SOLUCIÓN PARCIAL 6.

EJERCICIO 1

i) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^n(n+1)}$ $\quad \lim_{n\to\infty} \left| \frac{\cancel{(-1)^n}(-1)\cancel{x^n}\cdot x}{\cancel{2^n}(n+2)2} \cdot \frac{\cancel{2^n}(n+1)}{\cancel{(-1)^n}\cancel{x^n}} \right|$

$\lim_{n\to\infty} \left| \frac{(-1)\,x\,(n+1)}{2(n+2)} \right| = \lim_{n\to\infty} \frac{n+1}{2n+4}^{1/2} |x|$ 0.5

$\frac{1}{2}|x| < 1 \rightarrow |x| < 2 \qquad -2 < x < 2.$

R.C. = 2.

0.5

■ $x=-2.$

$\sum_{n=0}^{\infty} \frac{\cancel{(-1)^n}\cancel{(-1)^n}\cancel{2^n}}{\cancel{2^n}(n+1)} = \sum_{n=0}^{\infty} \frac{1}{n+1} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

Serie armónica: diverge.

0.5

■ $x=2$

$\sum_{n=0}^{\infty} \frac{(-1)^n \cancel{2^n}}{\cancel{2^n}(n+1)} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ serie armónica alternada: converge.

0.5

I.C. $]-2, 2]$

0.25 no

0.5

ii) $\sum_{n=0}^{\infty} \frac{\sqrt{n+1}\,(x+2)^n}{n!}$

$\lim_{n\to\infty} \left| \frac{\sqrt{n+2}\,\cancel{(x+2)^n}(x+2)}{(n+1)\cancel{n!}} \cdot \frac{\cancel{n!}}{\sqrt{n+1}\,\cancel{(x+2)^n}} \right|$ 0.5

$\lim_{n\to\infty} \left| \frac{\sqrt{n+2}\,(x+2)}{(n+1)\sqrt{n+1}} \right| = \underbrace{\frac{\sqrt{n+2}}{(n+1)^{3/2}}}_{0} |x+2|$

$\lim_{n\to\infty} \frac{(n+2)^{1/2}}{(n+1)^{3/2}} = \frac{n^{1/2}}{n^{3/2}} = \frac{1}{n} = \frac{1}{\infty} = 0$

$L = 0|x+2| = 0$

R.C. = ∞ 0.5

I.C. = $]-\infty, +\infty[$

EJERCICIO 2

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$

$x \rightarrow -t^2$

$e^{-t^2} = 1 - t^2 + \frac{t^4}{2!} - \frac{t^6}{3!} + \frac{t^8}{4!} + \dots$ 0.5

multiplicar por "t"

$te^{-t^2} = t - t^3 + \frac{t^5}{2!} - \frac{t^7}{3!} + \frac{t^9}{4!} - \dots$ 0.5

sumando t^3

$t^3 + te^{-t^2} = t - \cancel{t^3} + \frac{t^5}{2!} - \frac{t^7}{3!} + \frac{t^9}{4!} - \dots + \cancel{t^3}$

$\int_1^2 t^3 + te^{-t^2}\,dt = \int_1^2 t + \frac{t^5}{2!} - \frac{t^7}{3!} + \frac{t^9}{4!}\,dt$ 0.5

$\int_1^2 t^3 + te^{-t^2}\,dt = \frac{t^2}{2} + \frac{t^6}{6\cdot 2!} - \frac{t^8}{8\cdot 3!} + \frac{t^{10}}{10\cdot 4!} \Big|_1^2$

$= \frac{\cancel{4}^{2}}{\cancel{2}} + \frac{64}{12} - \frac{256}{48} + \frac{1024}{240} - \frac{1}{2} - \frac{1}{12}$

$+ \frac{1}{48} - \frac{1}{240}$

$\int_1^2 t^3 + te^{-t^2}\,dt = \frac{57}{10} \approx 5.7$ 0.5

EJERCICIO 3

$f(x) = (5-x)^{1/3}$ $\qquad f(4) = (5-4)^{1/3} = 1$ 0.4

$f'(x) = \frac{1}{3}(-1)(5-x)^{-2/3}$ $\qquad f'(4) = \frac{-\frac{1}{3}}{(5-x)^{2/3}} = -\frac{1}{3}$ 0.4

$f''(x) = \frac{-\frac{1}{3}\left(-\frac{2}{3}\right)(-1)}{(5-x)^{+5/3}}$ $\qquad f''(4) = \frac{-\frac{2}{9}}{(5-x)^{5/3}} = -\frac{2}{9}$ 0.4

$f'''(x) = \frac{-\frac{2}{9}\left(-\frac{5}{3}\right)(-1)}{(5-x)^{8/3}}$ $\qquad f'''(4) = \frac{-\frac{10}{27}}{(5-x)^{8/3}} = -\frac{10}{27}$ 0.4

$f(x) = 1 - \frac{1}{3}(x-4) - \frac{1}{9}(x-4)^2 - \frac{5}{81}(x-4)^3 + \dots$ 0.4

EJERCICIO 4.

$f(x) = \frac{2x-1}{x+1}$ $\qquad f(0) = \frac{-1}{1} = -1$ ✓ 0.4

$f'(x) = \frac{2(x+1) - (2x-1)(1)}{(x+1)^2} = \frac{\cancel{2x}+2-\cancel{2x}+1}{(x+1)^2} = \frac{3}{(x+1)^2}$

$f'(0) = 3.$ ✓ 0.4 $\qquad f' = 3(x+1)^{-2}$

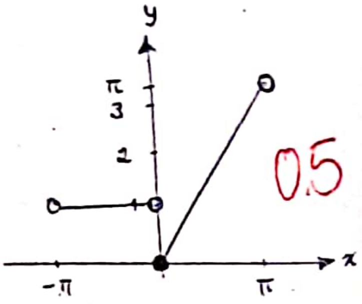
$f''(x) = 3(-2)(x+1)^{-3}$ $\qquad f''(0) = -6$ ✓ 0.4

$f(x) = \frac{2x-1}{x+1} = -1 + 3x - 3x^2 + 3x^3 - \dots$ 0.4

$f'''(x) = \frac{18}{(x+1)^4} \rightarrow$ 0.4

EJERCICIO 5

$$f(x) = \begin{cases} 1, & -\pi < x < 0 \\ x, & 0 \le x < \pi \end{cases}$$



0.5

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0} dx + \int_{0}^{\pi} x\,dx\right]$$

$$= \frac{1}{2\pi}\left[x\Big|_{-\pi}^{0} + \frac{x^2}{2}\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[0+\pi + \frac{\pi^2}{2} - 0\right] = \frac{1}{2\pi}\left[\frac{2\pi+\pi^2}{2}\right]$$

$$\boxed{= \frac{1}{4}[2+\pi] = a_0} \approx 1.285$$ 0.5

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0}\cos(nx)\,dx + \int_{0}^{\pi} \underbrace{x\cos(nx)\,dx}\right]$$

$$= \frac{1}{\pi}\left[\cancel{\frac{\sin(nx)}{n}}\Big|_{-\pi}^{0} + \cancel{\frac{x\sin(nx)}{n}} + \frac{\cos(nx)}{n^2}\Big|_{0}^{\pi}\right]$$

$x \quad \cos(nx)$

$1 \quad \sin(nx)/n$

$0 \quad -\cos(nx)/n^2$

$$= \frac{x\sin(nx)}{n} + \frac{\cos(nx)}{n^2}$$

$\sin(0) = \sin(\pi) = \sin(-\pi) = 0$

$$= \frac{1}{\pi}\left[\frac{\overset{(-1)^n}{\cos(n\pi)}}{n^2} - \frac{\overset{1}{\cos(0)}}{n^2}\right]$$

$$\boxed{a_n = \frac{1}{\pi n^2}\left[(-1)^n - 1\right]}$$ 0.5

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0}\sin(nx)\,dx + \int_{0}^{\pi}\underbrace{x\sin(nx)\,dx}\right]$$

$$= -\frac{x\cos(nx)}{n} + \frac{\sin(nx)}{n^2}$$

$x \quad \sin(nx)$

$1 \quad -\cos(nx)/n$

$0 \quad -\sin(nx)/n^2$

$$= \frac{1}{\pi}\left[-\frac{\cos(nx)}{n}\Big|_{-\pi}^{0} + \left(-\frac{x\cos(nx)}{n} + \cancel{\frac{\sin(nx)}{n^2}}\right)\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[-\frac{\overset{1}{\cancel{\cos(0)}}}{n} + \frac{\cos(-n\pi)}{n}\right] + \left[-\frac{\pi\cos(n\pi)}{n} + 0\right]$$

$$\boxed{= \frac{1}{\pi n}\left[-1 + (-1)^n - \pi(-1)^n\right] = b_n}$$ 0.5

$$f(x) = \frac{1}{4}[2+\pi] + \sum_{n=1}^{\infty} \frac{[(-1)^n - 1]}{\pi n^2}\cos(nx) + \frac{[-1+(-1)^n - \pi(-1)^n]}{\pi n}\sin(nx)$$

Evaluada en $n = \underbrace{1,2,3}, 4, \dots$

$$f(x) = \frac{[2+\pi]}{4} + \frac{-2}{\pi}\cos(x) + \frac{-2+\pi}{\pi}\sin(x) + -\frac{1}{2}\sin(2x) + \frac{-2}{9\pi}\cos(3x) + \frac{-2+\pi}{3\pi}\sin(3x)$$