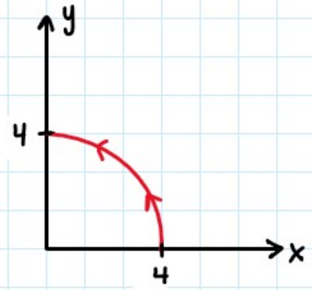
EJEMPLO: Evalue : a) $\int_C 1 + xy^2 \cdot dx$ b) $\int_C 1 + xy^2 \cdot dy$ c) $\int_C 1 + xy^2 \cdot ds$

Dónde C es la cuarta parte de una circunferencia de radio 4 centrada en el origen (I cuadrante) en sentido anti-horario.



$x = 4\cos t \longrightarrow dx = -4\operatorname{sen} t\, dt$

$y = 4\operatorname{sen} t \longrightarrow dy = 4\cos t \cdot dt$

$0 \leq t \leq \pi/2$

a) $\int_C 1 + xy^2 \cdot dx = \int_{\boxed{0}}^{\boxed{\pi/2}} \left[1 + (4\cos t)(4\operatorname{sen} t)^2\right] (-4\operatorname{sen} t\, dt)$

$\downarrow 4 \quad \downarrow 16 \quad \downarrow -4$

$$= \int_0^{\pi/2} \underbrace{-4\operatorname{sen} t}_{①} - \underbrace{256 \operatorname{sen}^3 t \cos t}_{②} \cdot dt$$

$u = \operatorname{sen} t$

$du = \cos t\, dt$

$-256 \int u^3 \cdot du$

$-256 \dfrac{u^4}{4}$

$-64 \operatorname{sen}^4 t.$

$$= 4\cos t - 64 \operatorname{sen}^4 t \Big|_0^{\pi/2}$$

$$= \underbrace{4\cancel{\cos(\pi/2)}}_{0} - \underbrace{4\cancel{\cos(0)}}_{1} - \underbrace{64 \cancel{\operatorname{sen}^4 (\pi/2)} + 64 \cancel{\operatorname{sen}^4 (0)}}_{1^4 = 1 \qquad 0^4 = 0} = -4 - 64 = \underline{-68}$$

b) $\int_C 1 + xy^2 \cdot dy$ c) $\int_C 1 + xy^2 \cdot ds$

$\cos^2 t = \dfrac{1 + \cos(2t)}{2}$

$\operatorname{sen}^2 t = \dfrac{1 - \cos(2t)}{2}$

$x = 4\cos t \longrightarrow dx = -4\operatorname{sen} t\, dt$

$y = 4\operatorname{sen} t \longrightarrow dy = 4\cos t \cdot dt$

$0 \leq t \leq \pi/2$

$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

b) $\int_0^{\pi/2} \left[1 + (4\cos t)(4\operatorname{sen} t)^2\right] (4\cos t \cdot dt) = \int_0^{\pi/2} 4\cos t + 256 \operatorname{sen}^2 t \cos^2 t \cdot dt$

$\left(\frac{dx}{dt}\right)^2 = 16\operatorname{sen}^2 t + \left(\frac{dy}{dt}\right)^2 = 16\cos^2 t$

$16\,[\underbrace{\cancel{\operatorname{sen}^2 t + \cos^2 t}}_{1}]$

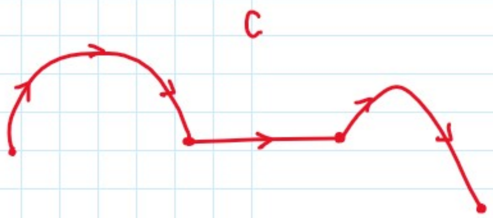
$ds = \sqrt{16} \cdot dt$

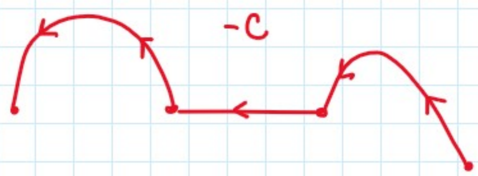
$ds = 4 \cdot dt$

c) $\int_0^{\pi/2} \left[1 + (4\cos t)(4\operatorname{sen} t)^2\right] (4 \cdot dt)$

$$= \int_0^{\pi/2} 4 + 256 \operatorname{sen}^2 t \cos t \cdot dt$$

C





$\int_C \boxed{\sim\sim}\, dx = +4$

$\int_C \boxed{\sim\sim}\, dy = -5$

$\int_C \boxed{\sim\sim}\, ds = +1/2$

$\int_{-C} \boxed{\sim\sim}\, \boxed{dx} = -4$

$\int_{-C} \boxed{\sim\sim}\, \boxed{dy} = +5$

$\int_{-C} \boxed{\sim\sim}\, ds = +1/2$

← NO CAMBIA →

EJEMPLO: Evaluar $\int_C (x+2)\, ds$, dónde C es la curva representada por:

$\vec{r}(t) = \underbrace{t}_{x}\,\hat{i} + \underbrace{\frac{4}{3} t^{3/2}}_{y}\,\hat{j} + \underbrace{\frac{1}{2} t^2}_{z}\,\hat{k}\,; \quad 0 \le t \le 2$

funciones...

$\vec{r}(t) = \underline{\quad}_x\,\hat{i} + \underline{\quad}_y\,\hat{j} + \underline{\quad}_z\,\hat{k}$

campos...

$\vec{F} = \underline{\quad}_M\,\hat{i} + \underline{\quad}_N\,\hat{j} + \underline{\quad}_P\,\hat{k}$

$x = t \longrightarrow dx = 1 \cdot dt$

$y = \frac{4}{3} t^{3/2} \longrightarrow dy = \cancel{\frac{4}{3}} \cdot \cancel{\frac{3}{2}} \cdot t^{1/2} = 2t^{1/2} \cdot dt$

$z = \frac{1}{2} t^2 \longrightarrow dz = \frac{1}{\cancel{2}} \cancel{2} t = t \cdot dt$

$\sqrt{(4t) \cdot t^2} = \sqrt{4t} \cdot \sqrt{t^2}$

$\sqrt{4t + t^2} =$'(

$ds = \sqrt{1^2 + (2t^{1/2})^2 + (t)^2}\, dt = \sqrt{1 + 4t + t^2}\, dt$

$$\int_0^2 (t+2) \cdot \underbrace{\sqrt{1+4t+t^2}}_{u}\, dt$$

$u = 1 + 4t + t^2$

$du = 4 + 2t \cdot dt$

$du = 2(2+t) \cdot dt$

$\frac{du}{2} = (t+2)\, dt$

$$\int \frac{\sqrt{u}\, du}{2}$$

$$= \frac{1}{\cancel{2}} \cdot \frac{u^{3/2}}{\frac{3}{\cancel{2}}} = \frac{1}{3} u^{3/2}$$

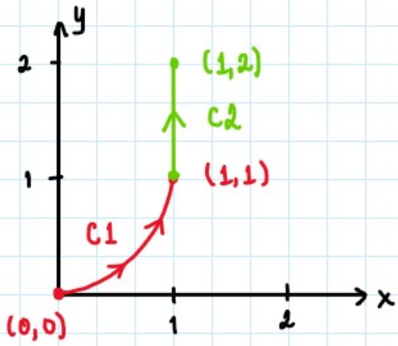
$$= \frac{1}{3} \left[1 + 4t + t^2\right]^{3/2} \Big|_0^2$$

$$= \frac{1}{3}\left\{\left[1+\cancel{4(2)}^{\,8}+\cancel{2^2}^{\,4}\right]^{3/2} - \left[1+\cancel{4(0)}^{\,0}+\cancel{0^2}^{\,0}\right]^{3/2}\right\}$$

$$= \frac{1}{3}\left\{[13]^{3/2} - 1\right\}$$

EJEMPLO: Evalue: $\int_C 2x.ds$. Dónde C consiste en el arco C_1 formado por: $y=x^2$ desde $(0,0)$ hasta $(1,1)$ seguido de C_2 que es un segmento rectilíneo de $(1,1)$ hasta $(1,2)$.

(x,y) $\qquad$ (x,y)



$$\int_C 2x\,ds = \int_{C1} 2x.ds + \int_{C2} 2x.ds$$

- Analizando C1.

• $x = t \longrightarrow dx = 1.dt$

$y = t^2 \longrightarrow dy = 2t.dt$

$0 \le t \le 1$

$ds = \sqrt{1^2 + (2t)^2}\ dt$

$ds = \sqrt{1+4t^2}\ dt$

• $x = t$ $\qquad$ $x = \text{Sen}\, t$

$y = \sqrt{1+t}$ $\qquad$ • $y = t$

$$\int_0^1 2t\sqrt{1+4t^2}\,.dt$$

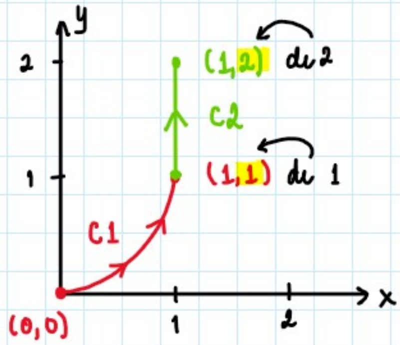
$u = 1+4t^2$ $\qquad$ $du = 4(2t)\,dt$

$du = 8t.dt$ $\qquad$ $\frac{du}{4} = 2t.dt$

$$\int \frac{\sqrt{u}\ du}{4} = \frac{1}{\cancel{4}_{2}}\,\frac{u^{3/2}}{\frac{3}{\cancel{2}}} = \frac{1}{6}\left[1+4t^2\right]^{3/2}\Big|_0^1$$

$$= \frac{1}{6}\left\{\left[1+\cancel{4(1)^2}^{\,4}\right]^{3/2} - \left[\cancel{1+0}^{\,1}\right]^{3/2}\right]$$

$$= \frac{1}{6}\left\{[5]^{3/2} - 1\right\}\Big|_{C1}$$



- Analizando C2

$x = 1 \longrightarrow dx = 0$

• $y = t \longrightarrow dy = 1.dt$

$1 \le t \le 2$

$ds = \sqrt{0^2 + 1^2} = 1.dt$

$ds = dt$

$$\int_c 2x.ds = \int_1^2 2(1)\,dt = 2t\Big|_1^2 = 2\left[\cancel{2-1}^{\,1}\right] = 2\Big|_{C2}$$

$$\int_C 2x\,ds = \int_{C1} 2x.ds + \int_{C2} 2x.ds = \frac{1}{6}\left\{[5]^{3/2} - 1\right\} + 2.$$

$x = \boxed{z^2+1}$, $y = \boxed{z+2}$ $(1, 2, 0) \longrightarrow (5, 4, 2)$

x, z y, z

Forma 1:

$$\begin{cases} x = t^2+1 \\ y = t+2 \qquad 0 \le t \le 2. \\ z = t \end{cases}$$

Forma 2: $x = z^2+1$ $y = z+2$ $z^2 = z^2$

$x - 1 = z^2$ $\boxed{y-2=z}$ $x-1 = (y-2)^2$

$(y-2)^2 = z^2$ $x = (y-2)^2+1$

$\pm\sqrt{x-1} = z$

$$\begin{cases} x = (t-2)^2+1 \\ y = t \\ z = t-2 \end{cases}$$

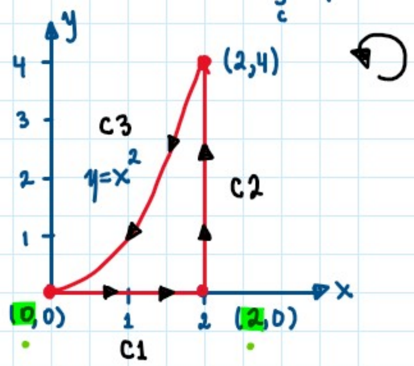
$2 \le t \le 4$

EJERCICIO ASIGNADO: Evaluar $\int_C x^2 - y + 3z.ds$, dónde C es el segmento rectilíneo comprendido desde $(0,0,0)$ hasta $(1,2,1)$

R/ $\frac{5}{6}\sqrt{6}$

- Integrales de línea de curvas cerradas.

EJEMPLO: Evaluar $\oint_C y^2.dx - x^2 dy$, dónde C es la curva presente:



$$\oint_C = \int_{C1} + \int_{C2} + \int_{C3}$$

- C1.

$x = t \longrightarrow dx = dt$

$y = 0 \longrightarrow dy = 0$

$0 \le t \le 2$

$$\int y^2.dx - x^2.dy$$

$$\int_0^2 \cancel{0^2 dt}^{\,0} - \cancel{t^2 0}^{\,0} = 0.$$

$C1 = 0$

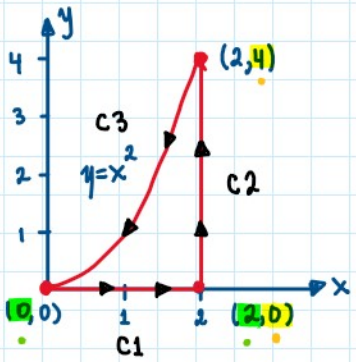
- Parametrizando "y"

1.P. ¿Cómo se llama función? $y=0 \longrightarrow dy=0$

2.P. Derivar en rectangulares.

3.P Sustituir en la integral $\int \cancel{y}^2 \cdot dx - x^2 \cdot \cancel{dy} = \int_0^2 \cancel{0^2} \cdot dx - x^2 \cdot \cancel{0} = 0.$

en terminos de x



- C2:

$x=2 \longrightarrow dx=0$

$y=t \longrightarrow dy=dt$

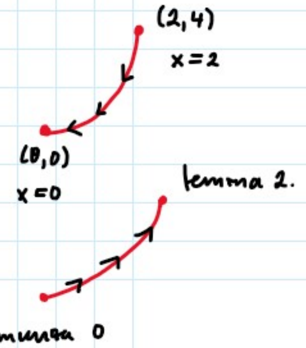
$0 \le t \le 4$

$\int y^2 \cdot dx - x^2 \cdot dy$

$\int_0^4 \cancel{t^2 \cdot (0)} - (2)^2 \cdot dt = -4 \int_0^4 dt = -4 \cdot t \Big|_0^4$

$= -4(4-0) = -4(4) = -16.$ $C2 = -16$

- C3: $y=x^2$



$x=t$

$y=t^2$

$0 \le t \le 2$

OPCIÓN INTEGRAR $\int_2^0$ ó INTEGREMOS $\int_0^2$ al final cambios signos. *

$x=t \longrightarrow dx=dt$

$y=t^2 \longrightarrow dy=2t\,dt$

$0 \le t \le 2$

$\int y^2 \cdot dx - x^2 \cdot dy$

$\int_0^2 (t^2)^2 dt - (t)^2 (2t \cdot dt) = \int_0^2 t^4 - 2t^3 \cdot dt = \frac{t^5}{5} - \frac{\cancel{2}t^4}{\cancel{4}\,2} \Big|_0^2 = \left(\frac{2^5}{5} - \frac{2^4}{2} - \cancel{\frac{0^5}{5}} + \cancel{\frac{0^4}{2}} \right)$

$= \frac{32}{5} - \frac{8}{1} = \frac{32-40}{5} = -\frac{8}{5}$

$\oint_c = 0 - \frac{16}{1} + \frac{8}{5} = \frac{-80+8}{5} = \frac{-72}{5}$

$* + \frac{8}{5}$ $C3: +\frac{8}{5}$

EJERCICIO: Calcule: $\oint_c xy\,dx + x^2 dy + z^2 dz$ donde C es la curva de intersección entre: $x^2 + \left(y-\frac{1}{2}\right)^2 = \frac{1}{4}$ con $y=z$. Se pide graficar la curva C.

Circunf. "xy" $C(0, 1/2)$ $r = 1/2$

recta. → plano.

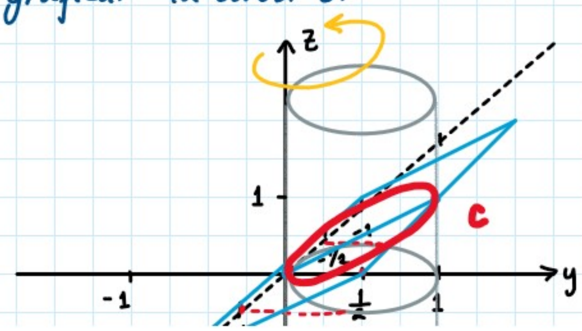
↳ Superficie cilíndrica

$y=z$

$z=0 \rightarrow y=0$ $(0,0)$

$z=1 \rightarrow y=1$ $(1,1)$

$x = 1/2 \cos t$



$x = \tfrac{1}{2}\cos t$

$y = \tfrac{1}{2}\sin t + \tfrac{1}{2}$

$\boxed{y = z}$

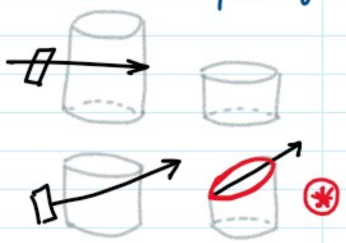
$z = \tfrac{1}{2}\sin t + \tfrac{1}{2}$

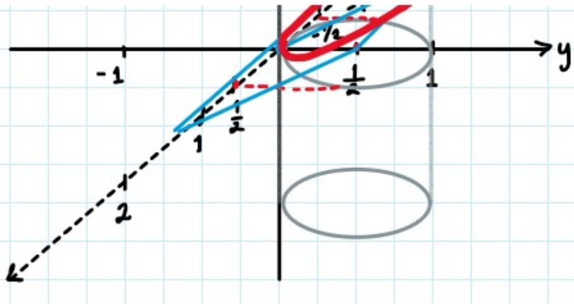
$0 \le t \le 2\pi$

$z=0 \to y=0 \quad (0,0)$

$z=1 \to y=1 \quad (1,1)$

plano "yz"





$$\oint_C xy\,dx + x^2\,dy + z^2\,dz$$

$x = \tfrac{1}{2}\cos t \longrightarrow dx = -\tfrac{1}{2}\sin t\,dt$

$y = \tfrac{1}{2}\sin t + \tfrac{1}{2} \longrightarrow dy = \tfrac{1}{2}\cos t\,dt$

$z = \tfrac{1}{2}\sin t + \tfrac{1}{2} \longrightarrow dz = \tfrac{1}{2}\cos t\,dt$

$0 \le t \le 2\pi$

$(\sin t\cos t)$

$\left[\tfrac{1}{4}\sin^2 t + \tfrac{1}{2}\sin t + \tfrac{1}{4}\right]$

$$\int_0^{2\pi} \underbrace{(\tfrac{1}{2}\cos t)(\tfrac{1}{2}\sin t + \tfrac{1}{2})(-\tfrac{1}{2}\sin t\,dt)} + \underbrace{(\tfrac{1}{2}\cos t)^2(\tfrac{1}{2}\cos t\,dt)} + \underbrace{(\tfrac{1}{2}\sin t + \tfrac{1}{2})^2(\tfrac{1}{2}\cos t\,dt)}$$

$$\underbrace{\cancel{-\tfrac{1}{8}\sin^2 t\cos t} - \tfrac{1}{8}\sin t\cos t} + \underbrace{\tfrac{1}{8}\cos^3 t} + \underbrace{\cancel{\tfrac{1}{8}\sin^2 t\cos t} + \tfrac{1}{4}\sin t\cos t + \tfrac{1}{8}\cos t}$$

$$= \tfrac{1}{8}\sin t\cos t + \tfrac{1}{8}\cos^3 t + \tfrac{1}{8}\cos t$$

$$\tfrac{1}{8}\int_0^{2\pi} \sin t\cos t + \cos^3 t + \cos t\,.\,dt = (\dots)$$

- Integral de linea de campo vectorial.

$W = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$

EJERCICIO: Hallar el trabajo invertido realizado por el campo: $\vec{F}(x,y,z) = -\tfrac{1}{2}x\,\hat{\imath} - \tfrac{1}{2}y\,\hat{\jmath} + \tfrac{1}{4}\hat{k}$, sobre una particula que se mueve a lo largo de la hélice

$\int_C \vec{F}\cdot\vec{T}\,ds = \int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt$

$\int_C \vec{F}\cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt$

$\vec{r}(t) = \underbrace{\cos t}_{x}\,\hat{\imath} + \underbrace{\sin t}_{y}\,\hat{\jmath} + \underbrace{t}_{z}\,\hat{k}$ desde $(1,0,\boxed{0})$ hasta $(-1,0,\boxed{3\pi})$

$x = \cos t$

$y = \sin t$

$z = t$

$$W = \int_a^b \underbrace{\vec{F}(\vec{r}(t))} \cdot \underbrace{\vec{r}\,'(t)}\,.\,dt$$

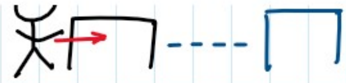
$0 \le t \le 3\pi$

$\underbrace{\ }_{=}$ W ✓

$\vec{F}(x,y,z) = -\tfrac{1}{2}x\,\hat{\imath} - \tfrac{1}{2}y\,\hat{\jmath} + \tfrac{1}{4}\hat{k}$

- $\vec{F}(\vec{r}(t)) = -\tfrac{1}{2}\cos t\,\hat{\imath} - \tfrac{1}{2}\sin t\,\hat{\jmath} + \tfrac{1}{4}\hat{k}$ ←

- $\vec{F}(\vec{r}(t)) = -\frac{1}{2}\cos t\,\hat{\imath} - \frac{1}{2}\sin t\,\hat{\jmath} + \frac{1}{4}\hat{k}$
- $\vec{r}\,'(t) = -\sin t\,\hat{\imath} + \cos t\,\hat{\jmath} + \hat{k}$



$$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) = \cancel{\tfrac{1}{2}\sin t\cos t} - \cancel{\tfrac{1}{2}\sin t\cos t} + \frac{1}{4} = \frac{1}{4}$$

$$\int_0^{3\pi} \frac{1}{4}\,dt = \frac{1}{4}t\Big|_0^{3\pi} = \frac{3\pi}{4}\ \text{u.W.}$$

EJERCICIO: Evalúe $\int_C \vec{F}\cdot d\vec{r}$, donde $\vec{F}(x,y,z) = xy\,\hat{\imath} + yz\,\hat{\jmath} + xz\,\hat{k}$ y C es la curva cúbica torcida: $x = t$, $y = t^2$, $z = t^3$; $0 \le t \le 1$

$$\vec{r}(t) = \underbrace{t}_{x}\,\hat{\imath} + \underbrace{t^2}_{y}\,\hat{\jmath} + \underbrace{t^3}_{z}\,\hat{k}$$

$$\int_C \vec{F}\cdot d\vec{r} = \int_a^b \underbrace{\vec{F}(\vec{r}(t))}\cdot \underbrace{\vec{r}\,'(t)}\,dt$$

$$\vec{F}(\vec{r}(t)) = \underset{x}{t}\,(\underset{y}{t^2})\,\hat{\imath} + \underset{y}{t^2}(\underset{z}{t^3})\,\hat{\jmath} + \underset{x}{t}\,(\underset{z}{t^3})\,\hat{k} = t^3\,\hat{\imath} + t^5\,\hat{\jmath} + t^4\,\hat{k}$$

$$\vec{r}\,'(t) = \hat{\imath} + 2t\,\hat{\jmath} + 3t^2\,\hat{k}$$

$$\vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t) = t^3 + 2t^6 + 3t^6 = t^3 + 5t^6 \longrightarrow \int_0^1 t^3 + 5t^6\,.\,dt$$

$$= \frac{t^4}{4} + \frac{5t^7}{7}\Big|_0^1 = \frac{1}{4} + \frac{5}{7} - 0 - 0 = \frac{7+20}{28} = \frac{27}{28}$$

ASIGNACIÓN: Calcule la integral de campo vectorial: $\vec{F}(x,y) = \underbrace{(x^2 - 2xy)}_{M}\hat{\imath} + \underbrace{(y^2 - 2xy)}_{N}\hat{\jmath}$ a lo largo de la parabola: $\underbrace{y = x^2}_{\text{Rectangulares.}}$ desde $(\boxed{-1},1)$ hasta $(\boxed{1},1)$.

R/ $-\frac{14}{15}$

- Parametricas.
  - $x = t$
  - $y = t^2$
- Función vectorial.
  - $\vec{r}(t) = t\,\hat{\imath} + t^2\,\hat{\jmath}$
  - $\vec{r}\,'(t) = \hat{\imath} + 2t\,\hat{\jmath}$

$$\vec{F}(\vec{r}(t)) = (t)^2 - 2(t)(t^2)\,\hat{\imath} + (t^2)^2 - 2(t)(t^2)\,\hat{\jmath}$$

$$\vec{F}(\vec{r}(t)) = (t^2 - 2t^3)\,\hat{\imath} + (t^4 - 2t^3)\,\hat{\jmath}$$

$$\int_{-1}^{1} t^2 - 2t^3 + 2t^5 - 4t^4\,.\,dt = (\dots) = -\frac{14}{15}$$

$$\int M\,dx + N\,dy + \cancel{P\,dz}^{\,0}$$

- Parametricas

$$\int_c M\,dx + N\,dy + \cancel{P\,dz}^{\,0}$$

$$\int_c (x^2-2xy)\,dx + (y^2-2xy)\,dy = \frac{-14}{15}$$

- Paramétricas

$x = t \rightarrow dx = dt$

$y = t^2 \rightarrow dy = 2t \cdot dt$

□ Integrales de línea independientes de la trayectoria (aplicando el teorema fundamental del cálculo)

EJEMPLO: Hallar el trabajo realizado por:

$\vec{F}(x,y) = \frac{1}{2}xy\,\hat{i} + \frac{1}{4}x^2\,\hat{j}$ sobre una partícula que se mueve de $(0,0)$ a $(1,1)$ a lo largo de $y=x$.

a) Desarrollando I.L. de campo vectorial.

$x = t$ $y = t$ $0 \le t \le 1$

$\vec{r}(t) = t\,\hat{i} + t\,\hat{j}$

$\vec{r}\,'(t) = \hat{i} + \hat{j}$

$$W = \int_c \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$$

$$\vec{F}(\vec{r}(t)) = \frac{1}{2}(t)(t)\,\hat{i} + \frac{1}{4}(t)^2\,\hat{j} = \frac{1}{2}t^2\,\hat{i} + \frac{1}{4}t^2\,\hat{j}$$

$$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) = \frac{1}{2}t^2 + \frac{1}{4}t^2 = \frac{3}{4}t^2$$

$$\frac{3}{4}\int_0^1 t^2\,dt = \frac{1}{4}t^3 \Big|_0^1 = \frac{1}{4}(1^3 - 0^3) = \frac{1}{4} \text{ u.W.}$$

b) Desarrollando I.L. independiente de trayectoria.

¿ $\vec{F}$ es conservativo?

$$\vec{F}(x,y) = \underbrace{\frac{1}{2}xy}_{M}\,\hat{i} + \underbrace{\frac{1}{4}x^2}_{N}\,\hat{j}$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

$$\frac{\partial M}{\partial y} = \frac{1}{2}x \checkmark \qquad \frac{\partial N}{\partial x} = \frac{1}{4}(2x) = \frac{1}{2}x \checkmark$$

$\vec{F}$ es conservativo

☑ aplicar este método.

$$\int \frac{1}{2}xy \cdot dx = \frac{x^2y}{4} + C(y)$$

$$\int \frac{1}{4}x^2 \cdot dy = \frac{x^2y}{4} + C(x)$$

$$\boxed{f = \frac{x^2y}{4} + C}$$ ← mueve de $(0,0)$ a $(1,1)$

$x=1, y=1$ $x=0, y=0$

$$W = \underset{\text{final}}{f(1,1)} - \underset{\text{inicial}}{f(0,0)} = \frac{1^2 \cdot 1}{4} + \cancel{C} - \left[\frac{0^2 \cdot 0}{4} + \cancel{C}\right] = \frac{1}{4} - 0 = \frac{1}{4} \text{ u.W.}$$

EJEMPLO: Evalue la I.L. $\int_c \vec{F}\cdot d\vec{r}$, dónde C es la curva definida por:

$\vec{r}(t) = e^t \cdot \operatorname{sen} t\ \hat{\imath} + e^t \cos t\ \hat{\jmath}$; $0 \le t \le \pi$. Además $\vec{F}(x,y) = \underbrace{(3+2xy)}_{M}\hat{\imath} + \underbrace{(x^2-3y^2)}_{N}\hat{\jmath}$

Utilizando T.F. para I.L.

inicio $\vec{r}(0) = \cancel{e^0}\,\cancel{\operatorname{sen} 0}\,\hat{\imath} + \cancel{e^0}\,\cancel{\cos 0}\,\hat{\jmath} = 0\hat{\imath} + 1\hat{\jmath}$ $(0,1)$ inicial.

fin $\vec{r}(\pi) = e^\pi \cancel{\operatorname{sen}\pi}\,\hat{\imath} + e^\pi \cancel{\cos\pi}\,\hat{\jmath} = 0\hat{\imath} - e^\pi\hat{\jmath}$ $(0,-e^\pi)$ final.

¿$\vec{F}$ es conservativa?

$\frac{\partial M}{\partial y} = 2x$ ✓ $\frac{\partial N}{\partial x} = 2x$ ✓ $\vec{F}$ es conservativa ☑

$\int (3+2xy)\,dx = 3x + x^2y + C(y)$

$\int (x^2-3y^2)\,dy = x^2y - y^3 + C(x)$

$$\boxed{f = 3x + x^2y - y^3 + C}$$ ← Evaluar...

$\int_c \vec{F}\cdot d\vec{r} = f(0,-e^\pi) - f(0,1) = \cancel{3(0)} + \cancel{(0)^2(-e^\pi)} - (-e^\pi)^3 \cancel{+C} - [\cancel{3(0)} + \cancel{0^2(1)} - 1^3 \cancel{+C}]$

$$\int_c \vec{F}\cdot d\vec{r} = e^{3\pi} + 1$$

EJEMPLO: Sabiendo que $\vec{F}(x,y,z) = \underbrace{2xy}_{M}\hat{\imath} + \underbrace{(x^2+z^2)}_{N}\hat{\jmath} + \underbrace{2yz}_{P}\hat{k}$ es conservativo.

Determine $\int_c \vec{F}\cdot d\vec{r}$ donde C es una curva suave desde $(1,1,0)$ hasta $(0,2,3)$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ ☑ $\frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$ ☑ $\frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}$ ☑ $\vec{F}$ es conservativa.

$2x = 2x$ $2z = 2z$ $0 = 0$

$\int 2xy \cdot dx$

$\int x^2+z^2 \cdot dy$ ⇨ $\boxed{f = x^2y + yz^2 + C}$

$\int 2yz \cdot dz$

$\int_c \vec{F}\cdot d\vec{r} = f(0,2,3) - f(1,1,0) = (\dots) = 17$

EJERCICIO: Evalue: $\int_c \underbrace{(2x\operatorname{sen} y)}_{M}dx + \underbrace{(x^2\cos y - 3y^2)}_{N}dy$ dónde C es una curva suave que varia desde $(-1,0)$ hasta $(5,1)$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ ✓

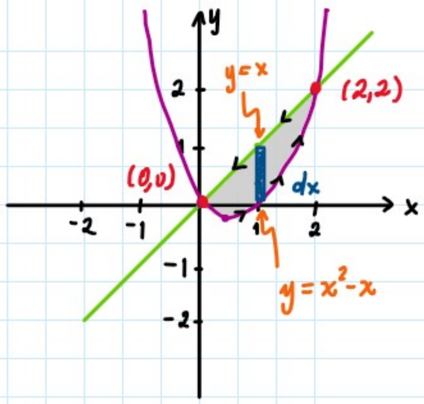
$f = x^2 \sin y - y^3 + C$

$\int_C = 25 \sin(1) - 1$

Teorema de Green.

EJEMPLO: Verifique el teorema de Green para: $\oint_C (y-x)\,dx + (2x-y)\,dy$

donde C es la curva formada por la acotación: $y=x \wedge y=x^2-x$



$y=x \to$ recta $(0,0) \wedge (1,1)$

$y = \underbrace{x^2 - x}$ → parabola.

$\underbrace{x^2 - x + \frac{1}{4}}_{TCP} - \frac{1}{4}$

$\left(x-\frac{1}{2}\right)^2$

$y = \left(x-\frac{1}{2}\right)^2 - \frac{1}{4}$

$x=\frac{1}{2}$

$y=-\frac{1}{4}$

Puntos de intersecc.

$y=y$

$x = x^2 - x$

$0 = x^2 - x - x$

$0 = x^2 - 2x$

$0 = x(x-2)$

$\boxed{x=0}$ ✓ $\quad y=x \quad y=0$

$x-2=0 \quad \boxed{x=2}$ ✓ $\quad y=x \quad y=2$

$\oint_C \underbrace{(y-x)}_{M}\,dx + \underbrace{(2x-y)}_{N}\,dy$

$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$

$dA = dx.dy$

$dA = dy.dx$ ✓

$\frac{\partial N}{\partial x} = 2 \quad - \quad \frac{\partial M}{\partial y} = 1 \quad = 1 \longrightarrow \iint_R 1.\,dA$

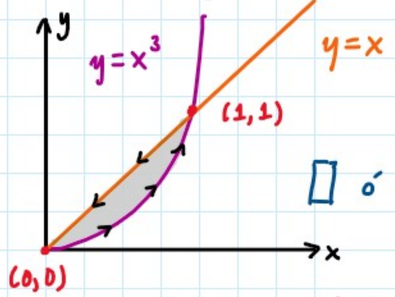
$$\int_{x=\boxed{0}}^{x=\boxed{2}} \int_{\boxed{x^2-x}=y}^{\boxed{x}=y} dy\,dx$$

$$= \int_0^2 \int_{x^2-x}^{x} dy.dx = \int_0^2 y\Big|_{x^2-x}^{x} dx = \int_0^2 x-(x^2-x)\,dx = \int_0^2 x - x^2 + x.dx = \int_0^2 2x - x^2.dx$$

$$= x^2 - \frac{x^3}{3}\Big|_0^2 = 4 - \frac{8}{3} - 0 + 0 = \frac{4}{1} - \frac{8}{3} = \frac{12-8}{3} = \frac{4}{3}$$

EJERCICIO: Calcule $\oint_C \underbrace{y^3}_{M}\,dx + \underbrace{(x^3+3xy^2)}_{N}\,dy$, donde C es la curva de intersección en el primer cuadrante entre: $y=x \wedge y=x^3$.

1.P. Graficar y encontrar puntos de intersección.



$$y=y \qquad x=x^3 \qquad 0=x^3-x \longrightarrow 0=x(x^2-1) \qquad x=0 \quad x^2=1 \quad x=\pm 1$$

$\square$ ó $\square$

$$\underbrace{\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)}$$

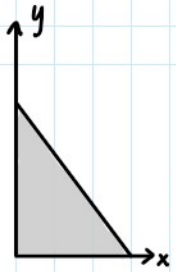
$$\iint_R 3x^2\, dA$$

2.P. Derivadas parciales.

$$\frac{\partial N}{\partial x} = 3x^2 + \cancel{3y^2}$$

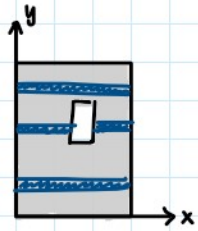
$$\frac{\partial M}{\partial y} = \cancel{3y^2}$$

$$(\ldots) = \frac{1}{4}$$



$$\longrightarrow \iint_R 3\cdot dA \qquad 3\iint_R dA \qquad 3\,A(R) \qquad 3\cdot\frac{b\times h}{2}$$

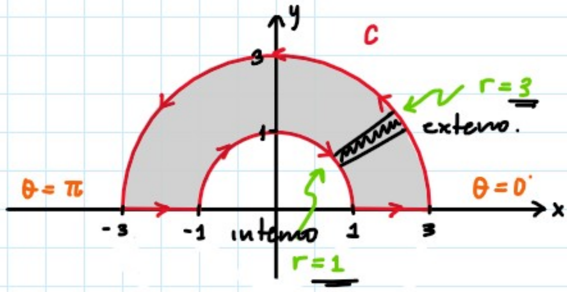
$$\hookrightarrow \iint_R x \text{ ó } y\cdot dA \quad \times \quad =\text{'(}$$



$$\longrightarrow \iint_R \left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right) dA \;\; \square \;\; - \iint_R \left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)\cdot dA \;\; \square$$

$\hookrightarrow$ 4 integrales...

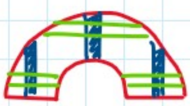
EJEMPLO: Evalue $\oint \underbrace{[\arctan(x) + y^2]}_{M} dx + \underbrace{[\ln(y) - x^2]}_{N} dy$ dónde C es la curva.



$$\frac{\partial N}{\partial x} = -2x \qquad \frac{\partial M}{\partial y} = 2y$$

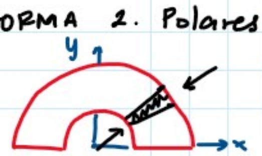
$$\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right) = -2x-2y = -2(x+y)$$

FORMA 1. Rectangulares.



$3\iint \ldots$

FORMA 2. Polares.



$1\iint$

FORMA 3. (Hueco)

$2\iint$

$$\iint_R -2(x+y).\,dA \leftarrow \text{T.G.}$$

$$-2\iint_R (x+y).\,dA.$$

CONVERTIR:

$\underbrace{x^2+y^2}=9$ $\underbrace{x^2+y^2}=1$

$r^2=9$ $r^2=1$

$r=3$ ✓ $r=1$ ✓

Integrando: $x+y$

$r\cos\theta + r\operatorname{Sen}\theta$

$r\,[\cos\theta+\operatorname{Sen}\theta]$ ✓

$dA = r.\,dr.\,d\theta$ ✓

$$-2\int_{\boxed{0}}^{\boxed{\pi}}\int_{\boxed{1}}^{\boxed{3}} (r\,[\cos\theta+\operatorname{Sen}\theta]).\,r.\,dr.\,d\theta.$$

$$-2\int_0^{\pi}\int_1^3 r^2\,[\cos\theta+\operatorname{Sen}\theta]\,dr.\,d\theta$$

$$\int r^2.\,dr = \frac{r^3}{3}\Big|_1^3 = \frac{3^3}{3} - \frac{1^3}{3} = 9-\frac{1}{3}$$

$$= \frac{26}{3}$$

$$-2\left(\frac{26}{3}\right)\int_0^{\pi} \cos\theta + \operatorname{Sen}\theta.\,d\theta$$

$$-\frac{52}{3}\left[\operatorname{Sen}\theta - \cos\theta\right]\Big|_0^{\pi} = -\frac{52}{3}\left[\overset{0}{\cancel{\operatorname{Sen}\pi}} - \overset{-1}{\cancel{\cos\pi}} - \overset{0}{\cancel{\operatorname{Sen}0}} + \overset{\boxed{1}}{\cancel{\cos 0}}\right]$$

$\boxed{+1}$

$$= -\frac{52}{3}[2] = -\frac{104}{3}$$

EJERCICIO: Calcule el trabajo realizado por: $\vec{F}(x,y) = y^3\,\hat{i} + (x^3+3xy^2)\,\hat{j}$ al mover la particula alrededor de una circunferencia de radio 3 en el origen.