Decision Support

From decision theory to decision aiding methodology

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Abstract

The paper presents the author's partial and personal historical reconstruction of how decision theory is evolving to a decision aiding methodology. The presentation shows mainly how "alternative" approaches to classic decision theory evolved. In the paper it is claimed that all such decision "theories" share a common methodological feature, which is the use of formal and abstract languages as well as of a model of rationality. Different decision aiding approaches can thus be defined, depending on the origin of the model of rationality used in the decision aiding process. The concept of decision aiding process is then introduced and analysed. The paper's ultimate claim is that all such decision aiding approaches can be seen as part of a decision aiding methodology.
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1. Introduction

Quite often I get asked what my job is. When I reply that I work in decision aiding, people remain perplexed and quite often ask "aiding what decisions?".

Indeed decision making is an activity that every person does every day. We all make decisions constantly, from the simplest "should I take my umbrella" [244] to the more complex "how should the international disarmament treaty be applied?" [163] and at all levels, individual: ("should I divorce?" [332]), organisational ("how do we schedule the crew shifts?" [58]), inter-organisational ("which trace for the highway?" [234]). Quite often,

during such decision processes we ask for help, advice, or support from friends, experts, consulting companies, etc. Several questions arise. Is it conceivable that a decision aiding methodology could exist independently from any specific domain, one which could be used in all such situations? Can an expert in decision aiding exist who is not an expert in any particular domain? What would be the difference between such an expert and a psychotherapist, a physician, a lawyer, an expert in logistics or your best friend?

What characterises decision aiding, both from a scientific and a professional point of view, is the fact that it is both "formal" and "abstract". By the first term I mean the use of formal languages, ones which reduce the ambiguity of human communication. By the second term I mean the use of languages that are independent from a specific domain of discourse.

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The basic idea is that the use of such languages implies the adoption of a model of ''rationality'' a key concept in decision aiding. Does it make sense to use such a language always and in any context? Obviously not. Being abstract and formal presents several disadvantages:

– it is much less effective with respect to human communication;
– it has a cost (not necessarily monetary);
– reducing ambiguity might not be desirable;
– it imposes a limiting framework on people's intuition and creativity.

Nevertheless, there are also several advantages, which in some circumstances can be interesting (see also [44]):

– it allows the participants in a decision process to talk the same language, a fact that improves transparency of the process and possibly increases participation (for an example see [15]);
– it allows the identification of the underlying structure of a decision problem (if there is any) and therefore allows the re-use of procedures and models (for nice examples see any textbook of Operational Research, for instance [341]);
– it is not affected by the biases of human reasoning that are due to education or tradition (for examples the reader is referred to [251]);
– it may help to avoid the common errors that are due to an informal use of formal methods; a typical case being the use of averages as a universal grading procedure (see [44] for a critical discussion of this issue).

In general terms, a formal and abstract language allows us to better analyse, understand, explain, justify a problem and/or a solution. It should be noted that organisations, companies, institutions, enterprises, ourselves, ask for and use formal methods of decision aiding. Students are promoted using the average of their grades. Traffic restrictions are applied based on a pollution index. Credit demands are rejected because of the client's credit rating. Production is scheduled, highways are designed, networks are administrated, using formal methods of decision support. In reality decision aiding is present in many aspects of our everyday life. People do not necessarily use this term, but there is always a formal and abstract language which is used in all the above examples. Therefore, when I talk about deci-

sion aiding I will always mean the use of a formal and abstract language in order to handle problem situations faced by individuals and/or organisations.

In this paper I will first present a brief history of the evolution of this domain from a scientific and a professional point of view (next section). Such a historical reconstruction pretends neither to be complete nor rigorously organised. Several readers might feel disappointed that some very important scientific achievements are not recognised. Indeed this is an essay which reflects my very personal point of view and is biased by at least three factors:

– scientific; I am not an expert in all areas of decision theory and operational research and I tend to emphasise in my presentation what I know better;
– professional; the real world experiences of decision aiding that I had the opportunity to conduct do not cover all different aspects of practicing decision aiding, so that I have a partial vision of this complex reality;
– geographical; being an European (western) I have not been exposed to the bulk of the contributions produced in decision theory and operational research just behind the corner (see for instance [185,177]) and this is a severe limitation.

In Section 3, I will present and discuss different decision aiding approaches that have been introduced during the 60 years of existence of this discipline: normative, descriptive, prescriptive and constructive approaches. I will try to explain the differences among these approaches by examining the origin of their particular ''models of rationality''. In Section 4, I will place myself within a constructive decision aiding approach and I will discuss how a decision aiding process is structured. In order to do that I will examine the ''artifacts'' produced by such a process: the representation of a problem situation, the definition of a problem formulation, the construction of an evaluation model and the formulation of a final recommendation. Such a presentation will allow me to differentiate decision aiding from other areas of scientific investigation such as automatic decision making.

The ultimate message I wish to deliver with this essay is that decision aiding is a human activity that can be (and actually has been) the subject of scientific investigation. Different ''decision theories'' have been developed with specific characteristics. At the

same time different "decision aiding practices" have been developed either as a result of testing theoretical conjectures or as a result of aiding real decision makers (individuals, organisation or collective entities) in their work. There is no one-to-one correspondence between "theories" and "practices". Nevertheless, I consider that all such "theories" and "practices" define a whole which I will call "decision aiding methodology". The reader should note that in the text I use the term "methodology" in a very precise way: *reasoning about methods*. I claim that we have several methods, but we should establish a common methodology for decision aiding purposes. Such reflections are discussed in the conclusions section. At the end of the paper I provide a long, but definitely partial list of references, an exhaustive presentation of the literature being impossible.

2. Some history

2.1. Genesis and youth

We can fix the "origin" of decision aiding as starting sometime just before the second world war, in the studies conducted by the British army on their new radar system installation and their efforts to break the German secret communication code (1936–37). The reader can get a flavour of this period in [52,178]. It is the first time the term "operational research" (in the USA "operations research") appears. The problem of how decisions are or ought to be taken by individuals, organisations and institutions was previously discussed by Aristotle [11] and more recently, during the 18th century (see [31] on probability [104], on combinatorial problems [37,71], on voting and social choice procedures) and also at the beginning of the 20th century ([237] on economic problems under multiple dimensions [107,302], on the scientific management of enterprises [80,81,181,248], on probability theory [308], on decidability). In all these contributions the concept of decision is central. I may just mention that, in order to argue for their thesis that probability only exists in terms of subjective belief, both Ramsey and de Finetti, have used what today is known as comparison of lotteries and the associated preferences of a decision maker. "*If the option of α for certain is indifferent with that of β if p is true and γ if p is false, we can define the subject's degree of belief in p as the ratio of the difference between α and γ to that between β and γ. This amounts roughly to defining the degree of belief in p by the odds at*

which the subject could bet on p, the bet being conducted in terms of differences of values as defined" [248, pp. 179–180].

In any case, it is the undeniable success of operational research in supporting military and intelligence activities of the allies that grounded the idea that decision making (and I extend to decision aiding) can be studied using a scientific approach and that general models of decision support were possible. Towards the end of the 1940s, beginning of the 1950s, several fundamental contributions appeared: (see [77,169] for linear programming [221,222,325], for decision and game theory [309], on algorithmics and the definition of "machines" able to solve "any problem"). It is during that period that the first scientific societies of operational research (in the United Kingdom in 1948, in the United States in 1950) and the first scientific journals appeared [33]. Also the first real world applications of this new discipline (in non-military applications) appeared (see [78]) as well as the first companies specialising in "decision aiding" (but this term was not used at that time). The best known example is the RAND corporation. Within RAND, operational research was developed into a "science" to be applied to the multiple problems of the new post-war industrialisation.

Such first contributions and experiences were characterised by the search for formal structures underlying precise decision problems and the use of mathematics and logic as modelling language. For an interesting presentation of the origins of these contributions as have been perceived by their authors themselves, see [197]. The first steps in this direction strengthened the idea that complex decision problems can be modelled through the use of a simple rationality model (maximise an utility function of the decision maker's decision variables, a function which is expected to faithfully represent the decision maker's preferences). Von Neumann's and Morgenstern's and Nash's contributions showed under what conditions such functions exist. Further on, the linear programming algorithm developed by Dantzig (the famous Simplex algorithm) introduced the first tools by which such problems could be effectively solved (even for large dimensions). Turing and also Wiener [340] and Ashby [13] went further to consider the possibility of formulating a general theory of computation and conceived "general problem solver" machines.

At that time, some critical contributions to this paradigm started to appear (although they were not always conceived as criticism). In 1951 Arrow

[12] published his famous impossibility theorem, showing that aggregating the preferences of rational individuals, under conditions considered natural (universality, independence, respect of unanimity, non-dictatorship), is impossible (if the result has to be "rational", that is a complete order). Arrow's result closed the discussion opened by Borda and Condorcet (in the sense that we know there is no universal preference aggregation procedure [42, 322,323]). At the same time it paved the way to the huge literature on social choice theory (the reader can see: [175,176,229,230,276,278,301]).

In 1953, Allais [7] published his famous paradox, where he showed that the axioms, introduced by von Neumann and Morgenstern as necessary and sufficient conditions for the existence of an utility function (and implicitly assumed necessary in order to exhibit a rational behaviour), are systematically violated in the behaviour of real decision makers when they are confronted by very simple choices (the reader can also see [73,209]). Such an empirical falsification of the expected utility theory opened another research direction on integrating the findings of cognitive science into decision theory (see for instance [8]).

Already in 1947, Simon [282] observed decision processes occurring within real organisations and concluded that the behaviour of real decision makers is far from the postulates of decision theory, at least as this theory was formulated at that time. During the 1950s, Simon [283–285] developed his "bounded rationality" theory, which states that a decision maker facing a choice behaves on the basis of a local satisfaction criterion, in the sense that he will choose the first solution which he subjectively considers as satisfactory without trying to attain an unrealistic (and useless) optimal solution. Actually Simon considers decision theory to be based on three implicit hypotheses (see the discussion in [219]):

– decision makers always know their problems well;
– such problems can always be formulated as an effectiveness (or efficiency) problem;
– the information and the resources necessary to find a solution are always available.

None of these hypotheses is true in reality (following Simon):

– decision makers never have a very precise idea of their problem;

– often their problems can be formulated as the search for a "satisfying compromise";
– solving a problem is always constrained by the available resources and time.

The innovation introduced by Simon is radical. Decision theory as had been developed up to that moment always considered the rationality model as existing independently from the decision maker and his decision process. Simon put at the center of his reflection the decision process (the mental activities of a decision maker) and postulated that a rationality model has to be found within such a process and not outside of it. The problem with this hypothesis is that, while an "exogenous rationality model" is compatible with an optimisation model (indeed the classic rationality model is based on optimisation), this is not always the case with a subjectively established model (at least not automatically or necessarily). Simon's work opened several research directions, both towards the creation of new decision aiding approaches (see for instance [193]) and towards what today is known as "artificial intelligence" (see [286]). It should be noted that the idea of looking for a satisfying solution has an immediate application to the problem of finding an acceptable compromise when the decision is subject to the presence of multiple criteria (see [324]).

At the end of the 1950s, beginning of the 1960s, several "classic books" appeared. These books were used to train generations of researchers and practitioners [30,59,67,72,79,106,116,125,153,203,318].

The 1950s and the 1960s saw significant increases in research, university classes and applications in different domains. Typical big clients of such studies were the companies managing networks (water distribution, telecommunications, electricity suppliers, railways, airlines). In addition, several consulting companies, specialising in operational research and decision support appeared. It should be remembered that these years were the ones where the world was trying to reconstruct itself after the war and tremendous resources were invested in trying to find viable and efficient solutions to important industrial and economic problems. Unsurprisingly there have been stories of success and failure. Such experiences allowed the first critical approaches to the now well established "classic" decision theory to become stronger (for early discussions see [2,5,66,74,182]).

At the beginning of the 1960s, Zadeh published his famous paper [342] about fuzzy sets. The paper introduced a new perspective on the treatment of

uncertainty, ambiguity and linguistic variables. Zadeh's innovation had a major impact on the future of the discipline, since it concerns a fundamental aspect of formal languages: set theory. The extension of set theory through the introduction of a ''membership function'', a ''measure'' of an element's membership to a given set, allowed the increase of both the expressivity and the flexibility of formal languages and therefore of the decision aiding models using them.

Another domain which introduced major contributions to the development of alternative approaches to decision theory is cognitive science and psychology (see [100,108,198,199,280,291]). Allais' intuition to experimentally validate the axioms of decision theory was followed by several researchers. I just quote here the work done by Tversky (see [310,311,313]). He showed that the properties, intuitively considered as rational for preference relations, are more a theoretical imposition, not necessarily corresponding to the behaviour of real decision makers. Tversky showed that preference can well be intransitive [311] and that similarity can be non-symmetric [313].

Such results emphasised the necessity of pursuing a more thorough study of the fundamental structures on which decision aiding models rely, namely the structure of preference relations [96,201,275] and of the functions which represent them (value or utility functions, see [111,183]). For further work on this subject the reader can see [113,243,252,256] (see also the recent survey [235]).

Remaining within the influence of the psychological studies, it should be observed that during the 1960s appeared a psychotherapy movement known as ''relational psychotherapy'' based on an approach claimed by the authors to be ''constructive'' (see [22,138,333]). Within such an approach, the importance of how a problem is formulated was emphasised as well as the importance of the relationship between the one who asks for help and the one who provides such help (the patient and the therapist in their terminology). This approach also emphasised the fact that a problem is not something given within a decision process: the process of defining and solving a problem is the same. Under such a perspective the solution of a problem is a construction and not the result of a search in a space of solutions nor a classic inference from a set of sentences (see the classical dichotomy in artificial intelligence literature [289]).

Let us remain in the 1960s and the first organisational studies concerning the behaviour of decision makers and the structuring of decision processes within real complex organisations. It was again Simon who gave a significant impulse to the research in this direction (see [75,102,208]). In these works, it was shown that the behaviour of an organisation (supposed to be composed by rational decision makers) does not correspond to the rational behaviour as described by decision theory (the reader can see an extreme model in [70]: the famous model where organisations are seen as garbage cans). The problem, already observed by Weber [336] in his studies during the 1920s on the bureaucracies, is that within an organisation different forms of rationality may co-exist (see [287]). Later on, related research was condensed in Mintzberg's work (see [213,214], also [211]).

During the 1960s the concept of ''decision'' and ''value'' was the focus of interesting research in philosophy which posed the question: is it possible to define the concept of ''good'' in a formal way? Von Wright ([327], see also [147]), published his ''Logic of Preference'' within which the sentence ''x is preferred to y'' is considered true if all the worlds where x is true are preferred to the worlds where y is true. This research direction was followed by [64,65,148,149,162] and by the work done in [249,250]. Von Wright continued the development of his theory in [328] (see also Huber in [155]). From this research direction was further developed what today is known as deontic logic (see [154] and more recently [10,231], for a criticism see [220]).

Coming back to more formal aspects of operational research and decision aiding it should be noted that during the 1960s, the first works concerning algorithmic complexity appeared. Hartmanis and Stearns [151] were the first to pose the problem in the form we know it today. On this basis Karp [170] proposed the classification currently in use. This gave the formal basis used by Garey and Johnson [124] in order to compile their famous ''encyclopedia'' (see also [236]). This research opened a big issue in optimisation. Several algorithms used to solve classic operational research problems (and others) appeared to be less useful in practice, since, in presence of large instances of the problem, the resources required to reach an optimal solution are immense and independent of the computer used. We just mention the problem of satisfying a logical clause and the famous ''travelling salesman problem'' (see [114,218] and moreover [200] which introduced one of the most widely used algorithm in combinatorial optimisation: Branch and Bound;

for a survey see [192]). Looking for an optimal solution, besides its cognitive, theoretical and epistemological problems, became also a practical problem.

The research program of artificial intelligence (see for example [227]), oriented towards the creation of "thinking machines" and the establishment of "general problem solving" procedures was boosted by the work of Newell and Simon [225,226]. The idea of looking for a satisfying solution (instead of an optimal one) was a partial reply to the problem of the resources required to arrive at a conclusion for any decision process. The question was re-formulated under a more "logical" approach by McCarthy and Hayes (see [210]), who opened the way to what today is known as non-monotonic reasoning and by Minsky [212] who suggested the use of new techniques to represent "knowledge", the latter being seen as the source of efficient problem solving.

Finally, during the 1960s, appear the first works on the problem of evaluating alternative decisions using multiple criteria, where the criteria could be conflicting. In reality this is the case in most decision situations. It was the choice (and sometimes the necessity) of researchers and of practitioners that pushed them to simplify problems using a single decision criterion. In 1955 Charnes et al. proposed the idea of "Goal Programming" [60]. This work was further developed in [59] opening the way to what today is known as "multi-objective programming" (for the first papers see [28,128,129,280, 344]). Bernard Roy presented his ideas on this issue for the first time in 1966 and then in 1968 [29,257] opening the way to an approach known as "outranking based" methods. Raiffa produced his famous RAND report on these types of problems in 1969 [247]. In 1972 the first international conference in this domain (see [68]) took place and in 1976 Keeney and Raiffa published their reference book [174] extending utility theory (see [111]) in the presence of multiple criteria.

The presence of multiple criteria poses a fundamental question. The concept of "vector optimum" makes little sense from a mathematical point of view (at least in the natural terms of minimising the value of a function). The only "objective" definition that can be introduced is the one of "efficient" solution [237]. A solution is considered efficient if there are no other solutions at least as good as the current one and strictly better under at least one criterion (dominance). The problem is that the set of efficient solutions can be extremely large and therefore useless from a practical point of view. Technically the different approaches can be distinguished by the procedure used to explore the set of efficient solutions in order to find the "best compromise" (again a concept with no precise mathematical definition). On the one hand we have approaches based on the establishing of a function that aggregates the different criteria in a single criterion (a multi-attribute utility function), the problem thus becoming again an optimisation one. On the other hand we have approaches based on the idea that the criteria can be seen as individuals having preferences and to use methods originating in social theory (for instance voting procedures) in order to obtain a global preference relation representing the whole set of criteria, where graph theory is used to obtain a final solution (since such comprehensive preference relation can be seen as a graph).

I will give some more details later on in this historical section about the differences among these two approaches. However, it should be noted that it quickly appeared that there were deeper differences than just the technical ones. These concerned how decision aiding is conceived and implemented rather than the technical procedures and the use of a specific method (for an interesting discussion on this issue see [262]). I will discuss such issues in Section 3.

I will conclude this first part of the history noting that at, the end of the 1960s, beginning of the 1970s, operational research and decision theory were having a period of strong development both in theory and in practice. This development together with the establishment of a dominant paradigm allowed the appearance of critical approaches which occupied the scene during the period I will call the "maturity period". It is interesting to note that

– the discussion about alternative "decision theories" has been rooted in practical problems; it is the quest for better helping real decision makers involved in real decision processes that pushed the research to explore "innovative" or "alternative" approaches within the discipline;
– the development of OR and Decision Theory created "branches" (such as game theory, mathematical programming, decision analysis, etc. just to give some examples) which started having their own independent evolution.

2.2. Maturity

In the following of this partial reconstruction of the evolution of decision theory I will focus on some

research directions that I briefly introduced in the previous part. As emphasised previously, the entrance of decision aiding into "maturity" [45] implied the establishment of different "branches", a specialisation that I will observe under the following directions:

– the structuring and formulation of decision problems;
– the contribution of cognitive sciences;
– the intersection of artificial intelligence and decision theory;
– the treatment of uncertainty;
– the development of multiple criteria decision analysis.

Such directions have been followed either within the OR and decision theory communities or by researchers coming from other fields sharing with OR an interest towards the concept of decision support. I will try to show that these directions do not diverge, but rather that they have several common points and potential areas of convergence.

As in other empirical sciences, operational research and decision theory entered their first official "crisis" for a practical reason. Towards the end of the 1960s, the British OR society wanted to create a kind of "chartered directory of OR professionals". The reason was simple: provide the practitioners of the domain a quality label allowing the discipline and its practice to be better promoted. Not surprisingly ORSA (in the USA) published, almost at the same time, its suggestion about the "the guidelines of OR practice" (see [232]). The initiative was followed by several questions: what are the boundaries of the "discipline" and how to fix them? Using the existing methods? Who decides whether a decision support method belongs to the discipline? Given a new method, how will it be legitimated to enter these boundaries? The difficulty in finding convincing answers to these questions let appear the differences between diverse decision theories and their critics. For the history, this debate reached a conclusion(?) only very recently (the British society finally modified its statutes in order to create the above mentioned directory in 2001!!!).

The reader can get an idea of this discussion in the famous articles of Ackoff [3,4]. A reconstruction of this discussion is also available in the introduction of [254] and more recently in [179]. An interesting perspective on the discussion about the "operational research crisis" is also in [43].

During the 1970s (mainly in the United Kingdom) there appeared new approaches to decision aiding, based on work done within the Tavistock Institute [103,120,298,303] and by Beer [23,24]. The reader can see a presentation of such approaches in [254]. I just quote here the better known: "Soft Systems Methodology" [62], "Strategic Choice" [119,120], "Cognitive Mapping" [98,99], "Robustness Analysis" [253,255].

Let us recall that in classic decision theory a decision problem is formulated in a unique way. It is always a problem of maximising a function on the outcomes of all potential actions. There is no alternative to this formulation and the decision maker has to adapt the information available and the perception he has of the problem to the axioms of the theory. In contrast, the new approaches claimed that the most important part within a decision aiding process is the one concerning the structuring and formulation of the decision problem. This practice was already being followed in certain psychotherapy methodologies [334]. Within such new approaches the attention is focussed on the interactions between the client(s) and the analyst(s). Several techniques were proposed in order to arrive at a definition of a representation of the problem situation on which all the participants could agree (see also the work done in [18,186,187,219]). What these approaches suggest is that once the decision makers have understood their problem, solving it is a secondary issue and in most cases a simple one. Little attention is indeed paid to how the problem can be formulated in logical/mathematical terms (this aspect has been criticised on several occasions). However, it cannot be denied that structuring and formulating a problem remains one of the most critical parts within a decision aiding process as several real world experiences have shown (see [16,26,63,254,296]). I will discuss this issue in more detail in Section 4.

As I have already mentioned in the previous part, decision theory has also been criticised on a cognitive basis. Several experiences (conducted mainly in the laboratory) have shown that decision makers do not behave as decision theory axioms pretend. Such experiences have also shown that the frame within which and the precise way a decision problem is formulated have a great influence on the behaviour of the decision maker. For instance asking for preferences between two alternatives presented in terms of gains or losses gives totally different answers. More generally the cognitive context of the decision process is fundamental for the

final result. For the first experiments conducted in this direction the reader can see [168,315,316].

A first tentative reply to these theoretical and practical problems was the extension of utility theory through the introduction of "belief coefficients" which were expected to take into account the cognitive context. The theory is now known as "prospect theory" [168]. Although the complete axiomatisation of this theory is still to be done (see for instance [330]), it has been the subject of a large research area that is still extremely active today [34,61,133,165, 166,171,202,204,290,326,331].

Another tentative answer developed at the same time (not necessarily in opposition to the previous one) had as an objective to identify "decision strategies", the procedures used by decision makers when facing a problem situation. One of the first to observe such behaviours was Tversky [312]. Similar types of studies can also be found in [21,216,217,130]. The common pattern of this research is always the same: the identification of regularities in the behaviour of the decision makers, such as the progressive elimination of alternatives or the research for dominance structures. The reader can see reviews of this approach in [299] and in [20]. What such approaches basically contributed was the centering of decision aiding on the decision maker, his cognitive effort and the decision context. For the first time decision aiding was focussed on the decision process and not on the decision theory.

The reader will recognise in the approach presented in the previous paragraph some of Simon's cognitive criticism. Simon's contribution found a fertile area of expansion in Artificial Intelligence. One of the principal points of view of Simon was that decision theory payed (at that time) little attention to the "process" of problem solving and the necessary resources. In Simon's mind Artificial Intelligence could fill this gap. Indeed a large part of the research conducted in this area concerned (and concerns) well known decision problems. It is interesting therefore to observe how these two research areas evolved.

One common area of interest between artificial intelligence and operational research concerned optimisation and planning algorithms with mutual benefits for both research areas. Indeed the problem of establishing a plan in order to "solve a problem" has been often viewed in artificial intelligence as the exploration of a tree structure where the nodes represent "states of the world" and the arcs represent transitions from one state to another [110,

228,266]. The goal of such an exploration is to establish a path from a state of the world which represents the present situation to a state of the world which represents the solution. Such an exploration is mostly based on the estimation of the length of the path remaining in order to reach the desired state of the world (the solution), as can be seen in the famous A^* algorithm [150]. Indeed the state space tree has a structure similar to the one generated by a Branch and Bound algorithm and it is explored using similar principles. In doing that, AI researchers used and use concepts derived from integer and dynamic programming methods developed in OR (see for instance [36,41]). At the same time AI researchers developed the so called "constraint satisfaction" based methods (see [304,319]). Such methods have been largely applied in typical operational research problems (see [9,19,35,57,93]). Practically the two communities were sharing (and still do share) a common concern: efficiently solve problems which are or can be considered decision problems. From this point of view the development of heuristics for the solution of hard optimisation problems common to both communities should be noted (see [1,136,137,139,242]). A partial bibliography on the above issues can be seen in [161].

Another interesting interaction was developed around what today is known as "qualitative decision theory". The issue here is to extend decision theory through the use of symbolic approaches not requiring the imposition of further hypotheses in order to quantify information (see [38–40,54– 56,88,89,91,95,194,195,300,337]). The problem is how to formulate a theory where the preferences are simply order relations and uncertainty is purely qualitative. The reader can see an exhaustive presentation and discussion of this issue in [92]. The result is that, if we want to remain within the frame of Savage's axioms, such a theory is too weak. Indeed as pointed out in [92] the decision rules obtained within such an approach are either not decisive or overconfident, thus not interesting from an operational point of view. The reasons for such a "negative result" are related to the impossibility results present in social choice theory (the resulting decision rule is likely dominance).

Last, but not least, a field of interesting research has been established in "preferential entailment". Doyle [84], and Shoham [281], have observed that a reasoning system with only simple inferencing capabilities was not able to take into account preferences which are considered a fundamental element of

human capability to solve problems. Their suggestion was to enhance inference systems, namely the ones able to perform non-monotonic reasoning, with an ordering relation among the possible interpretations of a logical clause in order to obtain "preferred" consequences instead of only "true" ones. Such an idea has been followed by several other researchers (under different perspectives: [6,53,85–87,121,122, 184,196]). Nevertheless, once more, as Doyle and Wellman have shown [90] (see also [92]), the problem of aggregating such orders remains within the frame of Arrow's impossibility theorem.

Such results may appear negative. However, they also open interesting research perspectives, such as relaxing the axiomatic frame within which to look for a solution (see for instance [338] or exploring the so called "non-rationalisable" choice functions; see [277,279]).

Another major innovation within the frame of decision theory has been the introduction of fuzzy sets and more particularly of possibility theory (see [94,343]). In order to get a general view of how these formalisms contributed to decision theory the reader can see two more reference books on this subject [115,292].

I will focus my attention on two specific contributions.

- The consideration of preference relations as fuzzy subsets (see [115,164]). This allowed to relate such concepts to the already existing literature on valued binary relations and graphs (see for instance [83]). The use of fuzzy sets theory has thus been extended to other decision aiding concepts: choice sets, kernels, etc. (see [32,180]). The reader can see Chapters 1 and 2 in [292] and for a recent review [235].
- The development of new aggregation procedures. Aggregating "uncertainty measures" or "fuzzy measures" is similar to aggregating preferences (see [240] and Chapter 7 in [44]). As a consequence a literature has been developed on the use of new aggregation operators (mainly based on the use of fuzzy integrals: [141,142]).

More generally speaking, possibility theory introduced the use of formalisms for representing uncertainty different from probability. The motivation for that was the consideration that the additive property of probability was problematic and the consequent conceptual discussion when subjective estimation of uncertainty is considered (see for

example [224]). The ordinal nature of possibility distributions allowed their use in a more flexible way for several different domains of decision aiding (see for instance [268,269,295]) although it did not solve all conceptual problems related to uncertainty modelling. That said the reader should remind that since the late 1980s there has been a large discussion on innovating the whole field of decision under uncertainty and risk: see [69,105,131,132,134,135, 159,160,158,205–207,223,274,246].

I will conclude the discussion on handling uncertainty by recalling the contributions based on the use of other logic formalisms that allow the inevitable uncertainties, ambiguities and inconsistencies which characterise a decision aiding process to be taken into account (see [117,241,305–307]).

In the first part of this historical reconstruction, I have argued that the formulation of a decision problem as an optimisation one is a simplification of reality. Decision problems are almost always situations where we find several different dimensions, several points of view, several participating actors and stakeholders, each of them carrying within the decision process his values, his preferences and his criteria. The optimisation simplification does not always allow to consider the complexity of the decision process.

Remember that, from a technical point of view, multiple criteria decision aiding methods can be grouped in two categories, based on how the set of the potential alternatives is explored (for a recent survey see [109]):

1. the establishment of an utility function synthesising the different criteria;
2. the use of pairwise comparison procedures and majority principles for establishing a final recommendation.

Within the first category we find methods based on the construction of a multi-attribute utility function (see [174]) and the methods which interactively explore the set of efficient solutions of a multi-objective program (see [123,321]). Possibly specific heuristics apply to these types of problems (particularly in the case of difficult ones such as in combinatorial optimisation, see [317]). The reader can see an excellent reference survey in [101]. The construction of the utility function can be obtained either directly (see for instance [326]) or indirectly (through for instance the AHP method [267], the UTA method [157], or the MACBETH method [17]).

Within the second category we find the methods known as "outranking methods". This name was given by Roy [258] to the preference relation representing the concept "at least as good as". Such methods are based on the principle: when we compare x to y under multiple criteria, x will be at least as good as y if it is the case for a weighted majority of criteria and there are no strong "blocking minorities" (for a discussion see [306]). The reader can get more details on these methods in [260,264,272, 273,324]. Recently the possibility of constructing such a relation from holistic evaluations of the alternatives, provided by the decision maker, was shown in [144,239]. More generally the extension of the theory of rough sets through the use of dominance based decision rules (see [145,293]) allows to take into account purely ordinal evaluations. For an axiomatic characterisation of this approach in terms of conjoint measurement and on the equivalence between "outranking based" models and "decision rules" based ones the reader can see [143,294].

Such different methods each present advantages and disadvantages. The construction of an utility function is more restrictive (in the sense of the conditions to be fulfilled) and requires a considerable cognitive effort on the part of the decision maker (not necessarily intuitive). On the other hand it allows to obtain a rich result and is axiomatically well founded. The "outranking methods" are much more flexible (since there are less conditions to respect), but they risk obtaining a very poor result and are sometimes difficult to justify from an axiomatic point of view. The reader can see an interesting discussion on this issue in [27] and in Chapter 6 in [44] as well as in Chapters 4–6 of [51].

However, the separation in categories of the above methods can be misleading. Adopting a conjoint measurement point of view Bouyssou and Pirlot (see [46–50]) have shown that it is possible to give a common axiomatisation to all such methods. Moreover, authors coming from different backgrounds (see [27,76,118,140,172,173,259,261–263, 297]) have often claimed that if any differences exist among the methods these depend in reality more on how the decision aiding process is implemented and less on the specific method adopted.

Concluding this historical reconstruction I may summarise my claims as follows:

- Despite the specialisation of the last years Operational Research and Decision Theory can be still viewed as a unique "discipline" (as its founders did more than 60 years ago).

- OR and Decision Theory are deeply rooted in practicing decision support, aiding real decision makers in real problem situations, involved in real decision processes. Even the more abstract theoretical results have originated from precise practical problems and the research for models fitting real demands. Theoretical soundness has always been accompanied by empirical validation.

- The evolution of the discipline has been influenced certainly by the vitality of the research conducted within it, but it has greatly benefit from "cultural contamination" from other disciplines such as philosophy, psychology, organisation theory, political science, logic, mathematics and computer science.

- There is an increasing interest in issues such as how to structure and formulate a problem, how to conduct and implement a decision aiding process, how to handle the relations with the client of the decision support requested, how to train young people to undergo the profession of OR and/or Decision Analyst. The issue here is to take the development of "decision theories" (which are in excellent shape) one step further towards what I call a "decision aiding methodology": *a body of knowledge and a coherent structure of reasoning about theories and practices concerning deciding and aiding to decide*. Within such a methodology it is possible to distinguish different approaches and in the following section I will try to sketch their principal differences.

3. Different decision aiding approaches

In order to be able to help someone to make a decision we must be able to elaborate his preferences. Indeed what we know are his "problems" and his "desires" (the problem situation). What I am talking about here is an elaboration based on the use of a formal language. Moving from the problem situation to a decision aiding model and the actions such a model might imply requires the use of what I call a "model of rationality", a tool enabling the translation of "informal" information (which is also naturally ambiguous) to a formal representation (where even ambiguity is represented in an unambiguous way). The question is where this model of rationality comes from.

I am going to support my presentation through an example. In the following I will use the term "client" to represent the person or collective entity that asks for decision support. The client is potentially, but not necessarily, a decision maker. Consider a client with a health problem who has a number of diagnoses and a certain number of proposed treatments. Assume that there is some uncertainty associated to the diagnoses and therefore to the outcomes of the treatments. What do we suggest to this client to do?

The classic approach in decision theory is straightforward. To each diagnosis (the states of the nature) is associated a probability and to each treatment (the potential actions) the respective outcomes. Using any of the standard protocols for constructing the client's value function on the set of the outcomes we are able to define an utility function (including uncertainty), which, when maximised, identifies the solution which should be adopted (since by definition, it is the one which maximises the client's expected utility). The existence of such a function is guaranteed by a certain umber of axioms that represent what, following the theory, should be the principles of a rational decision maker's behaviour [270]. Preferences are supposed to be transitive (and complete), since the presence of cycles would imply that the decision maker will be ready to infinitely increase what he is ready to pay for any of the solutions and this is considered inconsistent with the idea of rationality. Similarly preferences about uncertain outcomes ought to be independent from probabilities[1] (thus allowing to construct a mathematical expectation). It should be noted that there has been no observation of the client behaviour nor the question has been posed of what other decision makers do in similar situations. It is the decision maker who has to adapt himself and his behaviour to the axioms. Otherwise he is not rational and the information and his preferences ought to be modified. This type of approach is usually called *normative*.

It should be noted that although the model handles uncertainty, there is no uncertainty at all associated to the model itself: the diagnoses are all the possible diagnoses and so are the treatments. The only issue is to find the best choice for the client. As with laws or ethical norms the legitimation of

the model of rationality is external to the problem situation. The model of rationality is a law of behaviour imported into the decision process. The reader can see several classics on this approach such as: [111,112,203,270,329]. For a nice discussion on how "rational" such an approach is, the reader can see [215].

However, it might be the case that the client's behaviour does not respect the axioms of the classic decision theory. As an alternative one could look for a model of rationality based on empirical legitimation instead of a theoretical one: if other decision makers followed a certain strategy in order to make a decision under similar circumstances, why not apply the same to the present one? This is the basic idea of the approach usually called *descriptive*: define models and decision strategies based on the observation of real decision makers (see [156,167, 245, 326]).

Once again it should be noted that we impose a model of rationality which is independent of the problem situation. Nevertheless, there are more degrees of freedom. The client's personality is considered as a source of information. The problem is not necessarily formulated as an optimisation one (several alternatives are possible). On the other hand, as for the normative approach, we are sure about the problem and the model: we are looking for the best treatment for the client given the diagnoses, the treatments and the uncertainties of the outcomes. I might recall that some of these ideas can be found at the origin of the research on expert systems (see the nice discussion in [152]).

The problem is that we can find ourselves in a situation where the client cannot be associated with any model of rationality more or less ready made. He might exhibit intransitive and/or incomplete preferences. His perception of the uncertainty might escape any effort to quantify or to measure it. Moreover, the client might be aware that he has to "improve" the structure of his preferences, but perhaps there is no time or resources available to do that (or even there is no will to do it). Nevertheless, we have to suggest a recommendation and we have to do it *here and now*. An approach could be to look for a contingent rationality model without searching for it outside the decision process, but within it. Obviously the validity of such a model is strictly local, but its legitimation is clear: the client himself. I will call such an approach *prescriptive*.

Identifying such a model of rationality has to obey the constraints of the formal language we are

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[1] $P \succ Q, 0 < \alpha < 1 \Rightarrow \alpha P + (1 - \alpha)R \succ \alpha Q + (1 - \alpha)R$, see [111, p. 107].

using and take into account what the procedures can and cannot do with the available information (see the discussion in [44]). The reader can consult [25,27,172,191,314,320,324,335] for a discussion of such an approach. The fact that we do not impose a model of rationality, but that we look for it within the problem situation, allows us to be more pragmatic and not to force the client to accept a model of rationality imposed from outside. However, we have to recognise two hypotheses within such an approach: the first is that the client's problem is what has been presented to us and the second is that the client has a model of rationality (possibly a very personal one). The issue is to identify it.

The reality of decision aiding is that quite often the client does not have a very clear idea of the problem, at least not clear enough to allow the identification of a model of rationality. Are we sure these are all the possible diagnoses? Did we really consider all the possible treatments? Is it certain that the problem is to find a treatment for the client? What if at the end we discover that the best thing to do for the client is to take a long vacation (possibly together with the analyst)? In other terms, finding the solution of a well formulated problem is always possible. The risk is to find a solution to a problem the client does not have. The problem is that nobody really knows what the problem is. In such situations we might adopt an approach which I will call *constructive*: we have to construct the problem and its solution at the same time.

Within such an approach we do not have to look just for what is the method that best adapts to the client's problem. Together with the client we have to establish a representation of the problem situation, formulate a formal problem with the consensus of the client and then establish an evaluation model which will help to formulate the final recommendation. There is a fundamental learning dimension in such a process. The models we are going to formulate are the result of a mutual learning process: the client learns to reason about his problem in an abstract and formal way (the point of view of the analyst) and the analyst learns to reason about the client's problem following the client's point of view. Nothing can be considered as given or fixed and everything has to be constructed. The reader can see in [27,44,62,127,146,188–190,254,264,271,333] more references to such an approach.

In Table 1 (borrowed from [82]) I present the principal differences of the different approaches.

I will add some remarks ending this presentation.

1. It is clear (to me) that the differences among the approaches do not concern the methods used to solve a decision problem. It is possible to use a constructive approach and a combinatorial optimisation procedure if this fits to the situation. On the other hand imposing on the client the use of a method (as flexible as possible) corresponds, to my opinion, in using a normative or a descriptive approach since the legitimation of this choice is external to the client.

2. There is no unique model of rationality and rational behaviour. A client exhibiting "cyclic preferences" is not less rational than another client perfectly consistent with decision theory's axioms. Transitivity of preferences is necessary only if we interpret the sentence "x is preferred to y" as "I am ready to pay more for x than for y". If we interpret the same sentence as "there are more reasons in favour of x than in favour of y" (see [271,306]) then it is possible to understand that "cyclic preferences" (in this case) are due to the existence of a cyclic structure of arguments (it is exactly the case with Condorcet's paradox [71]).

3. The presence of inconsistency in the client's arguments is not necessarily a problem, it can be seen as a source of information for conducting the decision aiding process.

4. Conducting a decision aiding process is a decision aiding itself. Asking the question: "where do you want to go this evening?" implies that the set of alternatives is constrained to only external locations, the possibility of remaining at home not being considered. Asking "do you prefer to hear classic music or jazz" implies that the subject wants to hear music, silence not being considered. This type of implicit hypotheses enters the decision model just by the way in which the decision aiding process is conducted and should be an important source of reflection in our profession.

In the following I will focus on this last concept (the decision aiding process) in order to see how its structuring allows decision theory to operate in practice.

4. The decision aiding process

As already noted, Simon has suggested that a "decision" is not an "act", but a process. Following such a suggestion, rationality cannot be conceived with respect to an objective (substantial rationality), but with respect to the process itself (procedural

Table 1
Differences among approaches

Approach	Characteristics	Process to obtain the model
Normative	Exogenous rationality, ideal economic behaviour	To postulate
Descriptive	Exogenous rationality, empirical behaviour models	To observe
Prescriptive	Endogenous rationality, coherence with the decision situation	To unveil
Constructive	Learning process, coherence with the decision process	To reach a consensus

rationality). Rationality becomes a local coherence (with respect to a certain temporal instance of the process) and therefore is bounded (see [285,288]). In the following I will use a descriptive model of the decision process presented in [234].

In my presentation I will assume that the client is involved within one or more decision processes and that his demand for decision support refers to one of these decision processes. I will group the activities associated to such a support under the name of "*decision aiding process*" and I will identify the following elements:

– at least two participants, the client and the analyst;
– at least two objects of the process: the client's concerns and the analyst's motivations;
– a set of resources including the client's knowledge on his concern's domain, the analyst's methodological knowledge and the time;
– a converging object (a meta-object) consisting in a shared (among the participants) representation of the client's concerns (one or more artifacts [97,286]).

I consider the decision aiding process as a distributed cognition process. Nevertheless, my point of view will be operational and not cognitive. I make the hypothesis that the participants actively try to create a shared representation. I will try to analyse the artifacts such a process generates (for an expanded discussion see [238,296]):

– a representation of the problem situation;
– a problem formulation;
– an evaluation model;
– a final recommendation.

4.1. The problem situation

A representation of the problem situation is the result of an effort aimed at replying to questions of the type:

– who has a problem?
– why is this a problem?
– who decides on this problem?
– what is the commitment of the client on this problem?
– who is going to pay for the consequences of a decision?

The construction of such an artifact allows, on the one hand, the client to better understand his position within the decision process for which he asked the decision support and on the other hand, the analyst to better understand his role within this decision process.

From a formal point of view, a representation of the problem situation is a triple:

$$\mathscr{P} = \langle \mathscr{A}, \mathscr{O}, \mathscr{S} \rangle,$$

where

– $\mathscr{A}$ is the set of participants to the decision process;
– $\mathscr{O}$ is the set of stakes each participant brings within the decision process;
– $\mathscr{S}$ is the set of resources the participants commit on their stakes and the other participants' stakes.

Such a representation is not fixed once and for all within the decision aiding process, but usually will evolve. Actually one reason that such a representation is constructed is to help to understand the misunderstandings during the client–analyst interaction.

4.2. The problem formulation

For a given representation of the problem situation the analyst might propose to the client one or more "problem formulations". This is a crucial point of the decision aiding process. The representation of the problem situation has a descriptive or explicative scope. The construction of the problem formulation introduces what I have called a model of rationality. A problem formulation reduces the

reality of the decision process within which the client is involved to a formal and abstract problem. The result is that one or more of the client's concerns are transformed into formal problems on which we can apply a method (already existing, adapted from an existing one or created ad-hoc) of the type studied in decision theory.

Example 4.1. Consider the case of a client having the problem "to buy new buses in order to improve the service offered to the customers". Different problem formulation are possible:

– choose one among the potential suppliers;
– choose one among the offers received (a supplier may have done more than one);
– choose combinations of offers.

The choice of one among the above formulations is not neutral. The first is focussed on the suppliers rather than the offers and enables us to think about the will to establish a more strategic relationship with one of them. The second one is a more contingent formulation and introduces the implicit hypothesis that all buses will be bought from the same supplier. The third is also a contingent problem formulation, but also considers the possibility of buying from different suppliers. Obviously choosing one of the above formulations will strongly influence the outcome of the decision aiding process and the final decision.

From a formal point of view a problem formulation is a triple:

$$\Gamma = \langle \mathbb{A}, \mathbb{V}, \Pi \rangle,$$

where

– $\mathbb{A}$ is the set of potential actions the client may undertake within the problem situation as represented in $\mathscr{P}$;
– $\mathbb{V}$ is the set of points of view under which the potential actions are expected to be observed, analysed, evaluated, compared, including different scenarios for the future;
– Π is the problem statement, the type of application to perform on the set $\mathbb{A}$, an anticipation of what the client expects (the reader can see more details on this point in [14,233,265]; for a detailed example see, [296]).

Obtaining the client's consensus on a problem formulation can lend insight, since instead of having an "ambiguous" description of the problem we have an abstract and formal problem. Several decision aiding approaches will stop here, considering that formulating (and understanding) a problem is equivalent to solving it, thus limiting decision aiding in helping to formulate problems, the solution being a personal issue of the client. Other approaches instead will consider the problem formulation as given. Within a constructive approach the problem formulation is one among the products of the decision aiding process. It is expected to be used in order to construct the evaluation model.

4.3. The evaluation model

By "evaluation model" I indicate what traditionally are the decision aiding models conceived through any operational research, decision theory or artificial intelligence method. Classic decision aiding textbooks focus their attention on the construction of this model. In a normative approach there is no freedom, the structure of the model being predefined. Within other approaches more degrees of freedom are possible, at least as far as some of the model's parameters are concerned.

An evaluation model is an *n*-tuple:

$$\mathscr{M} = \langle A, D, E, H, \mathscr{U}, \mathscr{R} \rangle,$$

where

– A is a set of alternatives to which the model will apply;
– D is a set of dimensions (attributes) under which the elements of A are observed, measured, described, etc. (such a set can be structured, for instance through the definition of an hierarchy);
– E is a set of scales associated to each element of D;
– H is a set of criteria (if any) under which each element of A is evaluated in order to take into account the client's preferences (recall that a criterion is a preference model);
– $\mathscr{U}$ is a set of uncertainty measures associated to D and/or H;
– $\mathscr{R}$ is a set of operators enabling to obtain synthetic information about the elements of A or of $A \times A$, namely aggregation operators (acting on preferences, measures, uncertainties, etc.).

The reader can observe that a large part of the existing decision aiding models and methods can

be represented through the above description. Besides, such a description allows to focus our attention on a number of important remarks:

1. It is easy to understand why the differences among the decision aiding approaches do not depend on the adopted decision aiding method. The fact that we work with only one evaluation dimension or a single criterion can be the result of applying a constructive approach. We can conduct a decision aiding process constructively and end by using a combinatorial optimisation algorithm. What is important is to show that such tools are a consequence of the decision aiding process and not to choose them before the problem has been formulated or the evaluation model constructed.
2. The technical choices (typology of the measurement scales, different preference or difference models, different aggregation operators) are not neutral. Even when the client can formulate his problem clearly and is convinced about its formulation (possibly using one of the techniques aiding in formulating problems), the choice of a certain technique, procedure or operator can have important consequences that are not discussed at the moment when the problem has been formulated (for a critical discussion see [44]). Characterising such techniques, procedures and operators is therefore crucial, since it allows a control of their applicability to the problem as has been formulated during the decision aiding process.
3. The evaluation models are subject to validation processes, namely (see [189]):
   – conceptual validation (verify the suitability of the concepts used);
   – logical validation (verify the logical consistency of the model);
   – experimental validation (verify the results using experimental data);
   – operational validation (verify the implementation and use of the model in everyday life).

4.4. The final recommendation

The final recommendation represents the return to reality for the decision aiding process. Usually the evaluation model will produce a result, let us call it Φ. The final recommendation should translate such a result from the abstract and formal language in which Φ is formulated to the current language of the client and the decision process where he is involved. Some elements are very important in constructing this artifact:

– the analyst has to be sure that the model is formally correct;
– the client has to be sure that the model represents him, that he understands it and that he should be able to use its conclusions (the client should feel he is the "owner" of the results, besides being satisfied by them);
– the recommendation should be "legitimated" with respect to the decision process for which the decision aiding has been asked.

We should pay some attention to this last observation. The decision aiding process is an activity that introduces a certain distance between the participants on the one hand and the reality of the decision process and its organisational dimension on the other hand. Returning back to reality requires a check as to whether the results are legitimated. We should check whether such results are accepted or not by the participants to the decision process and understand the reasons for their position (such reasons can be completely independent from the decision process itself). Being able to put in practice the final recommendation definitely depends on such legitimation. No legitimation, no implementation (see [66]).

Concluding the presentation of the decision aiding process through its artifacts I would like to make two remarks.

• Not all such artifacts are necessarily created in all decision aiding processes. There might be cases where the evaluation model is not constructed, the client being satisfied by being enabled to correctly formulate his problem. A final recommendation might not always be formulated. This presentation tries to give an account of the possible outcomes of a decision aiding process.
• The identification of artifacts within the decision aiding process in the form of "checklists" is of course a simplification of the complexity of real decision aiding situations. The scope of such a presentation is mainly conceived for "training" purposes: what a "novice" analyst should check when involved in a decision aiding process. Experienced analysts may conduct a decision aiding process in a more "holistic" way, but the analysis of such an approach goes beyond the scope of this paper.

5. Conclusion

In this paper I have tried to outline a personal perspective on the evolution of decision theory. In such a presentation I mainly focussed on the appearance and growth of several alternative approaches to the so called "classic decision theory" and I did that in relation both with empirical validation and with the evolution of related scientific domains such as cognitive sciences, organisation theory and artificial intelligence.

A first hypothesis I tried to develop in the paper is that such "alternative approaches" are ultimately related to the "classic decision theory", since they all share the fundamental idea that decision making and decision aiding are human activities that can be scientifically investigated and that it is possible to use an abstract and formal language in aiding decision makers to handle the issues arising within the decision processes they are involved in. I would rather adopt the plural "decision theories" to emphasise the existence of several different theoretical approaches and methods studying and implementing the above idea (see [339]).

A second hypothesis I tried to develop in the paper is that decision aiding is a broader concept than the one of decision theory, since it includes not only the theoretical aspects of this activity, but also the practices and the behaviours that can be observed along what I call the decision aiding process. One of the characteristics of the "decision theories" evolution is the appearance of approaches aiming at including large parts of the decision aiding process in their field of investigation.

I do believe that decision aiding is practiced everyday by individuals and organisations and that, in a proportion of cases that is larger than what the number of relevant scientific publications would suggest, decision theories are correctly used. It should be noted that the level of analysis of such decision aiding practices is far less important if compared to the extremely fine theoretical achievements the "decision theories" can show. Under such a perspective our scientific area, despite its maturity, is still lacking the establishment of "best practices analysis" as it is the case for similar professions including lawyers, physicians and psychotherapists.

A third hypothesis I tried to develop in the paper concerns what decision theory and operational research owns to other scientific fields such as mathematics, cognitive science, organisation theory, artificial intelligence and more general computer science. Several critical advances in our field have been possible, thanks to the integration of findings obtained in such areas. At the same time, the focus of our research on concepts such as decision, preference, optimal and compromise solution, etc. provided invaluable contributions to these areas (not only, OR and decision theory are applied in fields such as molecular biology, archaeology, engineering, sociology and political science). The ultimate interdisciplinary nature of decision theory and decision aiding can be best observed through its practice.

Last, but not least, I tried to suggest my personal classification of the "decision aiding approaches". In order to do that I tried to establish the differences among such approaches on the basis of the origin of the "model of rationality" used in order to construct the artifacts of the decision aiding process. Under such a perspective I also tried to outline a model of the decision aiding process, sufficiently large to include most of the existing methods and techniques. I thus hope to contribute to the establishment of a "decision aiding methodology" (from the greek μεθοδολογία: λόγος μεθόδων, reasoning about methods).

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