SOLUCIÓN PARCIAL 1

EJERCICIO #1:

$x = z^2+1$, $y = z+2$ $\quad (1,2,0) \to (5,4,2)$

$x-1 = z^2$, $y-2 = z$

$(y-2)^2 = z^2$

$z^2 = z^2$

$x-1 = (y-2)^2$

$x = (y-2)^2+1$

0.5 $\begin{cases} x = (t-2)^2+1 \\ y = t \\ z = t-2 \end{cases}$

0.5 $\quad 2 \le t \le 4$

EJERCICIO #2:

$x = t^2 - t$, $y = t^2+1$ $\qquad P(2,2)$

$2 = t^2 - t \qquad 2 = t^2+1$

$t^2 - t - 2 = 0 \qquad 1 = t^2$

$(t-2)(t+1) = 0 \quad \boxed{t = \pm 1}$ ✓

$\boxed{t=2,\ t=-1}$

x ✓

$t = -1$ ✓ 0.5

a) $\dfrac{dy}{dx} = \dfrac{2t}{2t-1}\Big|_{t=-1}$

$m = \dfrac{2(-1)}{2(-1)-1} = \dfrac{-2}{-3} = \dfrac{2}{3}$

$y - 2 = \frac{2}{3}(x-2)$

$y = \frac{2}{3}(x-2)+2$ ó $\frac{2}{3}x + \frac{2}{3}$

$\begin{cases} x = t \\ y = \frac{2}{3}t + \frac{2}{3} \end{cases}$ 0.5

b) $R_{TH} \to \dfrac{dy}{dt} = 0 \qquad 2t = 0 \to t = 0.$ $\begin{cases} x = 0^2 - 0 = 0 \\ y = 0^2+1 = 1 \end{cases}$

0.5 $P(0,1)$

$R_{TV} \to \dfrac{dx}{dt} = 0 \qquad 2t-1 = 0 \to t = 1/2$ $\begin{cases} x = (\frac{1}{2})^2 - \frac{1}{2} \\ y = (\frac{1}{2})^2+1 \end{cases}$

0.5 $P(-\frac{1}{4}, \frac{5}{4})$

c) $\dfrac{d^2y}{dx^2} = \dfrac{\frac{-2}{(2t-1)^2}}{2t-1}$ $\qquad \dfrac{d}{dt}y' = \dfrac{2(2t-1)-2(2t)}{(2t-1)^2} = \dfrac{-2}{(2t-1)^2}$

$\dfrac{d^2y}{dx^2} = \dfrac{-2}{(2t-1)^3}$ ← 0.5 $\qquad 2t-1 = 0 \to \underline{t = 1/2}$

$-\infty \qquad t = 1/2 \qquad +\infty$

0	1
2 (+) C.Arr.	−2 (−) C.Abajo

C. Arriba $]-\infty, 1/2[$

C. Abajo: $]1/2, +\infty[$ 0.5

EJERCICIO 3

$x = 3\cos(\frac{t}{3})$, $y = 3\sin(\frac{t}{3})$

$z = \frac{1}{2}t^2 + 6 \qquad 0 \le t \le 1$

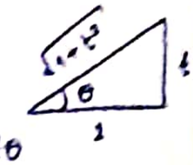
$\dfrac{dx}{dt} = -\sin(\frac{t}{3}) \qquad (\dfrac{dx}{dt})^2 = \sin^2(\frac{t}{3})$ ↗ 1

+

$\dfrac{dy}{dt} = \cos(\frac{t}{3}) \qquad (\dfrac{dy}{dt})^2 = \cos^2(\frac{t}{3})$

+

$\dfrac{dz}{dt} = t \qquad (\dfrac{dz}{dt})^2 = t^2$



$L = \int_0^1 \sqrt{1+t^2}\,dt$ 1.0

$t = \tan\theta$

$dt = \sec^2\theta\,d\theta$

$\sec\theta = \sqrt{1+t^2}$

$= \int \sec\theta \cdot \sec^2\theta\,d\theta$

$u = \sec\theta \qquad dv = \sec^2\theta$

$du = \sec\theta \cdot \tan\theta\,d\theta \qquad v = \tan\theta$

$= \int \sec^3\theta\,d\theta$

$= \sec\theta\tan\theta - \int \sec\theta \cdot \tan^2\theta\,d\theta \qquad \boxed{\tan^2\theta = \sec^2\theta - 1}$

$= \sec\theta\tan\theta - \int \sec^3\theta\,d\theta + \int \sec\theta\,d\theta$

$\int \sec^3\theta\,d\theta = \frac{1}{2}\left(\sec\theta \cdot \tan\theta + \ln|\sec\theta + \tan\theta|\right)$

$L = \frac{1}{2}\left[\sec\frac{\pi}{4}\tan\frac{\pi}{4} + \ln|\sec\frac{\pi}{4} + \tan\frac{\pi}{4}| - \sec 0\tan 0 - \ln|\sec 0 + \tan 0|\right]$ 1.0

$L = \frac{1}{2}\left[\sqrt{2}\,(1) + \ln|\sqrt{2}+1| - \cancel{1(0)}^{0} - \cancel{\ln|1+0|}^{0}\right]$

$L = \frac{1}{2}\left[\sqrt{2} + \ln|\sqrt{2}+1|\right]$ u.L.

0.5

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\sec\frac{\pi}{4} = \sqrt{2}$

$\cos 0 = 1$

$\sec 0 = 1$

$\tan\frac{\pi}{4} = 1$

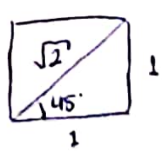
$\tan 0 = 0$

$0 = \tan\theta \to \theta = \tan^{-1}(0)$

$\theta = 0$

$1 = \tan\theta \to \theta = \tan^{-1}(1)$

$\theta = \pi/4$



$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\tan\frac{\pi}{4} = \frac{1}{1} = 1$

0.5

EJERCICIO 4

$\vec{h}(t) = t\hat{i} + 2\sin(t)\hat{j} + 2\cos(t)\hat{k}$

$\vec{h}'(t) = \hat{i} + 2\cos(t)\hat{j} - 2\sin(t)\hat{k}$

$\vec{h}''(t) = -2\sin(t)\hat{j} - 2\cos(t)\hat{k}$

$\vec{u}(t) = \frac{1}{t}\hat{i} + 2\sin(t)\hat{j} + 2\cos(t)\hat{k}$

$\vec{u}'(t) = -\frac{1}{t^2}\hat{i} + 2\cos(t)\hat{j} - 2\sin(t)\hat{k}$

$D_t[\vec{h}'(t)\cdot\vec{u}(t)] = \vec{h}''(t)\cdot\vec{u}(t) + \vec{h}'(t)\cdot\vec{u}'(t)$

$\vec{h}''(t)\cdot\vec{u}(t) = -4\sin^2(t) - 4\cos^2(t) = -4$ 0.5

$\vec{h}'(t)\cdot\vec{u}'(t) = -\frac{1}{t^2} + \cancel{4\cos^2(t) + 4\sin^2(t)}^{\,4} = -\frac{1}{t^2} + 4$ 0.5

$D_t[\vec{h}'(t)\cdot\vec{u}(t)] = -\frac{1}{t^2}$ 0.5

$\vec{R}(t) = \underbrace{\int \ln(-2t)dt}_{①}\,\hat{i} + \underbrace{\int \sec^2(t)\,dt}_{②}\,\hat{j} + \underbrace{\int te^{-t}dt}_{③}\,\hat{k}$

① $u = \ln(-2t)$ $\quad dv = dt$

$du = \frac{\cancel{2}}{\cancel{2}t} = \frac{1}{t}dt \quad v = t$ 0.5

$= t\ln(-2t) - \int dt = t\ln(-2t) - t + C_1$

② $\tan(t) + C_2$

③ $t \;\oplus\; e^{-t}$ $\quad = -te^{-t} - e^{-t} + C_3$

$1 \;\ominus\; -e^{-t}$ 0.5

$0 \quad e^{-t}$

$\vec{R}(t) = (t\ln(-2t) - t + C_1)\hat{i} + (\tan(t) + C_2)\hat{j} + (-te^{-t} - e^{-t} + C_3)\hat{k}$

$\vec{R}(-1) = (-\ln(2) + 1 + C_1)\hat{i} + (\tan(-1) + C_2)\hat{j}$

$+ (\cancel{e^1 - e^1}^{\,0} + C_3)\hat{k}$

$\vec{R}(-1) = 0\hat{i} - \sqrt{2}\hat{j} - \hat{k}$

$C_1 = \ln(2) - 1$ $\qquad C_3 = -1$

$C_2 = -\tan(-1) - \sqrt{2}$

$\vec{R}(t) = (t\ln(-2t) - t + \ln(2) - 1)\hat{i}$

$+ (\tan(t) - \tan(-1) - \sqrt{2})\hat{j}$

$+ (-te^{-t} - e^{-t} - 1)\hat{k}$ 0.5