SOLUCIÓN PARCIAL 3

EJERCICIO #1

$$\int_C \frac{2x^3\sqrt{y}}{z}\, ds$$

(paraboloide) $z = 6 - x^2 - 4y^2$

$z = 2$ (plano)

Intersección $z = z$

$6 - x^2 - 4y^2 = 2$

$4 = x^2 + 4y^2$

$1 = \frac{x^2}{4} + y^2$

elipse: S.M $x = 2$

S.M $y = 1$

• parametrización.

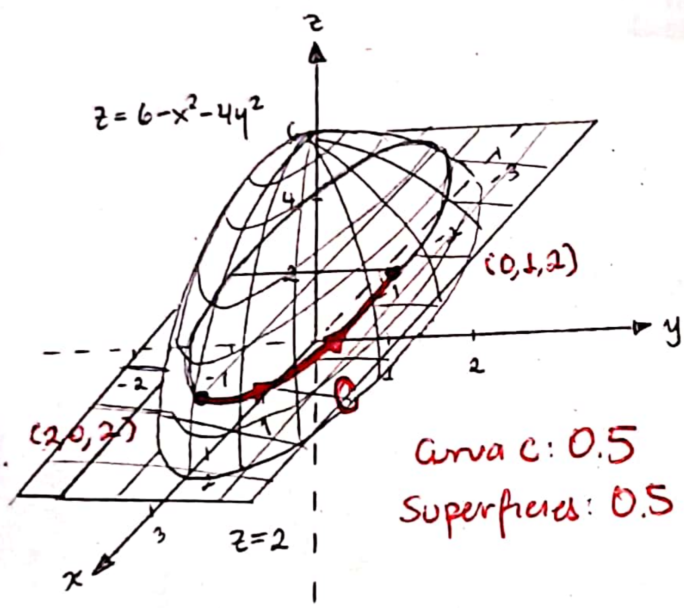
$x = 2\cos t \rightarrow dx = -2\sin t\, dt$

$y = \sin t \rightarrow dy = \cos t\, dt$

$z = 2 \longrightarrow dz = 0.dt$

$0 \leq t \leq \pi/2$

$(2,0,2)$ $\quad$ $(0,1,2)$ $\quad$ 0.5



Curva C: 0.5

Superficies: 0.5

$$ds = \sqrt{4\sin^2 t + \cos^2 t}\, dt$$

$$\int \frac{\cancel{2}(2\cos t)^3\sqrt{\sin t}}{\cancel{2}} \sqrt{4\sin^2 t + \cos^2 t}\, dt$$

$$= 8\int_0^{\pi/2} \cos^3 t \sqrt{\sin t}\, \underbrace{\sqrt{4\sin^2 t + \cos^2 t}}\, dt$$

ó

$$8\int_0^{\pi/2} \cos^3 t \sqrt{\sin t\,[4 - 3\cos^2 t]}\, dt$$

0.5

EJERCICIO #2

$\vec{F}(x,y) = y^3\,\hat{i} + (4 - x^2)\,\hat{j}$ $\qquad 0 \leq t \leq 2$

$y = (x-1)^2$; $0 \leq x \leq 2 \rightarrow \vec{r}(t) = t\,\hat{i} + (t-1)^2\,\hat{j}$

$\vec{F}(\vec{r}(t)) = (t-1)^6\,\hat{i} + (4 - t^2)\,\hat{j}$

$\vec{r}\,'(t) = \hat{i} + 2(t-1)\,\hat{j}$ $\qquad$ 0.5

$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) = (t-1)^6 + 8(t-1) - 2t^2(t-1)$

$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) = (t-1)^6 + 8t - 8 - 2t^3 + 2t^2$

$$\int_0^2 (t-1)^6 - 2t^3 + 2t^2 + 8t - 8\; dt \qquad 0.5$$

$u = (t-1)$

$du = dt$

$\int u^6\, du = \frac{u^7}{7}$

$$= \left.\frac{(t-1)^7}{7} - \frac{t^4}{2} + \frac{2}{3}t^3 + 4t^2 - 8t\right|_0^2$$

$$= \frac{1}{7} - 8 + \frac{16}{3} + \cancel{16} - \cancel{16} \qquad 0.5$$

$$-\left[-\frac{1}{7} - 0\right]$$

0.5

$$= \frac{2}{7} - 8 + \frac{16}{3} = \frac{6 - 168 + 112}{21} = \frac{-50}{21} \text{ u.W.}$$

EJERCICIO #3

$$\vec{F} = \underbrace{\left(\frac{1}{y} - \frac{2z}{x^2}\right)}_{M}\hat{i} - \underbrace{\left(\frac{1}{z} + \frac{x}{y^2}\right)}_{N}\hat{j} + \underbrace{\left(\frac{2}{x} + \frac{y}{z^2}\right)}_{P}\hat{k}$$

$\frac{\partial M}{\partial y} = -\frac{1}{y^2}$ $\qquad$ $\frac{\partial N}{\partial x} = -\frac{1}{y^2}$ ✓

$\frac{\partial M}{\partial z} = -\frac{2}{x^2}$ $\qquad$ $\frac{\partial P}{\partial x} = -\frac{2}{x^2}$ ✓

$\frac{\partial N}{\partial z} = \frac{1}{z^2}$ $\qquad$ $\frac{\partial P}{\partial y} = \frac{1}{z^2}$ ✓

$\therefore \vec{F}$ es conservativo $\qquad$ 0.5

$\int\left(\frac{1}{y} - \frac{2z}{x^2}\right)dx = \frac{x}{y} + \frac{2z}{x} + C(y,z)$

$\int -\left(\frac{1}{z} + \frac{x}{y^2}\right)dy = -\frac{y}{z} + \frac{x}{y} + C(x,z)$

$\int\left(\frac{2}{x} + \frac{y}{z^2}\right)dz = \frac{2z}{x} - \frac{y}{z} + C(x,y)$ $\qquad$ 1.0

$$f = \frac{x}{y} + \frac{2z}{x} - \frac{y}{z} + \boxed{C}$$

$W = f(4,2,-2) - f(2,-1,1)$

$W = 2 + \cancel{(-1)} \cancel{+1} - [\cancel{-2+1} + 1]$

$W = 2\ u.W$ 1.0

al realizar la integral con el orden alreves el trabajo sería negativo:

$-2u.W$ (y no depende de la trayectoria) 0.5

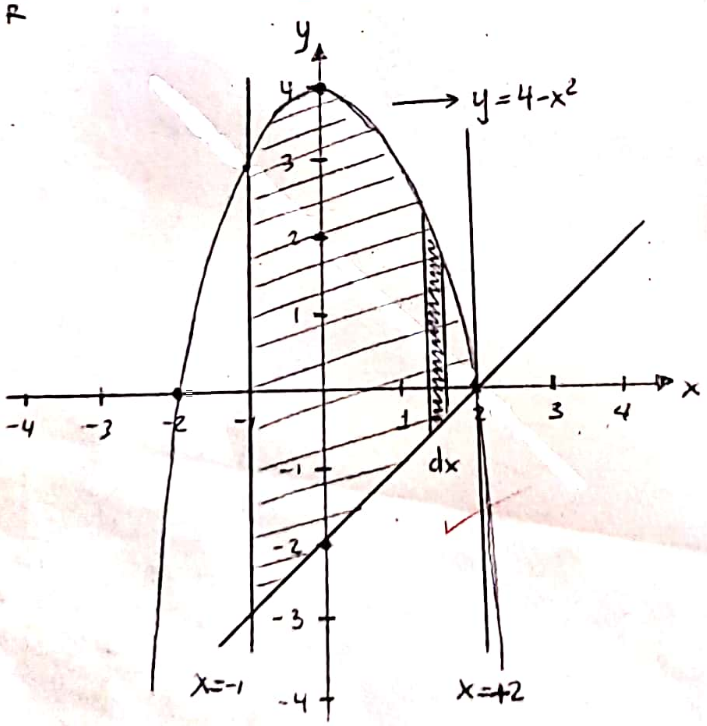
EJERCICIO 4:

$$\oint \underbrace{\frac{x^2 y}{x^2+1}}_{M} dx \underbrace{- \arctan(x)}_{N} dy$$

$$\frac{\partial N}{\partial x} = -\frac{1}{x^2+1} \qquad \frac{\partial M}{\partial y} = \frac{x^2}{x^2+1}$$

$$\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \frac{-1-x^2}{x^2+1} = -\frac{\cancel{x^2+1}}{\cancel{x^2+1}} = -1$$

$-\iint_R dA \longrightarrow$ 0.5



$y = 4 - x^2$ ✓
(parabola)
$x=2 \to y=0$
$x=0 \to y=4$

$y = x-2$ ✓
$x=0 \to y=-2$
$x=2 \to y=0$

1.0
(0.2 c/ gráfica)
+0.2 área.

$$-\int_{-1}^{2}\int_{x-2}^{4-x^2} dy.dx = -\int_{-1}^{2} (4-x^2)-(x-2)\,dx$$ 0.5

$$= -\int_{-1}^{2} -x^2 - x + 6 . dx = -\left[-\frac{x^3}{3} - \frac{x^2}{2} + 6x\right]\Bigg|_{-1}^{2}$$

$$= -\left[\frac{-8}{3} - 2 + 12 - \left(\frac{1}{3} - \frac{1}{2} - 6\right)\right]$$

$$= -\left[-\frac{8}{3} + 10 - \frac{1}{3} + \frac{1}{2} + 6\right]$$

$$= -\left[16 - 3 + \frac{1}{2}\right] = -\left[13 + \frac{1}{2}\right] = -\frac{27}{2}$$

0.5

Cambiaría ya que se debe restar $-\iint_R dA$ pero del hueco el cual es una circunferencia de radio $1/2$

$$4x^2 + 4y^2 = 1 \rightarrow \frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{4}} = 1$$

$$x^2 + y^2 = \frac{1}{4}$$ ✓

$$-\iint_R dA = -A(R) = -\pi\left(\frac{1}{2}\right)^2 = -\frac{\pi}{4}$$

El resultado sería: $-\frac{27}{2} + \frac{\pi}{4}$. 0.5