SERIES DE POTENCIA

- Series de constantes: $\sum_{n=1}^{\infty} \frac{n^2+1}{n} = 2 + \frac{5}{2} + \frac{10}{3} + \dots$

- Series de potencia: $\sum_{n=1}^{\infty} \frac{\cos(nx)}{n} = \cos(x) + \frac{\cos(2x)}{2} + \frac{\cos(3x)}{3} + \dots$
  (variables)

- Serie de potencia en "x"

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \dots + a_n (x-0)^n$$

Centro: 0

- Serie de potencia en "x-c"

$$\sum_{n=0}^{\infty} b_n (x-c)^n = b_0 + b_1 (x-c) + b_2 (x-c)^2 + \dots + b_n (x-c)^n$$

Centro: C

Ejemplos:

i) $\sum_{n=0}^{\infty} \frac{\ln(n)\, x^n}{n!}$ S.P. en "x" $C=0$

ii) $\sum_{n=0}^{\infty} \frac{(x+1)^n}{n^3}$ $(x-\underbrace{(-1)}))^n$ S.P. en "x-c" $C=-1$

iii) $\sum_{n=0}^{\infty} \frac{(x-e)^n}{e^n . n}$ S.P. en "x-c" $C=e$

INTERVALOS DE CONVERGENCIA PARA SERIE DE POTENCIA EN X

$$Sea \lim_{n\to\infty} \left| \frac{a_{n+1}\, x^{n+1}}{a_n\, x^n} \right| = L$$

CENTRO

i. **Si $L = \infty$ entonces la serie converge en un solo punto** $x = 0$

ii. **Si $L = 0$ entonces la serie converge $\forall\, x$ o bien el intervalo de convergencia =** $]-\infty, \infty[$ **o** R

iii. **Si $L \neq 0$ entonces buscaremos que intervalo de "x" genera que:** si $L < 1$ entonces la serie es absolutamente convergente.

INTERVALOS DE CONVERGENCIA PARA SERIE DE POTENCIA EN (X-C)

$$Sea \lim_{n\to\infty} \left| \frac{b_{n+1}\, (x-c)^{n+1}}{b_n\, (x-c)^n} \right| = L$$

CENTRO

i. **Si $L = \infty$ entonces la serie converge en un solo punto** $x = c.$

ii. **Si $L = 0$ entonces la serie converge $\forall\, x$ o bien el intervalo de convergencia =** $]-\infty, \infty[$ **o** R

iii. **Si $L \neq 0$ entonces buscaremos que intervalo de "x" genera que:** si $L < 1$ entonces la serie es absolutamente convergente.

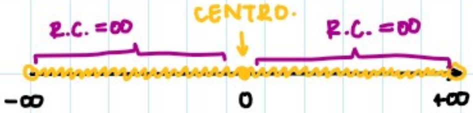
EJEMPLOS: Para las siguientes series determine el $\underbrace{\text{intervalo}}$ y radio de convergencia.

a) $\underbrace{\sum_{n=0}^{\infty} \frac{x^n}{n!}}$ $\Big\}\, a_n$ $\qquad \frac{x^{n+1}}{(n+1)!} \Big\}\, a_{n+1}$

S.P. en "x" $C = 0.$

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right| = \left| \frac{\cancel{x^n}.\, x}{(n+1).\cancel{n!}} \cdot \frac{\cancel{n!}}{\cancel{x^n}} \right| = \left| \frac{x}{n+1} \right| = \underbrace{\frac{1}{n+1}}_{0} |x| = 0|x| = 0$$

$$\lim_{n\to\infty} \frac{1}{n+1} = \frac{1}{n} = \frac{1}{\infty} = 0.$$



I.C. $=]-\infty, +\infty[$ ó $\mathbb{R}$

R.C. $= \infty$

b) $\underbrace{\sum_{n=0}^{\infty} \frac{(x-2)^n}{(n+1)\, 3^n}}$ $\Big\}\, b_n$ $\qquad \frac{(x-2)^{n+1}}{(n+2)\, 3^{n+1}} \Big\}\, b_{n+1}$

$(n+1)$
$\downarrow$
$(\underbrace{n+1+1})$
$(n+2)$

S.P. en "x-c" $C = 2$

$$\lim_{n\to\infty}\left|\frac{(x-2)^{n+1}}{(n+2)3^{n+1}}\cdot\frac{(n+1)3^n}{(x-2)^n}\right| = \left|\frac{\cancel{(x-2)^n}(x-2)}{(n+2)\cancel{3^n}\cdot 3}\cdot\frac{(n+1)\cancel{3^n}}{\cancel{(x-2)^n}}\right| = \left|\frac{(x-2)(n+1)}{(n+2)(3)}\right| = \underbrace{\frac{(n+1)}{3(n+2)}}_{\frac{1}{3}}|x-2|$$

$$\lim_{n\to\infty}\frac{n+1}{3(n+2)} = \frac{n+1}{3n+6} = \frac{\cancel{n}}{3\cancel{n}} = \frac{1}{3}$$

$$L = \frac{1}{3}|x-2| < 1$$

$$\frac{1}{3}|x-2| < 1$$

$$|x-2| < 3 \quad \leftarrow \text{R.C.}$$

P1: Despejar el valor abs.

$$-3 < x - \cancel{2} < 3$$
$$+2 \qquad \cancel{+2} \qquad +2$$

P2: Desarrollar el valor abs.

P3: Despejar la "x"

$$-1 < x < 5 \leftarrow \text{I.C.: } [-1,5[$$

- $x = -1$

$$\text{b) } \sum_{n=0}^{\infty}\frac{(x-2)^n}{(n+1)3^n} \to \sum_{n=0}^{\infty}\frac{(-1-2)^n}{(n+1)3^n} = \frac{(-3)^n}{(n+1)3^n} = \frac{(-1)^n\cancel{(3)^n}}{(n+1)\cancel{3^n}} = \frac{(-1)^n}{(n+1)}$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}$$ sí converge, -1 ✓
sí diverge, -1 X

✓ suma de fracciones parciales
✓ S.G.
✓ S.P.
✓ S. Alternada.
✓ Pruebas o criterios de conv.

P1. ✓
P2. ✓
P3. ✓
P4. ✓
P5. ✓

$$n=0 \to \frac{(-1)^0}{0+1} = 1$$

$$n=1 \to \frac{(-1)^1}{1+1} = -\frac{1}{2}$$

$$n=2 \to \frac{(-1)^2}{2+1} = +\frac{1}{3}$$

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\dots$$

Serie armónica alternada

[CONVERGE]

- $x = 5$

$$\text{b) } \sum_{n=0}^{\infty}\frac{(x-2)^n}{(n+1)3^n} \to \sum_{n=0}^{\infty}\frac{(5-2)^n}{(n+1)3^n} = \frac{\cancel{3^n}}{(n+1)\cancel{3^n}} = \frac{1}{n+1}$$

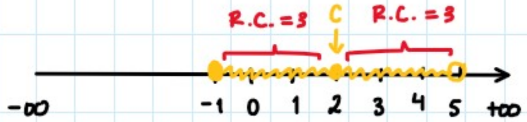
$$\underbrace{\sum_{n=0}^{\infty}\frac{1}{n+1}}$$ Sí converge, 5 ✓
Sí Diverge, 5 X

✓ Serie armónica → diverge.
✓ Criterio comp. límite → diverge.

✓ Serie armónica → diverge.
✓ Criterio comp. límite → diverge.
✓ Criterio integral → diverge.

R.C. = 3
I.C. = [-1 , 5[

[DIVERGE]



Representación de una función mediante series de potencia.

$$f(x) = \frac{1}{1-x} = \boxed{?} + \boxed{?} + \boxed{?} + \boxed{?} + \boxed{?} + \boxed{?}$$

FUNCIÓN BASE: $\frac{1}{1-x} = \underbrace{\sum_{n=0}^{\infty} x^n}_{\text{serie potencia}} = 1 + x + x^2 + x^3 + x^4 + \ldots$

Operaciones de las funciones base:

BASE: $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \ldots$

$x \to x^2$

$$\frac{1}{1-x^2} = 1 + (x^2) + (x^2)^2 + (x^2)^3 + (x^2)^4 + \ldots$$

$$\frac{1}{1-x^2} = 1 + x^2 + x^4 + x^6 + x^8 + \ldots$$

$x \to -x$

$$\frac{2}{1+x} = ?$$

$$\frac{1}{1-(-x)} = 1 + (-x) + (-x)^2 + (-x)^3 + (-x)^4 + \ldots$$

$$2\left(\frac{1}{1+x}\right) = \left(1 - x + x^2 - x^3 + x^4 - \ldots\right)2$$

$$\frac{2}{1+x} = 2 - 2x + 2x^2 - 2x^3 + 2x^4 - \ldots$$

EJEMPLO: Realizar las siguientes representaciones.

a) $\frac{d^2}{dx^2}\left(\frac{1/2}{1-x^2}\right)$

BASE: $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \ldots$

$x \to x^2$: x^2, $(x^2)^2$, $(x^2)^3$, $(x^2)^4$

P1. $x \to x^2$

$$\frac{1}{1-x^2} = 1 + x^2 + x^4 + x^6 + x^8 + \ldots$$

$$\frac{1}{1-x^2} = 1 + x^2 + x^4 + x^6 + x^8 + \dots$$

P2. Multiplicar $\frac{1}{2}$

$$\frac{(1/2)}{1-x^2} = \frac{1}{2} + \frac{1}{2}x^2 + \frac{1}{2}x^4 + \frac{1}{2}x^6 + \frac{1}{2}x^8 + \dots$$

P3. Derivando...

$$\frac{d}{dx}\left(\frac{1/2}{1-x^2}\right) = 0 + x + 2x^3 + 3x^5 + 4x^7 + \dots$$

P4. Derivando ...

$$\frac{d^2}{dx^2}\left(\frac{1/2}{1-x^2}\right) = 1 + 6x^2 + 15x^4 + 28x^6 + \dots$$

b) $\int_0^1 e^{-x^2}.dx$ tomando como base: $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

P1. $x \to -x^2$

$$e^{-x^2} = 1 + (-x^2) + \frac{(-x^2)^2}{2!} + \frac{(-x^2)^3}{3!} + \dots$$

$$e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \dots$$

P2. Integrar.

$$\int_0^1 e^{-x^2}.dx = \int_0^1 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \dots dx$$

$$\int_0^1 e^{-x^2}.dx = x - \frac{x^3}{3} + \frac{x^5}{5.2!} - \frac{x^7}{\underbrace{7.3!}_{3(2)(1)}} + \dots \Big|_0^1 = 1 - \frac{1}{3} + \frac{1}{10} - \frac{1}{42} - 0 = \frac{26}{35} \approx 0.742$$

SERIES DE TAYLOR Y MACLAURIN

S. Taylor: $f(x) = \sum_{n=0}^{\infty} \frac{f^n(c)}{n!} \underbrace{(x-c)^n}_{\text{S.P. en "x-c"}}$ derivadas.

$\downarrow$

$c = 1/2, 1, -3, \dots$

S. Maclaurin: $f(x) = \sum_{n=0}^{\infty} \frac{f^n(0)}{n!} \underbrace{x^n}_{\text{S.P. en "x"}}$ derivadas.

$\downarrow$

$c = 0.$

EJEMPLOS: Encontrar la serie de Taylor de: $f(x) = \frac{1}{x}$ centrada en $c = -1$. Utilizando 5

EJEMPLOS: Encontrar la serie de Taylor de: $f(x)=\frac{1}{x}$ centrada en $c=-1$. Utilizando 5 términos

P1. Evaluar funciones y derivadas.

func original.

$0! = 1$

$f(x)=\frac{1}{x}$ ó x^{-1} $\longrightarrow$ $f(-1)=\frac{1}{-1}=-1$ ✓ ← n=0

$f'(x)=-1\cdot x^{-2}=\frac{-1}{x^2}$ $\longrightarrow$ $f'(-1)=\frac{-1}{(-1)^2}=-1$ ✓ ← n=1

$f''(x)=-1(-2)x^{-3}=\frac{2}{x^3}$ $\longrightarrow$ $f''(-1)=\frac{2}{(-1)^3}=-2$ ✓ ← n=2

$f'''(x)=2(-3)x^{-4}=\frac{-6}{x^4}$ $\longrightarrow$ $f'''(-1)=\frac{-6}{(-1)^4}=-6$ ✓ ← n=3

$f^{IV}(x)=-6(-4)x^{-5}=\frac{24}{x^5}$ $\longrightarrow$ $f^{IV}(-1)=\frac{24}{(-1)^5}=-24$ ✓ ← n=4

P2. Formas S.T.

$$\sum_{n=0}^{\infty}\frac{f^{n}(c)}{n!}(x-c)^n$$

$\frac{f^{0}(-1)}{0!}(x-(-1))^0$ ← n=0

$\frac{f^{1}(-1)}{1!}(x-(-1))^1$ ← n=1

$\frac{f^{2}(-1)}{2!}(x-(-1))^2$ ← n=2

$$\frac{1}{x}=\frac{-1}{\cancel{0!}}\cancel{(x-(-1))^0}+\frac{-1}{\cancel{1!}}(x-(-1))^1+\frac{-2}{\cancel{2!}}(x-(-1))^2+\frac{-6}{\cancel{3!}}(x-(-1))^3+\frac{-24}{\cancel{4!}}(x-(-1))^4$$

1, 1, 1, 2, 6, 24

$(x-(-1))=(x+1)$

$$\frac{1}{x}=-1-(x+1)-(x+1)^2-(x+1)^3-(x+1)^4-\dots$$

EJEMPLO: Encuentre la serie de Maclaurin para: $f(x)=\frac{1}{2}(1+\cos(2x))$ Utilizando 3 términos.

asumir: $c=0$

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{n}(0)}{n!}x^n$$

$f(x)=\frac{1}{2}(1+\cos(2x))$ $\longrightarrow$ $f(0)=\frac{1}{2}(1+\cancel{\cos 0}^{1})=1$ ✓ ← n=0

$f'(x)=\frac{1}{\cancel{2}}(-\cancel{2}\sin(2x))$ $\longrightarrow$ $f'(0)=-\cancel{\sin(0)}^{0}=0$ ✗ ← n=1 ✗

$=-\sin(2x)$

$f''(x)=-2\cos(2x)$ $\longrightarrow$ $f''(0)=-2\cancel{\cos 0}^{1}=-2$ ✓ ← n=2

$f'''(x)=+4\sin(2x)$ $\longrightarrow$ $f'''(0)=4\cancel{\sin(0)}^{0}=0$ ✗ ← n=3 ✗

$f^{IV}(x)=8\cos(2x)$ $\longrightarrow$ $f^{IV}(0)=8\cancel{\cos(0)}^{1}=8$ ✓ ← n=4

$$f(x)=\frac{1}{2}(1+\cos(2x))=\frac{1}{\cancel{0!}}\cancel{x^0}+\frac{-\cancel{2}}{\cancel{2!}}x^2+\frac{\cancel{8}^{1}}{\cancel{4!}_{\cancel{24}\,3}}x^4+\dots$$

$$\frac{1}{2}(1+\cos(2x))=1-x^2+\frac{1}{3}x^4+\dots$$

SERIES DE FOURIER. → funcionan con funciones periodicas.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$$

¿Cuantos valen los coeficientes de Fourier?

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx \qquad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\cos(nx)\,dx \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\sin(nx)\,dx$$

$-\pi \leftrightarrow \pi$

EJEMPLO: Encuentre la serie de Fourier para la función:

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \pi - x, & 0 \le x < \pi \end{cases}$$

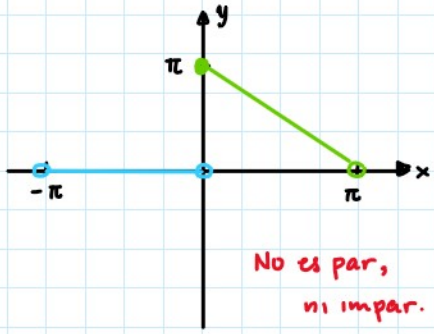
$x = 0 \quad \pi - 0 \to y = \pi$

$x = \pi \quad \pi - \pi \to y = 0$

Desarrolle para $n = 1, 2$ y 3.

$(0, \pi) \quad (\pi, 0)$

PASO 1. Graficar



No es par, ni impar.

PASO 2. Encontrar los coeficientes de Fourier.

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\left[\cancel{\int_{-\pi}^{0} 0\,dx}^{\,0} + \int_{0}^{\pi} \pi - x\,dx\right]$$

$$\frac{1}{2\pi}\int_0^{\pi} \pi - x\,dx = \frac{1}{2\pi}\left[\pi x - \frac{x^2}{2}\right]\Big|_0^{\pi} = \frac{1}{2\pi}\left[\pi(\pi) - \frac{\pi^2}{2} - 0 + 0\right] = \frac{1}{2\cancel{\pi}}\left(\frac{\pi^{\cancel{2}}}{2}\right) = \frac{\pi}{4}$$

π^2

$$\boxed{a_0 = \frac{\pi}{4}}$$

$$\boxed{a_0 = \frac{\pi}{4}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\cos(nx)\,dx = \frac{1}{\pi}\left[\cancel{\int_{-\pi}^{0} 0\cdot\cos(nx)\,dx}^{\,0} + \int_0^{\pi} (\pi - x)\cos(nx)\,dx\right]$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} (\pi - x)\cos(nx)\,dx\right] = \frac{1}{\pi}\int_0^{\pi} \underbrace{\pi\cos(nx)}_{✓\ ①} - \underbrace{x\cos(nx)}_{⊛}\,dx$$

derivar	integrar
x	$\cos(nx)$
1	$\mathrm{Sen}(nx)/n$
0	$-\cos(nx)/n^2$

(+, −)

$$⊛ = \underbrace{\frac{x\cdot\mathrm{Sen}(nx)}{n} + \frac{\cos(nx)}{n^2}}$$

$$= \frac{1}{\pi}\left[\frac{\pi\,\mathrm{Sen}(nx)}{n} - \frac{x\,\mathrm{Sen}(nx)}{n} - \frac{\cos(nx)}{n^2}\right]\Bigg|_0^{\pi}$$

$$= \frac{1}{\pi}\left[\cancel{\frac{\pi\sin(n\pi)}{n}}^{\,0} - \cancel{\frac{\pi\sin(n\pi)}{n}}^{\,0} - \cancel{\frac{\cos(n\pi)}{n^2}}^{\,(-1)^n}\right.$$

$$\left. - \cancel{\frac{\pi\sin(0)}{n}}^{\,0} + \cancel{\frac{0\cdot\sin(0)}{n}}^{\,0} + \cancel{\frac{\cos(0)}{n^2}}^{\,1}\right]$$

$$= \frac{1}{\pi}\left[-\frac{(-1)^n}{n^2} + \frac{1}{n^2}\right] = \frac{1}{\pi n^2}\left[-(-1)^n + 1\right]$$

$\mathrm{Sen}(0) = 0$ ✓

$\mathrm{Sen}(nx) = ?$

$n=1$	$\sin(x)$	$\sin(\pi) = 0$
$n=2$	$\sin(2x)$	$\sin(2\pi) = 0$
$n=3$	$\sin(3x)$	$\sin(3\pi) = 0$

$\mathrm{Sen}(n\pi) = 0$ ✓

$$a_n = \frac{1}{\ } \left[1 - (-1)^n\right]$$

$$\boxed{a_n = \frac{1}{\pi n^2}\left[1-(-1)^n\right]}$$

$n=3 \quad sen(3x) \quad sen(3\pi)=0$

$\cos(nx) = ?$ $\quad \cos(n\pi) = (-1)^n$ ✓

$n=1 \quad \cos(x) \quad \cos(\pi) = -1$

$n=2 \quad \cos(2x) \quad \cos(2\pi) = 1$

$n=3 \quad \cos(3x) \quad \cos(3\pi) = -1$

$sen\left(\frac{n\pi}{2}\right)$ ó $\cos(3n\pi)$

? ?

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cdot sen(nx)\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} \cancel{0 \cdot sen(nx)\,dx}^{\,0} + \int_{0}^{\pi}(\pi - x)\,sen(nx)\,dx\right]$$

$$b_n = \frac{1}{\pi}\int_{0}^{\pi} \underbrace{\pi\, sen(nx)}_{1} - \underbrace{x \cdot sen(nx)}_{*} \cdot dx \qquad (\ldots) \qquad \boxed{b_n = \frac{1}{n}}$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n\, sen(nx)$$

$$\boxed{a_0 = \frac{\pi}{4}} \qquad \boxed{a_n = \frac{1}{\pi n^2}\left[1-(-1)^n\right]}$$

$$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty} \underbrace{\frac{[1-(-1)^n]}{\pi n^2}\cos(nx) + \frac{1}{n}\, sen(nx)}_{n=1,\,2\ y\ 3.} \qquad \boxed{b_n = \frac{1}{n}}$$

$$f(x) = \frac{\pi}{4} + \frac{[1-\overset{2}{(-1)^1}]}{\pi \cdot 1^2}\cos(1 \cdot x) + \frac{\cancel{1}}{\cancel{1}}\, sen(1x) + \cancel{\frac{[1-\overset{+1}{(-1)^2}]}{\pi\, 2^2}}^{\,0}\cos(2x) + \frac{1}{2}\, sen(2x) + \frac{[1-\overset{-1}{(-1)^3}]}{\pi\, 3^2}\cos(3x) + \frac{1}{3}\, sen(3x)$$

$$f(x) = \frac{\pi}{4} + \frac{2}{\pi}\cos(x) + sen(x) + \frac{1}{2}\, sen(2x) + \frac{2}{9\pi}\cos(3x) + \frac{1}{3}\, sen(3x) + \ldots$$

1. Dadas las siguientes series de potencia, determine su intervalo y radio de convergencia.

a) $\sum_{n=0}^{\infty} \frac{2n\ x^{n+1}}{(2n+1)!}$ b) $\sum_{n=0}^{\infty} \frac{(-3)^n (x-1)^n}{\sqrt{n+1}}$

2. Aplique series de potencias, derivación, integración o alguna operación pertinente para calcular el valor de la siguiente integral definida:

$$\frac{d}{dx}\left(-\frac{sen(x^2)}{2x}\right)$$

Tomando como base: $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$

3. Encuentre los primeros cuatro términos de la Serie de Maclaurin para la función: $f(x) = (x^2+4)^3$
4. Sea g(x) una función periódica de periodo 2π definida por: $g(x) = 3 - 2x$

   Grafique g(x) y luego desarrolle en términos de una serie de Fourier evaluando desde n=1 hasta n=3.

¿Serie base: $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$?

S.P. en "x" $\leftrightarrow$ S. Geometrica. $\longrightarrow$ $S = \frac{a}{1-r}$ $a \to$ 1er termino. $1-r \to$ razón incre/decre.

$$\sum_{n=0}^{\infty} a_n x^n \equiv \sum_{n=0}^{\infty} a r^n$$

$$= a_0 + a_1 x + a_2 x^2 + a_3 x^3 - \ldots$$

$$a + ar + ar^2 + ar^3 + \ldots$$

$$\frac{a}{1-r} = \frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$$

"a=1" "r=x"

$a_n \to a$

$x \to r$

Series de Taylor.

¿ $f(x) = \sum_{n=0}^{\infty} \frac{f^n(c)}{n!}(x-c)^n$?

S.P. en "x-c" $\to \sum_{n=0}^{\infty} b_n (x-c)^n$

$$f(x) = b_0 + b_1 \underbrace{(x-c)}_{(c-c)} + b_2 \underbrace{(x-c)^2}_{(c-c)} + b_3 \underbrace{(x-c)^3}_{(c-c)} + \ldots$$

$$f'(x) = b_1 + 2b_2 \underbrace{(x-c)}_{(c-c)} + 3b_3 \underbrace{(x-c)^2}_{(c-c)} + \ldots$$

$$f''(x) = 2(1)b_2 + 3(2)b_3 \underbrace{(x-c)}_{(c-c)} + \ldots$$

$$f'''(x) = 3(2)(1)b_3 + \ldots$$

Evaluar $x=c$

$f(c) = 1b_0 \to f(c) = 0!\, b_0 \to b_0 = \frac{f(c)}{0!}$

$f'(c) = 1b_1 \to f'(c) = 1!\, b_1 \to b_1 = \frac{f'(c)}{1!}$

$f''(c) = 2(1)b_2 \to f''(c) = 2!\, b_2 \to b_2 = \frac{f''(c)}{2!}$

$f'''(c) = 3(2)(1)b_3 \to f'''(c) = 3!\, b_3 \to b_3 = \frac{f'''(c)}{3!}$

$$f(x) = b_0 + b_1(x-c) + b_2(x-c)^2 + b_3(x-c)^3 + \ldots$$

$$f(x) = \frac{f(c)}{0!} + \frac{f'(c)}{1!}(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \ldots$$

Taylor. $f(x) = \sum_{n=0}^{\infty} \frac{f^n(c)}{n!}(x-c)^n$

c=0 Maclaurin. $\longrightarrow f(x) = \sum_{n=0}^{\infty} \frac{f^n(0)}{n!} x^n$

Taylor.

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(c)}{n!}(x-c)^n$$

$C=0$ Madaume_. $\longrightarrow$ $f(x) = \sum_{n=0} \frac{f(0)}{n!} x^n$

$0! = 1$

$3! = 3 \cdot 2 \cdot 1$ (-1, -1)

$3! = 3 \cdot 2!$

$1! = 1$

$1! = 1 \cdot 0!$ (-1)

$\frac{\cancel{1!}\,1}{\cancel{1}\,1} = 0!$

$\boxed{1 = 0!}$

$\frac{2}{0!} = \infty$

$\frac{2}{0} = \infty$ $\quad \frac{2}{0!} = \frac{2}{1} = 2$

SERIES DE FOURIER PARA FUNCIONES PARES E IMPARES.

1. Función que no es par ni impar.

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\mathrm{sen}(nx)dx$$

2. Función par.

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx \longrightarrow a_0 = \frac{1}{\pi}\int_0^{\pi} f(x).dx \checkmark$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx \longrightarrow a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx.dx \checkmark$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\mathrm{sen}(nx)dx \longrightarrow b_n = 0 \times$$

3. Función impar.

$$\cancel{a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx} \text{ impar} \checkmark$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx \checkmark \longrightarrow a_0 = 0 \times \ y \ a_n = 0 \times$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\mathrm{sen}(nx)dx \qquad b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\,\mathrm{Sen}(nx).dx \checkmark$$

Finalmente la serie trigonométrica:

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n\cos(nx) + b_n\mathrm{sen}(nx))$$

Donde los coeficientes se determinan mediante las formulas de Euler se llaman: SERIES DE FOURIER, y los coeficientes adoptan el nombre de: COEFICIENTES DE FOURIER

$f(x) = x^2 \longrightarrow$ par.

$f(-x) = (-x)^2 = +x^2$ ✓

f. par.

$y=x^2$

$f(x) = x^1 \longrightarrow$ impar.

$f(-x) = -x$ $\quad$ f. impar

$y=x$