

\mathcal{U} -Bootstrap percolation

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Outline of the presentation

- 1 Introduction to \mathcal{U} -bootstrap and examples
- 2 Percolation and universality classes
- 3 Stable directions and critical densities
- 4 Behaviour around phase transition
- 5 Return to examples
- 6 Continuity and noise sensitivity
- 7 KCM
- 8 Conclusion and open questions

Definition and example of r -neighbour bootstrap

Definition for general \mathcal{U}
Example for 1 or 2 rules families

Definition and simulation of r -neighbour

Definition and simulation of OP

Definition and simulation of spiral

Definition and simulation of DTBP

Motivation for the model
Random initial set

Definition of $\theta_q(n)$ and θ_q

Phase transition

Classification of local homogeneous monotone cellular automatas

Review of previous examples

A geometric criterion to find the class of a model
Stable/unstable directions

Universality classes based on stable directions

Application to r -neighbours (and explicit threshold for $r = 2$)

Sketch of proof for universality classes

For supercritical and critical models, behaviour on \mathbb{Z}^d is trivial.
Need new tools for subcritical models
Definition of $\tilde{\theta}$ and \tilde{q}_c .

Definition of critical densities $u \mapsto d_u$

Theorem

Theorem 3.1

$$\tilde{q}_c = \sup_u d_u$$

(Sketch of ?) proof of theorem 3.1

Bounds on \tilde{q}_c

Use of OP to obtain bounds based on the trivial bound for DTBP

Interest of th 3.1 in some situations

Application for spiral model

Theorem

Theorem 3.5

$$\tilde{q}_c$$

is where exponential decay happens

Proof of th 3.5

Answers to questions 12 and 14 of Bolobas

Noise sensitivity and continuity

Theorem

Theorem 3.6

Definition of KCM

Theorem

Theorem 3.7

Proof of theorem 3.7

Methods introduced allow to

- 1 Recover and generalize previously known results
- 2 Study in depth and generality subcritical models
- 3 Characterize the phase transition using \tilde{q}_c
- 4 Obtain bounds on phase transition for subcritical models
- 5 Answer previously open questions

But they do not (yet) permit to answer the following questions

- 1 $q_c = \tilde{q}_c$?
- 2 continuity of $u \mapsto d_u$
- 3 dynamics on the torus