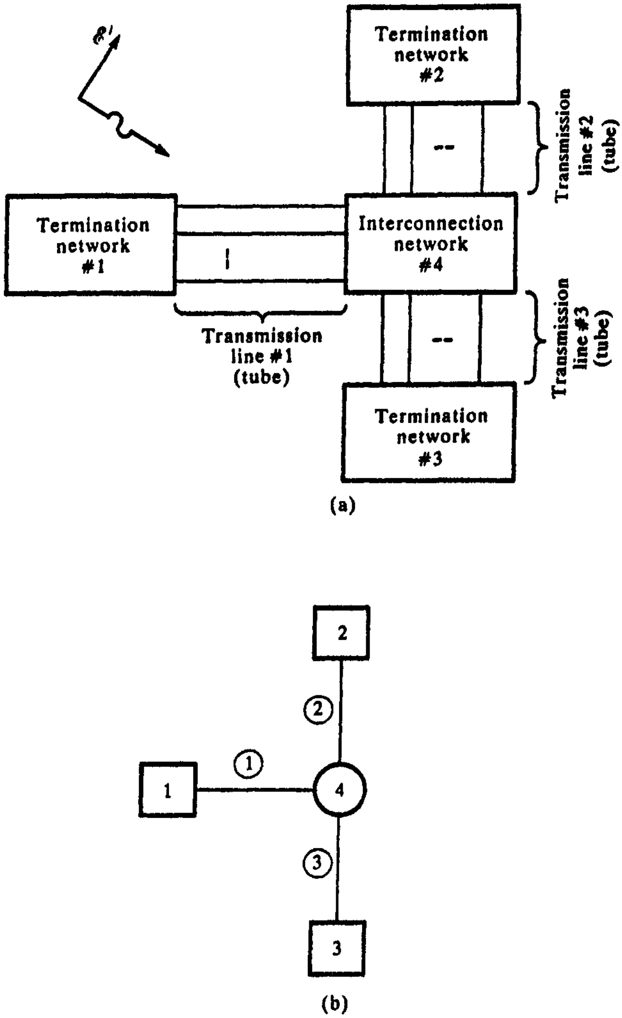


FIGURE 8.1 Illustration of a transmission-line network: (a) tube and network definitions, (b) representation with a graph.

currents via Kirchhoff's laws and the element characteristics within the termination/interconnection networks. One obvious representation method is to use SPICE subcircuit models for the tubes developed in Chapters 5 and 7 and interconnect and/or terminate the nodes of those subcircuit models in the resulting SPICE code. This method is very straightforward using the programs **SPICEMTL.FOR** or **SPICEINC.FOR** described in Appendix A to provide the SPICE subcircuit models of each tube. The advantages of this method are that it is straightforward to implement, and dynamic as well as nonlinear loading and elements within the termination/interconnection networks, such as diodes and transistors, are already available in the SPICE code and can be readily used to build the termination/interconnection networks to complete the overall characterization. So a wide variety of practical terminations can be analyzed without the need for developing either the models of complicated elements or the numerical integration routines to give a time-domain analysis. The disadvantage of this method is that it is so far applicable only to *lossless* lines.

An approximate method is to use lumped-circuit iterative models of each tube such as the lumped-pi or lumped-T models and use any lumped-circuit analysis program such as SPICE to analyze the resulting interconnection. Losses can be incorporated into this result, but the method is restricted to tubes that are electrically short. Time-domain results can again be reasonably approximated if the rise/fall times of the source waveforms are sufficiently longer than the tube one-way delays.

It is also possible to construct an exact model of the line using the admittance or impedance parameter characterizations of each tube [4, 5]. These methods have the advantages of simple construction of the overall equations which are to be solved to give the tube terminal voltages and currents. Losses such as skin-effect losses which vary as $\sqrt{f}$ can be approximated as lumped circuits or analyzed directly by determining the frequency response of the network.

With the exception of the SPICE subcircuit method, all methods ultimately must face the problem of the systematic interconnection of the tube models. Computer implementation of the interconnection of the tubes for a large network is not a simple task and must be designed so that a user can easily and unambiguously describe the interconnections to the resulting computer code. Of course all standard lumped-circuit analysis codes such as SPICE must address this problem of systematic and unambiguous implementation of the element interconnections via user input to the code, and the characterization of transmission-line networks is similar in that respect. Characterization of the tubes via the admittance parameters as in [4] was designed so that a systematic interconnection process will be effected. Another novel method is the use of the *scattering parameters* for the tubes [1, 2]. This leads to the so-called BLT equations (apparently named for the authors). The implementation of the BLT equations directly in the time domain was described in [3].

All of the above methods must address both the frequency-domain as well as the time-domain analysis of the network. The time-domain analysis of the network can be obtained in the usual fashion using the time-domain to frequency-domain transformation discussed earlier wherein the source waveform is decomposed into its spectral components and each component is passed through the previously computed frequency-domain transfer function. The time-domain result is the inverse Fourier transform of this. As before, the time-domain to frequency-domain method can readily handle skin-effect losses that are difficult to characterize in the time domain, but it suffers from the fundamental restriction that the network must be linear, i.e., the line and all terminations must be *linear*, since superposition is used.

8.1 REPRESENTATION WITH THE SPICE MODEL

Perhaps one of the more straightforward methods of characterizing and analyzing the crosstalk on transmission-line networks is with the SPICE equivalent circuit developed in Chapter 5 or for incident field illumination in Chapter 7. Each tube is characterized by its SPICE subcircuit model generated with the programs **SPICEMTL.FOR** or **SPICEINC.FOR** described in Appendix A. These subcircuit models are then interconnected and the terminations added to produce the final SPICE model of the network. The method is straightforward using the above codes to generate the subcircuit models but is restricted to lossless tubes. Again, this method can handle, in a straightforward way, dynamic and/or nonlinear loads in the termination/interconnection networks.

In order to illustrate the methods of this chapter we will use the example shown in Fig. 8.2. The network consists of three tubes. Tube #1 contains four wires, whereas tubes #2 and #3 contains two wires. All tubes will consist of bare wires above a ground plane as illustrated in Fig. 8.3. Tube #1 is of length 2 m and tubes #2 and #3 are of length 1 m. The cable is suspended 1 cm above an infinite, perfectly conducting ground plane, and the wires have radii of 7.5 mils. A source, $V_S(t)$, in network #1 drives line #1 of tube #1. This source is in the form of a ramp waveform with a risetime of 1 ns as shown in Fig. 8.3(c). The tubes are terminated in various resistive terminations at termination networks #1, #2, and #3. Interconnection network #4 contains a variety of terminations, open circuit, short circuit, series impedance, shunt impedance and direct connection, to illustrate the versatility of the method. The desired output will be the voltage, $V_{out}(t)$, across the termination of wire #1 of tube #3 at termination network #3. The graph of this transmission-line network is shown in Fig. 8.1(b). Each termination or interconnection network has the number of that node included within the symbol. The number of each *tube* is noted on that branch of the graph.

Figure 8.4 illustrates the resulting construction of the overall SPICE network with node numbering. The SPICE subcircuit models of the tubes are constructed using **SPICEMTL.FOR**, and the per-unit-length parameters are computed using **WIDESEP.FOR**.

8.2 REPRESENTATION WITH LUMPED-CIRCUIT ITERATIVE MODELS

The next method is to approximately characterize each tube with a lumped-circuit iterative structure such as a lumped-pi structure. These characterizations are obtained using the **SPICELPI.FOR** code. The resulting overall SPICE model of the network is virtually identical to that of Fig. 8.4 with the only exception being that the subcircuit models of the tubes are lumped-pi structures. Figure 8.5 shows the comparison of the predictions of the output voltage, $V_{out}(t)$, obtained with the SPICE model of Fig. 8.4 and the lumped-pi structure using

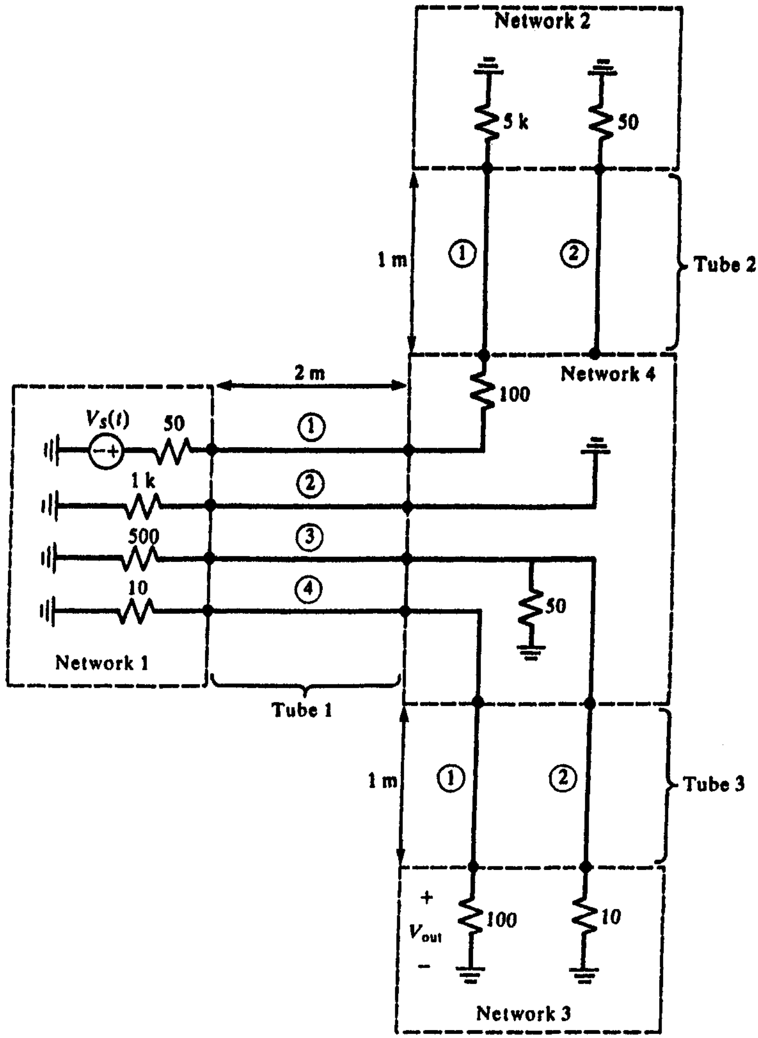


FIGURE 8.2 An example of a transmission-line network to illustrate and compare numerical results.

only one lumped-pi section to represent each tube. The correlation is obviously very poor due to the fact that the tube one-way delays are on the order of 10 ns which is not significantly smaller than the waveform rise time of 1 ns. Figure 8.6 shows this correlation for a risetime of 100 ns which is much better.

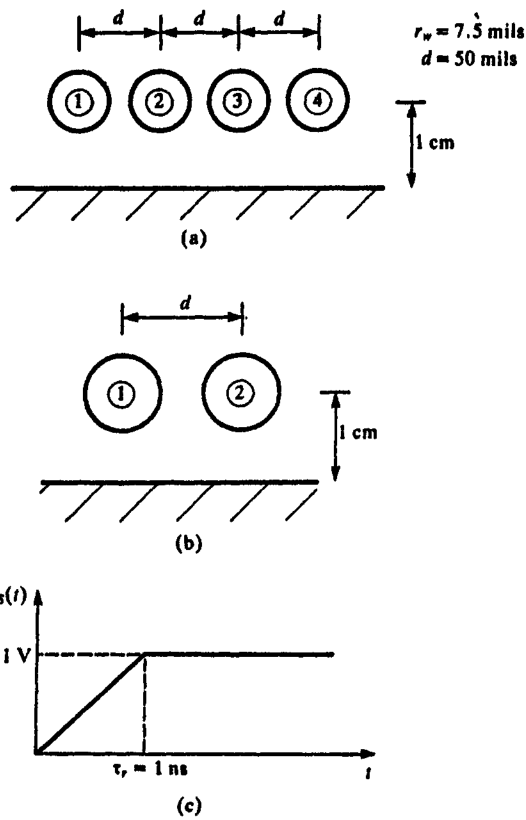


FIGURE 8.3 Cross-sectional dimensions of the tubes of the transmission-line network of Fig. 8.2: (a) tube 1, (b) tubes 2 and 3, (c) V_S versus t.

8.3 REPRESENTATION VIA THE ADMITTANCE OR IMPEDANCE PARAMETERS

The use of the admittance parameters to characterize the tubes was described in [4]. This leads to a straightforward way of incorporating the termination and interconnection networks since we essentially need to add admittances in order to construct the admittance matrix of the overall network.

The frequency-domain chain parameters of a uniform line are

$$\hat{\mathbf{V}}(\mathscr{L}) = \hat{\mathbf{\Phi}}_{11}\hat{\mathbf{V}}(0) + \hat{\mathbf{\Phi}}_{12}\hat{\mathbf{I}}(0) + \hat{\mathbf{V}}_{FT} \tag{8.1a}$$

$$\hat{\mathbf{I}}(\mathscr{L}) = \hat{\mathbf{\Phi}}_{21}\hat{\mathbf{V}}(0) + \hat{\mathbf{\Phi}}_{22}\hat{\mathbf{I}}(0) + \hat{\mathbf{I}}_{FT} \tag{8.1b}$$

where $\hat{\mathbf{V}}_{FT}$ and $\hat{\mathbf{I}}_{FT}$ are due to any incident field excitation of the line. The frequency-domain admittance parameters are derived in Chapter 4 from these

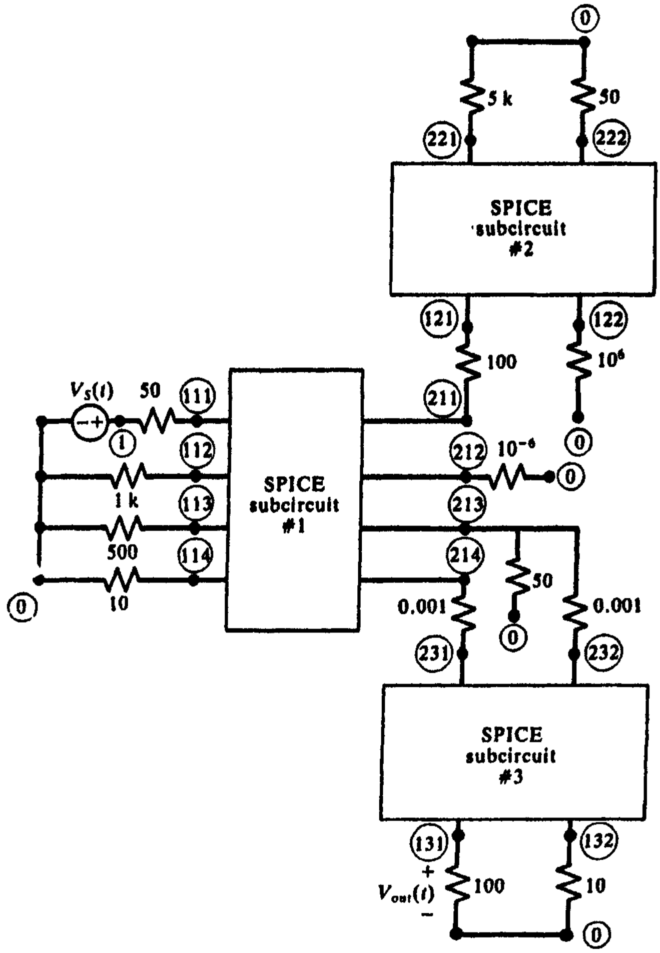


FIGURE 8.4 Illustration of the SPICE model of the transmission-line network of Fig. 8.2.

to yield

$$\hat{\mathbf{I}}(0) = \hat{\mathbf{Y}}_S\hat{\mathbf{V}}(0) + \hat{\mathbf{Y}}_M\hat{\mathbf{V}}(\mathscr{L}) + \hat{\mathbf{I}}_{S0} \tag{8.2a}$$

$$-\hat{\mathbf{I}}(\mathscr{L}) = \hat{\mathbf{Y}}_M\hat{\mathbf{V}}(0) + \hat{\mathbf{Y}}_S\hat{\mathbf{V}}(\mathscr{L}) + \hat{\mathbf{I}}_{SL} \tag{8.2b}$$

where

$$\begin{aligned}\hat{\mathbf{Y}}_S &= -\hat{\mathbf{\Phi}}_{12}^{-1}\hat{\mathbf{\Phi}}_{11} = -\hat{\mathbf{\Phi}}_{22}\hat{\mathbf{\Phi}}_{12}^{-1} \\ &= \hat{\mathbf{T}}(\mathbf{e}^{\hat{\gamma}\mathscr{L}} - \mathbf{e}^{-\hat{\gamma}\mathscr{L}})^{-1}(\mathbf{e}^{\hat{\gamma}\mathscr{L}} + \mathbf{e}^{-\hat{\gamma}\mathscr{L}})\hat{\mathbf{T}}^{-1}\hat{\mathbf{Y}}_C\end{aligned} \tag{8.3a}$$

$$\begin{aligned}\hat{\mathbf{Y}}_M &= \hat{\mathbf{\Phi}}_{12}^{-1} \\ &= -2\hat{\mathbf{T}}(\mathbf{e}^{\hat{\gamma}\mathscr{L}} - \mathbf{e}^{-\hat{\gamma}\mathscr{L}})^{-1}\hat{\mathbf{T}}^{-1}\hat{\mathbf{Y}}_C\end{aligned} \tag{8.3b}$$

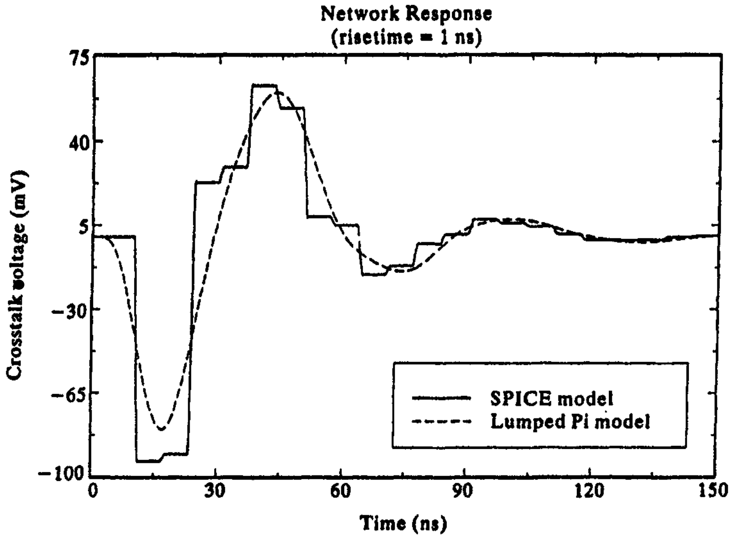


FIGURE 8.5 Comparison of crosstalk voltage at the termination of conductor #1 of tube #3 for the transmission-line network of Fig. 8.2 for a risetime of 1 ns using the SPICE model and using one lumped-pi section to represent each tube.

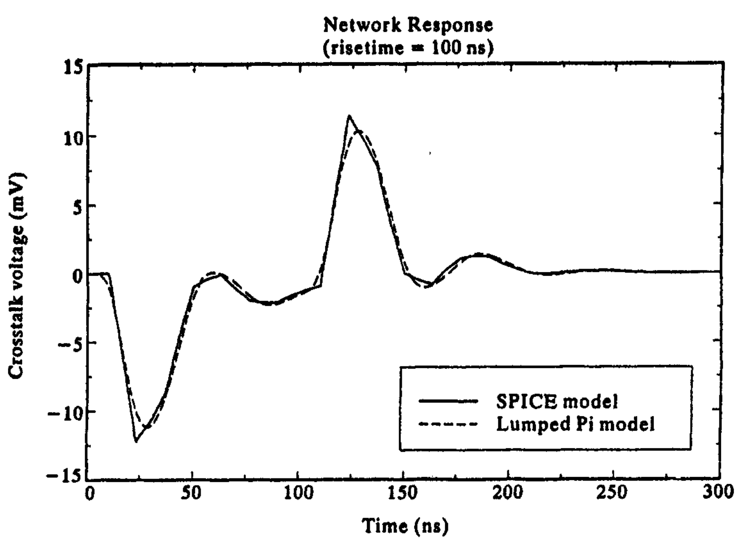


FIGURE 8.6 The predictions of Fig. 8.5 for a risetime of 100 ns showing the adequacy of the lumped-pi representation.

$$\hat{\mathbf{I}}_{S0} = -\hat{\mathbf{Y}}_M \hat{\mathbf{V}}_{FT} \tag{8.3c}$$

$$\hat{\mathbf{I}}_{SL} = -\hat{\mathbf{Y}}_S \hat{\mathbf{V}}_{FT} - \hat{\mathbf{I}}_{FT} \tag{8.3d}$$

Observe that the currents are defined as directed *into each end of the tube*. The admittance parameters show that the tube is reciprical as it should be. The various parameters in these are as defined in Chapter 4 where the per-unit-length impedance and admittance parameters are diagonalized as

$$\hat{\mathbf{T}}^{-1}\underbrace{(\mathbf{G} + j\omega\mathbf{C})}_{\hat{\mathbf{Y}}}\underbrace{(\mathbf{R}(f) + j\omega\mathbf{L})}_{\hat{\mathbf{Z}}}\hat{\mathbf{T}} = \hat{\boldsymbol{\gamma}}^2 \tag{8.4}$$

and the characteristic admittance matrix is

$$\begin{aligned}\hat{\mathbf{Y}}_C &= \hat{\mathbf{Z}}_C^{-1} \\ &= \hat{\mathbf{T}}\hat{\boldsymbol{\gamma}}\hat{\mathbf{T}}^{-1}\hat{\mathbf{Z}}^{-1} \\ &= \hat{\mathbf{T}}\hat{\boldsymbol{\gamma}}^{-1}\hat{\mathbf{T}}^{-1}\hat{\mathbf{Y}}\end{aligned} \tag{8.5}$$

The only potential disadvantage to the admittance parameter description of the tubes is that the parameters do not exist for frequencies where the tube is some multiple of a half-wavelength.

The tubes are characterized by the above admittance parameters with the following notation illustrated in Fig. 8.7(a). Consider the i-th tube connecting the j-th network and the k-th network at its endpoints. Denote the vector of currents and voltages at the ends of the tube as $\hat{\mathbf{I}}_i^j$, $\hat{\mathbf{V}}_i^j$, $\hat{\mathbf{I}}_i^k$, $\hat{\mathbf{V}}_i^k$ where the subscript denotes the tube and the superscript denotes the network at that end:

$$\hat{\mathbf{V}}_{\text{tube}}^{\text{termination/interconnection network}}$$

$$\hat{\mathbf{I}}_{\text{tube}}^{\text{termination/interconnection network}}$$

The admittance parameters (8.2) and (8.3) become

$$\hat{\mathbf{I}}_i^j = \hat{\mathbf{Y}}_{Si}\hat{\mathbf{V}}_i^j + \hat{\mathbf{Y}}_{Mi}\hat{\mathbf{V}}_i^k + \hat{\mathbf{I}}_{Si}^j \tag{8.6a}$$

$$\hat{\mathbf{I}}_i^k = \hat{\mathbf{Y}}_{Mi}\hat{\mathbf{V}}_i^j + \hat{\mathbf{Y}}_{Si}\hat{\mathbf{V}}_i^k + \hat{\mathbf{I}}_{Si}^k \tag{8.6b}$$

The characterization of the termination and interconnection networks must be general enough to include open and short circuits as well as lumped sources and impedances and direct connections within the termination/interconnection networks. As discussed in Chapter 4, a general way of characterizing these is in the form of a combination of generalized Thévenin and generalized Norton equivalents [1–4, 6, 7]. Consider characterizing the m-th interconnection network which has the i-th, j-th, and k-th tubes interconnected by it as

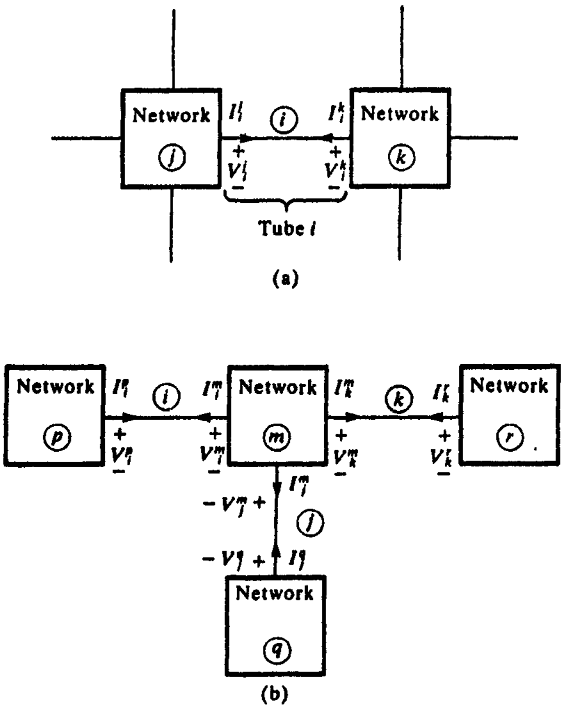


FIGURE 8.7 Definitions of the tube voltages and currents for (a) an individual tube and (b) an interconnection network.

illustrated in Fig. 8.7(b). The tube voltages and currents can be interrelated as

$$\hat{\mathbf{Y}}_i^m\hat{\mathbf{V}}_i^m + \hat{\mathbf{Z}}_i^m\hat{\mathbf{I}}_i^m + \hat{\mathbf{Y}}_j^m\hat{\mathbf{V}}_j^m + \hat{\mathbf{Z}}_j^m\hat{\mathbf{I}}_j^m + \hat{\mathbf{Y}}_k^m\hat{\mathbf{V}}_k^m + \hat{\mathbf{Z}}_k^m\hat{\mathbf{I}}_k^m = \hat{\mathbf{P}}^m \tag{8.7}$$

The total number of equations in (8.7) *equals the number of conductors incident at the termination/interconnection network* (*node*). For the example of Fig. 8.2 this is $4 + 2 + 2 = 8$. The fact this representation is completely general can be proven from the fact that it can be derived from a chain parameter representation of the ports of the network which always exits for any linear network. The representation in (8.7) has the sole purpose of enforcing Kirchhoff's voltage law (KVL), Kirchhoff's current law (KCL), and the element relations that are imposed by the particular interconnections within the interconnection network. Figure 8.8 illustrates some common examples. Figure 8.8(a) illustrates the k-th conductor of the i-th tube terminating in an open circuit within the m-th network. The constraint here is that the current is zero:

$$[\hat{\mathbf{I}}_i^m]_k = 0 \tag{8.8}$$

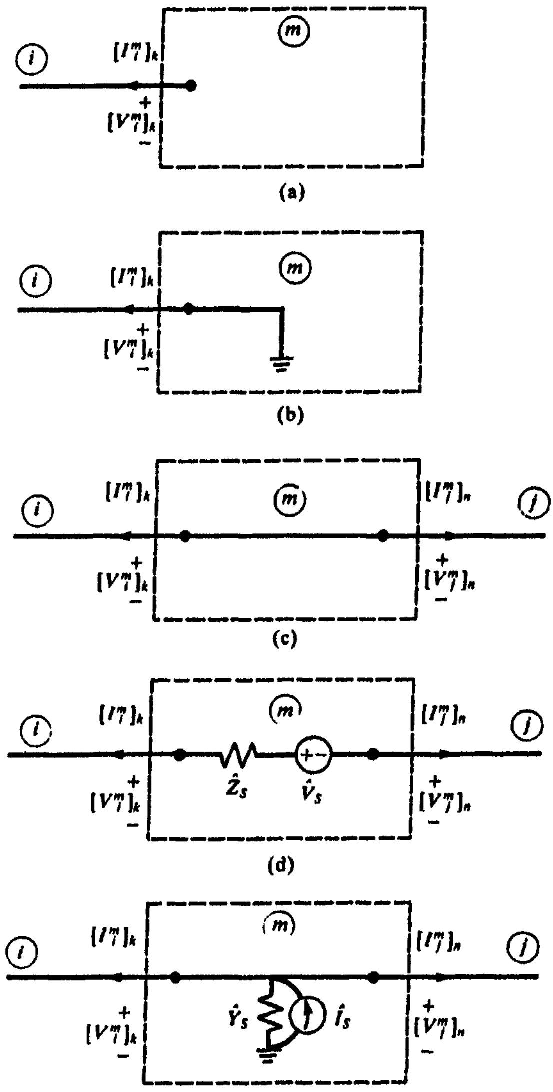


FIGURE 8.8 Illustration of the determination of the network characterizations for (a) an open circuit, (b) a short circuit, (c) a direct connection, (d) a Thévenin equivalent, and (e) a Norton equivalent.

Therefore a one appears in the column of $\hat{\mathbf{Z}}_i^m$ corresponding to the current of that conductor of that tube in $\hat{\mathbf{I}}_i^m$. Figure 8.8(b) illustrates the k-th conductor of the i-th tube terminating in a short circuit within the m-th network. The constraint here is that the voltage is zero:

$$[\hat{\mathbf{V}}_i^m]_k = 0 \tag{8.9}$$

Therefore a one appears in the column of $\hat{\mathbf{Y}}_i^m$ corresponding to the voltage of that conductor of that tube in $\hat{\mathbf{V}}_i^m$. Figure 8.8(c) illustrates a direct connection between the k-th conductor of tube i and the n-th conductor of tube j within the m-th network. The constraints here are that the voltage of the conductor of the i-th tube and the voltage of the conductor of the j-th tube are equal and the sum of the currents of the conductor of the i-th tube and the conductor of the j-th tube equals zero:

$$[\hat{\mathbf{V}}_i^m]_k - [\hat{\mathbf{V}}_j^m]_n = 0 \tag{8.10a}$$

$$[\hat{\mathbf{I}}_i^m]_k + [\hat{\mathbf{I}}_j^m]_n = 0 \tag{8.10b}$$

The first constraint is imposed by placing a one in the column of $\hat{\mathbf{Y}}_i^m$ corresponding to the voltage of that conductor of that tube in $\hat{\mathbf{V}}_i^m$ and by placing a *negative* one in the column of $\hat{\mathbf{Y}}_j^m$ corresponding to the voltage of that conductor of that tube in $\hat{\mathbf{V}}_j^m$. The second constraint is imposed by placing a one in the column of $\hat{\mathbf{Z}}_i^m$ corresponding to the current of that conductor of that tube in $\hat{\mathbf{I}}_i^m$ and by placing a one in the column of $\hat{\mathbf{Z}}_j^m$ corresponding to the voltage of that conductor of that tube in $\hat{\mathbf{I}}_j^m$. Multiple connections of conductors can be similarly handled. For example, consider the case of three conductors, k of tube i, n of tube j, and p of tube l, connected at a common point within the network. KCL requires that the sum of the currents at that interconnection equals zero: $[\hat{\mathbf{I}}_i^m]_k + [\hat{\mathbf{I}}_j^m]_n + [\hat{\mathbf{I}}_l^m]_p = 0$. Similarly, KVL requires that the differences of two of the three pairs of the voltages that are interconnected equal zero: $[\hat{\mathbf{V}}_i^m]_k - [\hat{\mathbf{V}}_j^m]_n = 0$, $[\hat{\mathbf{V}}_i^m]_k - [\hat{\mathbf{V}}_l^m]_p = 0$. Figure 8.8(d) illustrates a series connection of an impedance and a lumped voltage source. The constraints are that the currents are equal and the voltages are related by the element relations:

$$[\hat{\mathbf{I}}_i^m]_k + [\hat{\mathbf{I}}_j^m]_n = 0 \tag{8.11a}$$

$$[\hat{\mathbf{V}}_i^m]_k - [\hat{\mathbf{V}}_j^m]_n + \hat{Z}_S[\hat{\mathbf{I}}_i^m]_k = \hat{V}_S \tag{8.11b}$$

The first constraint is imposed by placing a one in the column of $\hat{\mathbf{Z}}_i^m$ corresponding to the current of that conductor of that tube in $\hat{\mathbf{I}}_i^m$ and by placing a one in the column of $\hat{\mathbf{Z}}_j^m$ corresponding to the voltage of that conductor of that tube in $\hat{\mathbf{I}}_j^m$. The second constraint is imposed by placing a one in the column of $\hat{\mathbf{Y}}_i^m$ corresponding to the voltage of that conductor of that tube in $\hat{\mathbf{V}}_i^m$, by placing a *negative* one in the column of $\hat{\mathbf{Y}}_j^m$ corresponding to the voltage of that conductor of that tube in $\hat{\mathbf{V}}_j^m$, placing $\hat{Z}_S$ in the column of $\hat{\mathbf{Z}}_i^m$ corresponding to the current of that conductor of that tube in $\hat{\mathbf{I}}_i^m$, and placing $\hat{V}_S$ in $\hat{\mathbf{P}}^m$ in the row corresponding to the equation being written. Figure 8.8(e) illustrates a parallel connection of an admittance and a lumped current source. The constraints are that the voltages are equal and the currents are related by the element relations:

$$[\hat{\mathbf{V}}_i^m]_k - [\hat{\mathbf{V}}_j^m]_n = 0 \tag{8.12a}$$

$$[\hat{\mathbf{I}}_i^m]_k + [\hat{\mathbf{I}}_j^m]_n + \hat{Y}_S[\hat{\mathbf{V}}_i^m]_k = \hat{I}_S \tag{8.12b}$$

The first constraint is imposed by placing a one in the column of $\hat{\mathbf{Y}}_i^m$ corresponding to the voltage of that conductor of that tube in $\hat{\mathbf{V}}_i^m$ and by placing a *negative* one in the column of $\hat{\mathbf{Y}}_j^m$ corresponding to the voltage of that conductor of that tube in $\hat{\mathbf{V}}_j^m$. The second constraint is imposed by placing a one in the column of $\hat{\mathbf{Z}}_i^m$ corresponding to the current of that conductor of that tube in $\hat{\mathbf{I}}_i^m$, by placing a one in the column of $\hat{\mathbf{Z}}_j^m$ corresponding to the current of that conductor of that tube in $\hat{\mathbf{I}}_j^m$, placing $\hat{Y}_S$ in the column of $\hat{\mathbf{Y}}_i^m$ corresponding to the voltage of that conductor of that tube in $\hat{\mathbf{V}}_i^m$, and placing $\hat{I}_S$ in $\hat{\mathbf{P}}^m$ in the row corresponding to the equation being written.

The final element of the process is the combination of the admittance parameters of the tubes and the constraint relations imposed by the termination/interconnection networks. A simple example will illustrate that result. Consider the m-th network interconnecting tubes i, j, and k as shown in Fig. 8.7(b) where the i-th tube connects to termination network p at the other end, the j-th tube connects to termination network q at the other end, and the k-th tube connects to termination network r at the other end. The tube admittance characterizations at the m-th end are

$$\hat{\mathbf{I}}_i^m = \hat{\mathbf{Y}}_{Si}\hat{\mathbf{V}}_i^m + \hat{\mathbf{Y}}_{Mi}\hat{\mathbf{V}}_i^p + \hat{\mathbf{I}}_{Si}^m \tag{8.13a}$$

$$\hat{\mathbf{I}}_j^m = \hat{\mathbf{Y}}_{Sj}\hat{\mathbf{V}}_j^m + \hat{\mathbf{Y}}_{Mj}\hat{\mathbf{V}}_j^q + \hat{\mathbf{I}}_{Sj}^m \tag{8.13b}$$

$$\hat{\mathbf{I}}_k^m = \hat{\mathbf{Y}}_{Sk}\hat{\mathbf{V}}_k^m + \hat{\mathbf{Y}}_{Mk}\hat{\mathbf{V}}_k^r + \hat{\mathbf{I}}_{Sk}^m \tag{8.13c}$$

The network characterization is given in (8.7). Substituting (8.13) gives

$$\begin{aligned}(\hat{\mathbf{Y}}_i^m + \hat{\mathbf{Z}}_i^m\hat{\mathbf{Y}}_{Si})\hat{\mathbf{V}}_i^m + (\hat{\mathbf{Y}}_j^m + \hat{\mathbf{Z}}_j^m\hat{\mathbf{Y}}_{Sj})\hat{\mathbf{V}}_j^m + (\hat{\mathbf{Y}}_k^m + \hat{\mathbf{Z}}_k^m\hat{\mathbf{Y}}_{Sk})\hat{\mathbf{V}}_k^m \\ + \hat{\mathbf{Z}}_i^m\hat{\mathbf{Y}}_{Mi}\hat{\mathbf{V}}_i^p + \hat{\mathbf{Z}}_j^m\hat{\mathbf{Y}}_{Mj}\hat{\mathbf{V}}_j^q + \hat{\mathbf{Z}}_k^m\hat{\mathbf{Y}}_{Mk}\hat{\mathbf{V}}_k^r = \hat{\mathbf{P}}^m - \hat{\mathbf{Z}}_i^m\hat{\mathbf{I}}_{Si}^m - \hat{\mathbf{Z}}_j^m\hat{\mathbf{I}}_{Sj}^m - \hat{\mathbf{Z}}_k^m\hat{\mathbf{I}}_{Sk}^m\end{aligned} \tag{8.14}$$

This provides a simple rule for constructing the overall admittance matrix which can be solved for the voltages at the ends of each tube:

$$
\underbrace{\begin{bmatrix}
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & (\hat{\mathbf{Y}}_l^m + \hat{\mathbf{Z}}_l^m \hat{\mathbf{Y}}_{Sl}) & (\hat{\mathbf{Y}}_j^m + \hat{\mathbf{Z}}_j^m \hat{\mathbf{Y}}_{Sj}) & (\hat{\mathbf{Y}}_k^m + \hat{\mathbf{Z}}_k^m \hat{\mathbf{Y}}_{Sk}) & \vdots & (\hat{\mathbf{Z}}_l^m \hat{\mathbf{Y}}_{Ml}) & (\hat{\mathbf{Z}}_j^m \hat{\mathbf{Y}}_{Mj}) & (\hat{\mathbf{Z}}_k^m \hat{\mathbf{Y}}_{Mk}) & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots
\end{bmatrix}}_{\hat{\mathbf{Y}}_T}
$$

$$
\times \underbrace{\begin{bmatrix} \vdots \\ \hat{\mathbf{V}}_l^m \\ \hat{\mathbf{V}}_j^m \\ \hat{\mathbf{V}}_k^m \\ \vdots \\ \hat{\mathbf{V}}_l^p \\ \hat{\mathbf{V}}_j^q \\ \hat{\mathbf{V}}_k^r \\ \vdots \end{bmatrix}}_{\hat{\mathbf{V}}_T} = \underbrace{\begin{bmatrix} \vdots \\ \hat{\mathbf{P}}^m - \hat{\mathbf{Z}}_l^m \hat{\mathbf{I}}_{Sl}^m - \hat{\mathbf{Z}}_j^m \hat{\mathbf{I}}_{Sj}^m - \hat{\mathbf{Z}}_k^m \hat{\mathbf{I}}_{Sk}^m \\ \vdots \end{bmatrix}}_{\hat{\mathbf{P}}_T} \tag{8.15}
$$

As an example, consider the transmission-line network in Fig. 8.2 with graph shown in Fig. 8.1(b). The admittance matrix becomes

$$
\begin{bmatrix}
(\hat{\mathbf{Y}}_1^1 + \hat{\mathbf{Z}}_1^1 \hat{\mathbf{Y}}_{S1}) & \hat{\mathbf{Z}}_1^1 \hat{\mathbf{Y}}_{M1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & (\hat{\mathbf{Y}}_2^2 + \hat{\mathbf{Z}}_2^2 \hat{\mathbf{Y}}_{S2}) & \hat{\mathbf{Z}}_2^2 \hat{\mathbf{Y}}_{M2} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & (\hat{\mathbf{Y}}_3^3 + \hat{\mathbf{Z}}_3^3 \hat{\mathbf{Y}}_{S3}) & \hat{\mathbf{Z}}_3^3 \hat{\mathbf{Y}}_{M3} \\
\hat{\mathbf{Z}}_1^4 \hat{\mathbf{Y}}_{M1} & (\hat{\mathbf{Y}}_1^4 + \hat{\mathbf{Z}}_1^4 \hat{\mathbf{Y}}_{S1}) & \hat{\mathbf{Z}}_2^4 \hat{\mathbf{Y}}_{M2} & (\hat{\mathbf{Y}}_2^4 + \hat{\mathbf{Z}}_2^4 \hat{\mathbf{Y}}_{S2}) & \hat{\mathbf{Z}}_3^4 \hat{\mathbf{Y}}_{M3} & (\hat{\mathbf{Y}}_3^4 + \hat{\mathbf{Z}}_3^4 \hat{\mathbf{Y}}_{S3})
\end{bmatrix}
$$

$$
\times \begin{bmatrix} \hat{\mathbf{V}}_1^1 \\ \hat{\mathbf{V}}_1^4 \\ \hat{\mathbf{V}}_2^2 \\ \hat{\mathbf{V}}_2^4 \\ \hat{\mathbf{V}}_3^3 \\ \hat{\mathbf{V}}_3^4 \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{P}}^1 \\ \hat{\mathbf{P}}^2 \\ \hat{\mathbf{P}}^3 \\ \hat{\mathbf{P}}^4 \end{bmatrix}
$$

Numbering each conductor of each tube as shown gives the following. First we examine termination network #1. The constraints are

$$
[\hat{\mathbf{V}}_1^1]_1 = V_S(t) - 50[\hat{\mathbf{I}}_1^1]_1
$$

$$
[\hat{\mathbf{V}}_1^1]_2 = -1\mathrm{k}[\hat{\mathbf{I}}_1^1]_2
$$

$$[\hat{\mathbf{V}}_1^1]_3 = -500[\hat{\mathbf{I}}_1^1]_3$$

$$[\hat{\mathbf{V}}_1^1]_4 = -10[\hat{\mathbf{I}}_1^1]_4$$

Therefore

$$\hat{\mathbf{Y}}_1^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \hat{\mathbf{Z}}_1^1 = \begin{bmatrix} 50 & 0 & 0 & 0 \\ 0 & 1k & 0 & 0 \\ 0 & 0 & 500 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} \quad \hat{\mathbf{P}}^1 = \begin{bmatrix} V_S(t) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The number of equations equals the number of conductors incident on this node: 4. Similarly, termination networks #2 and #3 are characterized by

$$[\hat{\mathbf{V}}_2^2]_1 = -5k[\hat{\mathbf{I}}_2^2]_1$$

$$[\hat{\mathbf{V}}_2^2]_2 = -50[\hat{\mathbf{I}}_2^2]_2$$

and

$$[\hat{\mathbf{V}}_3^3]_1 = -100[\hat{\mathbf{I}}_3^3]_1$$

$$[\hat{\mathbf{V}}_3^3]_2 = -10[\hat{\mathbf{I}}_3^3]_2$$

giving

$$\hat{\mathbf{Y}}_2^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \hat{\mathbf{Z}}_2^2 = \begin{bmatrix} 5k & 0 \\ 0 & 50 \end{bmatrix} \quad \hat{\mathbf{P}}^2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and

$$\hat{\mathbf{Y}}_3^3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \hat{\mathbf{Z}}_3^3 = \begin{bmatrix} 100 & 0 \\ 0 & 10 \end{bmatrix} \quad \hat{\mathbf{P}}^3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The number of equations equals the number of conductors incident on each node: 2. Interconnection network #4 has the following constraints. KVL imposes

$$[\hat{\mathbf{V}}_1^4]_1 - [\hat{\mathbf{V}}_2^4]_1 = -100[\hat{\mathbf{I}}_1^4]_1$$

$$[\hat{\mathbf{V}}_1^4]_2 = 0$$

$$[\hat{\mathbf{V}}_1^4]_3 - [\hat{\mathbf{V}}_3^4]_2 = 0$$

$$[\hat{\mathbf{V}}_1^4]_4 - [\hat{\mathbf{V}}_3^4]_1 = 0$$

KCL imposes

$$[\hat{\mathbf{I}}_1^4]_1 + [\hat{\mathbf{I}}_2^4]_1 = 0$$

$$[\hat{\mathbf{I}}_1^4]_3 + [\hat{\mathbf{I}}_3^4]_2 = -\tfrac{1}{50}[\hat{\mathbf{V}}_1^4]_3$$

$$[\hat{\mathbf{I}}_1^4]_4 + [\hat{\mathbf{I}}_3^4]_1 = 0$$

$$[\hat{\mathbf{I}}_2^4]_2 = 0$$

Observe that the total number of constraint equations for network #4 equals the total number of conductors incident on that node: $4+2+2=8$. This requirement must always be met for any set of constraint equations for a termination/interconnection network. The matrices in (8.7) become

$$\hat{\mathbf{Y}}_1^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{50} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \hat{\mathbf{Z}}_1^4 = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\hat{\mathbf{Y}}_2^4 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \hat{\mathbf{Z}}_2^4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\hat{\mathbf{Y}}_3^4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -1 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \hat{\mathbf{Z}}_3^4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

and

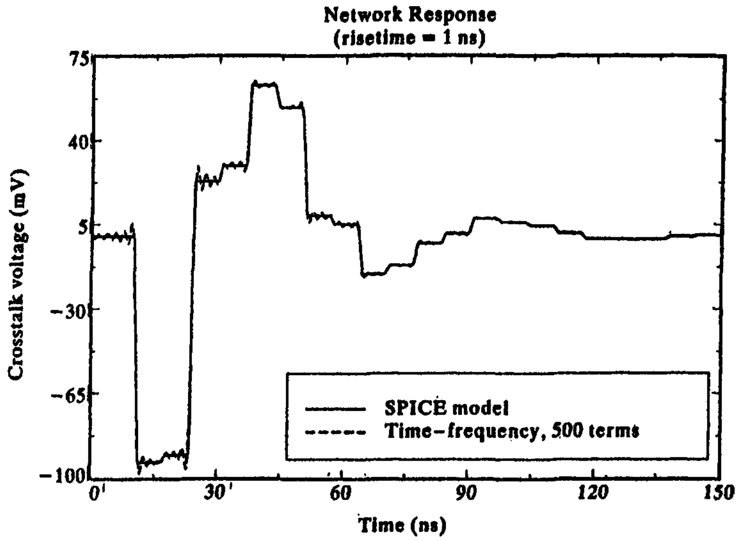


FIGURE 8.9 Comparison of crosstalk voltage at the termination of conductor #1 of tube #3 for the transmission-line network of Fig. 8.2 for a risetime of 1 ns using the SPICE model and the time-domain to frequency-domain transformation which utilizes the frequency-domain transfer function.

$$\hat{\mathbf{P}}^4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The predictions of this model are compared to those of the SPICE model for a risetime of 1 ns in Figure 8.9. The predictions of the time-domain to frequency-domain transformation are obtained by first determining the frequency-domain transfer function with this model at the spectral harmonics of the input. The ramp waveform of Fig. 8.3(c) is modeled as a trapezoidal waveform with identical rise and fall times and a 1 MHz repetition rate. This is decomposed into its spectral components and combined with the frequency-domain transfer function computed with the above admittance parameter

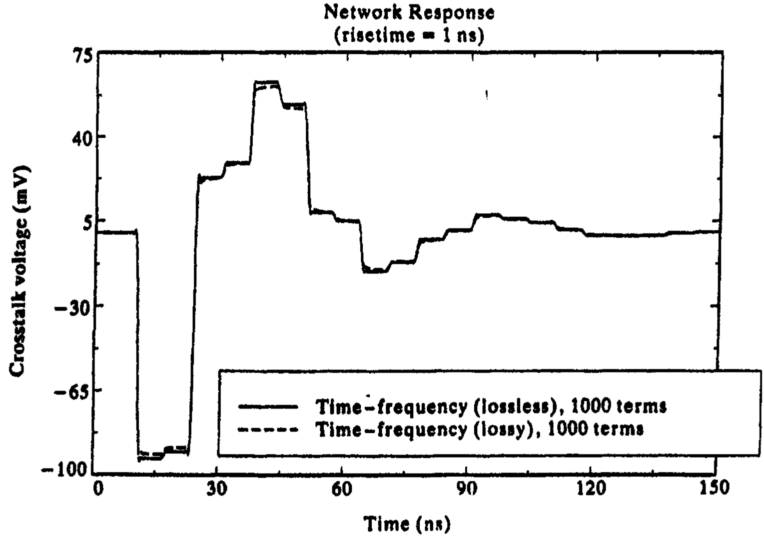


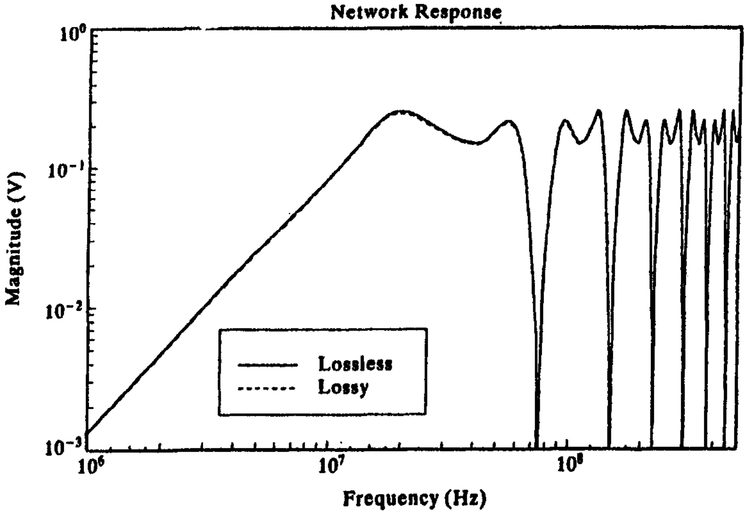
FIGURE 8.10 Comparison of crosstalk voltage at the termination of conductor #1 of tube #3 for the transmission-line network of Fig. 8.2 for a risetime of 1 ns using the time-domain to frequency-domain transformation with and without losses.

model at 500 harmonics. The resulting spectral components of the output voltage are combined using **TIMEFREQ.FOR** giving $V_{out}(t)$. The spectrum of this waveform rolls off at -40 dB/decade above $1/\pi\tau_r = 318.3$ MHz so a final frequency of 500 MHz is marginally sufficient and some ringing appears on the waveform at the transitions. The frequency-dependent losses of the conductors are included in the transfer function and the results recomputed and shown in Fig. 8.10 using 1000 harmonics. This upper limit of 1000 MHz for the frequency-domain transfer function is a factor of 3 higher than the point where the spectrum rolls of at -40 dB/decade. This gives better predictions than the use of 500 harmonics. Observe that the wire losses appear to have a minor effect on the output voltage waveshape. Figure 8.11 shows the frequency response of the transfer function obtained with this method with and without losses. This further confirms that the wire losses have little effect in this problem. Time-domain results can be directly obtained using the admittance matrix characterization and convolution as described in [5].

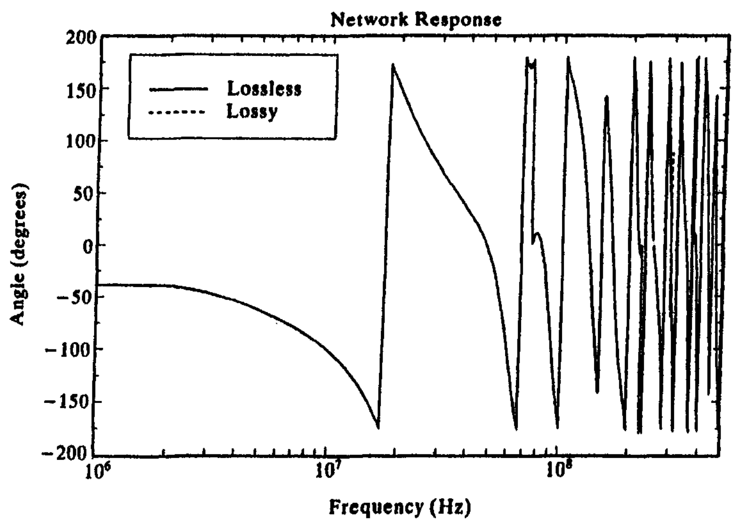
The admittance parameters are not, of course, the only way of characterizing the tubes. The dual is the impedance parameter characterization:

$$\hat{\mathbf{V}}(0) = \hat{\mathbf{Z}}_S\hat{\mathbf{I}}(0) + \hat{\mathbf{Z}}_M(-\hat{\mathbf{I}}(\mathscr{L})) + \hat{\mathbf{V}}_{S0} \tag{8.16a}$$

$$\hat{\mathbf{V}}(\mathscr{L}) = \hat{\mathbf{Z}}_M\hat{\mathbf{I}}(0) + \hat{\mathbf{Z}}_S(-\hat{\mathbf{I}}(\mathscr{L})) + \hat{\mathbf{V}}_{SL} \tag{8.16b}$$



(a)



(b)

FIGURE 8.11 The frequency-domain crosstalk voltage V_{out} at the termination of conductor #1 of tube #3 for the transmission-line network of Fig. 8.2 with and without losses: (a) magnitude, (b) phase.

The chain parameters in (8.1) can be manipulated to yield

$$\begin{aligned}\hat{\mathbf{Z}}_S &= -\hat{\mathbf{\Phi}}_{21}^{-1}\mathbf{\Phi}_{22} = -\mathbf{\Phi}_{11}\mathbf{\Phi}_{21}^{-1} \\ &= \hat{\mathbf{Z}}_C\hat{\mathbf{T}}(e^{\hat{\gamma}\mathscr{L}} - e^{-\hat{\gamma}\mathscr{L}})^{-1}(e^{\hat{\gamma}\mathscr{L}} + e^{-\hat{\gamma}\mathscr{L}})\hat{\mathbf{T}}^{-1}\end{aligned} \tag{8.17a}$$

$$\begin{aligned}\hat{\mathbf{Z}}_M &= -\hat{\mathbf{\Phi}}_{12}^{-1} \\ &= 2\hat{\mathbf{Z}}_C\hat{\mathbf{T}}(e^{\hat{\gamma}\mathscr{L}} - e^{-\hat{\gamma}\mathscr{L}})^{-1}\hat{\mathbf{T}}^{-1}\end{aligned} \tag{8.17b}$$

$$\hat{\mathbf{V}}_{S0} = \hat{\mathbf{Z}}_M\hat{\mathbf{V}}_{FT} \tag{8.17c}$$

$$\hat{\mathbf{V}}_{SL} = \hat{\mathbf{V}}_{FT} + \hat{\mathbf{Z}}_S\hat{\mathbf{I}}_{FT} \tag{8.17d}$$

The overall matrix to be solved can be obtained by substituting (8.16) for the tubes connected to node m which is characterized by (8.7) to give

$$\hat{\mathbf{Z}}_T\hat{\mathbf{I}}_T = \hat{\mathbf{P}}_T \tag{8.18}$$

This result has the same form as (8.14) and (8.15) with the following substitutions:

$$\hat{\mathbf{Y}}_l^m \Rightarrow \hat{\mathbf{Z}}_l^m \tag{8.19a}$$

$$\hat{\mathbf{Z}}_l^m \Rightarrow \hat{\mathbf{Y}}_l^m \tag{8.19b}$$

$$\hat{\mathbf{Y}}_{Sl} \Rightarrow \hat{\mathbf{Z}}_{Sl} \tag{8.19c}$$

$$\hat{\mathbf{Y}}_{Ml} \Rightarrow \hat{\mathbf{Z}}_{Ml} \tag{8.19d}$$

$$\hat{\mathbf{I}}_{Sl}^m \Rightarrow \hat{\mathbf{V}}_{Sl}^m \tag{8.19e}$$

$$\hat{\mathbf{V}}_l^m \Rightarrow \hat{\mathbf{I}}_l^m \tag{8.19f}$$

With the impedance parameters we solve for the currents incident on the nodes, $\hat{\mathbf{I}}_l^m$.

8.4 REPRESENTATION WITH THE BLT EQUATIONS

An alternative to the above methods is the use of the *scattering parameter* representation of the tubes [1–3]. Consider a tube having *lumped excitation* at some point, $z = \tau$, along its length illustrated in Fig. 8.12. These lumped excitations may represent point sources such as the direct attachment of a lightning stroke or can be extended to include distributed incident-field excitation as we will show. The general frequency-domain solution of the MTL equations for this tube can be written as in Chapter 4 for the tube segments to the left and right of the sources as

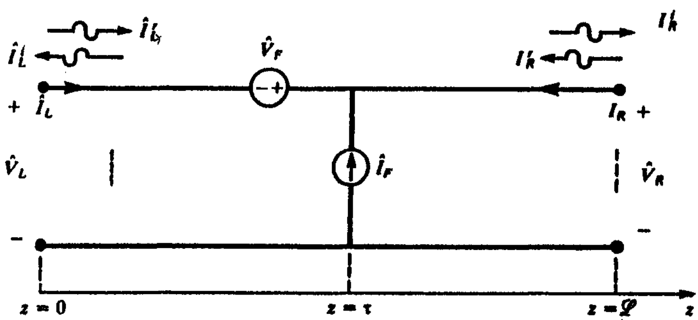


FIGURE 8.12 Determination of the scattering parameters for a tube having lumped excitation at a point on the tube.

$$\hat{\mathbf{V}}(z) = \hat{\mathbf{Z}}_C\hat{\mathbf{T}}(\mathbf{e}^{-\hat{\gamma}z}\hat{\mathbf{I}}_L^+ + \mathbf{e}^{\hat{\gamma}z}\hat{\mathbf{I}}_L^-) \tag{8.20a}$$

$$\hat{\mathbf{I}}(z) = \hat{\mathbf{T}}(\mathbf{e}^{-\hat{\gamma}z}\hat{\mathbf{I}}_L^+ - \mathbf{e}^{\hat{\gamma}z}\hat{\mathbf{I}}_L^-) \tag{8.20b}$$

for $0 \le z < \tau$ and

$$\hat{\mathbf{V}}(z) = \hat{\mathbf{Z}}_C\hat{\mathbf{T}}(\mathbf{e}^{-\hat{\gamma}z}\hat{\mathbf{I}}_R^+ + \mathbf{e}^{\hat{\gamma}z}\hat{\mathbf{I}}_R^-) \tag{8.21a}$$

$$\hat{\mathbf{I}}(z) = \hat{\mathbf{T}}(\mathbf{e}^{-\hat{\gamma}z}\hat{\mathbf{I}}_R^+ - \mathbf{e}^{\hat{\gamma}z}\hat{\mathbf{I}}_R^-) \tag{8.21b}$$

for $\tau < z \le \mathscr{L}$. The subscripts on the undetermined-constant vectors, L and R, represent the left and right segments, respectively. The characteristic impedance matrix is the inverse of the characteristic admittance matrix given in (8.5), and the propagation matrix is determined as in (8.4). Evaluating these at $z = \tau$ yields

$$-\hat{\mathbf{Z}}_C\hat{\mathbf{T}}(\mathbf{e}^{-\hat{\gamma}\tau}\hat{\mathbf{I}}_L^+ + \mathbf{e}^{\hat{\gamma}\tau}\hat{\mathbf{I}}_L^-) + \hat{\mathbf{Z}}_C\hat{\mathbf{T}}(\mathbf{e}^{-\hat{\gamma}\tau}\hat{\mathbf{I}}_R^+ + \mathbf{e}^{\hat{\gamma}\tau}\hat{\mathbf{I}}_R^-) = \hat{\mathbf{V}}_F \tag{8.22a}$$

$$-\hat{\mathbf{T}}(\mathbf{e}^{-\hat{\gamma}\tau}\hat{\mathbf{I}}_L^+ - \mathbf{e}^{\hat{\gamma}\tau}\hat{\mathbf{I}}_L^-) + \hat{\mathbf{T}}(\mathbf{e}^{-\hat{\gamma}\tau}\hat{\mathbf{I}}_R^+ - \mathbf{e}^{\hat{\gamma}\tau}\hat{\mathbf{I}}_R^-) = \hat{\mathbf{I}}_F \tag{8.22b}$$

Adding and subtracting these yields

$$\hat{\mathbf{I}}_L^+ - \hat{\mathbf{I}}_R^+ = -\tfrac{1}{2}\mathbf{e}^{\hat{\gamma}\tau}\hat{\mathbf{T}}^{-1}\hat{\mathbf{Z}}_C^{-1}(\hat{\mathbf{V}}_F + \hat{\mathbf{Z}}_C\hat{\mathbf{I}}_F) \tag{8.23a}$$

$$\hat{\mathbf{I}}_L^- - \hat{\mathbf{I}}_R^- = -\tfrac{1}{2}\mathbf{e}^{-\hat{\gamma}\tau}\hat{\mathbf{T}}^{-1}\hat{\mathbf{Z}}_C^{-1}(\hat{\mathbf{V}}_F - \hat{\mathbf{Z}}_C\hat{\mathbf{I}}_F) \tag{8.23b}$$

The currents at the line endpoints can be logically decomposed into *incident* and *reflected* waves by evaluating (8.20b) at $z = 0$ and (8.21b) at $z = \mathscr{L}$ as

$$\hat{\mathbf{I}}_L = \hat{\mathbf{I}}(0) = \hat{\mathbf{I}}_L^r + \hat{\mathbf{I}}_L^i \tag{8.24a}$$

$$\hat{\mathbf{I}}_R = -\hat{\mathbf{I}}(\mathscr{L}) = \hat{\mathbf{I}}_R^i + \hat{\mathbf{I}}_R^r \tag{8.24b}$$

where

$$\hat{\mathbf{I}}_L^r = \hat{\mathbf{T}}\hat{\mathbf{I}}_L^+ \tag{8.25a}$$

$$\hat{\mathbf{I}}_L^i = -\hat{\mathbf{T}}\hat{\mathbf{I}}_L^- \tag{8.25b}$$

$$\hat{\mathbf{I}}_R^i = -\hat{\mathbf{T}}\mathbf{e}^{-\hat{\gamma}\mathscr{L}}\hat{\mathbf{I}}_R^+ \tag{8.25c}$$

$$\hat{\mathbf{I}}_R^r = \hat{\mathbf{T}}\mathbf{e}^{\hat{\gamma}\mathscr{L}}\hat{\mathbf{I}}_R^- \tag{8.25d}$$

where superscripts i and r denote *incident* and *reflected*, respectively. This is a sensible designation if we designate the incident wave as the portion incoming at the termination and the reflected wave as the portion outgoing from the junction and also observe that a component containing $\mathbf{e}^{-\hat{\gamma}z}$ is traveling to the right and a component containing $\mathbf{e}^{\hat{\gamma}z}$ is traveling to the left. To conform with the results of the previous section, the tube currents are directed into the tube at both ends. Combining (8.23) and (8.25) yields the reflected components in terms of the incident quantities as

$$\hat{\mathbf{I}}_L^r = -(\hat{\mathbf{T}}\mathbf{e}^{\hat{\gamma}\mathscr{L}}\hat{\mathbf{T}}^{-1})\hat{\mathbf{I}}_R^i - \tfrac{1}{2}(\hat{\mathbf{T}}\mathbf{e}^{\hat{\gamma}\tau}\hat{\mathbf{T}}^{-1})(\hat{\mathbf{I}}_F + \hat{\mathbf{Z}}_C^{-1}\hat{\mathbf{V}}_F) \tag{8.26a}$$

$$\hat{\mathbf{I}}_R^r = -(\hat{\mathbf{T}}\mathbf{e}^{\hat{\gamma}\mathscr{L}}\hat{\mathbf{T}}^{-1})\hat{\mathbf{I}}_L^i - \tfrac{1}{2}(\hat{\mathbf{T}}\mathbf{e}^{\hat{\gamma}(\mathscr{L}-\tau)}\hat{\mathbf{T}}^{-1})(\hat{\mathbf{I}}_F - \hat{\mathbf{Z}}_C^{-1}\hat{\mathbf{V}}_F) \tag{8.26b}$$

(There are sign differences between these results and those of [1–3] due to our choice of having the total currents directed into the tube at both ends.) In matrix notation these become

$$\begin{bmatrix} \hat{\mathbf{I}}_L^r \\ \hat{\mathbf{I}}_R^r \end{bmatrix} = \hat{\mathbf{R}}\begin{bmatrix} \hat{\mathbf{I}}_L^i \\ \hat{\mathbf{I}}_R^i \end{bmatrix} + \hat{\mathbf{I}}_T \tag{8.27}$$

where the *tube propagation matrix* $\hat{\mathbf{R}}$ is

$$\hat{\mathbf{R}} = \begin{bmatrix} \mathbf{0} & \hat{\mathbf{\Gamma}} \\ \hat{\mathbf{\Gamma}} & \mathbf{0} \end{bmatrix} \tag{8.28a}$$

with entries

$$\hat{\mathbf{\Gamma}} = -\hat{\mathbf{T}}\mathbf{e}^{\hat{\gamma}\mathscr{L}}\hat{\mathbf{T}}^{-1} \tag{8.28b}$$

The current source vector due to the lumped sources at $z = \tau$ is

$$\hat{\mathbf{I}}_T = \begin{bmatrix} -\tfrac{1}{2}(\hat{\mathbf{T}}\mathbf{e}^{\hat{\gamma}\tau}\hat{\mathbf{T}}^{-1})(\hat{\mathbf{I}}_F + \hat{\mathbf{Z}}_C^{-1}\hat{\mathbf{V}}_F) \\ -\tfrac{1}{2}(\hat{\mathbf{T}}\mathbf{e}^{\hat{\gamma}(\mathscr{L}-\tau)}\hat{\mathbf{T}}^{-1})(\hat{\mathbf{I}}_F - \hat{\mathbf{Z}}_C^{-1}\hat{\mathbf{V}}_F) \end{bmatrix} \tag{8.29a}$$

In the case of distributed excitation such as is the case for incident field illumination of the tube, the source vector becomes simply

$$\hat{\mathbf{I}}_T = \int_0^{\mathscr{L}} \begin{bmatrix} -\tfrac{1}{2}(\hat{\mathbf{T}}\mathbf{e}^{\hat{\gamma}\tau}\hat{\mathbf{T}}^{-1})[\hat{\mathbf{I}}_F(\tau) + \hat{\mathbf{Z}}_C^{-1}\hat{\mathbf{V}}_F(\tau)] \\ -\tfrac{1}{2}(\hat{\mathbf{T}}\mathbf{e}^{\hat{\gamma}(\mathscr{L}-\tau)}\hat{\mathbf{T}}^{-1})(\hat{\mathbf{I}}_F(\tau) - \hat{\mathbf{Z}}_C^{-1}\hat{\mathbf{V}}_F(\tau)] \end{bmatrix} d\tau \tag{8.29b}$$

The total voltages and currents at the left end of the tube become, by evaluating (8.20) at $z = 0$ and substituting (8.25a) and (8.25b),

$$\hat{\mathbf{V}}(0) = \hat{\mathbf{V}}_L = \hat{\mathbf{Z}}_C(\hat{\mathbf{I}}_L^r - \hat{\mathbf{I}}_L^i) \tag{8.30a}$$

$$\hat{\mathbf{I}}(0) = \hat{\mathbf{I}}_L = \hat{\mathbf{I}}_L^r + \hat{\mathbf{I}}_L^i \tag{8.30b}$$

Similarly, the total voltages and currents at the right end of the tube become, by evaluating (8.21) at $z = \mathscr{L}$ and substituting (8.25c) and (8.25d),

$$\hat{\mathbf{V}}(\mathscr{L}) = \hat{\mathbf{V}}_R = \hat{\mathbf{Z}}_C(\hat{\mathbf{I}}_R^r - \hat{\mathbf{I}}_R^i) \tag{8.31a}$$

$$-\hat{\mathbf{I}}(\mathscr{L}) = \hat{\mathbf{I}}_R = \hat{\mathbf{I}}_R^r + \hat{\mathbf{I}}_R^i \tag{8.31b}$$

Observe that, as in the case of the admittance parameters, the total currents are defined as being *directed into the tubes* at both ends. The *incident* components are defined as being the components traveling out of the tubes or into the network attached to that end. The *reflected* components are similarly defined as being the components traveling into the tubes or out of the network attached to that end. Hence the origin of the names *incident* and *reflected*: with respect to the network or node at which the end of the tube is incident.

These results can be derived from the chain parameter matrix representation of the line with incident-field illumination. Substituting the chain parameter matrix of the line given in (7.29) into (8.30) and (8.31) we obtain (8.26). This gives the current source vector in (8.29b) in terms of the total incident-field vectors, $\hat{\mathbf{I}}_{FT}$ and $\hat{\mathbf{V}}_{FT}$, given in (7.29) as

$$\hat{\mathbf{I}}_T = \begin{bmatrix} -\frac{1}{2}\hat{\mathbf{T}}e^{\hat{\gamma}\mathscr{L}}\hat{\mathbf{T}}^{-1}(\hat{\mathbf{I}}_{FT} + \hat{\mathbf{Y}}_C\hat{\mathbf{V}}_{FT}) \\ -\frac{1}{2}(\hat{\mathbf{I}}_{FT} - \hat{\mathbf{Y}}_C\hat{\mathbf{V}}_{FT}) \end{bmatrix} \tag{8.32}$$

We next form the *junction scattering matrix* for node m, $\hat{\mathbf{S}}_m$, which relates the reflected and incident current components at node m as

$$\hat{\mathbf{I}}_m^r = \hat{\mathbf{S}}_m\hat{\mathbf{I}}_m^i \tag{8.33}$$

The number of equations here equals the total number of conductors incident on that node. First consider a *termination node* where only one tube is incident as shown in Fig. 8.13. Suppose the network contains no lumped sources and is characterized by a generalized Thévenin equivalent as

$$\begin{aligned}\hat{\mathbf{V}}_m &= -\hat{\mathbf{Z}}_m\hat{\mathbf{I}}_m \\ &= -\hat{\mathbf{Z}}_m(\hat{\mathbf{I}}_m^r + \hat{\mathbf{I}}_m^i)\end{aligned} \tag{8.34}$$

(Recall that the total currents are defined as being directed into the tube ends and are therefore directed out of the termination networks.) The relations for

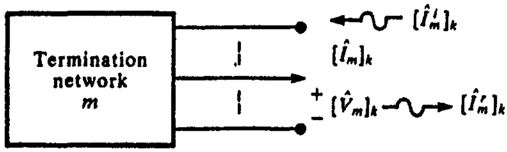


FIGURE 8.13 Definitions of incident and scattered waves for determining the scattering parameters of a termination network.

the voltage and current at the end of the tube that is incident on this node are

$$\hat{\mathbf{V}}_m = \hat{\mathbf{Z}}_{Cm}(\hat{\mathbf{I}}^r_m - \hat{\mathbf{I}}^i_m) \tag{8.35}$$

Solving (8.34) and (8.35) yields the *current scattering matrix*:

$$\hat{\mathbf{S}}_m = -(\hat{\mathbf{Z}}_m + \hat{\mathbf{Z}}_{Cm})^{-1}(\hat{\mathbf{Z}}_m - \hat{\mathbf{Z}}_{Cm}) \tag{8.36}$$

This has a direct parallel to the scalar *current reflection coefficient* for a two-conductor line [A.1].

The next form of the scattering matrix is for an interconnection network wherein there are two or more tubes incident. In the admittance parameter representation of the previous section we characterized these in a general sense as a combination of generalized Thévenin and generalized Norton equivalents so that series and parallel impedances and excitation sources could be included. The formulation of the BLT equations in [1–3] has the excitation sources along the tubes due to incident fields. We will derive the BLT equations with that assumption although we will later modify them for the more general case of lumped sources within the networks. Thus the interconnection networks will simply have:

1. Conductors directly connected.
2. Conductors terminated in short circuits.
3. Conductors terminated in open circuits as shown in Fig. 8.14.

First consider the direction connection of several conductors as shown in Fig. 8.14(a). This is characterized as

$$\mathbf{C}_I \hat{\mathbf{I}}_m = \mathbf{0} \tag{8.37a}$$

$$\mathbf{C}_V \hat{\mathbf{V}}_m = \mathbf{0} \tag{8.37b}$$

where $\hat{\mathbf{I}}_m$ contains the total currents of the conductors incident on the m-th node and $\hat{\mathbf{V}}_m$ contains the voltages of those conductors. The first relation in (8.37a) enforces KCL at the connection so that for each connection a one

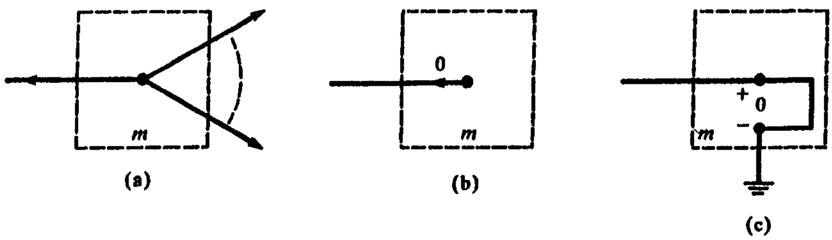


FIGURE 8.14 Interconnection networks: (a) direct connection, (b) open circuit, and (c) short circuit.

appears in the columns of $\mathbf{C}_\mathrm{I}$ corresponding to the currents of $\hat{\mathbf{I}}_m$ for the conductors that are connected. The second condition in (8.37b) enforces KVL at the connection so that for n conductors connected, there are $(n-1)$ equations enforcing $\hat{V}_a - \hat{V}_b = 0$ for $(n-1)$ pairs of the voltages. Thus a one appears in the column of $\mathbf{C}_V$ corresponding to the voltage of the pair in $\hat{\mathbf{V}}_m$ and a negative one appears in the column of $\mathbf{C}_V$ corresponding to the other voltage of the pair in $\hat{\mathbf{V}}_m$. The second situation is an open circuit as illustrated in Fig. 8.14(b). Enforcing KCL requires that we place a one in the column of $\mathbf{C}_\mathrm{I}$ corresponding to the current of $\hat{\mathbf{I}}_m$ that is constrained to zero by the open circuit. The third constraint is a short circuit illustrated in Fig. 8.14(c). Enforcing KVL requires that we place a one in the column of $\mathbf{C}_\mathrm{V}$ corresponding to the voltage of $\hat{\mathbf{V}}_m$ that is constrained to zero by the short circuit. The sum of the row dimensions of $\mathbf{C}_\mathrm{I}$ and $\mathbf{C}_\mathrm{V}$ must equal the total number of conductors incident on the node. The scattering matrix for this interconnection node can now be formulated. Decomposing the total currents and voltages into their incident and reflected components gives

$$-\mathbf{C}_\mathrm{I}\hat{\mathbf{I}}_m^r = \mathbf{C}_\mathrm{I}\hat{\mathbf{I}}_m^i \tag{8.38a}$$

$$\mathbf{C}_\mathrm{V}\hat{\mathbf{Z}}_{Cm}\hat{\mathbf{I}}_m^r = \mathbf{C}_\mathrm{V}\hat{\mathbf{Z}}_{Cm}\hat{\mathbf{I}}_m^i \tag{8.38b}$$

where $\hat{\mathbf{Z}}_{Cm}$ contains the characteristic impedance matrices of the tubes incident upon the node on the main diagonal and zeros elsewhere. Solving (8.38) yields the scattering matrix of the interconnection node as

$$\hat{\mathbf{S}}_m = \begin{bmatrix} -\mathbf{C}_\mathrm{I} \\ \mathbf{C}_\mathrm{V}\hat{\mathbf{Z}}_{Cm} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{C}_\mathrm{I} \\ \mathbf{C}_\mathrm{V}\hat{\mathbf{Z}}_{Cm} \end{bmatrix} \tag{8.39}$$

Arranging the tube characterizations for all tubes in the network and the scattering matrix representations for all nodes in the network yields the overall characterization

$$\hat{\mathbf{I}}_\mathrm{T}^r = \hat{\mathbf{R}}_\mathrm{T}\hat{\mathbf{I}}_\mathrm{T}^i + \hat{\mathbf{I}}_{\mathrm{TT}} \tag{8.40a}$$

$$\hat{\mathbf{I}}_\mathrm{T}^r = \hat{\mathbf{S}}_\mathrm{T}\hat{\mathbf{I}}_\mathrm{T}^i \tag{8.40b}$$

The overall tube propagation matrix, $\hat{\mathbf{R}}_T$, is $2n_T \times 2n_T$ where n_T is the total number of conductors in the overall transmission-line network and has the individual tube characterization matrices given in (8.27) on the main diagonal and zeros elsewhere. Similarly the overall junction scattering matrix, $\hat{\mathbf{S}}_T$, is $2n_T \times 2n_T$ and contains the individual scattering matrices for the interconnection and termination networks on the main diagonal and zeros elsewhere. Combining (8.40) gives

$$(\hat{\mathbf{S}}_T - \hat{\mathbf{R}}_T)\hat{\mathbf{I}}_T^i = \hat{\mathbf{I}}_{TT} \tag{8.41}$$

which are referred to as the BLT (Baum, Liu, and Tesche) equations. The total currents can be obtained by writing

$$\hat{\mathbf{I}}_T = \hat{\mathbf{I}}_T^i + \hat{\mathbf{I}}_T^r \tag{8.42}$$

Solving (8.40) and (8.42) simultaneously gives the total currents as

$$\hat{\mathbf{I}}_T = (\mathbf{1} + \hat{\mathbf{S}}_T)(\hat{\mathbf{S}}_T - \hat{\mathbf{R}}_T)^{-1}\hat{\mathbf{I}}_{TT} \tag{8.43}$$

This formulation can be extended to include series and parallel impedances and lumped sources in the termination/interconnection networks. In order to provide that extension, consider the general characterization of the m-th termination/interconnection node illustrated in Fig. 8.7(b) and given in (8.7):

$$\hat{\mathbf{Y}}_i^m\hat{\mathbf{V}}_i^m + \hat{\mathbf{Z}}_i^m\hat{\mathbf{I}}_i^m + \hat{\mathbf{Y}}_j^m\hat{\mathbf{V}}_j^m + \hat{\mathbf{Z}}_j^m\hat{\mathbf{I}}_j^m + \hat{\mathbf{Y}}_k^m\hat{\mathbf{V}}_k^m + \hat{\mathbf{Z}}_k^m\hat{\mathbf{I}}_k^m = \hat{\mathbf{P}}^m \tag{8.7}$$

This can be written in matrix form as

$$[\hat{\mathbf{Y}}_i^m \quad \hat{\mathbf{Y}}_j^m \quad \hat{\mathbf{Y}}_k^m]\begin{bmatrix}\hat{\mathbf{V}}_i^m \\ \hat{\mathbf{V}}_j^m \\ \hat{\mathbf{V}}_k^m\end{bmatrix} + [\hat{\mathbf{Z}}_i^m \quad \hat{\mathbf{Z}}_j^m \quad \hat{\mathbf{Z}}_k^m]\begin{bmatrix}\hat{\mathbf{I}}_i^m \\ \hat{\mathbf{I}}_j^m \\ \hat{\mathbf{I}}_k^m\end{bmatrix} = \hat{\mathbf{P}}^m \tag{8.44}$$

The currents and voltages can be decomposed into incident and reflected components according to (8.31) as

$$\hat{\mathbf{V}}_i^m = \hat{\mathbf{Z}}_{Ci}(\hat{\mathbf{I}}_i^{mr} - \hat{\mathbf{I}}_i^{mi}) \tag{8.45a}$$

$$\hat{\mathbf{I}}_i^m = \hat{\mathbf{I}}_i^{mr} + \hat{\mathbf{I}}_i^{mi} \tag{8.45b}$$

and likewise for tube j and tube k. Substituting (8.45) into (8.44) yields

$$\begin{bmatrix}\hat{\mathbf{I}}_i^{mr} \\ \hat{\mathbf{I}}_j^{mr} \\ \hat{\mathbf{I}}_k^{mr}\end{bmatrix} = \hat{\mathbf{S}}_m\begin{bmatrix}\hat{\mathbf{I}}_i^{mi} \\ \hat{\mathbf{I}}_j^{mi} \\ \hat{\mathbf{I}}_k^{mi}\end{bmatrix} + \hat{\mathbf{I}}_S^m \tag{8.46}$$

where

$$\hat{\mathbf{S}}_m = -\{[\hat{\mathbf{Z}}_i^m \quad \hat{\mathbf{Z}}_j^m \quad \hat{\mathbf{Z}}_k^m] + [\mathbf{Y}_i^m \quad \hat{\mathbf{Y}}_j^m \quad \hat{\mathbf{Y}}_k^m]\hat{\mathbf{Z}}_{Cm}\}^{-1} \times \{[\hat{\mathbf{Z}}_i^m \quad \hat{\mathbf{Z}}_j^m \quad \hat{\mathbf{Z}}_k^m] - [\hat{\mathbf{Y}}_i^m \quad \hat{\mathbf{Y}}_j^m \quad \hat{\mathbf{Y}}_k^m]\hat{\mathbf{Z}}_{Cm}\} \tag{8.47a}$$

and the collection of characteristic impedance matrices of the tubes incident on the termination/interconnection network is

$$\hat{\mathbf{Z}}_{Cm} = \begin{bmatrix} \hat{\mathbf{Z}}_{Ci} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{Z}}_{Cj} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \hat{\mathbf{Z}}_{Ck} \end{bmatrix} \tag{8.47b}$$

The current source vector is

$$\hat{\mathbf{I}}_S^m = \{[\hat{\mathbf{Z}}_i^m \quad \hat{\mathbf{Z}}_j^m \quad \hat{\mathbf{Z}}_k^m] + [\hat{\mathbf{Y}}_i^m \quad \hat{\mathbf{Y}}_j^m \quad \hat{\mathbf{Y}}_k^m]\hat{\mathbf{Z}}_{Cm}\}^{-1}\hat{\mathbf{P}}^m \tag{8.47c}$$

For example, consider the case of a termination network (only one tube incident on the node) and a generalized Thévenin representation of this network:

$$\underbrace{\mathbf{1}}_{\hat{\mathbf{Y}}_i^m}\hat{\mathbf{V}}_i^m + \underbrace{\hat{\mathbf{Z}}_m}_{\hat{\mathbf{Z}}_i^m}\hat{\mathbf{I}}_i^m = \underbrace{\hat{\mathbf{V}}_{Sm}}_{\hat{\mathbf{P}}^m} \tag{8.48}$$

The above representation becomes

$$\hat{\mathbf{S}}_m = -(\hat{\mathbf{Z}}_m + \hat{\mathbf{Z}}_{Ci})^{-1}(\hat{\mathbf{Z}}_m - \hat{\mathbf{Z}}_{Ci}) \tag{8.49a}$$

$$\hat{\mathbf{I}}_S^m = (\hat{\mathbf{Z}}_m + \hat{\mathbf{Z}}_{Ci})^{-1}\hat{\mathbf{V}}_{Sm} \tag{8.49b}$$

Thus the scattering parameter representation in (8.33) has been modified to include lumped sources in the termination/interconnection networks. In the case of an interconnection network containing only short circuits, open circuits, and/or direct connections, this representation becomes

$$[\hat{\mathbf{Y}}_i^m \quad \hat{\mathbf{Y}}_j^m \quad \mathbf{Y}_k^m] = \begin{bmatrix} \mathbf{0} \\ \mathbf{C}_V \end{bmatrix} \tag{8.50}$$

and

$$[\hat{\mathbf{Z}}_i^m \quad \hat{\mathbf{Z}}_j^m \quad \hat{\mathbf{Z}}_k^m] = \begin{bmatrix} \mathbf{C}_I \\ \mathbf{0} \end{bmatrix} \tag{8.51}$$

where $\mathbf{C}_V$ and $\mathbf{C}_I$ were originally defined in (8.37). Substituting (8.50) and (8.51) into (8.47a) yields (8.39).

The tube characterizations in (8.27) remain unchanged. Thus the overall representation of the network is of the form in (8.40):

$$\hat{\mathbf{I}}^r_{\mathrm{T}} = \hat{\mathbf{R}}_{\mathrm{T}} \hat{\mathbf{I}}^i_{\mathrm{T}} + \hat{\mathbf{I}}_{TT} \tag{8.52a}$$

$$\hat{\mathbf{I}}^r_{\mathrm{T}} = \hat{\mathbf{S}}_{\mathrm{T}} \hat{\mathbf{I}}^i_{\mathrm{T}} + \hat{\mathbf{I}}_{ST} \tag{8.52b}$$

where $\hat{\mathbf{S}}_{\mathrm{T}}$ is $2n_{\mathrm{T}} \times 2n_{\mathrm{T}}$ and contains the individual scattering matrices given in (8.47a) on the main diagonal and zeros elsewhere, and $\hat{\mathbf{I}}_{\mathrm{ST}}$ is $2n_{\mathrm{T}} \times 1$ and contains the $\hat{\mathbf{I}}^m_S$ in (8.47c). In a fashion similar to the earlier derivation we obtain the general form of the BLT equations for the total tube currents as

$$\hat{\mathbf{I}}_{\mathrm{T}} = [\mathbf{1} + \hat{\mathbf{S}}_{\mathrm{T}}][\hat{\mathbf{S}}_{\mathrm{T}} - \hat{\mathbf{R}}_{\mathrm{T}}]^{-1}\hat{\mathbf{I}}_{TT} - [\mathbf{1} + \hat{\mathbf{R}}_{\mathrm{T}}][\hat{\mathbf{S}}_{\mathrm{T}} - \hat{\mathbf{R}}_{\mathrm{T}}]^{-1}\hat{\mathbf{I}}_{\mathrm{ST}} \tag{8.53}$$

For example, consider the two-conductor line shown in Fig. 4.1 (without incident-field illumination). The total tube propagation matrix $\hat{\mathbf{R}}_{\mathrm{T}}$ is simply (8.28) since there is only one tube and becomes

$$\hat{\mathbf{R}}_{\mathrm{T}} = \begin{bmatrix} 0 & -e^{\hat{\gamma}\mathscr{L}} \\ -e^{\hat{\gamma}\mathscr{L}} & 0 \end{bmatrix}$$

Writing (8.44) at the source and at the load gives

$$\underbrace{1}_{\hat{Y}^S_1} \hat{V}^S_1 + \underbrace{\hat{Z}_S \hat{I}^S_1}_{\hat{Z}^S_1} = \underbrace{\hat{V}_S}_{\hat{P}^S}$$

$$\underbrace{1}_{\hat{Y}^L_1} \hat{V}^L_1 + \underbrace{\hat{Z}_L \hat{I}^L_1}_{\hat{Z}^L_1} = \underbrace{\hat{V}_L}_{\hat{P}^L}$$

Thus the scattering matrices at the source and load in (8.47a) become

$$\hat{S}_S = -(\hat{Z}_S + \hat{Z}_C)^{-1}(\hat{Z}_S - \hat{Z}_C) = -\hat{\Gamma}_S$$

$$\hat{S}_L = -(\hat{Z}_L + \hat{Z}_C)^{-1}(\hat{Z}_L - \hat{Z}_C) = -\hat{\Gamma}_L$$

where $\hat{\Gamma}_S$ and $\hat{\Gamma}_L$ are the *voltage reflection coefficients* at the source and load, respectively. The *current reflection coefficients* are the negatives of these [A.1]. Thus the overall scattering matrix is

$$\hat{\mathbf{S}}_{\mathrm{T}} = \begin{bmatrix} -\hat{\Gamma}_S & 0 \\ 0 & -\hat{\Gamma}_L \end{bmatrix}$$

Also the current source vectors in (8.47c) become

$$\hat{I}_S^S = (\hat{Z}_S + \hat{Z}_C)^{-1}\hat{V}_S$$
$$\hat{I}_S^L = (\hat{Z}_L + \hat{Z}_C)^{-1}0$$

so that the overall current source vector becomes

$$\hat{\mathbf{I}}_{ST} = \begin{bmatrix} \hat{Z}_S + \hat{Z}_C & 0 \\ 0 & \hat{Z}_L + \hat{Z}_C \end{bmatrix}^{-1} \begin{bmatrix} \hat{P}^S \\ \hat{P}^L \end{bmatrix} = \begin{bmatrix} \dfrac{\hat{V}_S}{(\hat{Z}_S + \hat{Z}_C)} \\ 0 \end{bmatrix}$$

Forming the BLT equation in (8.53) for the total currents yields

$$\begin{aligned} \begin{bmatrix} \hat{I}_1^S \\ \hat{I}_1^L \end{bmatrix} &= -(1 + \hat{\mathbf{R}}_T)(\hat{\mathbf{S}}_T - \hat{\mathbf{R}}_T)^{-1}\hat{\mathbf{I}}_{ST} \\ &= -\begin{bmatrix} 1 & -e^{\hat{\gamma}\mathscr{L}} \\ -e^{\hat{\gamma}\mathscr{L}} & 1 \end{bmatrix} \begin{bmatrix} -\hat{\Gamma}_S & e^{\hat{\gamma}\mathscr{L}} \\ e^{\hat{\gamma}\mathscr{L}} & -\hat{\Gamma}_L \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{\hat{V}_S}{(\hat{Z}_S + \hat{Z}_C)} \\ &= \frac{1}{1 - \hat{\Gamma}_S\hat{\Gamma}_L e^{-2\hat{\gamma}\mathscr{L}}} \begin{bmatrix} 1 - \hat{\Gamma}_L e^{-2\hat{\gamma}\mathscr{L}} \\ (\hat{\Gamma}_L - 1)e^{-\hat{\gamma}\mathscr{L}} \end{bmatrix} \frac{\hat{V}_S}{(\hat{Z}_S + \hat{Z}_C)} \end{aligned}$$

Recalling that $\hat{I}_1^S = \hat{I}(0)$ and $\hat{I}_1^L = -\hat{I}(\mathscr{L})$ these results are identical to those given in (4.26b).

The time-domain solution can be obtained from this formulation by first using the BLT equations to obtain the frequency-domain transfer function and then using the time-domain to frequency-domain transformation method as before. Again, that technique for obtaining the time-domain solution assumes linear loads and tubes since superposition is implicitly used.

8.5 DIRECT TIME-DOMAIN SOLUTIONS IN TERMS OF TRAVELING WAVES

The above solution methods concentrated on the frequency-domain solution of transmission-line networks. The time-domain solution can be obtained from this solution via the time-domain to frequency-domain technique, which is very straightforward and has been used on numerous occasions. Of course, the time-domain to frequency-domain technique requires *linear terminations*. In this final section we briefly investigate the direct time-domain solution via the traveling waves existing on the various tubes of the network.

To begin the discussion consider a tandem connection of two-conductor lines having a discontinuity as shown in Fig. 8.15(a). Each line is characterized by a characteristic impedance, Z_{Ci}, and time delay, T_i. Lossless lines and resistive terminations are assumed to simplify the discussion. The frequency-domain

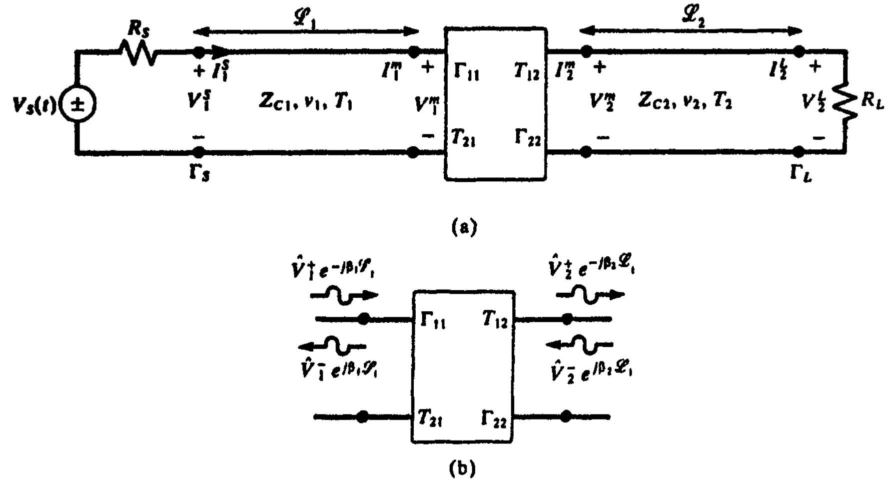


FIGURE 8.15 Determination of the scattering parameters at the junction of two different two-conductor lines: (a) line configuration, and (b) definition of incident and reflected voltage waves at the junction.

solutions on each line are of the form

$$\hat{V}_i(z) = \hat{V}_i^+ e^{-j\beta_i z} + \hat{V}_i^- e^{j\beta_i z} \tag{8.54a}$$

$$\hat{I}_i(z) = \frac{1}{Z_{Ci}} \hat{V}_i^+ e^{-j\beta_i z} - \frac{1}{Z_{Ci}} \hat{V}_i^- e^{j\beta_i z} \tag{8.54b}$$

in accordance with the general solution of the transmission-line equations for each tube. The discontinuity may be characterized as before with a form of *voltage* scattering parameter matrix that relates the incident and reflected voltage waves at that point as illustrated in Fig. 8.15(b). The incident waves are the portions of (8.54a) that are incoming at the junction, $\hat{V}_1^+ e^{-j\beta_1 \mathscr{L}_1}$ and $\hat{V}_2^- e^{j\beta_2 \mathscr{L}_1}$. The reflected waves are the portions of (8.54a) that are outgoing at the junction, $\hat{V}_1^- e^{j\beta_1 \mathscr{L}_1}$ and $\hat{V}_2^+ e^{-j\beta_2 \mathscr{L}_1}$.

$$\underbrace{\begin{bmatrix} \hat{V}_1^- e^{j\beta_1 \mathscr{L}_1} \\ \hat{V}_2^+ e^{-j\beta_2 \mathscr{L}_1} \end{bmatrix}}_{\hat{V}^{\text{reflected}}} = \begin{bmatrix} \Gamma_{11} & \mathrm{T}_{12} \\ \mathrm{T}_{21} & \Gamma_{22} \end{bmatrix} \underbrace{\begin{bmatrix} \hat{V}_1^+ e^{-j\beta_1 \mathscr{L}_1} \\ \hat{V}_2^- e^{j\beta_2 \mathscr{L}_1} \end{bmatrix}}_{\hat{V}^{\text{incident}}} \tag{8.55}$$

The Γ_{ii} elements are the *voltage reflection coefficients*, and the T_{ij} elements are the *voltage transmission coefficients*. Figure 8.16 illustrates the determination of these parameters for various discontinuities. Consider the direct connection shown in Fig. 8.16(a). The voltages and currents must be continuous. Equating

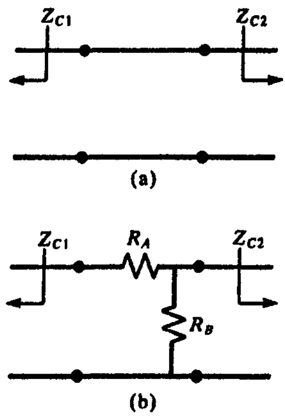


FIGURE 8.16 Illustration of the simplified determination of the scattering parameters for (a) a direct connection, and (b) a resistive interconnection network.

equations (8.54) for the left and right sections and putting them in the form of (8.55) yields

$$\Gamma_{11} = \frac{Z_{C2} - Z_{C1}}{Z_{C2} + Z_{C1}} \qquad T_{12} = \frac{2Z_{C1}}{Z_{C2} + Z_{C1}} \tag{8.56a}$$

$$T_{21} = \frac{2Z_{C2}}{Z_{C1} + Z_{C2}} \qquad \Gamma_{22} = \frac{Z_{C1} - Z_{C2}}{Z_{C1} + Z_{C2}} \tag{8.56b}$$

This result is directly analogous to the case of a uniform plane wave incident at an interface between two media [A.1]. It is a sensible result for the following reason. In the time domain, the wave incident from line #1 "sees" an impedance at the junction equal to the characteristic impedance of line #2 since it has not arrived at the termination of line #2 and therefore line #2 appears infinite in length. So the reflection coefficient can be calculated as though line #1 is terminated in the characteristic impedance of line #2. The transmission coefficient is easily calculated from the reflection coefficients since the total voltage incident on the junction from line #1 is the sum of the incident and reflected waves on that line which, because of the direct connection, must equal the transmitted voltage or $(1 + \Gamma_{11}) = T_{21}$. In a similar fashion, $(1 + \Gamma_{22}) = T_{12}$.

The BLT equations can be easily formed for this network although their solution is best suited to computer implementation. The overall tube propagation matrix is

$$\hat{\mathbf{R}}_T = \begin{bmatrix} 0 & -e^{j\beta_1\mathscr{L}} & 0 & 0 \\ -e^{j\beta_1\mathscr{L}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -e^{j\beta_2\mathscr{L}} \\ 0 & 0 & -e^{j\beta_2\mathscr{L}} & 0 \end{bmatrix}$$

Writing (8.7) at the source, the junction and the load gives

$$\underbrace{1}_{\hat{Y}_1^S}\hat{V}_1^S + \underbrace{R_S}_{\hat{Z}^S}\hat{I}_1^S = \underbrace{\hat{V}_S}_{\hat{P}^S}$$

$$\underbrace{\begin{bmatrix}1 & -1\\ 0 & 0\end{bmatrix}}_{\hat{\mathbf{Y}}^m}\begin{bmatrix}\hat{V}_1^m\\ \hat{V}_2^m\end{bmatrix} + \underbrace{\begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix}}_{\hat{\mathbf{Z}}^m}\begin{bmatrix}\hat{I}_1^m\\ \hat{I}_2^m\end{bmatrix} = \underbrace{\begin{bmatrix}0\\ 0\end{bmatrix}}_{\hat{\mathbf{P}}^m}$$

$$\underbrace{1}_{\hat{Y}_1^L}\hat{V}_1^L + \underbrace{\hat{R}_L}_{\hat{Z}_1^L}\hat{I}_1^L = \underbrace{0}_{\hat{P}^L}$$

where m designates the "middle node" (the junction). The junction scattering matrices are obtained from (8.47a) as

$$\hat{\mathbf{S}}_S = -(R_S + Z_C)^{-1}(R_S - Z_C) = -\Gamma_S \qquad \hat{\mathbf{I}}_S^S = (R_S + Z_C)^{-1}\hat{V}_S$$

$$\begin{aligned}\hat{\mathbf{S}}_m &= -\left\{\begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix} + \begin{bmatrix}1 & -1\\ 0 & 0\end{bmatrix}\begin{bmatrix}Z_{C1} & 0\\ 0 & Z_{C2}\end{bmatrix}\right\}^{-1}\left\{\begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix} - \begin{bmatrix}1 & -1\\ 0 & 0\end{bmatrix}\begin{bmatrix}Z_{C1} & 0\\ 0 & Z_{C2}\end{bmatrix}\right\}\\ &= -\begin{bmatrix}Z_{C1} & -Z_{C2}\\ 1 & 1\end{bmatrix}^{-1}\begin{bmatrix}-Z_{C1} & Z_{C2}\\ 1 & 1\end{bmatrix}\\ &= \begin{bmatrix}-\Gamma_{11} & -\mathrm{T}_{12}\\ -\mathrm{T}_{21} & -\Gamma_{22}\end{bmatrix}\end{aligned}$$

$$\hat{\mathbf{S}}_L = -(R_L + Z_C)^{-1}(R_L - Z_C) = -\Gamma_L \qquad \hat{\mathbf{I}}_S^L = (R_L + Z_C)^{-1}0$$

Thus the overall scattering matrix becomes

$$\hat{\mathbf{S}}_T = \begin{bmatrix}-\Gamma_S & 0 & 0 & 0\\ 0 & -\Gamma_{11} & -\mathrm{T}_{12} & 0\\ 0 & -\mathrm{T}_{12} & -\Gamma_{22} & 0\\ 0 & 0 & 0 & -\Gamma_L\end{bmatrix}$$

These reflection and transmission coefficients are the negative of the voltage reflection and transmission coefficients in (8.56) because we are considering currents and because of the current directions at the junctions (into the tubes). Similarly, the total current source vector is

$$\hat{\mathbf{I}}_{ST} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \frac{\hat{V}_S}{(R_S + Z_C)}$$

The BLT equations are then formed from (8.53) with $\hat{\mathbf{I}}_{TT} = \mathbf{0}$ (no incident-field illumination).

Figure 8.16(b) shows a connection consisting of a resistive network. For the above reasons we may similarly determine

$$\Gamma_{11} = \frac{R_2 - Z_{C1}}{R_2 + Z_{C1}} \qquad \mathrm{T}_{12} = \frac{Z_{C1}}{Z_{C1} + R_A}(1 + \Gamma_{22}) \tag{8.57a}$$

$$\mathrm{T}_{21} = \frac{R_B \parallel Z_{C2}}{R_A + R_B \parallel Z_{C2}}(1 + \Gamma_{11}) \qquad \Gamma_{22} = \frac{R_1 - Z_{C2}}{R_1 + Z_{C2}} \tag{8.57b}$$

where

$$R_1 = R_B \parallel (R_A + Z_{C1}) \qquad R_2 = R_A + R_B \parallel Z_{C2} \tag{8.57c}$$

The transmission coefficients are determined by first obtaining the total voltage incident on the junction, $(1 + \Gamma_{11})$ or $(1 + \Gamma_{22})$ and then using voltage division.

In the time domain, the source initially "sees" a termination of Z_{C1} so that the voltage wave sent out is, by voltage division,

$$V(t) = \frac{Z_{C1}}{R_S + Z_{C1}} V_S(t) \tag{8.58}$$

This wave travels down line #1 reaching the discontinuity at one time delay of T_1 where $\Gamma_{11}V(t - T_1)$ is reflected and $\mathrm{T}_{21}V(t - T_1)$ is transmitted across the junction. The reflected portion arrives at the source at $2T_1$ where a portion of it, $\Gamma_S\Gamma_{11}V(t - 2T_1)$, is sent back towards the junction where a portion of it is reflected, $\Gamma_{11}\Gamma_S\Gamma_{11}V(t - 3T_1)$, and a portion is transmitted, $\mathrm{T}_{21}\Gamma_S\Gamma_{11}V(t - 3T_1)$, etc. Meanwhile, the portion of the original wave that was transmitted across the junction, $\mathrm{T}_{21}V(t - T_1)$, arrives at the load where it is reflected as $\Gamma_L\mathrm{T}_{21}V(t - T_1 - T_2)$. This is sent back to the junction where a portion of it, $\Gamma_{22}\Gamma_L\mathrm{T}_{21}V(t - 2T_2)$, is reflected back toward the load and a portion, $\mathrm{T}_{12}\Gamma_L\mathrm{T}_{21}V(t - T_1 - 2T_2)$, is transmitted across the junction onto line #1. This process of continued reflections and transmissions continues, and the total voltage at any point on either line is the sum of the total waves at that point and time. Clearly the line voltages will be linear combinations of the initial transmitted wave, $V(t)$, delayed in time by various sums of multiples of the line delays, T_1 and T_2. A lattice diagram can be constructed as described in Chapter 5 to aid in determining these total voltages but the process is clearly very complicated owing to the multitude of reflections and transmissions. If either line is terminated in its characteristic impedance, the summation is simplified considerably, and a series solution can be developed. But for completely mismatched lines, the summation is tedious.

A symbolic solution can be obtained as in Chapters 5 and 6 if we use the time-shift or difference operator as before:

$$D_l^{\pm k} f(t) = f(t \pm kT_l) \tag{8.59}$$

in the BLT equations of (8.53). Thus

$$\mathbf{R}_\mathrm{T}(D) = \begin{bmatrix} 0 & -D_1 & 0 & 0 \\ -D_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -D_2 \\ 0 & 0 & -D_2 & 0 \end{bmatrix} \tag{8.60}$$

The BLT equations in (8.53) with $\hat{\mathbf{I}}_{TT} = \mathbf{0}$ (no incident-field illumination) can be written as

$$[\mathbf{R}_\mathrm{T}(D) - \mathbf{S}_\mathrm{T}][\mathbf{R}_\mathrm{T}(D) + \mathbf{1}]^{-1}\mathbf{I}_\mathrm{T}(t) = \mathbf{I}_{\mathrm{ST}}(t) \tag{8.61}$$

and $[\mathbf{R}_\mathrm{T}(D) + \mathbf{1}]^{-1}$ is simple to obtain. However, the literal solution of the BLT equations is very tedious even for this simple case. If we substitute a previously described SPICE model for each line, the summation of these waves is taken care of and nonresistive loads can be considered. A direct time-domain summation of the traveling waves was implemented using the scattering parameters of the termination/interconnection networks in [3] but, once again, keeping track of all the incident/reflected waves on the line is a very tedious task because of the multiple reflections/transmissions and the different mode velocities on each tube.

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PROBLEMS

8.1 Derive the admittance parameters (8.3).

8.2 Derive the admittance parameter relation for a transmission-line network (8.14).

8.3 Derive the impedance parameters (8.17).

8.4 Derive the relations given in (8.26) and (8.27).

8.5 Derive the scattering parameter relation for a termination network (8.36).

8.6 Derive the scattering parameter relation for an interconnection network (8.39).

8.7 Derive the BLT equations given in (8.41) and in (8.43).

8.8 Derive the scattering parameter matrix and current source vector given for the general case in (8.47).

8.9 Derive the BLT equations for the general case given in (8.53).

8.10 For the tandem connection of two-conductor lines shown in Fig. 8.15 derive series expressions for the time-domain source and load voltages in terms of delayed source-voltage waveforms for a matched load on line #2, i.e., $R_L = Z_{C2}$.

8.11 Consider the transmission-line network shown in Fig. P8.11. All conductors are #20 gauge wires ($r_w = 16$ mils) at a height of 1 cm above a ground plane. Solve for the voltage $V_{out}(t)$ where $V_S(t)$ is a 10 MHz trapezoidal pulse train with rise/fall times of 1 ns and 50% duty cycle. Compute this using:

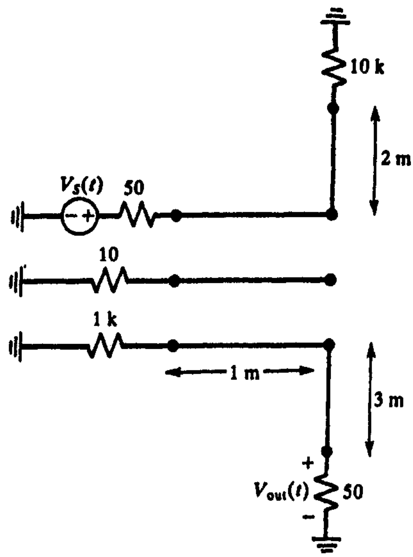

FIGURE P8.11

1. The SPICE model.
2. The lumped-pi model.
3. The admittance parameter model.
4. The impedance parameter model.
5. The BLT equations.