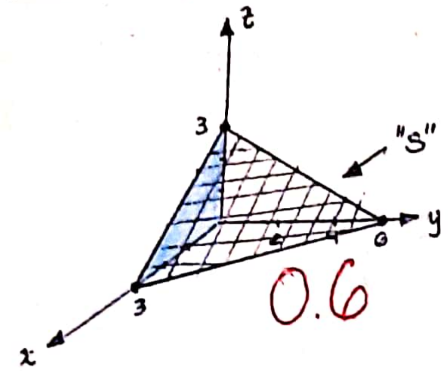
PARCIAL 4.

EJERCICIO #1

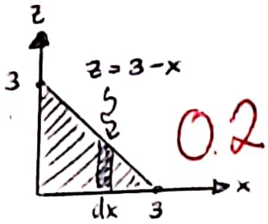
$S: 2x+y+2z=6$

P. Corte. $x=3$, $y=6$, $z=3$



0.6

proyección "xz"



0.2

$y = 6-2z-2x$

$$\iint_R [(x)(6-2x-2z)z - 2](3)\,dA$$

$$= 3\iint_R 6xz - 2x^2z - 2xz^2 - 2 \,.\, dA$$

0.4

$f_x=-2 \quad f_x^2=4$

$f_y=-2 \quad f_y^2=4$

$ds=\sqrt{1+4+4}\,dA$

$ds = 3dA$ 0.2

$$= 3\int_0^3\int_0^{3-x} 6xz-2x^2z-2xz^2-2\,.\,dz\,dx$$

$$= 3\int_0^3 \underbrace{3xz^2 - x^2z^2}_{xz^2(3-x)} - \frac{2xz^3}{3} - 2z \Big|_0^{3-x} dx$$

$$x z^2(3-x) - \frac{2}{3}xz^3 - 2z$$

$$= 3\int_0^3 x(3-x)^2(3-x) - \frac{2}{3}x(3-x)^3 - 2(3-x)\,dx$$

$$= 3\int_0^3 \frac{1}{3}x(3-x)^3 - 2(3-x)\,.\,dx$$

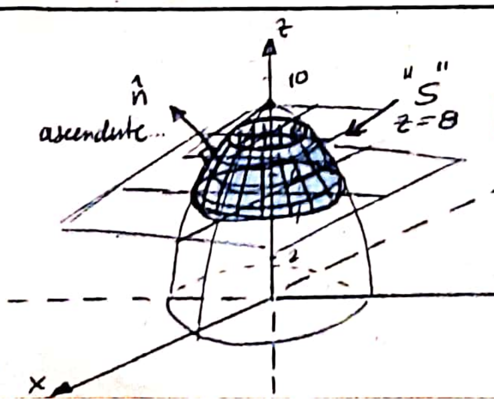
$$= 3\Big[\int_0^3 9x - 9x^2 + 3x^3 - \frac{1}{3}x^4 - 6 + 2x\,.\,dx$$

$$= \int_0^3 -x^4 + 9x^3 - 27x^2 + 33x - 18\,.\,dx$$

$$= -\frac{x^5}{5} + \frac{9x^4}{4} - 9x^3 + \frac{33}{2}x^2 - 18x \Big|_0^3$$

$$= -\frac{3^5}{5} + \frac{9(81)}{4} - 9(27) + \frac{33(9)}{2} - 18(3)\,.$$

$$= -\frac{297}{20}$$ 0.6



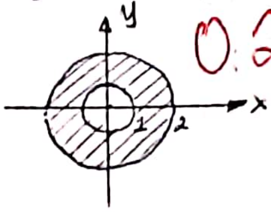
0.6

EJERCICIO 2: $z = 9 - x^2 - y^2$

$z=5 \to 5 = 9-x^2-y^2 \Rightarrow x^2+y^2=4 \quad C(0,0)\ r=2$

$z=8 \to 8 = 9-x^2-y^2 \Rightarrow x^2+y^2=1 \quad C(0,0)\ r=1$

proyección "xy"



0.2

$\vec{F} = 2x\hat{i} + 2y\hat{j} + z\hat{k}$

$z = 9-x^2-y^2$

$f_x = -2x \to -f_x = 2x$

$f_y = -2y \to -f_y = 2y$

$\langle 2x, 2y, 1\rangle$ 0.4

$\vec{F}\cdot\hat{n} = 4x^2 + 4y^2 + \cancel{z}$ $\quad 9-x^2-y^2$

$$\iint_R \underbrace{3x^2+3y^2}_{3r^2+9} + 9\,.\,dA$$ 0.4

$1 \le r \le 2$

$0 \le \theta \le 2\pi$

$dA = r\,dr\,d\theta$

$$\int_0^{2\pi}\int_1^2 3r^3 + 9r\,.\,dr\,d\theta = \int_0^{2\pi} \frac{3r^4}{4} + \frac{9r^2}{2}\Big|_1^2 d\theta$$

0.2

$$= \int_0^{2\pi} \left(12 + 18 - \frac{3}{4} - \frac{9}{2}\right).d\theta = \frac{99}{4}\int_0^{2\pi} d\theta = \frac{99}{4}(2\pi)$$

Flujo $= \frac{99}{2}\pi$ u.V. 0.2

EJERCICIO #3

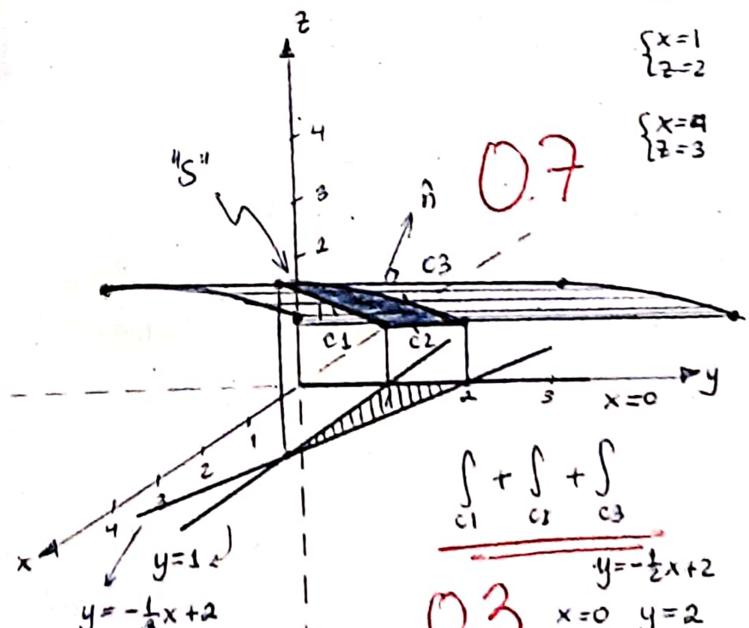
$z = 1+\sqrt{x}$, $x=0$, $y=1$, $y=-\frac{1}{2}x+2$.

$z = 1+\sqrt{x}$

$\begin{cases} x=0 \\ z=1 \end{cases}$

$\begin{cases} x=1 \\ z=2 \end{cases}$

$\begin{cases} x=4 \\ z=3 \end{cases}$



0.7

$$\int_{C1} + \int_{C2} + \int_{C3}$$

$y = -\frac{1}{2}x + 2$

0.3

$x=0 \quad y=2$

$x=2 \quad y=1$

$$\text{Rot}\,\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & -2x & 2xyz^2 \end{vmatrix} = (2xz^2)\hat{\imath} - (2yz^2)\hat{\jmath} + (-2-1)\hat{k}$$

$$\hookrightarrow -3\hat{k}$$

$z = 1+\sqrt{x}$

$\hookrightarrow f_x = \dfrac{1}{2\sqrt{x}}$

$\hookrightarrow f_y = 0$

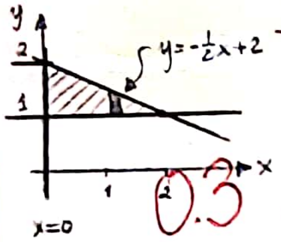
$$\langle 2xz^2, -2yz^2, -3\rangle \cdot \langle -\tfrac{1}{2\sqrt{x}}, 0, 1\rangle$$

$$= -\sqrt{x}\,z^2 - 3$$

$$-\iint_R \sqrt{x}\,z^2+3 \,.\, dA \qquad 0.7$$

$$z^2 = (1+\sqrt{x})^2 = 1+2\sqrt{x}+x$$

$$\sqrt{x}\,(1+2\sqrt{x}+x) = \sqrt{x}+2x+x^{3/2}$$



0.3

$$= -\int_0^2\int_1^{-\frac{1}{2}x+2} \sqrt{x}+2x+x^{3/2}+3 \,.\, dy\,.\,dx$$

$$-\int_0^2 (\sqrt{x}+2x+x^{3/2}+3)\,[-\tfrac{1}{2}x+\underbrace{2-1}_{+1}]\,dx \qquad 0.4$$

$$-\int_0^2 -\tfrac{1}{2}x^{3/2}+x^{1/2}-x^2+2x-\tfrac{1}{2}x^{5/2}+x^{3/2}-\tfrac{3}{2}x+3 \,.\, dx$$

$$-\int_0^2 -\tfrac{1}{2}x^{5/2}+\tfrac{1}{2}x^{3/2}+x^{1/2}-x^2+\tfrac{1}{2}x+3 \,.\, dx$$

$$= \frac{1}{\cancel{2}}\frac{x^{7/2}}{\frac{7}{\cancel{2}}} - \frac{1}{\cancel{2}}\frac{x^{5/2}}{\frac{5}{\cancel{2}}} - \frac{x^{3/2}}{\frac{3}{2}} + \frac{x^3}{3} - \frac{x^2}{4} - 3x \Big|_0^2$$

$$= \frac{x^{7/2}}{7} - \frac{x^{5/2}}{5} - \frac{2}{3}x^{3/2} + \frac{x^3}{3} - \frac{x^2}{4} - 3x \Big|_0^2$$

$$= \frac{8\sqrt{2}}{7} - \frac{4\sqrt{2}}{5} - \frac{4\sqrt{2}}{3} + \frac{8}{3} - 1 - 6$$

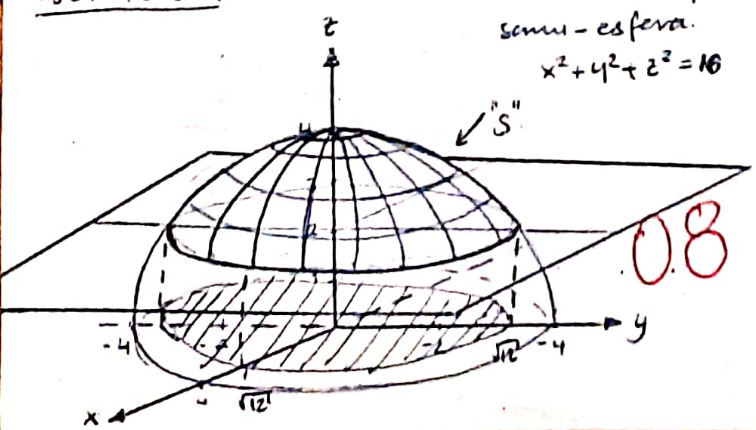
$$= -\frac{104}{105}\sqrt{2} - \frac{13}{3} \qquad 0.6$$

<u>EJERCICIO 4</u>:

$z = \sqrt{16-x^2-y^2}$

semi-esfera

$x^2+y^2+z^2=16$



0.8

$$\text{DIV}\,\vec{F} = -\cancel{\tfrac{1}{y}} - 4y + \cancel{\tfrac{1}{y}} + 2 = 2-4y$$

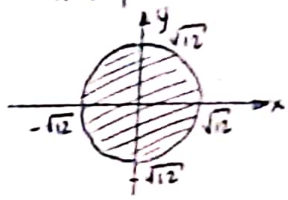
$$\iiint_D 2-4y\,dV \qquad 0.2$$

- RECTANGULARES

$$\int_{-\sqrt{12}}^{\sqrt{12}}\int_{-\sqrt{12-x^2}}^{\sqrt{12-x^2}}\int_2^{\sqrt{16-x^2-y^2}} 2-4y \,.\, dz\,.\,dy\,.\,dx \qquad 0.5$$

$x^2+y^2+\frac{z^2}{4}=16$, $z=2$

$x^2+y^2=12$



- CILINDRICAS

$$\int_0^{2\pi}\int_0^{\sqrt{12}}\int_2^{\sqrt{16-r^2}} 2r-4r^2\sin\theta \,.\, dz\,.\,dr\,.\,d\theta \qquad 0.5$$

$2-4y = 2-4r\,\text{sen}\,\theta.$

$dV = r\,.\,dz\,.\,dr\,.\,d\theta$

$0 \le r \le \sqrt{12}$ $\quad 0 \le \theta \le 2\pi$

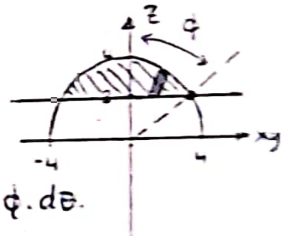
$\sqrt{16-x^2-y^2} \rightarrow \sqrt{16-r^2}$

- ESFERICAS

$P(0, \sqrt{12}, 2)$

$\phi = \cos^{-1}\left(\dfrac{2}{\sqrt{0+12+4}}\right)$

$\phi = \cos^{-1}(1/2) = \pi/3$



$$\int_0^{2\pi}\int_0^{\pi/3}\int_{2\sec\phi}^{4} (2-4\rho\sin\phi\sin\theta) \cdot \rho^2\sin\phi \; d\rho\,.\,d\phi\,.\,d\theta$$

0.5

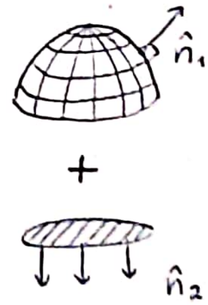
$x^2+y^2+z^2=16 \rightarrow \rho^2=16$

$\rho=4$

$z=2 \rightarrow \rho\cos\phi=2$

$\rho = 2\sec\phi$

$dV = \rho^2\,\text{sen}\,\phi\,.\,d\rho\,.\,d\phi\,d\theta.$



$$\iint_{S1} + \iint_{S2}$$

$\hat{n}_1 = \langle -f_x, -f_y, 1\rangle \qquad 0.3$

$z = \sqrt{16-x^2-y^2}$

$f_x = \dfrac{-x}{\sqrt{16-x^2-y^2}}$

$$\hat{n}_1 = \underbrace{\left\langle \frac{x}{\sqrt{16-x^2-y^2}}, \frac{y}{\sqrt{16-x^2-y^2}}, 1\right\rangle}$$

$f_y = \dfrac{-y}{\sqrt{16-x^2-y^2}}$

$$\hat{n}_2 = \underbrace{\langle 0, 0, -1\rangle} \qquad 0.2$$