PARCIAL 5

EJERCICIO 1

$a_n = \left\{ \sqrt{3}, \sqrt{\frac{8}{5}}, 1, \sqrt{\frac{12}{17}}, \sqrt{\frac{7}{13}} \right\}$

$\sqrt{\frac{6}{2}}, \sqrt{\frac{8}{5}}, \sqrt{\frac{10}{10}}, \sqrt{\frac{12}{17}}, \sqrt{\frac{14}{26}}$

decreciendo... 0.5

$a_n = \sqrt{\frac{2x+4}{x^2+1}}$ ó $\sqrt{\frac{2(x+2)}{x^2+1}}$ $\quad$ $a_n = \sqrt{\frac{2(n+2)}{n^2+1}}$

$\sqrt{\lim_{n\to\infty} \frac{2(n+2)}{n^2+1}} = 0$ ∴ converge en 0.

0.5

EJERCICIO 2

$a_n = \frac{2n^3}{n-3n^3}$ $\quad$ $\lim_{n\to\infty} \frac{2n^3}{n-3n^3} = \frac{2\cancel{n^3}}{-3\cancel{n^3}} = -2/3$ $\quad$ 0.5

∴ $\{a_n\}$ converge

$b_n = \left(1 - \frac{5}{2n}\right)^{-3n}$ $\quad$ $\lim_{n\to\infty} \left(1-\frac{5}{2n}\right)^{-3n} = e^{+15/2}$

$= \left(1 + \frac{1}{-\frac{2}{5}n}\right)^{-3n} = \left(1 + \frac{1}{-\frac{2}{5}n}\right)^{-3n \cdot \frac{-\frac{2}{5}}{-\frac{2}{5}}}$ 0.5

$= \left[\left(1 + \frac{1}{-\frac{2}{5}n}\right)^{-\frac{2}{5}n}\right]^{+\frac{15}{2}} = e^{+\frac{15}{2}}$

∴ $\{b_n\}$ converge. 0.5

EJERCICIO 3

Sucesiones alternadas ó donde el grado del numerador > denominador, por ejemplo:

$c_n = \frac{n^2+1}{n}$ $\quad$ $\lim_{n\to\infty} \frac{n^2+1}{n} = \frac{n^2}{n} = n = \infty$

∴ $\{c_n\}$ diverge. 0.5

EJERCICIO 4

✓ No es monotona. 0.5

✓ Acotada inferiormente en cero... 0.5

EJERCICIO 5

1, 3, 6, 10, 15, ... 0.5 $\quad a_{21} = \frac{21(22)}{2}$

$a_n = \frac{n^2+n}{2}$ ó $a_n = \frac{n(n+1)}{2}$

$a_{21} = 231$ 0.5

FIGURA 21. tendrá 231 puntos

EJERCICIO 6

a) $\sum_{n=1}^{\infty} \frac{2}{n^2(n+2)}$ $\quad$ Sucesión de sumas parciales... 0.3

$S_3 = \frac{2}{3} + \frac{1}{8} + \frac{2}{45} = \frac{301}{360}$ 0.2

b) $\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)\left(\frac{1}{3}\right)^n$ 0.3 Serie geometrica.

con. $a = \frac{3}{2}$

$r = \frac{1}{3}$ 0.2

$S_3 = \frac{3}{2} + \frac{1}{2} + \frac{1}{6} = \frac{13}{6}$

EJERCICIO 7.

a) 0.213213213...

$\frac{213}{1000} + \frac{213}{1000000} + \frac{213}{1000000000} + \dots$

$a = \frac{213}{1000}$ $\quad$ $r = \frac{1}{1000}$ $\quad$ $S = \frac{\frac{213}{1000}}{1-\frac{1}{1000}}$

$S = \frac{71}{333}$ 0.5

b) 1.8353535...

$\frac{18}{10} + \frac{35}{1000} + \frac{35}{100000} + \frac{35}{10000000} + \dots$

$a = \frac{35}{1000}$ $\quad$ $r = \frac{1}{100}$ $\quad$ $S = \frac{\frac{35}{1000}}{1-\frac{1}{100}}$

$S = \frac{7}{198}$

$\frac{18}{10} + \frac{7}{198} = \frac{1817}{990}$ 0.5

EJERCICIO 8. (comparación en el límite)

1) $\sum_{n=2}^{\infty} \underbrace{\frac{2}{\sqrt{n^3-n}}}_{a_n}$ $\quad$ $b_n = \frac{2}{n^{3/2}}$ $\quad$ Serie "p" con p = 3/2 converge. 0.3

$\lim_{n\to\infty} \frac{\cancel{2}}{\sqrt{n^3-n}} \cdot \frac{\sqrt{n^3}}{\cancel{2}} = \sqrt{\frac{n^3}{n^3-n}} = \sqrt{1} = 1$ 0.3

0.4 ∴ La serie $\sum \frac{2}{\sqrt{n^3-n}}$ converge

Escaneado con CamScanner

ii) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{1+n^3}$ (Criterio de la serie alternada)

$$\lim_{n\to\infty} \frac{3}{1+n^3} = \frac{3}{n^3} = \frac{3}{\infty} = 0 \checkmark$$ 0.3

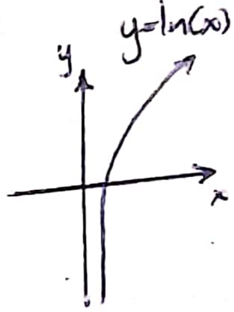
$$\frac{3}{1+(n+1)^3} \leq \frac{3}{1+n^3}$$ Cumple con las condiciones... 0.3

$n=1$ $\quad \frac{1}{3} < \frac{3}{2} \checkmark$

$\therefore \sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{1+n^3}$ converge. 0.4

iii) $\sum_{n=1}^{\infty} \frac{(n+1)!}{\ln(n+1)} 3^n$ (Criterio del cociente o la razón)

$$\lim_{n\to\infty} \frac{(n+2)!\, 3^{n+1}}{\ln(n+2)} \cdot \frac{\ln(n+1)}{(n+1)!\, 3^n}$$



$$\lim_{n\to\infty} \frac{(n+2)\cancel{(n+1)!}\,\cancel{3^n}\cdot 3 \ln(n+1)}{\ln(n+2)\,\cancel{(n+1)!}\,\cancel{3^n}}$$ 0.3

$$3 \lim_{n\to\infty} (n+2) \cdot \frac{\ln(n+1)}{\ln(n+2)} = \frac{\infty(\infty)}{\infty} = \frac{\infty}{\infty} \text{ (L'H)}$$ 0.3

$$3 \lim_{n\to\infty} \frac{\ln(\cancel{n+1})^{\infty} + \cancel{\frac{n+2}{n+1}}^{1}}{\cancel{\frac{1}{n+2}}^{0}} = \frac{\infty+1}{0} = \frac{\infty}{0} = \infty$$

$\therefore \sum_{n=1}^{\infty} \frac{(n+1)!}{\ln(n+1)} 3^n$ diverge. 0.4