Image credit: www.spacex.com

Project One

USING OPTIMIZATION AND GRADIENT METHODS TO REPRESENT THE LANDING OF A ROCKET

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Formulation

The representation of a rocket landing can be modeled in a discrete timeframe.

$$d(t+1) = d(t) + v(t)\,\Delta t,$$

$$v(t+1) = v(T) + a(t)\Delta t,$$

The closed-loop controller aspect of the model specifies control input as:

$$a(t) = f_\theta\big(x(t)\big)$$

$f_\theta(\cdot)$ from above is a neural network with parameters θ, which are derived from optimization. Implementing loss based on cumulative and short-term incentives, while also defining proper landing conditions we are optimizing:

$$\min_\theta \big|\big|\, x(T)\, \big|\big|^2$$

$$d(t+1) = d(t) + v(t)\,\Delta t,$$

$$v(t+1) = v(t) + a(t)\Delta t,$$

$$a(t) = f_\theta\big(x(t)\big), \forall t = [0, T-1]$$

$$\phi(t+1) = \ \phi(t) + \omega(t)\Delta t \ \forall\ t = [0, T-1]$$

$$\omega(t+1) = \omega(t) + \alpha(t)\,\Delta t \ \forall\ t = [0, T-1]$$

This formulation is unconstrained with respect to θ.

Using the Pytorch toolkit, the problem can be further enhanced with realistic conditions i.e. A landing platform, more advanced dynamics, or optimization given multiple initial conditions (position only).

Programming

Please see the GitHub Repository (with a large credit to Prof. Ren, et al) for the explicit code. Here we will outline the implementation of the additions to our model:

- Advanced Dynamics
  - Orientation

Advanced Dynamics:

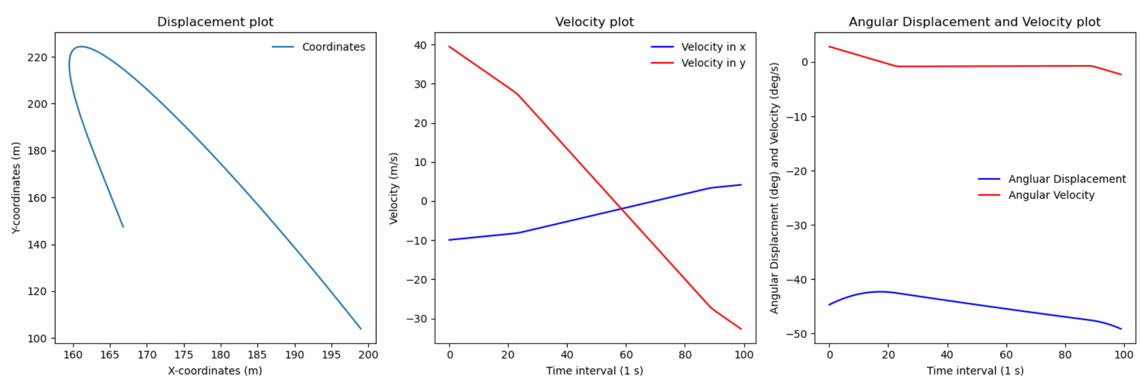
To accommodate orientation, a rotational velocity and angular position are applied to the state space. In addition, the network is supplied with an additional action; to rotate the vessel. For a physical representation, this is like using fins (like Falcon Heavy boosters returning to a landing platform) or angled side thrusters (more common in vessels made for out of atmosphere conditions). For the scope of the project, the addition to the action set will be more useful to think of as a thrusting system with a defined thrust constant (i.e., the thrust produced from the thruster at full power). This will be used to ensure the vessel is landing upright.

Result Analysis

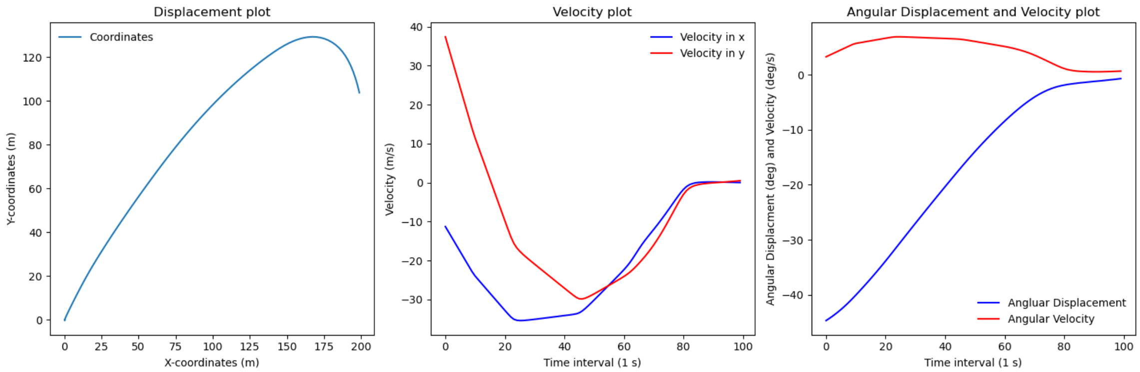
In conclusion to the evaluation of the dynamics and corresponding program, multiple initial states are input into the system by hand. The neural network optimizes the system to minimize the loss function (discussed above in Formulation). This means taking as little action as possible while also landing at its destination, (0,0) on the graph. Here we have an initial state with:

$$state_o = \left[x, y, \dot{x}, \dot{y}, \theta, \dot{\theta}\right] = [\,200\ m, 100\ m, -10\ \frac{m}{s}, 40\ \frac{m}{s}, -45°, 3\ \frac{deg}{s}\,]$$

This is a potentially likely condition for a craft. The vessel has a negative x component and a positive y component to the velocity, while rotating slowly but at an orientation of negative forty-five degrees with respect to the positive vertical axis. The vessel is also a large distance from the surface to show the large change in loss when the neural network optimizes itself. Here is the initial idea of how to land the craft from the network:



From the above figure, the first iteration is unsuccessful at landing the vessel with also a very large loss of twenty-six thousand units. As we approach the fortieth iteration, both linear and angular velocities approach zero and the trajectory curve approaches the origin:



Pictured above is the convergence of the network's solution. This solution is not unique to the system but rather to the initial conditions. As the initial state is a feasible and real solution, it can be concluded that the unconstrained optimization problem has been solved in sufficiently. In addition to the improved orientational forces (rotational velocity and angular orientation), more considerations involving drag, a set of acceptable final positions, or more involved dynamics like that of drag would be a welcome improvement to the design and will be added. Also note the thrust and gravitational coefficients were constructed to more easily assist in the convergence of the network, as having values very close to that of the gravitational force lead to large overshoots where, in a realistic setting, a violent impact would occur due to lack of time for the thrust to counteract gravity in a addition to the randomized initial conditions.