Dynamics of Mechanical Systems
Railway arc bridge

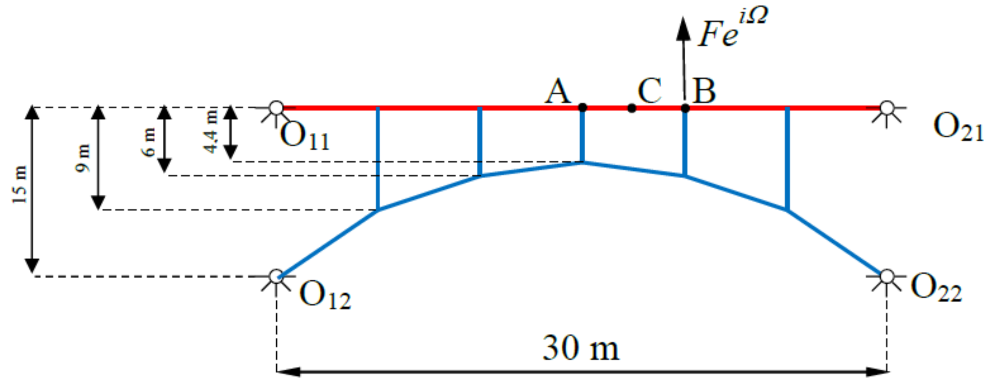


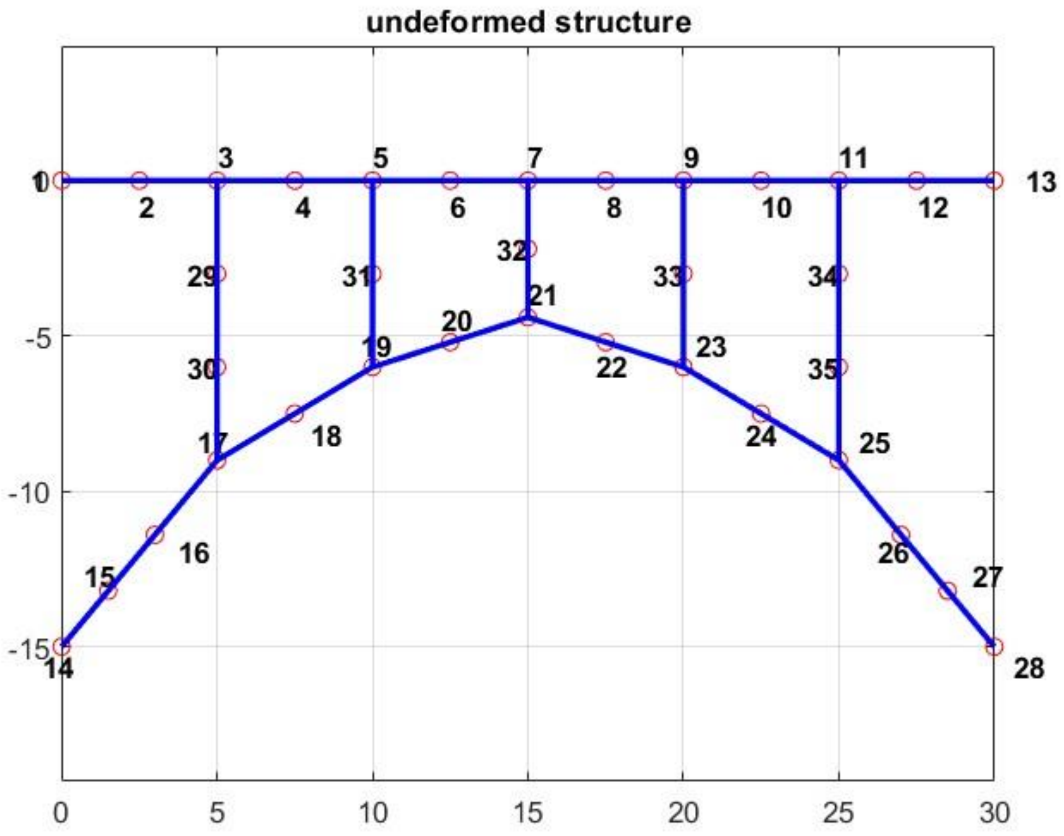
Fig.1

Consider the railway bridge shown in Fig.1. All beams are made of steel (E=2.06e11 N/m^2, ρ=7800 kg/m^3). Beams in blue have IPE300 cross section (A=5.381e-3m^2, I=8.356e-5 m^4) and beams in red have IPE400 cross section (A=8.446e-3 m^2, I=2.313e-4 m^4). Damping is defined according to the "proportional damping" assumption: [C]=α[M]+β[K], with α=0.8 s^{-1} and β=3.0e-5 s.

1. Define a FE model of the structure suitable for analysing its dynamic response in the **0-24 Hz** frequency range (***using a safety coefficient 2***). Plot the undeformed structure.

```
%point 1


E=2.06e11;                %[N/m^2]
rho=7800;                 %[Kg/m^3]
A_red=8.446e-3;           %[m^2]
A_blue=5.381e-3;          %[m^2]
I_red=2.313e-4;           %[m^4]
I_blue=8.356e-5;          %[m^4]


saf_coef=2;


freq_max=24;              %[Hz]
omega_max=2*pi*freq_max   %[rad/s]


Red_m=A_red*rho           %[kg/m]
Blue_m=A_blue*rho         %[kg/m]


Red_EA=A_red*E            %[N]
Blue_EA=A_blue*E          %[N]


Red_EI=E*I_red            %[Nm^2]
Blue_EI=E*I_blue          %[Nm^2]
```

```
%red part


L_max_red=sqrt((pi^2/(saf_coef*omega_max))*sqrt(Red_EI/Red_m))
Lk_red_test=5
omega_first_red=(pi/Lk_red_test)^2*sqrt((Red_EI)/Red_m)/saf_coef


%blue part


L_max_blue=sqrt((pi^2/(saf_coef*omega_max))*sqrt(Blue_EI/Blue_m))
Lk_blue_test=4.5
omega_first_blue=(pi/Lk_blue_test)^2*sqrt(Blue_EI/Blue_m)/saf_coef


a=;
b=;
Length=sqrt(a^2+b^2)


y1=;
y2=;
x1=;
x2=;
x=;
y=((x-x1)/(x2-x1))*(y2-y1)+y1
```



Total nodes number 35

Number of d.o.f. 97

Number of beam elements 38

Number of string elements 0

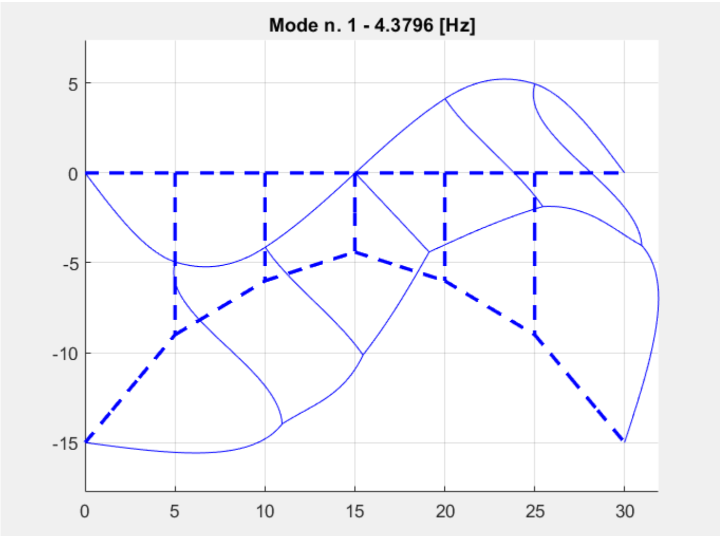
Number of tensile beam elements 0

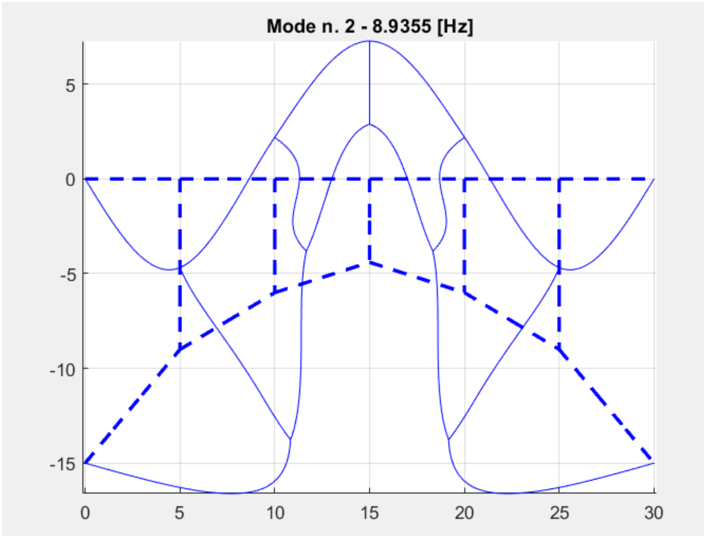
Number of concentrated masses 0

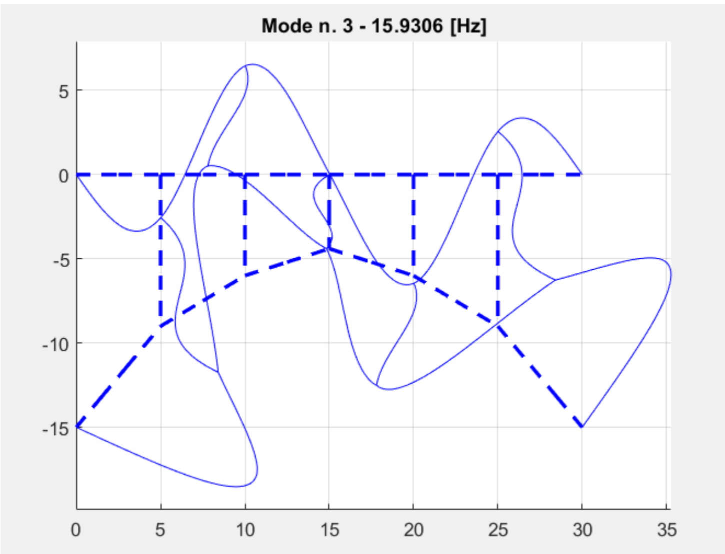
Number of concentrated springs 0

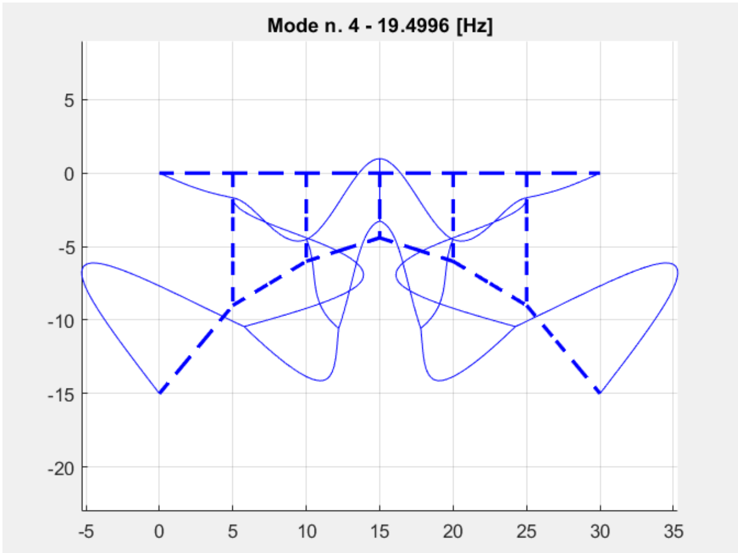
Total mass [kg] 5005.9694

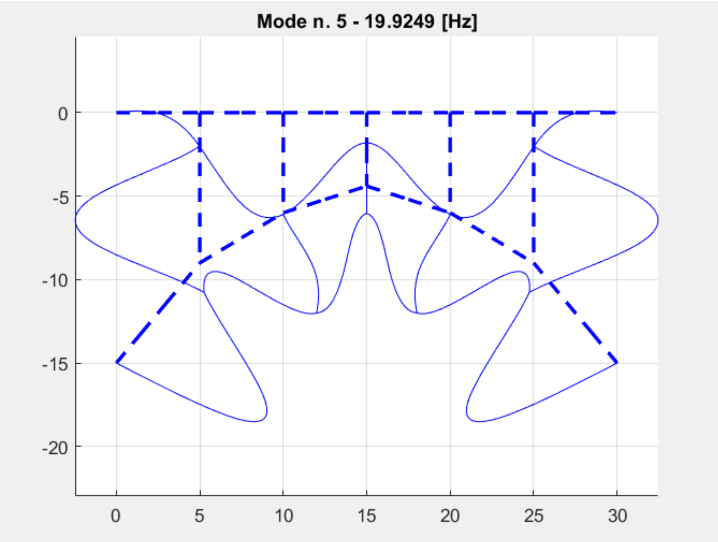
2. Compute the structure's natural frequencies and modes of vibration. Plot the modal shapes associated to the natural frequencies of the bridge up to **24 Hz**.

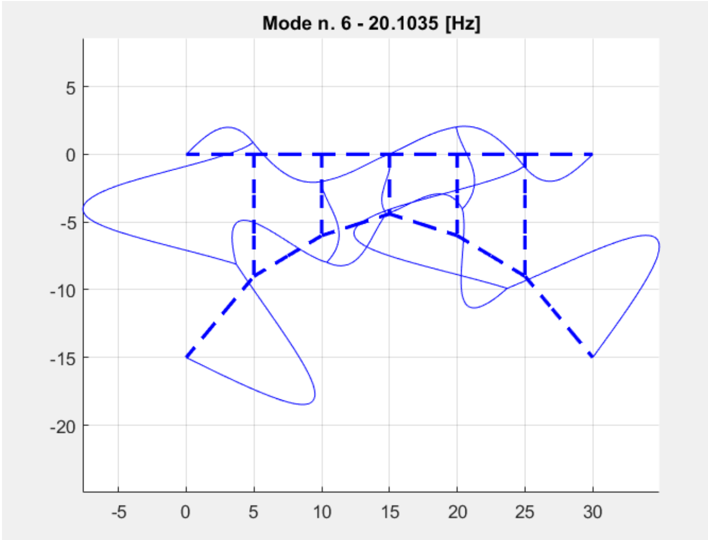










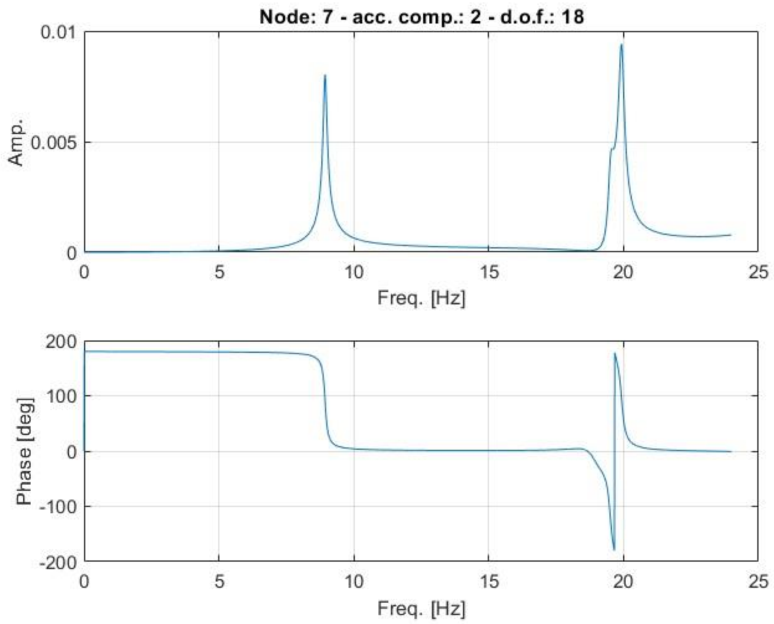


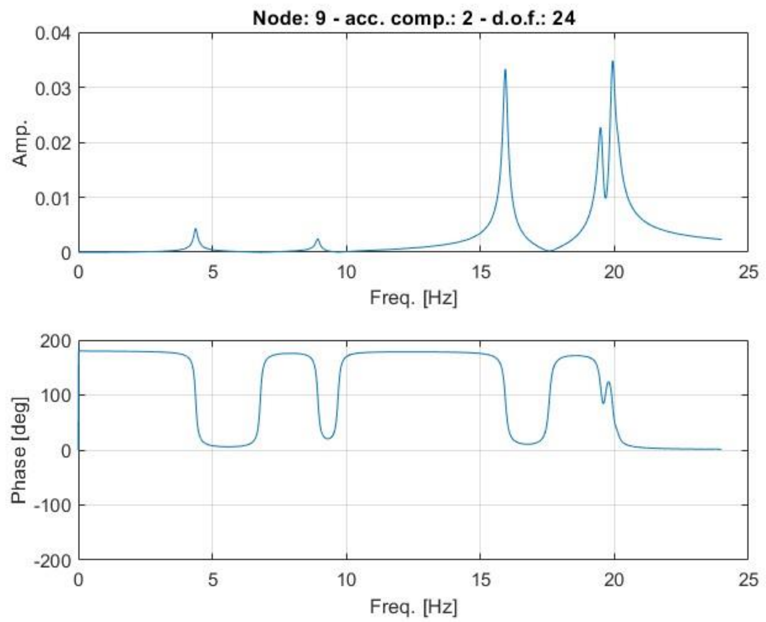
3. Compute the natural frequencies of the damped structure up to **24 Hz** and the related non-dimensional damping ratios .

```
%point 3

f1=4.3796;          %[Hz]
f2=8.9355;          %[Hz]
f3=15.9306;         %[Hz]
f4=19.4996;         %[Hz]
f5=19.9249;         %[Hz]
f6=20.1035;         %[Hz]

w1=2*pi*f1;         %[rad/s]

w2=2*pi*f2;         %[rad/s]

w3=2*pi*f3;         %[rad/s]

w4=2*pi*f4;         %[rad/s]

w5=2*pi*f5;         %[rad/s]

w6=2*pi*f6;         %[rad/s]

alpha=0.8;        %[s^(-1)]

beta=3e-5;        %[s]

A=[1/(2*w1) w1/2;
   1/(2*w2) w2/2;
   1/(2*w3) w3/2;
   1/(2*w4) w4/2;
   1/(2*w5) w5/2;
   1/(2*w6) w6/2;];

x=[alpha;
   beta;];
```

```
b=A*x
b = 0.0149
    0.0080
    0.0055
    0.0051
    0.0051
    0.0051
```

4. Compute the structure frequency response functions between an input force applied at position B in vertical direction and the output **vertical acceleration** evaluated at points A and B. Assume the input force to vary in the 0-24 Hz frequency range and set the frequency resolution to 0.01 Hz. Plot the Bode diagrams (in linear scales).



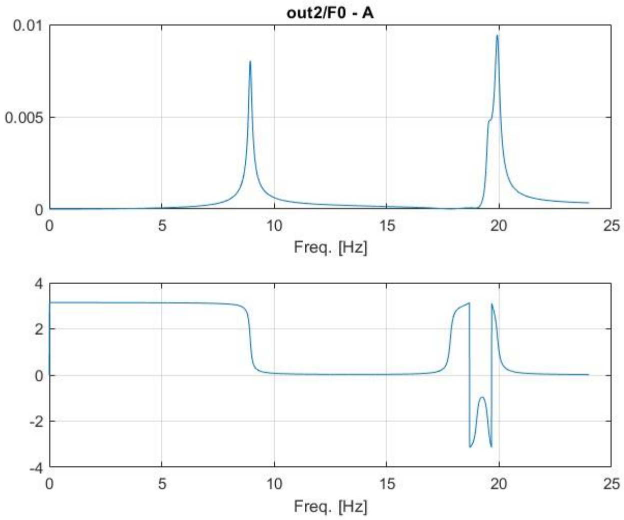


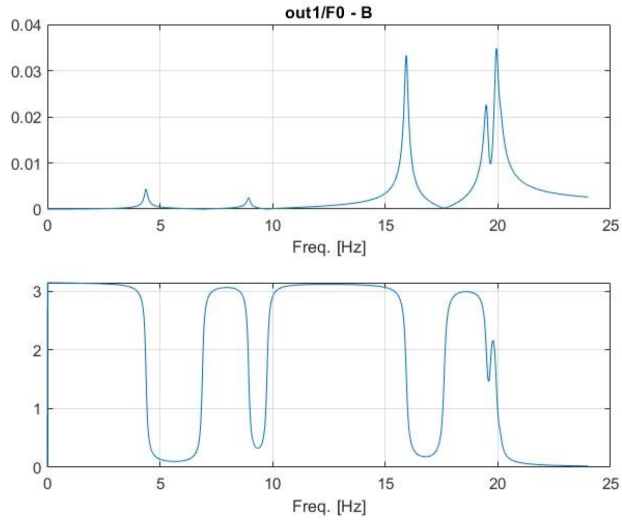
5. Compute the same FRF as point 4 developing a model in modal coordinates limited to the first three modes. Plot the Bode diagrams superimposed (with two different colours) to those of point 4.

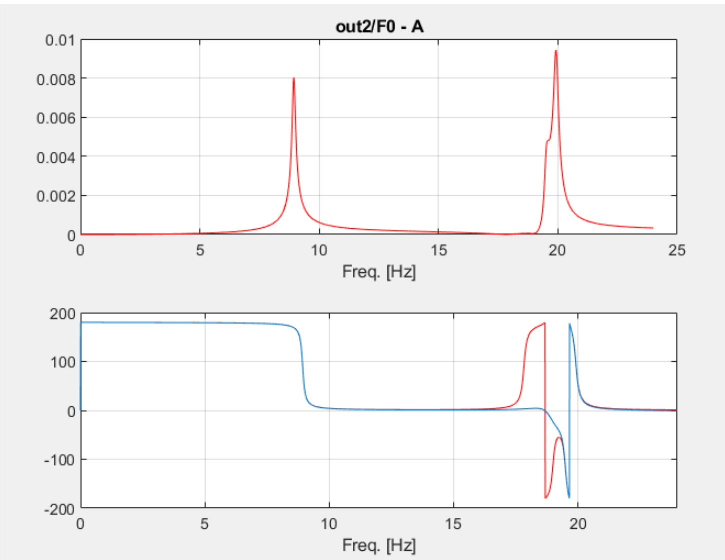
```
%point 5

load('C:\Users\Ricky\OneDrive\Desktop\PoliMi\Automation & Control
Engineering\First Semester_First Year\Dynamics of Mechanical
Systems\Yearwork\Bridge\1\Bridge_mkr.mat')
whos

ndgl=97;
MFF=M(1:ndgl,1:ndgl);
MFC=M(1:ndgl,ndgl+1:end);
CFF=R(1:ndgl,1:ndgl);
CFC=R(1:ndgl,ndgl+1:end);
KFF=K(1:ndgl,1:ndgl);
KFC=K(1:ndgl,ndgl+1:end);
idb

vett_F=zeros(ndgl,1);
vett_F(24)=1;

% natural frequencies and modes of vibration

[eigenvectors eigenvalues]=eig(MFF\KFF);

freq=sqrt(diag(eigenvalues))/(2*pi);

PSI_B=zeros(97,97);

PSI_B(24,83)=eigenvectors(24,83);
PSI_B(24,84)=eigenvectors(24,84);
PSI_B(24,90)=eigenvectors(24,90);
PSI_B(24,95)=eigenvectors(24,95);
PSI_B(24,96)=eigenvectors(24,96);
PSI_B(24,97)=eigenvectors(24,97);

PSI_A=zeros(97,97);

PSI_A(18,83)=eigenvectors(18,83);
PSI_A(18,84)=eigenvectors(18,84);
PSI_A(18,90)=eigenvectors(18,90);
PSI_A(18,95)=eigenvectors(18,95);
PSI_A(18,96)=eigenvectors(18,96);
PSI_A(18,97)=eigenvectors(18,97);

Mod_M=eigenvectors'*MFF*eigenvectors;
Mod_C=eigenvectors'*CFF*eigenvectors;
Mod_K=eigenvectors'*KFF*eigenvectors;
```

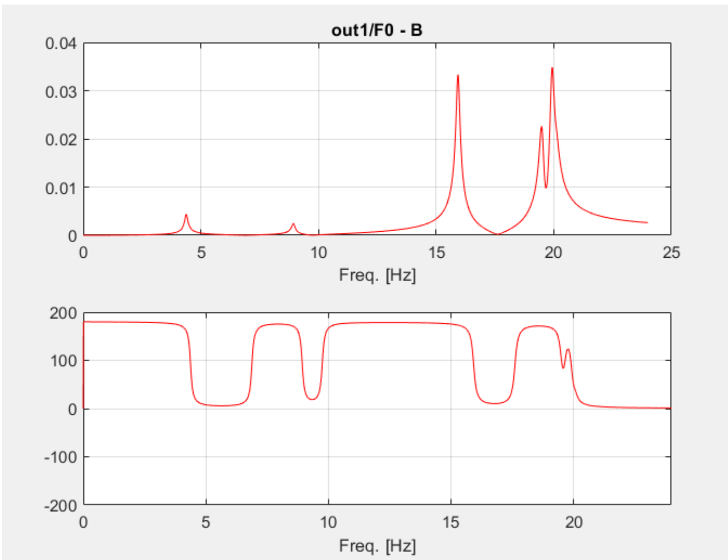
```
Mod_F=eigenvectors'*vett_F;

vett_f=0:0.01:24;
i=sqrt(-1);
for k=1:length(vett_f)
    ome=vett_f(k)*2*pi;
    A=-ome^2*Mod_M+i*ome*Mod_C+Mod_K;
    q=A\Mod_F;
    y_B=PSI_B*q;
    y_A=PSI_A*q;
    out1=-ome^2*y_B(24);
    out2=-ome^2*y_A(18);
    mod1(k)=abs(out1);
    fas1(k)=angle(out1);
    mod2(k)=abs(out2);
    fas2(k)=angle(out2);
end

figure(1)
subplot 211;plot(vett_f,mod1,"r");grid
title('out1/F0 - B');
xlabel('Freq. [Hz]');
subplot 212;plot(vett_f,fas1*(180/pi),"r");grid
xlabel('Freq. [Hz]');
axis([0 24 -200 200]);

% Abilito la modalità di sovrapposizione
hold on;

% Importo il secondo grafico
fig2 = openfig('B.fig', 'invisible');

% Sovrappongo il secondo grafico al primo
ax2 = gca;
copyobj(allchild(get(fig2, 'CurrentAxes')), ax2);

% Disattiva la modalità di sovrapposizione
hold off;

figure(2)
subplot 211;plot(vett_f,mod2,"r");grid
title('out2/F0 - A');
xlabel('Freq. [Hz]');
subplot 212;plot(vett_f,fas2*(180/pi),"r");grid
xlabel('Freq. [Hz]');
axis([0 24 -200 200]);

% Abilito la modalità di sovrapposizione
hold on;
```

```
% Importo il secondo grafico
fig4 = openfig('A.fig', 'invisible');

% Sovrappongo il secondo grafico al primo
ax4 = gca;
copyobj(allchild(get(fig4, 'CurrentAxes')), ax4);

% Disattiva la modalità di sovrapposizione
hold off;
```







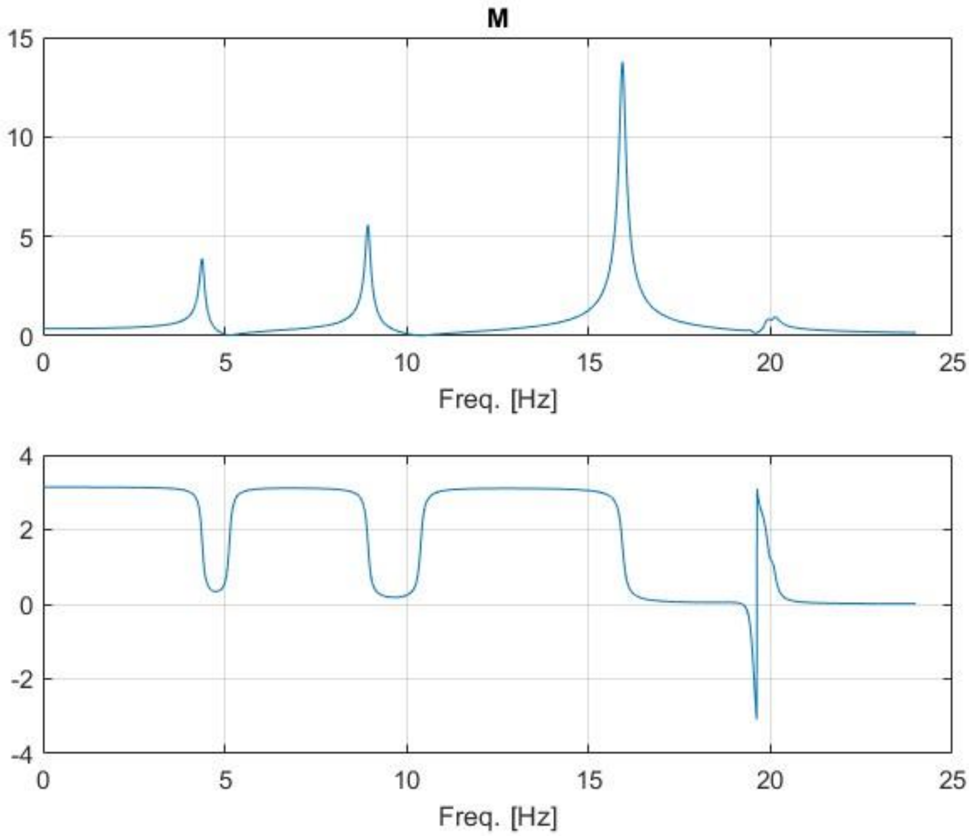


6. For the same input described at point 4, compute the FRF of the bending moment at point C, located on the bridge deck at mid-distance between points A and B.

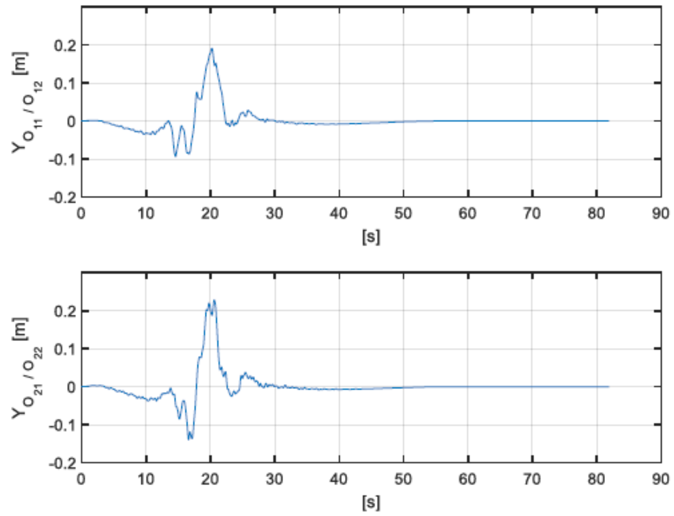
```
%point 6

E=2.06e11;                  %[N/m^2]
I_red=2.313e-4;             %[m^4]
```

```
load('C:\Users\Ricky\OneDrive\Desktop\PoliMi\Automation & Control
Engineering\First Semester_First Year\Dynamics of Mechanical
Systems\Yearwork\Bridge\1\Bridge_mkr.mat')
whos

ndgl=97;
MFF=M(1:ndgl,1:ndgl);
MFC=M(1:ndgl,ndgl+1:end);
CFF=R(1:ndgl,1:ndgl);
CFC=R(1:ndgl,ndgl+1:end);
KFF=K(1:ndgl,1:ndgl);
KFC=K(1:ndgl,ndgl+1:end);
idb
vett_F=zeros(ndgl,1);
vett_F(24)=1;

% natural frequencies and modes of vibration

[eigenvectors eigenvalues]=eig(MFF\KFF);

freq=sqrt(diag(eigenvalues))/(2*pi);

Mod_M=eigenvectors'*MFF*eigenvectors;
Mod_C=eigenvectors'*CFF*eigenvectors;
Mod_K=eigenvectors'*KFF*eigenvectors;
Mod_F=eigenvectors'*vett_F;

PSI_A=zeros(97,97);

PSI_A(24,83)=eigenvectors(24,83);
PSI_A(24,84)=eigenvectors(24,84);
PSI_A(24,90)=eigenvectors(24,90);
PSI_A(24,95)=eigenvectors(24,95);
PSI_A(24,96)=eigenvectors(24,96);
PSI_A(24,97)=eigenvectors(24,97);

PSI_B=zeros(97,97);

PSI_B(21,83)=eigenvectors(21,83);
PSI_B(21,84)=eigenvectors(21,84);
PSI_B(21,90)=eigenvectors(21,90);
PSI_B(21,95)=eigenvectors(21,95);
PSI_B(21,96)=eigenvectors(21,96);
PSI_B(21,97)=eigenvectors(21,97);

PSI_C=zeros(97,97);

PSI_C(25,83)=eigenvectors(25,83);
PSI_C(25,84)=eigenvectors(25,84);
PSI_C(25,90)=eigenvectors(25,90);
```

```
PSI_C(25,95)=eigenvectors(25,95);
PSI_C(25,96)=eigenvectors(25,96);
PSI_C(25,97)=eigenvectors(25,97);

PSI_D=zeros(97,97);

PSI_D(22,83)=eigenvectors(22,83);
PSI_D(22,84)=eigenvectors(22,84);
PSI_D(22,90)=eigenvectors(22,90);
PSI_D(22,95)=eigenvectors(22,95);
PSI_D(22,96)=eigenvectors(22,96);
PSI_D(22,97)=eigenvectors(22,97);

Lk=2.5;
e=0;
shape_func=[0;
              (12*e)/(Lk^3)-6/(Lk^2);
              Lk*((6*e)/(Lk^3)-4/(Lk^2));
              0;
              (-12*e)/(Lk^3)+6/(Lk^2);
              Lk*((6*e)/(Lk^3)-2/(Lk^2));];

vett_f=0:0.01:24;
i=sqrt(-1);
for k=1:length(vett_f)
    ome=vett_f(k)*2*pi;
    A=-ome^2*Mod_M+i*ome*Mod_C+Mod_K;
    q=A\Mod_F;
    y_B=PSI_A*q;
    y_A=PSI_B*q;
    theta_B=PSI_C*q;
    theta_A=PSI_D*q;
     x_M=[0;
         y_A(21);
         theta_A(22);
         0;
         y_B(24);
         theta_B(25)];
    M=E*I_red*shape_func'*x_M;
    mod1(k)=abs(M);
    fas1(k)=angle(M);
end

figure(1)
subplot 211;plot(vett_f,mod1);grid
title('M');
xlabel('Freq. [Hz]');
subplot 212;plot(vett_f,fas1);grid
xlabel('Freq. [Hz]');
```


M
15
10
5
0
0
5
10
15
20
25
Freq. [Hz]
4
2
0
-2
-4
0
5
10
15
20
25
Freq. [Hz]

7. Compute the bridge response due to a seismic motion of the ground, represented as a vertical displacement of points O_{11}/O_{12} (same displacement for these two points) and O_{21}/O_{22} (same displacement for these two points). The time histories of the displacements at these points are shown in the figure below:



The two time histories are provided in file *seismic_displ.txt*, available on WeBeeP. The file has three columns containing respectively: time, vertical displacement of points O_{11}/O_{12}, vertical displacement of points O_{21}/O_{22} displacement respectively. The outputs to be produced are:

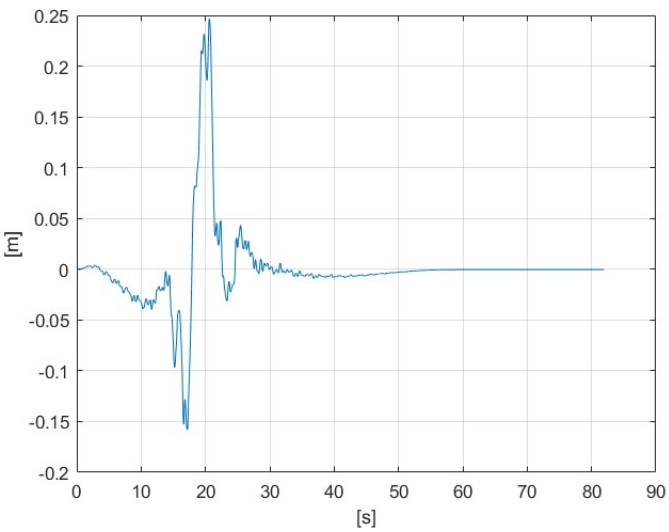
6.1) Vertical displacement of point A

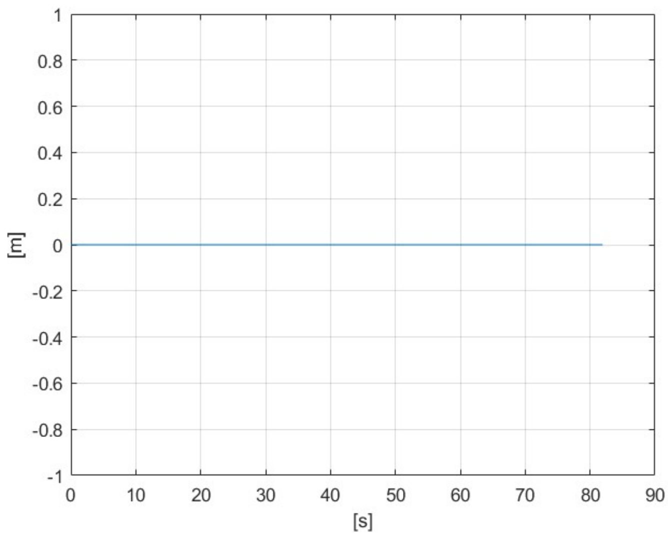
6.2) Vertical acceleration of point A

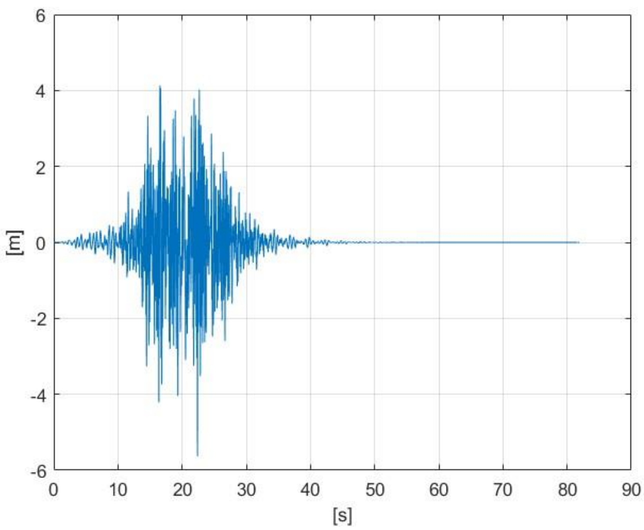
```
%point 7

load('C:\Users\Ricky\OneDrive\Desktop\PoliMi\Automation & Control
Engineering\First Semester_First Year\Dynamics of Mechanical
Systems\Yearwork\Bridge\1\Bridge_mkr.mat')
load('seismic_displ.txt')
whos

freq_max=24;            %[Hz]
omega_max=2*pi*freq_max  %[rad/s]
ndgl=97;
MFF=M(1:ndgl,1:ndgl);
MFC=M(1:ndgl,ndgl+1:end);
CFF=R(1:ndgl,1:ndgl);
CFC=R(1:ndgl,ndgl+1:end);
KFF=K(1:ndgl,1:ndgl);
KFC=K(1:ndgl,ndgl+1:end);
idb

t=seismic_displ(:,1);
s1=seismic_displ(:,2);
s2=seismic_displ(:,3);
```

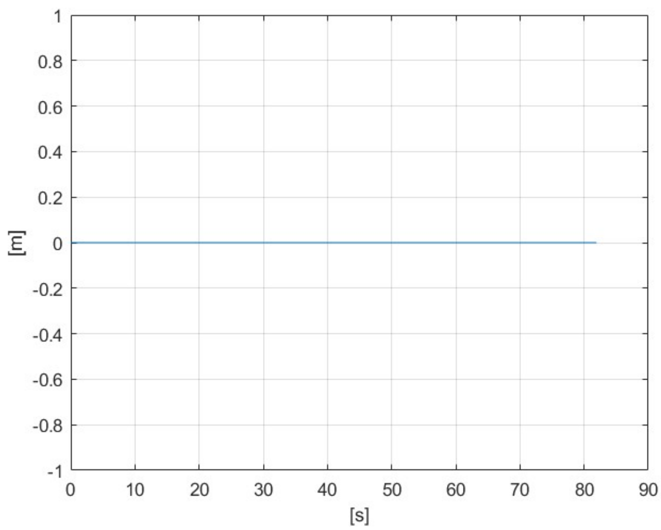
```
F_1=fft(s1);
F_1_n=F_1(1:4096,:);
F_2=fft(s2);
F_2_n=F_2(1:4096,:);

vett_f=1:1:4096;
i=sqrt(-1);
vect_xC0 = zeros(8,1);
v_disp=zeros(8192,1);
v_acce=zeros(8192,1);
N=8192;
for k=1:length(vett_f)
    ome=(vett_f(k)*2*pi)/81.91;
    if ome<omega_max
        vect_xC0(1,1)=0;
        vect_xC0(2,1)=F_1_n(k+1);
        vect_xC0(3,1)=0;
        vect_xC0(4,1)=F_1_n(k+1);
        vect_xC0(5,1)=0;
        vect_xC0(6,1)=F_2_n(k+1);
        vect_xC0(7,1)=0;
        vect_xC0(8,1)=F_2_n(k+1);
        A=-ome^2*MFF+i*ome*CFF+KFF;
        vect_FFC0=-(-ome^2*MFC+i*ome*MFC+KFC)*vect_xC0;
        vect_xF0=A\vect_FFC0;
        v_disp(k+1)=vect_xF0(18);
        v_disp(N-k+1)=conj(vect_xF0(18));
        v_acce(k+1)=-ome^2*vect_xF0(18);
        v_acce(N-k+1)=conj(-ome^2*vect_xF0(18));
    end
end

ome=0;
vect_xC0(1,1)=0;
vect_xC0(2,1)=F_1_n(1);
vect_xC0(3,1)=0;
vect_xC0(4,1)=F_1_n(1);
vect_xC0(5,1)=0;
vect_xC0(6,1)=F_2_n(1);
vect_xC0(7,1)=0;
vect_xC0(8,1)=F_2_n(1);
A=-ome^2*MFF+i*ome*CFF + KFF;
vect_FFC0=-(-ome^2*MFC+i*ome*MFC+KFC)*vect_xC0;
vect_xF0 = A\vect_FFC0;
inv(1)=vect_xF0(18);
inv1(1)=-ome^2*vect_xF0(18);

v_disp_t = ifft(v_disp);
v_acce_t = ifft(v_acce);
```

```
figure(5);
plot(t,real(v_disp_t));grid;xlabel('[s]');ylabel('[m]');
figure(6);
plot(t,imag(v_disp_t));grid;xlabel('[s]');ylabel('[m]');

figure(7);
plot(t,real(v_acce_t));grid;xlabel('[s]');ylabel('[m]');
figure(8);
plot(t,imag(v_acce_t));grid;xlabel('[s]');ylabel('[m]');
```









8. Define a change in the structure to reduce **by 30% at least** the maximum amplitude of the FRF of the vertical acceleration of point B for a unit force applied at B (see point 4.) **without increasing the total mass of the system by more than 5%**. The following modifications are not allowed: any change in the length of the span or vertical distance between points A and C; any change of material properties; use of additional constraints. If the section of one or more beam is changed, unified beam sections such as IPE, HPE etc. shall be used.

```
%point 8
E=2.06e11;                %[N/m^2]
rho=7800;                 %[Kg/m^3]
saf_coef=2;
I=0.8014e-6;              %[m^4]
m=6.00;                   %[kg]
A=764e-6;                 %[m^2]
EA=A*E                    %[N]
EI=E*I                    %[Nm^2]
freq_max=24;              %[Hz]
omega_max=2*pi*freq_max   %[rad/s]


L_max=sqrt((pi^2/(saf_coef*omega_max))*sqrt(EI/(A*rho)))
Lk_8_test=4.5
omega_first=(pi/Lk_8_test)^2*sqrt(EI/(A*rho))/saf_coef


a=;
b=;
Length=sqrt(a^2+b^2)


y1=;
y2=;
x1=;
x2=;
x=;
y=((x-x1)/(x2-x1))*(y2-y1)+y1
```

Total nodes number 49

Number of d.o.f. 139

Number of beam elements 59

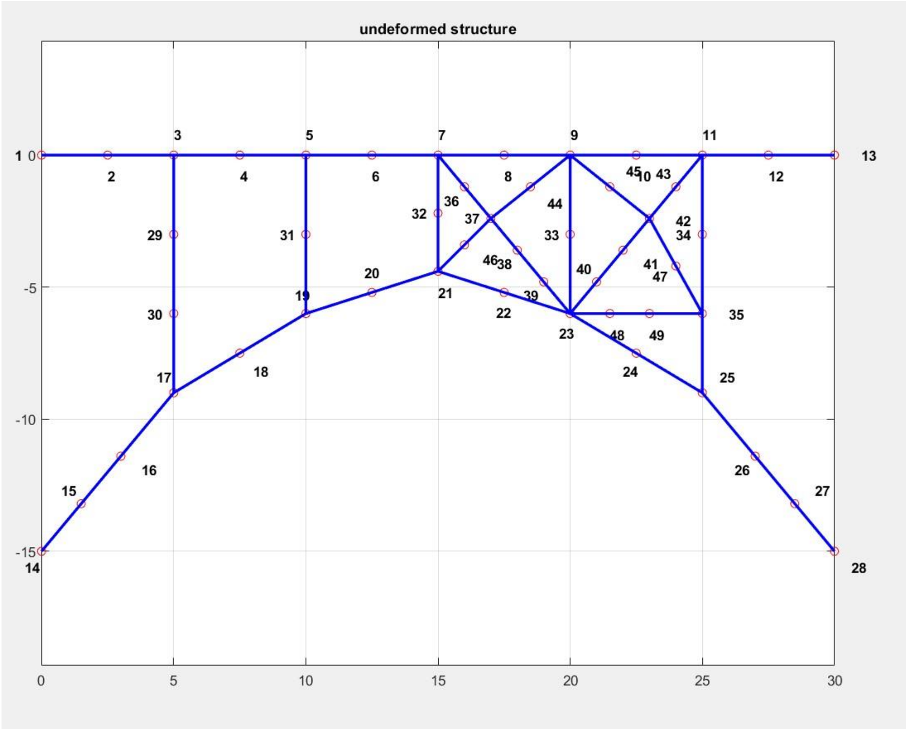
Number of string elements 0

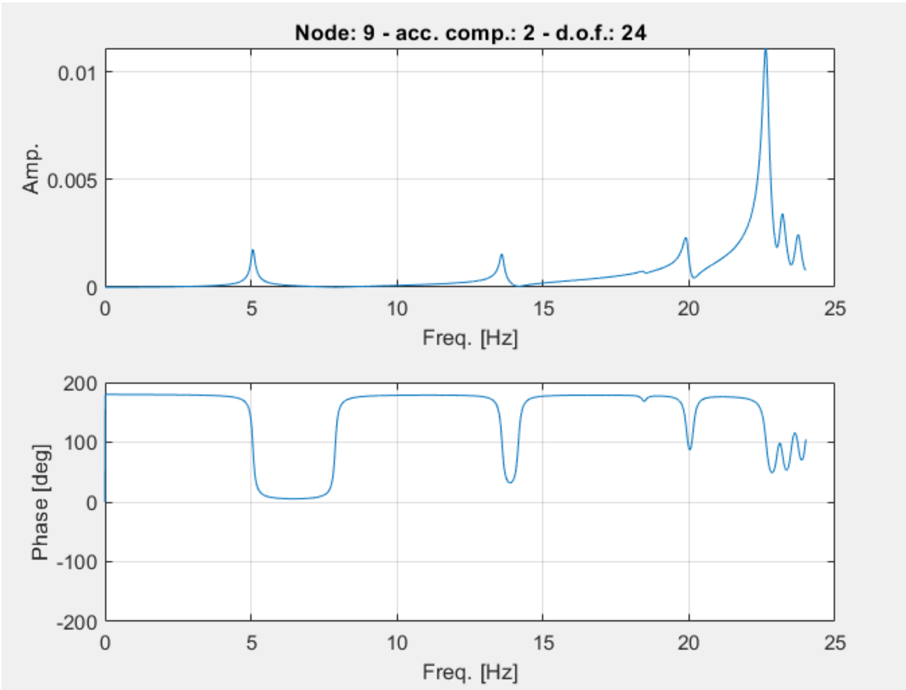
Number of tensile beam elements 0

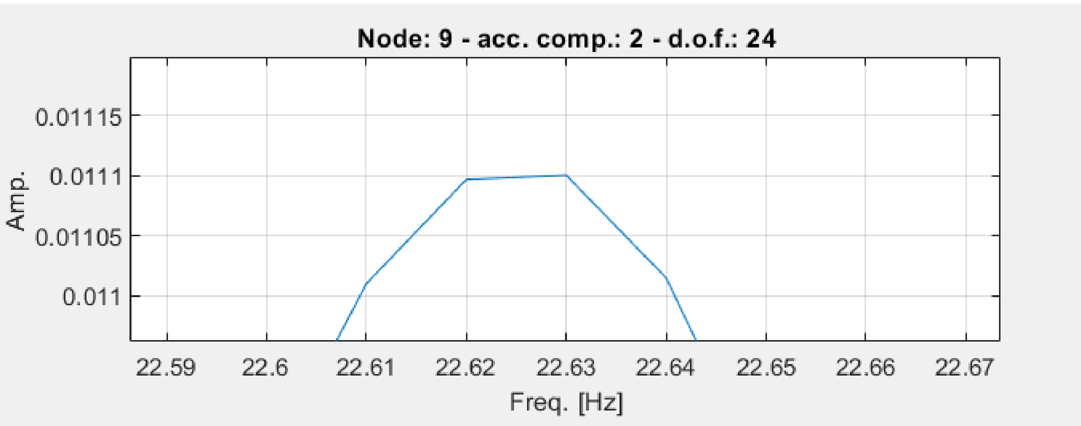
Number of concentrated masses 0

Number of concentrated springs 0

Total mass [kg] 5217.475


undeformed structure


Node: 9 - acc. comp.: 2 - d.o.f.: 24
Amp.
Freq. [Hz]
Phase [deg]
Freq. [Hz]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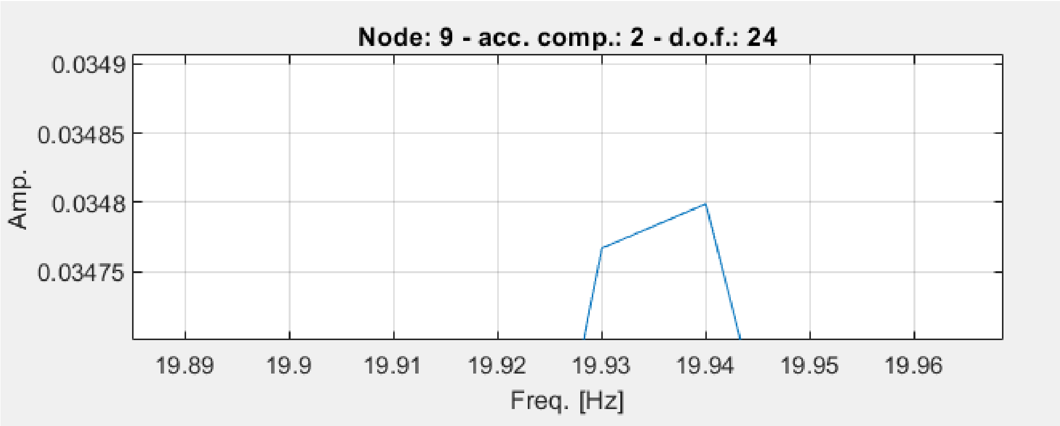
Amp.
Freq. [Hz]



The maximum amplitude of the FRF of the vertical acceleration of point B for a unit force applied at B is reduced by 68,1% !
The price per kg of IPE beams varies between €1.3 and €2.
I've increased the total mass of the system by about 212 kg with IPE80, so, considering the worst case, I'll spend 424€!

h mm	b mm	a mm	e mm	r mm	Peso kg/m	Sezione cm2	Momenti di inerzia Jx cm4	Momenti di inerzia Jy cm4	Moduli di resistenza Wx cm3	Moduli di resistenza Wy cm3	Raggi di inerzia ix cm	Raggi di inerzia iy cm
80	46	3,8	5,2	5	6,0	7,64	80,14	8,49	20,03	3,69	3,24	1,05