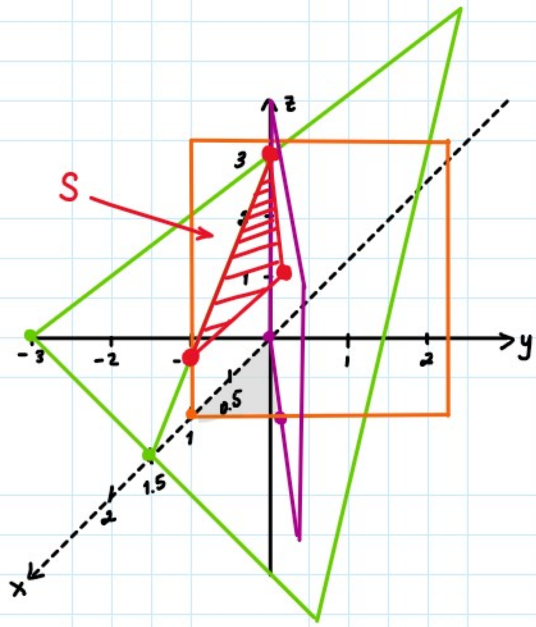
Integrales de superficie.

EJEMPLO: Evalue $\iint_S (xy+z)\,ds$; dónde S es la parte del plano: $\underbrace{2x-y+z=3}$ que esta sobre el borde limitado en el I octante por: $y=x \wedge x=1$. Grafique S.



P1. Graficar y enmarcar S.

$2x-y+z=3$

I. con ejes: "x" $(y=z=0)$

$2x=3 \rightarrow x=3/2=1.5$

"y" $(x=z=0)$

$-y=3 \rightarrow y=-3$

"z" $(x=y=0)$

$z=3$

$y=x \rightarrow$ recta

$(0,0) \wedge (1,1)$

$x=1 \rightarrow$ recta.

$2\cancel{x}^{1}-\cancel{y}^{1}+z=3$

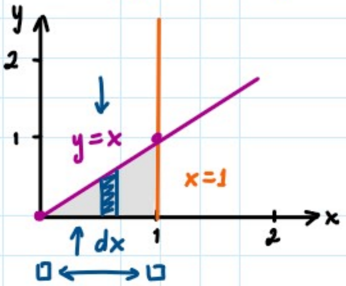
$1+z=3$

$z=3-1$

$z=2$

$(1,1,2)$

P2. Proyección "xy" $\rightsquigarrow$ despejar "z" a las superficie S.



$2x-y+z=3 \rightarrow z=3-2x+y \leftarrow$

$f_x=(-2)$ $\quad f_x^2=4$

$f_y=+1$ $\quad f_y^2=1$

$ds=\sqrt{1+f_x^2+f_y^2}\;dA$

$ds=\sqrt{1+4+1}\;dA=\sqrt{6}\,dA$

P3. Plantear la integral.

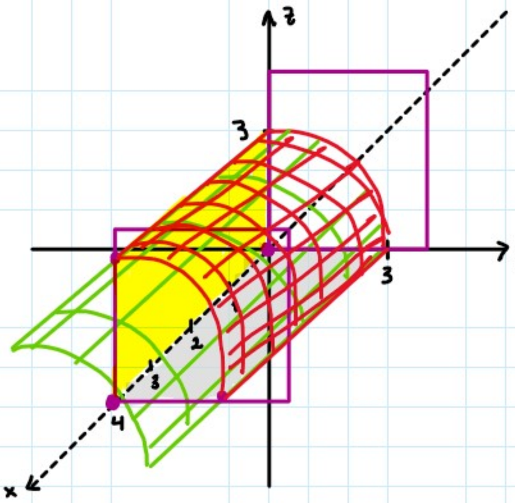
$$\iint_S (xy+z)\,ds \rightarrow \iint_R (xy+3-2x+y)\sqrt{6}\,dA = \sqrt{6}\int_0^1\int_0^x xy+3-2x+y\,.\,dy\,.\,dx$$

P4. Resolver. $\sqrt{6}\int_0^1 \left(\frac{xy^2}{2}+3y-2xy+\frac{y^2}{2}\right)\Big|_0^x dx$

$$\sqrt{6}\int_0^1 \frac{x^3}{2}+3x-2x^2+\frac{x^2}{2}\,dx = \sqrt{6}\int_0^1 \frac{x^3}{2}-\frac{3}{2}x^2+3x\,dx = \sqrt{6}\left[\frac{x^4}{8}-\frac{x^3}{2}+\frac{3x^2}{2}\right]\Big|_0^1$$

$$=\sqrt{6}\left[\frac{1}{8}\underbrace{-\frac{1}{2}+\frac{3}{2}}\right]=\frac{9}{8}\sqrt{6}$$

EJEMPLO: Evaluar la integral: $\iint_S (x+z)\,ds$ dónde S es la porción de: $y^2+z^2=9$ en el primer octante, limitado por $x=0 \wedge x=4$.



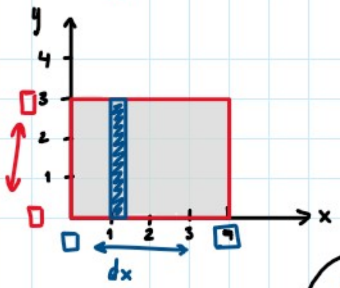
$y^2+z^2=9$

$x=0 \wedge x=4$

$C(0,0)$ $r=\sqrt{9}=3$

plano "yz"

Proyección: "xy" → despejar "z"



$y^2+z^2=9$

$z^2=9-y^2$

$z=\pm\sqrt{9-y^2}$

$(9-y^2)^{1/2}$

$ds=\sqrt{1+\cancel{f_x}^2+f_y^2}\,dA$

0

$f_x=0$ $\quad f_x^2=0$

$f_y=\frac{1}{\cancel{2}}\cdot(9-y^2)^{-1/2}(-\cancel{2}y)$

$f_y=\frac{-y}{\sqrt{9-y^2}}$

$f_y^2=\frac{y^2}{9-y^2}$

$ds=\sqrt{1+0+\frac{y^2}{9-y^2}}\,dA$

$\frac{1}{1}+\frac{y^2}{9-y^2}$

$\frac{9-\cancel{y^2}+\cancel{y^2}}{9-y^2}=\frac{9}{9-y^2}$

$ds=\sqrt{\frac{9}{9-y^2}}\,dA=\frac{3\,dA}{\sqrt{9-y^2}}$

$\iint_S (x+z)\,ds \rightarrow \iint_R \left(x+\sqrt{9-y^2}\right)\frac{3\,dA}{\sqrt{9-y^2}}$

$3\int_0^4\int_0^3 \left(x+\sqrt{9-y^2}\right)\frac{1}{\sqrt{9-y^2}}\cdot dy\cdot dx$

$3\int_0^4\int_0^3 \frac{x}{\sqrt{9-y^2}}+\cancel{\frac{\sqrt{9-y^2}}{\sqrt{9-y^2}}}^{\,1}\cdot dy\cdot dx$

$3\int_0^4 \lim_{h\to 3^-}\int_0^h \frac{x}{\sqrt{9-y^2}}+1\cdot dy\cdot dx$

Integral impropia

R// $= 36+12\pi$

1. Superficies abiertas.

$\hat{n}$ es ascendente

$\hat{n}$ es descendente.

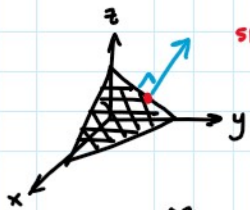
z

si el ejercicio

1. Superficies abiertas.

$\hat{n}$ es ascendente

$\hat{n}$ es descendente.



si el ejercicio no dice nada =′(

asumir: ascendente.

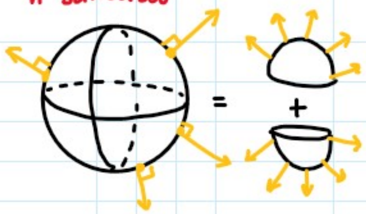
OPCIONES:

V. ascendentes y V. descendente.

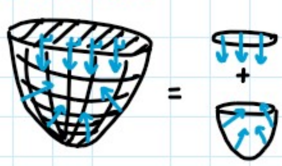
asumir.

2. Superficies cerradas.

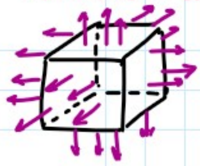
$\hat{n}$ saliendo



$\hat{n}$ entrando



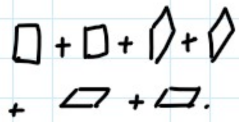
no dice nada el ejercicio =′(



OPCIONES: V. saliendo y V. entrando.

asumir.

asumir saliendo



$$-f_x\hat{\imath} - f_y\hat{\jmath} + \hat{k}$$

$\hat{n}$ arriba

$$f_x\hat{\imath} + f_y\hat{\jmath} - \hat{k}$$

$\hat{n}$ abajo.

$$\text{Flujo} = \iint_R \langle M, N, P\rangle \cdot \langle -f_x, -f_y, 1\rangle \, dA$$

$$\text{Flujo} = \iint_R \langle M, N, P\rangle \cdot \langle f_x, f_y, -1\rangle \, dA$$

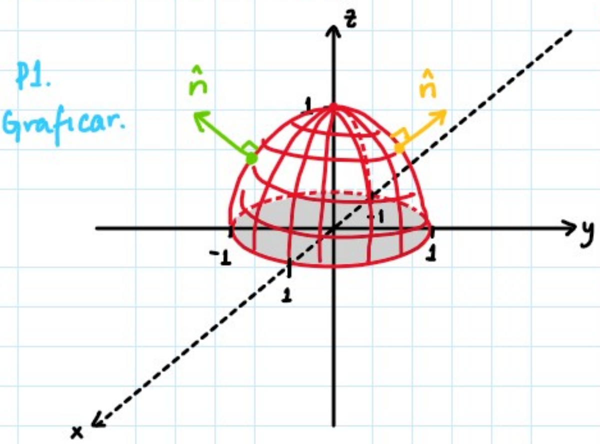
EJEMPLO 1. Calcule el flujo del campo vectorial $\vec{F}(x,y,z) = \underbrace{x}_{M}\hat{\imath} + \underbrace{y}_{N}\hat{\jmath} + \underbrace{z}_{P}\hat{k}$ a través de la superficie S dada por: $\underbrace{z = 1 - x^2 - y^2}_{\text{paraboloide elíptico}}$ que está sobre el plano "xy." haciendo $\hat{n}$ normal ascendente.

paraboloide elíptico.

eje "z".

V(0,0,1)

P1. Graficar.



Intersep con z=0 ← "xy"

$0 = 1 - x^2 - y^2$

$x^2 + y^2 = 1$

C(0,0) r=1

¿Cómo obtener $\hat{n}$?

Obligación: [despejar z]

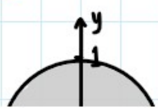
$z = 1 - x^2 - y^2$

$f_x = -2x \rightarrow -f_x = 2x$

$f_y = -2y \rightarrow -f_y = 2y$

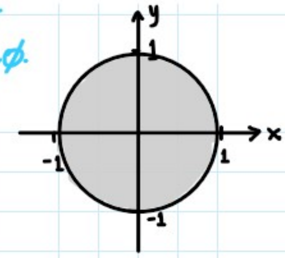
Proyección "xy"

P2. Encontrar $\hat{n}$ y proyecc.



P3. Realizar el producto punto y

P2. Encontrar $\hat{n}$ y proyecc.



P3. Realizar el producto punto y sustituir "z".

$$\text{Flujo} = \iint_R \langle M, N, P\rangle \cdot \langle -f_x, -f_y, 1\rangle \, dA$$

$$\langle x, y, z\rangle \cdot \langle 2x, 2y, 1\rangle$$

$$2x^2 + 2y^2 + z$$

$$= \iint_R 2x^2 + 2y^2 + \cancel{z}^{\,1-x^2-y^2} \, . \, dA$$

$$2x^2 + 2y^2 + 1 - x^2 - y^2$$

$$x^2 + y^2 + 1$$

$0 \le r \le 1$ ✓

$0 \le \theta \le 2\pi$ ✓

$$\underbrace{x^2+y^2}_{r^2} + 1$$

$r^2 + 1$ ✓

$dA = r \, dr . d\theta$ ✓

$$= \iint_R x^2 + y^2 + 1 \, . \, dA$$

P4. Plantear y resolver la I.D.

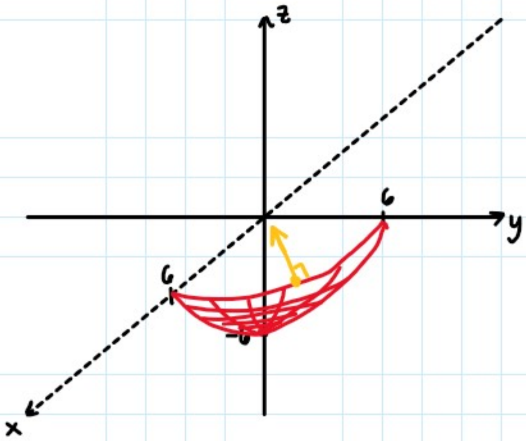
$$= \int_0^{2\pi} \int_0^1 \underbrace{(r^2+1)\, r}_{r^3 + r} \, dr . d\theta = \int_0^{2\pi} \frac{r^4}{4} + \frac{r^2}{2} \Big|_0^1 \, d\theta = \int_0^{2\pi} \left(\frac{1}{4} + \frac{1}{2}\right) . d\theta = \frac{3}{4} \int_0^{2\pi} d\theta$$

$$= \frac{3}{4}\theta \Big|_0^{2\pi} = \frac{3}{\cancel{4}_{\,2}} (\cancel{2}^{\,1}\pi) = \frac{3\pi}{2} \text{ u.V.}$$

EJEMPLO 2. Plantee la integral de flujo si $\vec{F}$ y S estan dados por:

$\vec{F}(x,y,z) = \underbrace{x}_{M}\hat{\imath} + \underbrace{y}_{N}\hat{\jmath} + \underbrace{z}_{P}\hat{k}$,   S : $\underbrace{x^2 + y^2 + z^2 = 36}$ en el V octante.

Esfera $(0,0,0)$ $r = \sqrt{36} = 6$.



asumir: $\hat{n}$ ascendente.

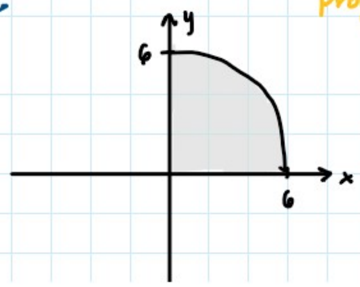
$$x^2 + y^2 + z^2 = 36$$

$$z = -\sqrt{36 - x^2 - y^2} = -(36 - x^2 - y^2)^{1/2}$$

$$f_x = \cancel{-}\frac{1}{\cancel{2}}(36 - x^2 - y^2)^{-1/2}(\cancel{-}\cancel{2}x) = \frac{x}{\sqrt{36 - x^2 - y^2}}$$

$$f_y = \frac{y}{\sqrt{36 - x^2 - y^2}} \text{ (por analogía)}$$

proyección "xy"



$$\text{Flujo} = \iint_R \langle M, N, P\rangle \cdot \langle -f_x, -f_y, 1\rangle \, dA$$

$$\langle x, y, z\rangle \cdot \left\langle \frac{-x}{\sqrt{36 - x^2 - y^2}}, \frac{-y}{\sqrt{36 - x^2 - y^2}}, 1 \right\rangle$$

$-x^2 - y^2$

$$\frac{-x^2-y^2}{\sqrt{36-x^2-y^2}} + z$$

$$\iint_R \frac{-x^2-y^2}{\sqrt{36-x^2-y^2}} + \cancel{z} \cdot dA \qquad z \to -\sqrt{36-x^2-y^2}$$

$$\frac{-x^2-y^2}{\sqrt{36-x^2-y^2}} - \frac{\sqrt{36-x^2-y^2}}{1}$$

$$= \frac{-x^2-y^2-(36-x^2-y^2)}{\sqrt{36-x^2-y^2}}$$

$$= \frac{-\cancel{x^2}-\cancel{y^2}-36+\cancel{x^2}+\cancel{y^2}}{\sqrt{36-x^2-y^2}} = \frac{-36}{\sqrt{36-x^2-y^2}}$$

$$-36\iint_R \frac{dA}{\sqrt{36-x^2-y^2}}$$

$0 \leq r \leq 6$
$0 \leq \theta \leq \pi/2$

$$\sqrt{36-x^2-y^2} = \sqrt{36-\underbrace{(x^2+y^2)}_{r^2}} = \sqrt{36-r^2}$$

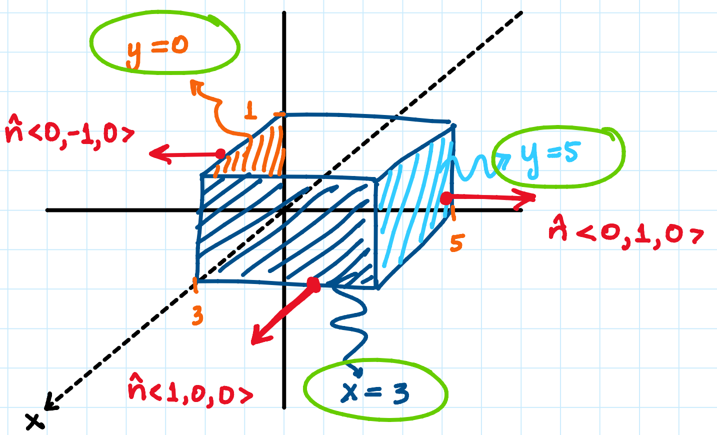
$dA = r.dr.d\theta$

$$-36\int_0^{\pi/2} \lim_{m\to 6^-} \int_0^m \frac{r\,dr\,d\theta}{\sqrt{36-r^2}}$$

CASO ESPECIAL.

$x = \#$, $y = \#$ ó $z = \#$

asumir vectores...



vector se asume:

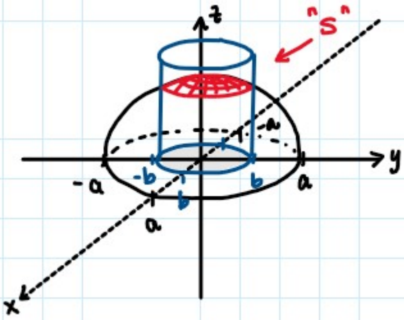
ÁREA DE UNA SUPERFICIE.

$$A(S) = \iint_S dS = \iint_R \sqrt{1 + f_x^2 + f_y^2}\ dA \leftarrow \text{proyección "xy"}$$

Resumen: S: esfera $x^2+y^2+z^2=a^2$ • $C(0,0,0)$ $r=a$

$[a > b.]$

limitada dentro del cilindro: $x^2+y^2=b^2 \leftarrow C(0,0)$ $r=b$.

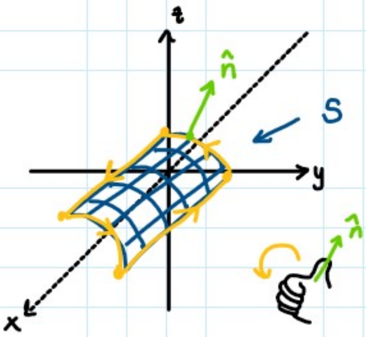


• $x^2+y^2+z^2=a^2 \longrightarrow z = \sqrt{a^2-x^2-y^2}$

$f_x = \sim\sim$ $\quad f_y = \sim\sim$

$$\iint_R \sqrt{1+f_x^2+f_y^2}\ .dA = (\dots) =$$

TEOREMA DE STOKES.



$\iint_S \dots = 5$

$\int_{C1} + \int_{C2} + \int_{C3} + \int_{C4} = \dots = 5$

$$\oint_C Mdx + Ndy + Pdz \equiv \iint_S Rot\vec{F} \cdot \hat{n}\, dS$$

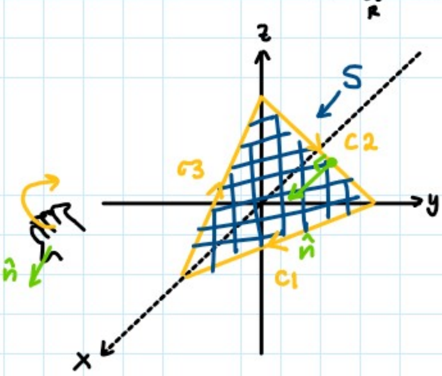
$\hookrightarrow \iint_R Rot\vec{F} \cdot \langle -f_x, -f_y, 1 \rangle\, dA$

VERIFIQUE: I.S. y I.L. ✓

Aplique, resuelva,... I.S.

$\iint_S = -3$

$\int_{C1} + \int_{C2} + \int_{C3} = \dots = -3$

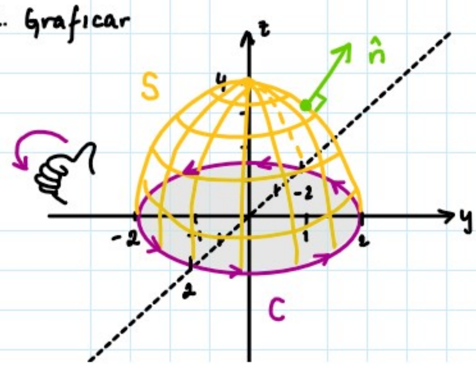


EJEMPLO: Verifique el Teorema de Stokes sabiendo que: $\vec{F}(x,y,z) = \langle y, 2z, x^2 \rangle$

y la superficie "S" es la parte de: • $z = 4-x^2-y^2$ sobre el plano "xy". asumir: $\hat{n}$ asc.

$$\underbrace{\oint_C Mdx + Ndy + Pdz}_{\text{I.L.}} = \underbrace{\iint_R Rot\vec{F} \cdot \langle -f_x, -f_y, 1 \rangle\, dA}_{\text{I.S.} \rightarrow \text{I.D.}}$$

P1. Graficar



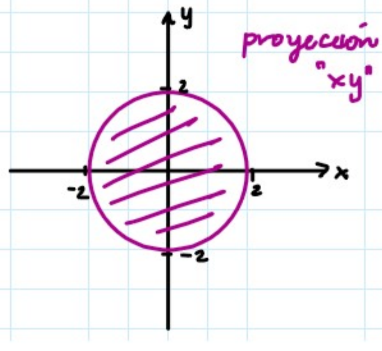
$z = 4-x^2-y^2$

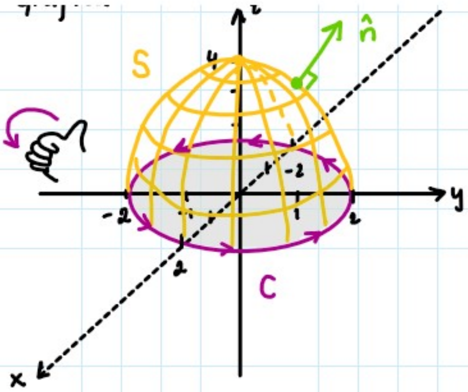
$V(0,0,4)$

Intersecc plano "xy" $\rightarrow z=0$

$0 = 4-x^2-y^2$

$x^2+y^2=4 \rightarrow C(0,0)$ $r=2$





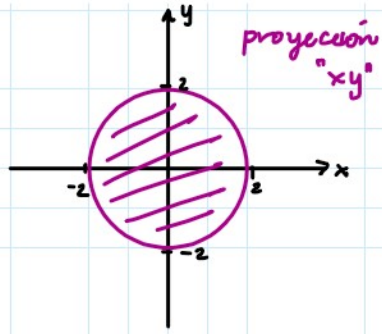
$z = 4 - x^2 - y^2$

$V(0,0,4)$

Intersec plano "xy" $\rightarrow z = 0$

$0 = 4 - x^2 - y^2$

$x^2 + y^2 = 4 \rightarrow C(0,0) \; r = 2$



- Por I.L. $\oint_C M\,dx + N\,dy + P.\,dz$

$\vec{F}(x,y,z) = \langle \underbrace{y}_{M}, \underbrace{2z}_{N}, \underbrace{x^2}_{P} \rangle$

$$\oint_C y.\,dx + \cancelto{0}{2z}dy + x^2 \cancelto{0}{dz}$$

$x = 2\cos t \longrightarrow dx = -2\sin t\,dt$

$y = 2\sin t \longrightarrow dy = 2\cos t\,dt$

$z = 0 \longrightarrow dz = 0.dt$

$$\oint_C y.\,dx$$

$0 \leq t \leq 2\pi$

$$\int_0^{2\pi} 2\sin t.\,(-2\sin t).\,dt = -4\int_0^{2\pi} \sin^2 t.\,dt = \frac{\cancelto{2}{-4}}{\cancelto{1}{2}}\int_0^{2\pi} 1 - \cos(2t)\;.\,dt$$

$$= -2\int_0^{2\pi} 1 - \cos(2t)\,dt = -2\left[t - \frac{\sin(2t)}{2}\right]\Bigg|_0^{2\pi} = -2\left[2\pi - \cancelto{0}{\frac{\sin(4\pi)}{2}} - 0 + \cancelto{0}{\frac{\sin 0}{2}}\right]$$

$= -4\pi$ | por I.L.

- Por I.S.

$\vec{F}(x,y,z) = \langle \underbrace{y}_{M}, \underbrace{2z}_{N}, \underbrace{x^2}_{P} \rangle$

$$\iint_R \underbrace{\text{Rot}\,\vec{F}}\cdot \underbrace{\langle -f_x, -f_y, 1\rangle}\,dA.$$

$$\text{Rot}\,\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & 2z & x^2 \end{vmatrix} = (0-2)\hat{\imath} - (2x - 0)\hat{\jmath} + (0-1)\hat{k}$$

$$= -2\hat{\imath} - 2x\hat{\jmath} - \hat{k} \quad \text{ó} \quad \langle -2, -2x, -1\rangle \,\bullet$$

$z = 4 - x^2 - y^2 \longrightarrow f_x = -2x \leftrightarrow -f_x = 2x$

$f_y = -2y \leftrightarrow -f_y = 2y$

$\langle -f_x, -f_y, 1\rangle \equiv \langle 2x, 2y, 1\rangle \,\bullet$

$\text{Rot}\,\vec{F}\cdot n = -4x - 4xy - 1$

$$\iint_R -4x - 4xy - 1.\,dA \longrightarrow \int_0^{2\pi}\int_0^{2}\left[-4r\cos\theta - 4r^2\sin\theta\cos\theta - 1\right] r.\,dr.\,d\theta$$

$$\iint_R -4x-4xy-1 \, . \, dA \longrightarrow \int_0 \int_0 \left[-4r\cos\theta - 4r^2\sin\theta\cos\theta - 1\right] r.dr.d\theta$$

$0 \le r \le 2$
$0 \le \theta \le 2\pi$

$-4x-4xy-1$

$-4r\cos\theta - 4r\cos\theta\, r\sin\theta - 1$
$-4r\cos\theta - 4r^2\sin\theta\cos\theta - 1$

$dA = r\,dr\,d\theta$

$$= \int_0^{2\pi}\int_0^2 -4r^2\cos\theta - 4r^3\sin\theta\cos\theta - r \, . \, dr.d\theta$$

$$= \int_0^{2\pi} -4\cos\theta . \frac{r^3}{3} - \cancel{4}\sin\theta\cos\theta \frac{r^4}{\cancel{4}} - \frac{r^2}{2} \Big|_0^2 . \, d\theta$$

$$= \int_0^{2\pi} -4\cos\theta\left(\frac{8}{3}\right) - \sin\theta.\cos\theta\,(16) - 2 \, . \, d\theta$$

$$= \int_0^{2\pi} -\frac{32}{3}\cos\theta - \underbrace{16\sin\theta.\cos\theta} - 2 \, . \, d\theta$$

$u = \sin\theta$
$du = \cos\theta\, d\theta$

$-16\int u.du = -\frac{16u^2}{2}$

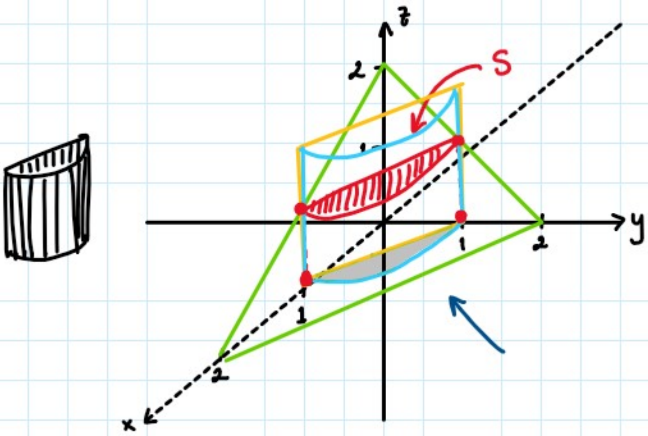
$= -8u^2 = -8\sin^2\theta$

$$= -\frac{32}{3}\cancel{\sin\theta}^{\,0} - 8\cancel{\sin^2\theta}^{\,0} - 2\theta \Big|_0^{2\pi}$$

$\sin(2\pi) = 0$
$\sin(0) = 0$

$$= -2\theta\Big|_0^{2\pi} = -2(2\pi - 0) = \boxed{-4\pi}$$ por I.S.

Dónde S: es el plano: $x+y+z=2$ en el primer octante limitado por: • $y=-x+1$ y • $x^2+y^2=1$.



$x+y+z=2$

P.C. $x=2$
$y=2$
$z=2$

• $y=-x+1$

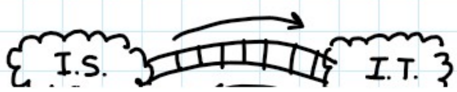
$x=0 \rightarrow y=-0+1=1$
$(0,1)$

$x=1 \rightarrow y=-1+1=0$
$(1,0)$

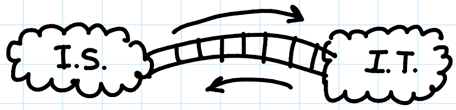
$x^2+y^2=1$

$C(0,0)$ $r=1$

TEOREMA DE LA DIVERGENCIA DE GAUSS.

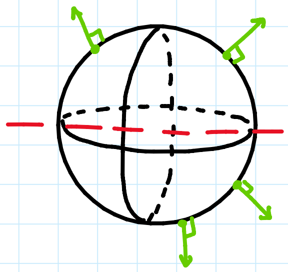


TEOREMA DE LA DIVERGENCIA DE GAUSS.

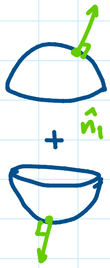


$\iint_S \vec{F}\cdot\hat{n}\, ds \equiv \iiint_D \text{Div}\vec{F}\,.dv$

* SUPERFICIE DEBE SER CERRADA.



I.S.



$\iint_{S1}$ $\hat{n}_1 <-f_x, -f_y, 1>$

$+$ $\iint_{S2}$ $+$

$\hat{n}_2 <f_x, f_y, -1>$

$= 20.$

VERIFIQUE: I.S. y I.T. ✓

Calcule, resuelve, utilice...

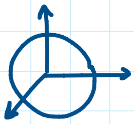
I.T. ✓

✓ rectangulares.

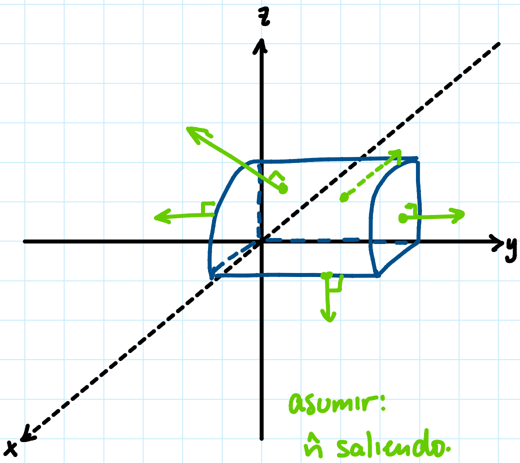
✓ cilíndricas.

✓ esféricas.

I.T.

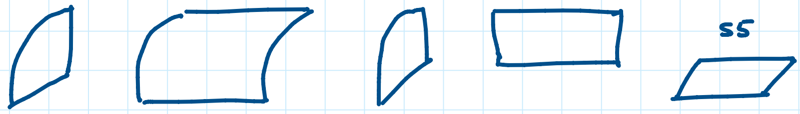


$\iiint_D = 20.$



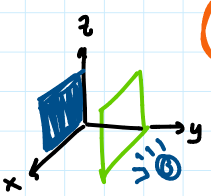
asumir: $\hat{n}$ saliendo.

(izq.) S1	S2	(der) S3	S4	S5
$\underbrace{y=0}$	$\underbrace{x^2+z^2=3}$	$\underbrace{y=4}$	$\underbrace{x=0}$	$\underbrace{z=0}$
$<0,-1,0>$	$z=\sqrt{3-x^2}$	$<0,1,0>$	$<-1,0,0>$	$<0,0,-1>$
	$<-f_x,-f_y,1>$			



$\iint_{S1} + \iint_{S2} + \iint_{S3} + \iint_{S4} + \iint_{S5} = -4$

$\iiint_D = -4.$

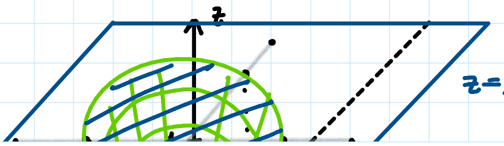


$\hat{n}$ obliga $\underbrace{\text{proyecc. } xy}$ $\leftrightarrow$ $<f_x, f_y, -1>$ ó $<-f_x,-f_y,1>$

$\underset{n}{<0,1,0>} \leftarrow y=1$

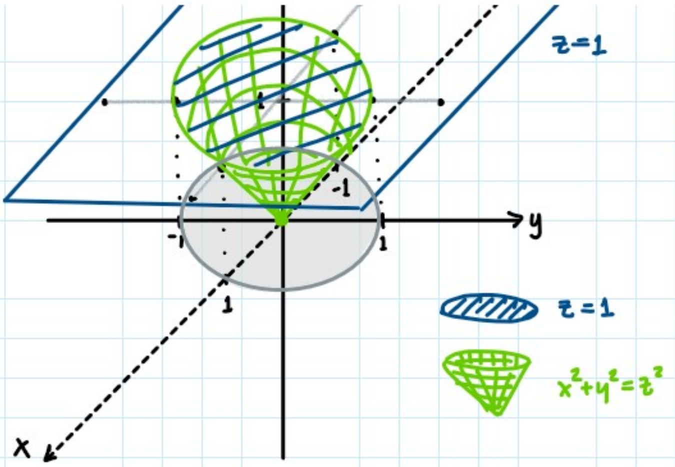
EJEMPLO: Verifique el teorema de la divergencia de Gauss si $\vec{F}$ y S están definidos por: $\vec{F} = <\underbrace{x-y}_{M}, \underbrace{x+z}_{N}, \underbrace{z-y}_{P}>$ S es la parte del $\underbrace{\text{cono}}$: $x^2+y^2=z^2$ limitado: $\underbrace{0 \le z \le 1}_{z=0 \quad z=1}$.

#1. Graficar...



Intersecc. $x^2+y^2=z^2$

$z=1$ ■ $z=0$ $x^2+y^2=0$ $(0,0)$



$z=1$

- $z=0$ $\quad x^2+y^2=0 \quad (0,0)$
- $z=1$ $\quad x^2+y^2=1^2 \quad C(0,0)\ r=1.$

- Por I.T. $\quad \iiint_D \operatorname{Div}\vec{F}\,dV \rightarrow 2\iiint_D dV$

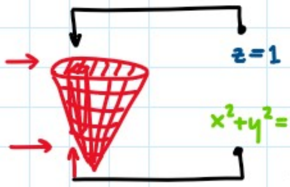
$\operatorname{Div}\vec{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z} = 1+0+1 = 2$

- CILINDRICAS. (polares... pero en 3D) $\quad dA = r.dr.d\theta \rightarrow dV = r.dz.dr.d\theta$

$r \le z \le 1$ ← analizar en 3D.

$0 \le r \le 1$

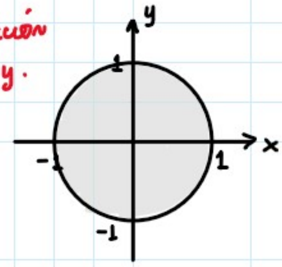
$0 \le \theta \le 2\pi$ } proyección "xy"



$\underbrace{\sqrt{x^2+y^2}}$ $= z$

$\sqrt{r^2} = z \qquad z=r$

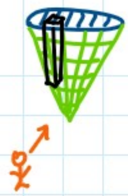
- proyección xy.

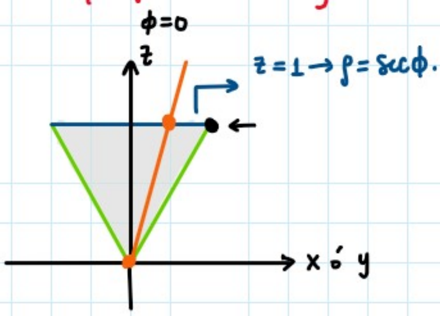


$$2\int_0^{2\pi}\int_0^1\int_r^1 r.dz.dr.d\theta = \frac{2\pi}{3} \quad (\dots)$$

- ESFERICAS. $\big(\underbrace{\rho \text{ ó } p \rightarrow \text{"radio"},\ \phi \rightarrow \text{ángulo } [0-\pi]}_{\text{proyección } z-\text{"xy"}},\ \theta \leftarrow \text{mismo que en polares}\big)$

- proyección "xy"





Conversiones.

$x = \rho \operatorname{sen}\phi \cos\theta$

$y = \rho \operatorname{sen}\phi \operatorname{sen}\theta$

$z = \rho\cos\phi$

$\phi = \cos^{-1}\left(\frac{z}{\rho}\right)$

$\rho^2 = x^2+y^2+z^2 \longrightarrow \rho = \sqrt{x^2+y^2+z^2}$

$dV = \rho^2 \operatorname{sen}\phi.d\rho.d\phi.d\theta$

$z=1 \qquad z = \rho\cos\phi$

$z = z$

$1 = \rho\cos\phi$

$\frac{1}{\cos\phi} = \rho$

$\sec\phi = \rho$

$0 \le \rho \le \sec\phi.$

$0 \le \phi \le \underbrace{\pi/4}_{\text{ángulo del cono.}}$

$0 \le \theta \le 2\pi$

$$\phi = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) = \cos^{-1}\left(\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+x^2+y^2}}\right) = \cos^{-1}\left(\frac{\sqrt{x^2+y^2}}{\sqrt{2(x^2+y^2)}}\right) = \cos^{-1}\left(\frac{\cancel{\sqrt{x^2+y^2}}}{\sqrt{2}\,\cancel{\sqrt{x^2+y^2}}}\right)$$

- $x^2+y^2 = z^2$ ← cono.
- $\sqrt{x^2+y^2} = z$

$\phi = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

- $x^2+y^2 = z^2 \leftarrow$ cono.
- $\sqrt{x^2+y^2} = z$

$z = \sqrt{x^2+y^2}$

$x=1,\ y=1,\ z=?\qquad z = \sqrt{1^2+1^2} = \sqrt{2}$

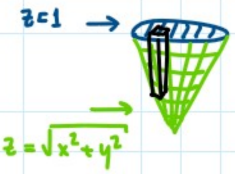
$$\phi = \cos^{-1}\left(\frac{\sqrt{2}}{\sqrt{1+1+2}}\right) = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

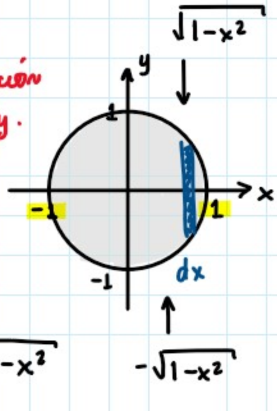
$$\phi = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$2\iiint_D dV = 2\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\sec\phi} \rho^2 \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta$$

■ RECTANGULARES.



• proyección xy.



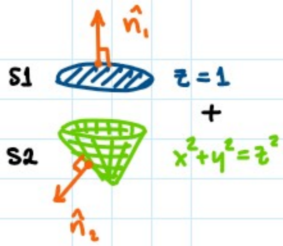
$x^2+y^2 = 1$

$y = \pm\sqrt{1-x^2}$

$$-1 \le x \le 1$$
$$-\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$$
$$\sqrt{x^2+y^2} \le z \le 1$$

$$2\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{1} dz\,dy\,dx$$

□ Por. I.S.



$$\iint_{S1} \vec{F}\cdot\hat{n}\,ds \qquad \hat{n}_1 \langle 0,0,1\rangle$$

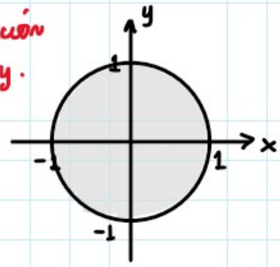
$$\iint_{S2} \vec{F}\cdot\hat{n}\,ds \qquad \hat{n}_2 \langle f_x, f_y, -1\rangle$$

$$\vec{F} = \langle \underbrace{x-y}_{M}, \underbrace{x+z}_{N}, \underbrace{z-y}_{P} \rangle$$

■ $\underbrace{\text{S1.}}_{z=1}$ $\quad \vec{F}\cdot n_1 = \langle x-y, x+z, z-y\rangle \cdot \langle 0,0,1\rangle = 0+0+z-y = z-y.$

$$\iint_R \overset{1}{\cancel{z}}-y.\,dA \qquad \iint_R \underbrace{1-y}_{1-r\operatorname{sen}\theta}.\,dA$$

• proyección xy.



$$\int_0^{2\pi}\int_0^1 (1-r\operatorname{sen}\theta)\,\underbrace{r.dr.d\theta}_{dA} = \int_0^{2\pi}\int_0^1 r-r^2\operatorname{sen}\theta.\,dr.\,d\theta.$$

$$= \int_0^{2\pi} \frac{r^2}{2} - \frac{r^3}{3} \sin\theta \Big|_0^1 . d\theta . = \int_0^{2\pi} \frac{1}{2} - \frac{1}{3} \sin\theta . d\theta$$

$0 \leq r \leq 1$
$0 \leq \theta \leq 2\pi$

$$= \frac{1}{2}\theta + \frac{1}{3}\cos\theta \Big|_0^{2\pi} = \frac{1}{\cancel{2}}(\cancel{2}\pi) + \frac{1}{3}\overset{1}{\cancel{\cos(2\pi)}} - 0 - \frac{1}{3}\overset{1}{\cancel{\cos(0)}} = \pi + \cancel{\frac{1}{3}} - \cancel{\frac{1}{3}} = \underline{\pi}_{\ S1.}$$

$dA = r.dr.d\theta$

▫ S2: $z = \sqrt{x^2+y^2}$

$R// \ \underline{-\frac{\pi}{3}}_{\ S2}$

$$\iint_{S1} + \iint_{S2} = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \text{ por I.S.}$$