A Computer Based Method for Computing the N-Dimensional Generalized ABCD Parameter Matrices of N-Dimensional Systems with Distributed Parameters

A. Aziz Bhatti

Department of Electrical Engineering
Memphis State University
Memphis, Tennessee 38152

ABSTRACT

This paper presents a new method for developing and solving, both in transient and steady state, the N-dimensional matrix networks of N-dimensional transmission line networks or communication circuits with distributed parameters. The method uses Cayley-Hamilton's theorem to compute the hyperbolic N-dimensional generalized ABCD parameter matrices with finite terms which are fundamental to the solution of N-dimenstional matrix networks. The square root function of the complex matrix $[W_2]$ is also computed with finite terms. As a result, truncation of matrices is eliminated. The method is extremely useful in the fault analysis of n-dimensional unbalanced coupled systems with distributed parameters. To date no such method is reported in the literature.

I. INTRODUCTION

An n-dimensional transmission line network or a communication circuit with distributed parameters constitutes a coupled and complex symmetric or asymmetric network. Such networks can be represented by a set of coupled non-singular square matrices which may not necessarily be diagonalizable.

In the study of transient and steady state analysis of n-dimensional coupled and unbalanced circuits with distributed parameters, often it becomes desirable to represent such networks as n-dimensional matrix networks. The solution and development of such N-dimensional matrix networks requires the computation of n-dimensional generalized ABCD parameter matrix tensors. A computer based method which utilizes Caley-Hamilton's theorem has been proposed in this paper to compute such ABCD parameter matrices. This paper considers the development of n-dimensional matrix networks of n-dimensional symmetric or asymmetric coupled circuits with distributed parameters [1,2,3] as opposed to those of one-dimensional scalar circuits treated in standard text books [4,6]. The proposed method which utilizes Cayley-Hamilton's theorm is straight forward, computationally efficient, and neither it involves the use of eigenvector based transformations necessary for diagonalization of parameter matrices nor does it require the evaluation of infinite series of hyperbolic functions of n-dimensional matrices as their arguments. The method computes the square root function of the complex matrix $[W_2]$ together with the hyperbolic n-dimensional generalized ABCD parameter matrices with finite terms, thereby eliminating the truncation of matrices and achieving an improved closed form solution. The method is extremely useful in the steady state and transient analysis of n-dimensional, unbalanced coupled systems with distributed parameters. To date no such method exists in the literature.

II. PROPOSED METHOD OF DEVELOPMENT

In practice, a considerable variety of physical problems involves the transient and steady state analysis of n-dimensional coupled electric circuits. The matrix partial differential equations describing the behavior of such circuits can be written as

$$\frac{-\partial v(x,t)}{\partial x} = \left[[R] + [L]\frac{\partial}{\partial t} \right] i(x,t) \qquad (1)$$

$$\frac{-\partial i(x,t)}{\partial x} = \left[[G] + [C]\frac{\partial}{\partial t} \right] v(x,t) \qquad (2)$$

Taking the Laplace transform and ignoring the initial conditions, equations (1 and 2) can be written as ordinary differential equations in the compact form as

$$\frac{-dI}{dx} = [Y]\ [V] \qquad (3)$$

$$\frac{-dV}{dx} = [Z]\ [I] \qquad (4)$$

TH0301-2/90/0000/0590$01.00 © 1990 IEEE

where

$$[Z] = [R] + s[L] \tag{5a}$$

$$[Y] = [G] + s[C] \tag{5b}$$

$$[Z] = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & & & \\ r_{n1} & r_{n2} & \cdots & r_{nn} \end{bmatrix} + s \begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1n} \\ L_{21} & L_{22} & \cdots & L_{2n} \\ \vdots & & & \\ L_{n1} & L_{n2} & \cdots & L_{nn} \end{bmatrix} \tag{6}$$

Matrix [Y] is similiarly defined. The [V] and [I] are n-dimensional column vectors and are functions of parameters and the distance x, and the Laplace operator s. From equations (3 and 4), it follows that

$$\frac{d^2I}{dx^2} = [Y][Z]\ [I] = [W_1]\ [I] \tag{7}$$

$$\frac{d^2V}{dx^2} = [Z][Y]\ [V] = [W_2] \tag{8}$$

The matrix product [Z][Y] does not equal the matrix product [Y][Z], except in special cases, whether the matrices are symmetric or not.

The matrix equations (7 and 8) are coupled. In the case of transient, the eigenvalues and eigenvectors vary with frequency, and the modal transformation matrices must be calculated at each frequency.

However, the method proposed here does not require the computation of model transformations, and therefore of eigenvectors. The steady state solution, with s=jw, of equations (7) and (8) leads to the different set of n-dimensional [A],[B],[C],[D] parameter matrices of a symmetric or asymmetric n-dimensional transmission line network. The both set of generalized parameter matrices are derived.

The solution of equations (3 and 7) may be written as

$$I = \exp(x[W_1]^{1/2})\ A_1 + \exp(-x[W_1]^{1/2})\ A_2 \tag{9}$$

$$V = [Y]^{-1}[W_1]^{1/2} \exp(x[W_1]^{1/2}\ A_1 - \exp(-x[W_1]^{1/2})\ A_2] \tag{10}$$

where A_1 and A_2 are n-dimensional column vector constants of integration, and

$$[W_1]^{1/2} = \sqrt{[Y][Z]} = [\alpha_1'] + s\ [\beta_1'] \tag{11}$$

The eigenvalues of matrices [Z], [Y], and [W] are complex and vary with frequency. The [W] is the propagation constant matrix, and [α'] is the attenuation constant matrix in nepers per unit length. While [β'] being a function of the displacement x and the Laplace variable s is the phase constant matrix in radians per unit length, and represents a delay factor matrix.

The solution of voltage and current vectors, from equations (9 and 10) in terms of generalized ABCD coupled n-dimensional parameter matrices can be written as

$$\begin{bmatrix} [V] \\ [I] \end{bmatrix} = \begin{bmatrix} [A] & [B] \\ [C] & [D] \end{bmatrix} \begin{bmatrix} [V_r] \\ [I_r] \end{bmatrix} \tag{12}$$

where

$$[A] = [Y]^{-1} \operatorname{Cosh}(x[W_1]^{1/2})\ [Y] \tag{13}$$

$$[B] = [[Y]^{-1}\ [Z]]^{1/2})\ \operatorname{Sinh}\ (x[W_1]^{1/2} \tag{14}$$

$$[C] = \operatorname{Sinh}\ (x[W_1]^{1/2})\ [[Z]^{-1}\ [Y]]^{1/2} \tag{15}$$

$$[D] = \operatorname{Cosh}\ (x[W_1]^{1/2}) \tag{16}$$

Similarly, the solution of equations (4 and 8) yields the following set of [A],[B],[C],[D] parameter matrices:

$$[A] = \operatorname{Cosh}\ (x[W_2]^{1/2}) \tag{17}$$

$$[B] = \operatorname{Sinh}\ (x[W_2]^{1/2})\ [[Z]^{-1}\ [Y]]^{1/2} \tag{18}$$

$$[C] = [[Z]^{-1}\ [Y]]^{1/2}\ \operatorname{Sinh}\ (x[W_2]^{1/2}) \tag{19}$$

$$[D] = [Z]^{-1} \operatorname{Cosh}\ (x[W_2]^{1/2})\ [Z] \tag{20}$$

where

$$[W_2]^{1/2} = \sqrt{[Z][Y]} = [\alpha_2'] + s\ [\beta_2'] \tag{21}$$

It may be noted that in general $[\alpha_2'] \neq [\alpha_1']$, and $[\beta_2'] \neq [\beta_1']$.

The [A], [B], [C], [D], $[W]^{1/2}$, and $[W]^{-1}$ are complex n-dimensional coupled non-singular square matrices.

PROPOSED METHOD OF SOLUTION

The [A], [B], [C], and [D], are complex n-dimensional coupled non-singular square matrices. These can be computed using the Cayley-Hamilton's theorem, and thereby avoiding the evaluation of infinite series representing the hyperbolic functions having n-dimensional coupled matrices as their arguments. Using Cayley-Hamilton's theorem, the matrix [A] in equation (17), for example, can be computed for a particular frequency as follows [5]:

$$[A] = \mathrm{Cosh}(x[W_2]^{1/2}) = \mathrm{Cosh}[P] \tag{22}$$

Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the eigenvalues of [P].

$$\begin{bmatrix} 1 & \lambda_1 & \lambda_1^2 & \cdots & \lambda_1^{n-1} \\ 1 & \lambda_2 & \lambda_2^2 & \cdots & \lambda_2^{n-1} \\ \cdots & & & & \\ \cdots & & & & \\ 1 & \lambda_n & \lambda_n^2 & \cdots & \lambda_n^{n-1} \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \cdots \\ \cdots \\ \alpha_{n-1} \end{bmatrix} = \begin{bmatrix} \mathrm{Cosh}\,\lambda_1 \\ \mathrm{Cosh}\,\lambda_2 \\ \cdots \\ \cdots \\ \mathrm{Cosh}\,\lambda_n \end{bmatrix} \tag{23}$$

$\alpha_0, \alpha_2, \ldots, \alpha_{n-1}$ are the Cayley-Hamilton's constants. Using equations (22 and 23), it follows from Cayley-Hamilton's theorem that

$$[A] = \mathrm{Cosh}[P] = \alpha_0 I + \alpha_1 P + \alpha_2 P^2 + \ldots + \alpha_{n-1} P^{n-1} \tag{24}$$

$$[A] = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \cdots & & & \\ \cdots & & & \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix} \tag{25}$$

Computation of Square-Root of $[W_2]$ Matrix

The square root of a complex matrix $[W_2]$ can be computed using the Cayley-Hamilton's theorm as follows [5]:

$$[W_2]^{1/2} = [P] \tag{26}$$

Let $\beta_1\ \beta_2, \ldots, \beta_n$ be the eigenvalue of $[W_2]$.

$$\begin{bmatrix} 1 & \beta_1 & \beta_1^2 & \cdots & \beta_1^{n-1} \\ 1 & \beta_2 & \beta_2^2 & \cdots & \beta_2^{n-1} \\ \cdots & & & & \\ \cdots & & & & \\ 1 & \beta_n & \beta_n^2 & \cdots & \beta_n^{n-1} \end{bmatrix} \begin{bmatrix} k_o \\ k_1 \\ \cdot \\ \cdot \\ k_{n-1} \end{bmatrix} = \begin{bmatrix} \beta_1^{1/2} \\ \beta_2^{1/2} \\ \cdot \\ \cdot \\ \beta_n^{1/2} \end{bmatrix} \tag{27}$$

$k_o, k_1, \ldots, k_{n-1}$, are the Cayley-Hamilton's constants. Using equations (26) and (27), it follows from Cayley-Hamilton's theorem that

$$[P] = [W_2]^{1/2} = k_o I + k_1 W_2 + k_2 W_2^2 + \ldots + k_{n-1} W_2^{n-1} \tag{28}$$

where

$$[P] = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \cdots & & & \\ \cdots & & & \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{bmatrix} \tag{29}$$

It may be noted that the power of the complex matrix [P] will also be computed by using Cayley-Hamilton's theorem [5]. The remaining [B], [C], and [D] generalized constants' parameter matrices can be similarly computed. System matrices having repeated eigenvalues will require differentiation and can be easily handled by this method. In such cases, the use of eigenvector based model transformations may produce Jordan System matrices which having coupled elements will be difficult to solve with conventional methods.

CONCLUSIONS

A new computer based method for computing the n-dimensional generalized ABCD parameter matrices of n-dimensional transmission line networks or communication circuits with distributed parameters has been presented. The method of solution does not require the computation of modal transformations, and therefore of eigenvectors. Proposed solution utilizes the Cayley-Hamilton's theorem, and does not involve the evaluation of infinite series representing the hyperbolic functions having n-dimensional coupled matrices as their arguments. In this method the generalized ABCD parameter matrices which are fundamental to the solution of such n-dimensional networks are computed with finite terms. The square root function of the complex $[W_2]$ matrix is computed by this method with finite terms. As a result, truncation of matrices is eliminated, and an improved closed form solution is achieved. The method is extremely useful in the steady state and transient analysis of n-dimenstional, unbalanced, coupled systems with distributed parameters.

Key Words - ABCD parameter matrices, N-dimensional circuits with distributed parameters, Matrix circuit, Cayley-Hamilton's theorm, Solution of large networks, N-phase transmission lines, Fault analysis, Hyperbolic functions of N-dimensional matrices.

REFERENCES

[1] Bhatti, A. A., and Chu, C. M.: Microcomputer based analysis of residual fault currents and recovery voltages under microcomputer based restructurable control of UHV lines. Submitted to IEEE Trans. for publication.

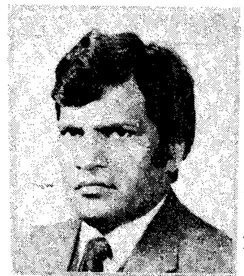
[2] Bhatti, A. A.: Computer based performance analysis of faulted multiconductor multicircuit fault-tolerant network with distributed parameters. Submitted to IEEE Trans. for publication.

[3] Bowman, W. I., et al., "Development of Equivalent Pi and T Matrix Circuits for Long Untransposed Transmission Lines", IEEE T-PAS-83, June, 1964, pp. 625-632.

[4] Central Station Engineers: Electrical transmission and distribution reference book, 4th edition, Westinghouse Electric Corporation, East Pittsburgh, Pennsylvania, USA.

[5] DeRusso, P. M., et al.: State variables for engineers, John Wiley & Sons, Inc., New York.

[6] Stevenson, W. D., "Elements of Power System Analysis", (Book) 4th Edition, McGraw-Hill, 1982.



Abdul Aziz Bhatti was born in Lahore, Pakistan. He received the FTCG in Electrical Engineering from City and Guilds of London Institute London, England the B. A. degree in mathematics from Punjab University, Lahore Pakistan, the M. S. degree in Electrical Engineering from Illinois Institute of Technology, Chicago, Illinois; the MSE degree in Computer Information and Control Engineering and the Ph.D. degree in Electrical and Computer Engineering both from the University of Michigan, Ann Arbor, Michigan.

He worked as a Project Engineer in Pakistan on distribution systems, EHV/UHV transmission liens and substations, in the USA on a number of positions involving consultancies, advanced engineering, research and applications in the areas of computer, control and power systems. From 1984 to 1987 he was a Senior Research Scientist at the High Technology Center of the Boeing Company, Seattle, WA. Since 1987, he is on the faculty of Electrical Engineering of Memphis State University, Memphis, TN. He is an author or co-author of numerous publications in the areas of computers, control, electric power systems, and artificial intelligence. His current interest includes: technology assessment, forecast, transfer, and management; parallel computer architectures, AI and knowledge-based machines, expert systems, intelligent computer networks, fault-tolerant computing, neural network computing, computer-aided design of intelligent and fault-tolerant VLSI chips, analysis and development of mircocomputer algorithms for protection and control of electric power systems. Dr. Aziz Bhatti is a member of several IEEE societies.