SUCESIONES.

Una sucesión es una función cuyo dominio es el conjunto de los enteros positivos y cuyo rango es el conjunto de los números reales.

Ejemplo demostrativo:

Dom.	1	2	3	4	5	...	n
	1^2	2^2	3^2	4^2	5^2		
Rang.	1	4	9	16	25	...	n^2

Dada la sucesión: 1, 4, 9, 16, 25, …

↳ lista de números con un patrón escondido.

Término General.
* Nos indica el patrón y define totalmente la sucesión.

¿Cuál es el séptimo termino de la sucesión?

T.G. → n^2 $\quad n=7 \quad 7^2 = 49$

NOTACIÓN.

$$\{f(n)\}, \{a_n\}, \{b_n\}, \{c_n\}, \{u_n\}$$

Una sucesión queda totalmente definida por su término general…

CASOS ESPECIALES.

$-1, 2, -3, 4, -5, 6, \dots$

$-, +, -, +, -, +, \dots$

inicia ⊖

$(-1)^n$ signo…

Dom. $n=1$	$n=2$	$n=3$	$n=4$	...
$(-1)^1$	$(-1)^2$	$(-1)^3$	$(-1)^4$	
-1	$+1$	-1	$+1$	

inicia ⊖

$+1, -2, +3, -4, +5, -6, \dots$

$+, -, +, -, +, -, \dots$

inicia ⊕

$(-1)^{n+1}$ signo… $\leftrightarrow (-1)^{n-1}$

Dom. $n=1$	$n=2$	$n=3$	$n=4$
$(-1)^{1+1}$	$(-1)^{1+2}$	$(-1)^{1+3}$	$(-1)^{1+4}$
$+1$	-1	$+1$	-1

inicia ⊕

EJEMPLOS: Encontrar el término general de:

a) $\left\{-\frac{1}{2}, \frac{3}{5}, -\frac{1}{2}, \frac{7}{17}, -\frac{9}{26}, \dots\right\}$ ($n=1$, $n=2$, $n=3$, $n=4$, $n=5$; numeradores $+2, +2, +6, +2$; denominadores $+3, -3, +15, +9$)

$-\frac{1}{2} \cdot \frac{5}{5} = -\frac{5}{10}$

$\boxed{(-1)^n}$ signo. $\quad \dfrac{\boxed{2n-1}}{\boxed{n^2+1}}$ num. / den.

$$\left\{-\frac{1}{2}, \frac{3}{5}, -\frac{5}{10}, \frac{7}{17}, -\frac{9}{26}, \dots\right\}$$

R// $(-1)^n \dfrac{2n-1}{n^2+1}$

- Signos: $-, +, -, +, -$ ⟶ $(-1)^n$
- Numerador: 1 ✓ 3 ✓ 5 ✓ 7 9

"n+2"
- $n=1 \to 1+2 = 3$ ✗
- $n=2 \to 2+2 = 4$ ✗
- $n=3 \to 3+2 = 5$ ✗

"2n"
- $n=1 \to 2(1) = 2$ ✗
- $n=2 \to 2(2) = 4$ ✗
- $n=2 \to 2(3) = 6$ ✗

"2n-1"
- $n=1 \to 2(1)-1 = 1$ ✓
- $n=2 \to 2(2)-1 = 3$ ✓
- $n=3 \to 2(3)-1 = 5$ ✓

$n+2$ $n=1 \rightarrow 1+2=3$ ✗ ; $n=2 \rightarrow 2+2=4$ ✗ ; $n=3 \rightarrow 3+2=5$ ✗

"$2n$" $n=1 \rightarrow 2(1)=2$ ✗ ; $n=2 \rightarrow 2(2)=4$ ✗ ; $n=3 \rightarrow 2(3)=6$ ✗

"$2n-1$" $n=1 \rightarrow 2(1)-1=1$ ✓ ; $n=2 \rightarrow 2(2)-1=3$ ✓ ; $n=3 \rightarrow 2(3)-1=5$ ✓

- **Denominador:** 2 ✓ 5 ✓ 10 ✓ 17 26

"n^2+1"

$n=1 \rightarrow 1^2+1=2$ ✓

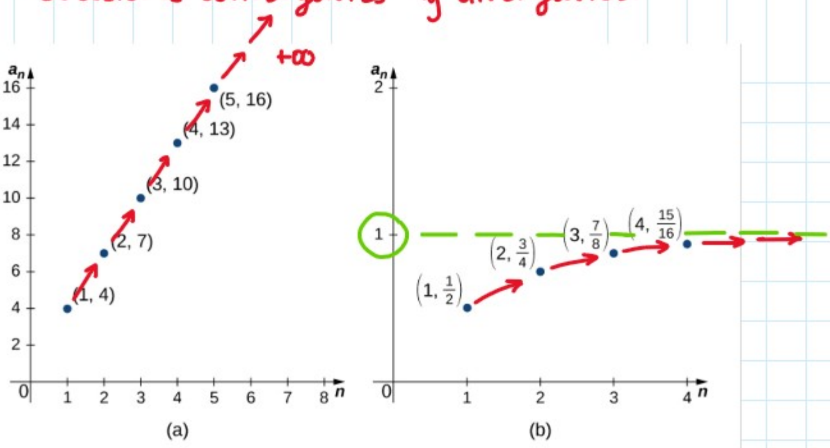
$n=2 \rightarrow 2^2+1=5$ ✓

$n=3 \rightarrow 3^2+1=10$ ✓

b) $\left\{ \operatorname{Sen}\left(\frac{\pi}{3}\right), \cancel{-\operatorname{Sen}(\pi)}^{\,0}, \operatorname{Sen}\left(\frac{9\pi}{5}\right), -\operatorname{Sen}\left(\frac{8\pi}{3}\right), \operatorname{Sen}\left(\frac{25\pi}{7}\right), \dots \right\}$

$$\underbrace{\boxed{(-1)^{n+1}}}_{\text{signo}} \operatorname{Sen}\left(\frac{\overset{\text{num.}}{\boxed{n^2}}\,\pi}{\underset{\text{den.}}{\boxed{n+2}}} \right)$$

Sucesiones convergentes y divergentes.



¿Cómo saber si la sucesión converge o diverge?

$$\lim_{n\to\infty} a_n = L$$

Sí $L = +\infty$ ó $-\infty$ $\therefore a_n$ diverge

Sí $L = (+), (-), 0$ $\therefore a_n$ converge.

$n=1 \rightarrow 4$ $n=1 \rightarrow 1/2$

$n=2 \rightarrow 7$ $n=2 \rightarrow 3/4$

(a) 4, 7, 10, 13, 16, ...   (b) 1/2, 3/4, 7/8, 15/16, ...

Sucesión divergente.   Sucesión convergente.

EJEMPLOS: Determine si las siguientes sucesiones convergen o divergen.

a) $a_n = \dfrac{8n^2+n}{-n^3+8}$

FORMA 1

$$\lim_{n\to\infty} \frac{8n^2+n}{-n^3+8} = \frac{8(\infty)^2+\infty}{-(\infty)^3+8} = \frac{\infty}{-\infty} \quad (L'H) \quad (\cdots) =$$

$$\lim_{n\to\infty} \frac{8n^2+n}{-n^3+8} = \frac{8(\infty)^2+\infty}{-(\infty)^3+8} = \frac{\infty}{-\infty} \quad (L'H) \quad (\cdots) =$$

FORMA 2

$$\frac{\frac{8n^2}{n^3}+\frac{n}{n^3}}{\frac{-n^3}{n^3}+\frac{8}{n^3}} = (\ldots) =$$

$\therefore \{a_n\}$ converge...

FORMA 3

$$\lim_{n\to\infty} \frac{8\cancel{n^2}}{-n^{\cancel{3}}} = \frac{8}{-n} = \frac{8}{-\infty} = 0$$

b) $b_n = \frac{(n+1)(n+2)}{2-n}$

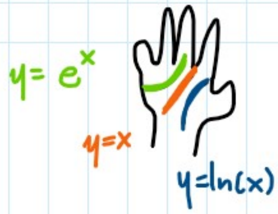
$n^2+2n+n+2$

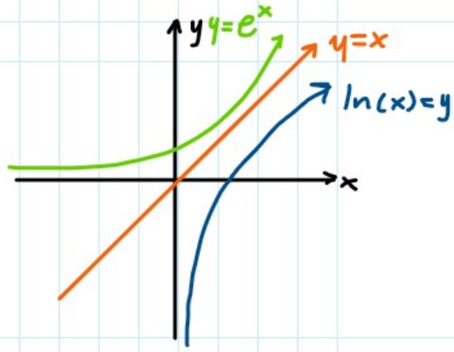
n^2+3n+2

$$\lim_{n\to\infty} \frac{n^2+3n+2}{2-n} = \frac{n^{\cancel{2}}}{-\cancel{n}} = -n = -\infty$$

$\therefore \{b_n\}$ diverge.

c) $c_n = \frac{\ln(n)}{e^n}$





$$\lim_{n\to\infty} \frac{\ln(n)}{e^n} = \frac{\ln(\infty)}{e^\infty} = \frac{\infty}{\infty} \quad (L'H)$$

$$(L'H) \lim_{n\to\infty} \left(\frac{\frac{1}{n}}{\frac{e^n}{1}}\right) = \frac{1}{n e^n} = \frac{1}{\infty . e^\infty} = \frac{1}{\infty(\infty)} = \frac{1}{\infty^2} = 0$$

$\therefore \{c_n\}$ converge.

$$\frac{\#}{\infty} = 0 \leftrightarrow \frac{\#}{0} = \infty$$

d) $d_n = \left(\frac{n+1}{n-1}\right)^n$

CASOS L'H $\longrightarrow$ $\frac{0}{0}$ ó $\frac{\infty}{\infty}$, $0.\infty$, $\infty-\infty$, 1^∞ ó ∞^0 ó 0^0

$$\lim_{n\to\infty} \left(\frac{n+1}{n-1}\right)^n = 1^n = 1^\infty \quad (L'H)$$

$$\sqrt{\lim_{n\to\infty} n} = \lim_{n\to\infty} \sqrt{n}$$

$$\lim_{n\to\infty} \left(\frac{n+1}{n-1}\right) = \frac{\cancel{n}}{\cancel{n}} = 1$$

$$R = \lim_{n\to\infty} \left(\frac{n+1}{n-1}\right)^n$$

$$\ln R = \lim_{n\to\infty} \ln\left(\frac{n+1}{n-1}\right)^n$$

$$\ln R = \lim_{n\to\infty} \underbrace{n}_{f(x)} . \underbrace{\ln\left(\frac{n+1}{n-1}\right)}_{g(x)} = \infty . 0 \quad (L'H)$$

$$n.\ln(1)$$
$$\infty . 0$$

$$f(x).g(x) \rightarrow \frac{f(x)}{\frac{1}{g(x)}} \text{ ó } \frac{g(x)}{\frac{1}{f(x)}}$$

$$n.\ln\left(\frac{n+1}{n-1}\right)$$

$$\frac{n}{\frac{1}{\ln\left(\frac{n+1}{n-1}\right)}} \text{ ó } \frac{\ln\left(\frac{n+1}{n-1}\right)}{\frac{1}{n}}$$

Reescribir... → ppdes. de log...

$$\lim_{n\to\infty} \frac{\ln\left(\frac{n+1}{n-1}\right)}{\frac{1}{n}} = \frac{\ln(n+1) - \ln(n-1)}{n^{-1}}$$

→ ppdes. de pot.-

(L'H)

$$\lim_{n\to\infty} \frac{\frac{1}{n+1} - \frac{1}{n-1}}{-n^{-2}} = \frac{\frac{n-1-(n+1)}{(n+1)(n-1)}}{-\frac{1}{n^2}}$$

$$\lim_{n\to\infty} \frac{\frac{\cancel{n}-1-\cancel{n}-1}{n^2-1}}{-\frac{1}{n^2}} = \frac{\frac{-2}{n^2-1}}{\frac{-1}{n^2}} = \frac{\cancel{-}2n^2}{\cancel{-}1(n^2-1)}$$

$$\lim_{n\to\infty} \frac{2n^2}{n^2-1} = \frac{2\cancel{n^2}}{\cancel{n^2}} = 2$$

$$\cancel{e}^{\cancel{\ln} R} = e^2$$

$$R = e^2 \qquad \therefore \{d_n\} \text{ converge.}$$

ASIGNACIÓN: Determine si la siguiente sucesión converge o diverge.

$$\left\{ \frac{1}{\sqrt{27}}, -\frac{\sqrt{5}}{16}, \frac{\sqrt{7}}{25}, -\frac{1}{12}, \frac{\sqrt{11}}{49}, -\frac{\sqrt{13}}{64}, \dots \right\}$$

SUCESIONES ACOTADAS.

$$\{n\} = 1, 2, 3, 4, 5, 6, 7, \dots, \infty$$

?

acotada inferiormente.

$$1 \le n < \infty$$

$$\left\{\frac{n}{n+1}\right\} = \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots, 1$$

$$\lim_{n\to\infty} \frac{n}{n+1} = \frac{\cancel{n}}{\cancel{n}} = 1$$

acotada.

$$\frac{1}{2} \leq n < 1$$

SUCESIONES PARTICULARES

Fibonacci

$$\underbrace{a_1 = 1}, \ \underbrace{a_2 = 1}, \ \underbrace{a_n = a_{n-1} + a_{n-2}}_{\forall n \geq 3}$$

- $n=3$

$$a_3 = a_{3-1} + a_{3-2}$$

$$a_3 = \cancel{a_2} + \cancel{a_1} = 2$$

$$1 + 1$$

$$\underbrace{a_3 = 2}$$

- $n=4$

$$a_4 = \cancel{a_3} + \cancel{a_2} = 3$$

$$2 + 1$$

$$\underbrace{a_4 = 3}$$

1, 1, 2, 3, ...

TECNICAS DE SUCESIONES.

2. $\{r^n\}$ dónde r es constante.

que es convergente: $-1 < r \leq 1$.

¿Será $\{(\frac{1}{2})^n\}$ convergente?

$\hookrightarrow r = \frac{1}{2}$ $\quad -1 < \frac{1}{2} \leq 1$ ✓

es convergente.

3. $\lim_{n\to\infty} |a_n| = 0 \leftrightarrow \lim_{n\to\infty} a_n = 0$

X

4. $\lim_{n\to\infty} \left(\underbrace{1}_{1.C.\ +1} + \frac{1}{n} \right)^{n} = e$

2.C. $+1$

3.C.

Resolver: $\lim_{n\to\infty} \left(1 + \frac{2}{n} \right)^{n}$

$$\left(1 + \frac{\frac{1}{2}\cdot \cancel{2}}{\frac{1}{2}\cdot n} \right)^{n} \qquad n \cdot \frac{2}{2} = \frac{n}{2}\cdot 2$$

$$\left(1 + \frac{1}{\frac{n}{2}} \right)^{n\cdot\frac{2}{2}} \qquad (a^{b})^{c} = a^{b\cdot c}$$

$$\left[\left(1 + \frac{1}{n/2} \right)^{n/2} \right]^{2}$$

$$\lim_{n\to\infty} \left(1 + \frac{2}{n} \right)^{n} = e^{2}$$

SERIES.

- Sucesión: $1, 3, 5, 7, \dots$
  ↳ $\{a_n\}$ ← siempre inician $n = 1$.
- Serie: $1 + 3 + 5 + 7 + \dots$
  ↳ $S_n = \sum^{\infty} u_n$ $\begin{cases} n=2 \\ n=1 \\ n=0 \end{cases}$

$S_n = 1 + 3 + 5 + 7 + \dots$

$S_1 = 1$

$S_2 = 1 + 3 = \underbrace{4}$

$S_3 = \underbrace{1 + 3}_{4} + 5 = 4 + 5 = \underbrace{9}$

$S_4 = \underbrace{1 + 3 + 5}_{9} + 7 = 9 + 7 = 16$

- SUCESIÓN DE SUMAS PARCIALES.

∞

PARCIALES.

$$\sum_{n=1}^{\infty} \underbrace{\frac{1}{n(n+1)}}_{\text{no es un termino general.}} = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \cdots + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \cdots \approx 1 \cong 0.89$$

$S_1 = \frac{1}{2}$

$S_2 = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

$S_3 = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} = \frac{3}{4}$

▸ Encontra $S_n \to$ T.G.

$$\frac{1}{K(K+1)} = \frac{A}{K} + \frac{B}{K+1}$$

$$\frac{1}{\cancel{K(K+1)}} = \frac{A(K+1) + BK}{\cancel{K(K+1)}}$$

$$1 = A(K+1) + BK$$

$$\underbrace{1} = \underbrace{AK} + \underbrace{A} + \underbrace{BK}$$

"K" $\quad 0 = A + B \to -A = B$

Sin "K" $\quad 1 = A$ ✓ $\quad -1 = B$ ✓

$$\frac{1}{K} - \frac{1}{K+1}$$

$$1 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{5}} - \frac{1}{6}$$

Evaluar: $K = 1, 2, 3, 4, \ldots, n$

$$\underbrace{1 - \cancel{\frac{1}{2}}}_{K=1} + \underbrace{\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}}_{K=2} + \underbrace{\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}}_{K=3} + \underbrace{\cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}}_{K=4} + \cdots + \underbrace{\cancel{\frac{1}{n}} - \frac{1}{n+1}}_{K=n}$$

$$S_n = \frac{1}{1} - \frac{1}{n+1}$$

$$S_n = \frac{n + \cancel{1} - \cancel{1}}{n+1} = \frac{n}{n+1} \leftarrow \text{T.G.}$$

¿Para qué sirve el T.G. $\to S_n$?

$n=1 \quad S_1 = \frac{1}{1+1} = \frac{1}{2}$

$n=2 \quad S_2 = \frac{2}{2+1} = \frac{2}{3}$

$n=2 \quad S_2 = \frac{2}{2+1} = \frac{2}{3}$

$n=3 \quad S_3 = \frac{3}{3+1} = \frac{3}{4}$

$S \rightarrow$ la suma total de los terminos...

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{n}{n+1} = \frac{\cancel{n}}{\cancel{n}} = 1 \qquad \underbrace{S=1}$$

$\updownarrow$

Serie es convergente.

▫ SERIES GEOMETRICAS.

$$\left.\begin{array}{l} \sum_{n=1}^{\infty} a.r^{n-1} = a + ar + ar^2 + ar^3 + \ldots \\ \sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \ldots \end{array}\right\} a + ar + ar^2 + ar^3 + \ldots$$

$a=2 \quad \underbrace{r=-3/4} \quad -1 < r < 1$ ✓

Serie converge. $\rightarrow S = \frac{a}{1-r}$

$a=-0.5 \quad \underbrace{r=4} \quad |r| \geq 1$

Serie diverge $\rightarrow S = \infty$

$$\sum_{n=1}^{\infty} \underbrace{3}_{a}\underbrace{(2)}_{r}^{n-1} = 3 + 3(2) + 3(2)^2 + 3(2)^3 + \ldots$$

a = primer termino.
r = razón incremento/decremento.

$$S_n = a + \cancel{ar} + \cancel{ar^2} + \cancel{\ldots} + \cancel{ar^{n-1}}$$

$$\underbrace{-r}S_n = \cancel{ar} + \cancel{ar^2} + \cancel{ar^3} + \cancel{\ldots} + ar^n$$

$$S_n - rS_n = a - ar^n$$

$$S_n(\underbrace{1-r}) = a(1-r^n)$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S = \lim_{n\to\infty} \frac{a(1-\overset{0}{r^n})}{1-r} \qquad \text{condiφ: } -1 < r < 1$$

$$S = \frac{a}{1-r}$$

Una serie geometrica converge

cuando: $-1 < r < 1$

y diverge: $|r| \geq 1$

EJEMPLO: $1^{\infty} = 1$ $\quad 1^{2} = 1$ $\quad 1^{5} = 1$

$$\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} \leftarrow \frac{1^{n-1}}{2^{n-1}} \leftarrow \left(\frac{1}{2}\right)^{n-1}$$

$$\sum_{n=1}^{\infty} \underset{a}{1} \cdot \Bigg(\underset{r}{\frac{1}{2}}\Bigg)^{n-1} \qquad a \cdot r^{n-1}$$

$r = \frac{1}{2}$

$$\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = \underbrace{\overset{a}{1}}_{\substack{n=1 \\ a}} + \underbrace{\frac{1}{2}}_{\substack{n=2 \\ ar}} + \underbrace{\frac{1}{4}}_{\substack{n=3 \\ ar^{2}}} + \dots$$

$$\frac{\cancel{a}r}{\cancel{a}} = \frac{\frac{1}{2}}{1} = \frac{1}{2} \qquad r = \frac{1}{2}$$

APLICACIONES DE LA SERIE GEOMETRICA

$$1.6232323\ldots = \frac{\boxed{?}}{\boxed{?}}$$

Transformar decimales periodicos a fracciones...

$$\boxed{1.6}\,\overbrace{23}\,\overbrace{23}\,\overbrace{23}\ldots$$

$10 \quad 100 \quad 1{,}000 \quad 10{,}000 \quad 100{,}000$

$$\frac{16}{10}$$

$$\frac{6}{10} = 0.6$$

$$\frac{2}{100} = 0.02$$

SERIE...

$$\underbrace{\frac{23}{1{,}000}}_{a} + \underbrace{\frac{23}{100{,}000}}_{ar} + \underbrace{\frac{23}{10{,}000{,}000}}_{ar^{2}} + \dots$$

$$\frac{16}{10} + \frac{23}{990}$$

$$\frac{16}{10} + \frac{23}{990}$$

$$= \frac{1607}{990}$$

$$\underbrace{\frac{23}{1,000}}_{a} + \underbrace{\frac{23}{100,000}}_{ar} + \underbrace{\frac{23}{10,000,000}}_{ar^2}$$

$$a = \frac{23}{1,000} \qquad r = \frac{1}{100}$$

$r = \dfrac{1}{100} \leftarrow$ 2 ceros. $\quad 23 \rightarrow$ 2 cifras

$$\frac{\cancel{a}r}{\cancel{a}} = \frac{\frac{\cancel{23}}{100,\cancel{000}}}{\frac{\cancel{23}}{1\cancel{000}}} = \frac{1}{100} = r$$

$$S = \frac{a}{1-r} = \frac{\frac{23}{1000}}{1 - \frac{1}{100}} = \underbrace{\frac{23}{990}}$$

17.812812812...

0.0545454...

SERIE P ó HIPERARMÓNICA

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \qquad \frac{1}{n^2} \leftarrow p$$

$P > 1$ entonces converge.

$P \leq 1$ entonces diverge.

$$\sum_{n=1}^{\infty} \frac{1}{n^1} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

$p = 1$

serie divergente.

Serie armónica.

SERIE ALTERNADA.

$+, -, +, -, +, \dots \leftarrow$ Suc.

$$2 - 1 + \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16}$$

Serie.

¿Será la serie convergente?

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \dots$$

Valor absoluto..

$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\dots$

Sí convergente; entonces.

es absolutamente convergente.

- Serie Geometrica. $a=1$, $r=\frac{1}{2}$ ✓

∴ converge

R// La serie es absolutamente convergente.

¿Será convergente la serie: $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\dots$?

Valor absoluto.

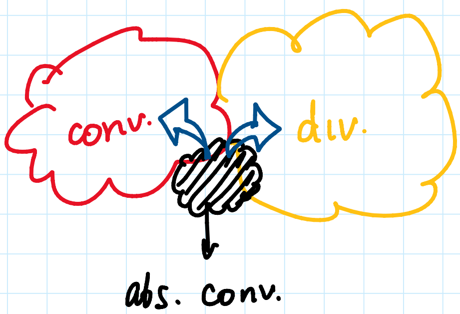
$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\dots$

Serie hiperarmónica. $\sum_{n=1}^{\infty}\frac{1}{n^2}$

$p=2$

∴ converge.

R// La serie es absolutamente convergente.



iii) $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\dots$

$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\dots$

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$

Serie "p" → armónica

divergente.

R// No es absolutamente convergente.

$\sum \frac{1}{n(n+1)}$ s. de. f. p.

$\sum 2(8)^{n-1}$ S. G.

$\sum \frac{1}{n^{4/7}}$ S. "p"

$- \square + \square - \square + \square -$ S. alternada

$\sum_{n=1}^{\infty} ne^{n}$

PRUEBAS O CRITERIO DE CONVERGENCIA Y DIVERGENCIA.

1. Prueba de comparación directa.

CONDIĆ: $a_n \leq b_n$

$\underbrace{\sum b_n}$ conv. → $\sum \underbrace{a_n}$ conv.

$\sum a_n$ div. → $\sum b_n$ div.

conocidas. (inventar) desconocidas. (ejercicio)

EJ. 1. $\sum_{n=1}^{\infty} \frac{n}{2^n (n+1)}$ } desconocida

← a_n

$\frac{n}{n+1} = \frac{\cancel{n}}{\cancel{n}} = 1$

$$\underbrace{\sum_{n=1}^{\infty} \frac{1}{2^n}}_{\text{conocida}} = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$$

$r=\frac{1}{2}$ ↳ converge.

b_n

$a_n \le b_n$ ✓ $n=2$

$$\frac{n}{2^n(n+1)} \le \frac{1}{2^n}$$

$$\frac{\cancel{2}}{2^{\cancel{2}}(2+1)} \le \frac{1}{2^2}$$

$$\frac{1}{6} < \frac{1}{4} \checkmark$$

$\therefore \sum_{n=1}^{\infty} \frac{n}{2^n(n+1)}$ converge.

Ej. 2 $\left.\sum_{n=1}^{\infty} \frac{n}{5n^2-4}\right\}$ desconocida

b_n

$$\frac{n}{5n^2-4} = \frac{\cancel{n}}{5n^{\cancel{2}}} = \frac{1}{5n}$$

$$\sum_{n=1}^{\infty} \frac{1}{5n} = \frac{1}{5}\underbrace{\sum_{n=1}^{\infty} \frac{1}{n^1}}_{\text{Conocida}} \; a_n$$

Serie armónica ↳ diverge.

$a_n \le b_n$ ✓ $n=1$

$$\frac{1}{5n} \le \frac{n}{5n^2-4}$$

$$\frac{1}{5(1)} \le \frac{1}{5(1)^2-4}$$

$$\frac{1}{5} < 1 \checkmark$$

$\therefore \sum_{n=1}^{\infty} \frac{n}{5n^2-4}$ diverge.

2. Prueba de comparación en el límite.

CONDIØ: $\lim_{n\to\infty} \frac{a_n}{b_n} = L$

$0 < L < \infty$ ✓

$\sum a_n \wedge \sum b_n$ convergente.

$\sum a_n \wedge \sum b_n$ divergente.

NOTA: a_n SIEMPRE es la desconocida.

Ej. 1. $\displaystyle \underbrace{\sum_{n=1}^{\infty} \frac{3n-2}{n^3-2n^2+11}}_{a_n}$

$$\frac{3n-2}{n^3-2n^2+11} = \frac{3\cancel{n}}{n^{\cancel{3}}} = \frac{3}{n^2}$$

$\displaystyle \underbrace{\sum_{n=1}^{\infty} \frac{3}{n^2}}_{b_n}$ ← S.P. con $p=2$
$p>1$ converge.

$$\lim_{n\to\infty} \frac{a_n}{b_n}$$ ← le voy a dar vuelta.

$$\lim_{n\to\infty} \frac{(3n-2)}{(n^3-2n^2+11)} \cdot \frac{n^2}{3}$$

$$\lim_{n\to\infty} \frac{3n^3-2n^2}{3n^3-6n^2+33} = \frac{\cancel{3n^3}}{\cancel{3n^3}} = 1$$

$L = 1$ ✓

$\therefore \displaystyle \sum_{n=1}^{\infty} \frac{3n-2}{n^3-2n^2+11}$ converge.

Ej. 2. $\displaystyle \underbrace{\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+19n}}}_{a_n}$

$$\frac{1}{\sqrt{n^2+19n}} = \frac{1}{\sqrt{n^2}} = \frac{1}{n}$$

$\displaystyle \underbrace{\sum_{n=1}^{\infty} \frac{1}{n}}_{b_n}$ serie armónica
↳ divergente.

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+19n}} \cdot \frac{n}{1}$$

$$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+19n}} = \frac{n}{\sqrt{n^2}} = \frac{\cancel{n}}{\cancel{n}} = 1$$

$L = 1$ ✓

$L = 1$ ✓

$\therefore \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+19n}}$ es divergente.

Prueba o criterio de la serie alternada

$$\sum_{n=1}^{\infty} (-1)^{n-1} \underbrace{a_n} \quad \frac{n}{n^2+8} \qquad \frac{\sqrt{n}}{2^n\, n+1}$$

CONDIØ: p̄ ser convergente.

a) $\lim_{n\to\infty} a_n = 0$ ✓

b) $a_{n+1} \le a_n$ ✓

EJ 1. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$ serie alternada...

$a_n = \frac{1}{2n-1}$

a) $\lim_{n\to\infty} \frac{1}{2n-1} = \frac{1}{2n} = \frac{1}{\infty} = 0$ ✓

b) $\underbrace{a_{n+1}}_{n=2} \le \underbrace{a_n}_{n=1}$

$\frac{1}{2(2)-1} \le \frac{1}{2(1)-1}$

$\frac{1}{3} < 1$ ✓

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$ es convergente

$\{a_n\} = \frac{(-1)^{n+1}}{2n-1}$ Sucesión alternada...

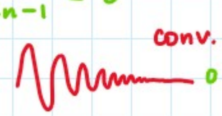
$\lim_{n\to\infty} \frac{(-1)^{n+1}}{2n-1}$

$\underbrace{(-1)^{n+1}}_{\text{signo intercalado}} \cdot \underbrace{\frac{1}{2n-1}}$

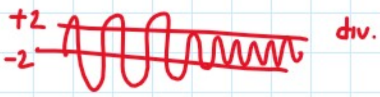
+ ó −

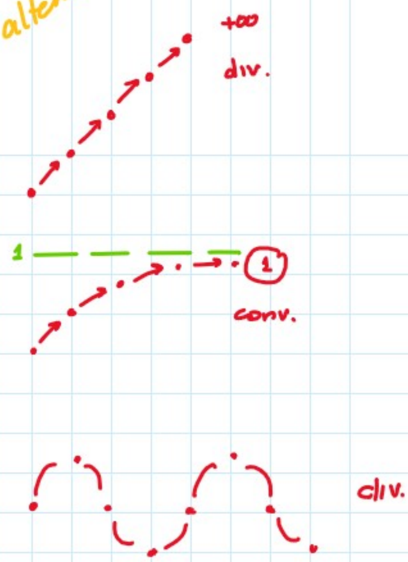
$\lim_{n\to\infty} \frac{1}{2n-1} = 0$

± 0



± 2





EJ. 2. $\sum_{n=1}^{\infty} \frac{(-1)^n\, n^2}{n^2+1}$

$a_n = \frac{n^2}{n^2+1}$

falla la prueba

a) $\lim_{n\to\infty} \frac{n^2}{n^2+1} = \frac{\cancel{n^2}}{\cancel{n^2}} = 1.$ ✗

∴ No es convergente.

EJ. 3. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$

Serie armónica → diverge.

armónica alternada converge.

$a_n = \frac{1}{n}$

a) $\lim_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0$ ✓

b) $\underbrace{a_{n+1}}_{n=4} \leq \underbrace{a_n}_{n=3}$

$\frac{1}{4} < \frac{1}{3}$ ✓

∴ $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$ es convergente.

■ Prueba del cociente o la razón.

Parte A. $\sum_{n=1}^{\infty} a_n$

CONDIC: $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$

$L < 1$ converge
$L > 1$ diverge
$L = 1$ Prueba falla =’(

EJ. 1 $\left.\sum_{n=1}^{\infty} \frac{2^n}{n!}\right\} a_n$

$\left.\frac{2^{n+1}}{(n+1)!}\right\} a_{n+1}$

$\lim_{n\to\infty} \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n}$

$\lim_{n\to\infty} \frac{\cancel{2^n} \cdot 2 \cdot \cancel{n!}}{(n+1) \cdot \cancel{n!} \, \cancel{2^n}} = \frac{2}{n+1}$

$5! = 5.4.3.2.1$

$7! = 7.6.5.4.3.2.1$

$3! = 3.2.1$

$7! = 7.6.5.4!$

$n! = n(n-1)(n-2)\ldots 1.$

$7! = 7.6!$

Simplificar... $\frac{5!}{7!} = \frac{\cancel{5!}}{7.6.\cancel{5!}} = \frac{1}{7.6} = \frac{1}{42}$

$\frac{(n+1)!}{(n-1)!} = \frac{(n+1).n.\cancel{(n-1)!}}{\cancel{(n-1)!}} = (n+1)(n)$

$$= \frac{2}{n} = \frac{2}{\infty} = 0$$

$L = 0$ $\quad \therefore \sum_{n=1}^{\infty} \frac{2^n}{n!}$ converge.

EJ. 2. $\displaystyle\sum_{n=1}^{\infty} \underbrace{\frac{2^n}{n^{20}}}_{a_n}$

$$\underbrace{\frac{2^{n+1}}{(n+1)^{20}}}_{a_{n+1}}$$

$$\lim_{n\to\infty} \frac{2^{n+1}}{(n+1)^{20}} \cdot \frac{n^{20}}{2^n}$$

$$\lim_{n\to\infty} \frac{\cancel{2^n} \cdot 2 \cdot n^{20}}{\cancel{2^n} \cdot (n+1)^{20}}$$

$$\lim_{n\to\infty} 2 \cdot \left(\frac{n}{n+1}\right)^{20}$$

$$2 \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{20}$$

$$2 \left(\lim_{n\to\infty} \cancelto{1}{\frac{n}{n+1}} \right)^{20}$$

$2(1)^{20} = 2 \qquad L = 2$

$\therefore \displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^{20}}$ diverge.

Parte B. $\displaystyle\sum_{n=1}^{\infty} (-1)^n a_n$

CONDIϕ: $\displaystyle\lim_{n\to\infty} \left| \frac{(-1)^{n+1} a_{n+1}}{(-1)^n a_n} \right|$

$L < 1$; absolutamente conv.
$L > 1$; divergente.
$L = 1$; prueba falla.

EJ. 1. $\displaystyle\sum_{n=1}^{\infty} \underbrace{(-1)^n \frac{n^3}{3^n}}_{a_n}$

$$\underbrace{(-1)^{n+1} \frac{(n+1)^3}{3^{n+1}}}_{a_{n+1}}$$

$$\lim_{n\to\infty} \left| \frac{\cancel{(-1)^{n+1}} (n+1)^3}{3^{n+1}} \cdot \frac{3^n}{\cancel{(-1)^n} n^3} \right|$$

$$\frac{(-1)^{n+1}}{(-1)^n} = \frac{\cancel{(-1)^n}(-1)}{\cancel{(-1)^n}} = -1$$

$$\lim_{n\to\infty} \frac{(n+1)^3 \cdot 3^n}{3^{n+1} \cdot n^3} = \frac{(n+1)^3 \cdot \cancel{3^n}}{\cancel{3^n} \cdot 3 \cdot n^3}$$

$$\frac{1}{3} \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^3 = \frac{1}{3}(1)^3 = \frac{1}{3}$$

$$L = \frac{1}{3}$$

$\therefore \sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$ es absolutamente convergente.

▫ Prueba o criterio de la integral.

CONDIØ: No se vaya a indeterminar

$$\sum_{n=1}^{\infty} a_n \longrightarrow \int_1^{\infty} a_x \cdot dx$$

Respuesta: # converge.
Respuesta: ∞ ó $-\infty$ diverge.

EJ. 1 $\sum_{n=1}^{\infty} \frac{1}{4n^2}$

$$\int_1^{\infty} \frac{1}{4x^2} \cdot dx$$

$$\frac{1}{4} \lim_{h\to\infty} \int_1^h \frac{dx}{x^2}$$

$$\frac{1}{4} \lim_{h\to\infty} \left[-x^{-1}\right]_1^h \qquad x^{-1} = \frac{1}{x}$$

$$\frac{1}{4} \lim_{h\to\infty} \left[-\frac{1}{\underset{\infty}{h}} + \frac{1}{1}\right]$$

$$\frac{1}{4}[-0+1] = \frac{1}{4}$$

$\therefore \sum_{n=1}^{\infty} \frac{1}{4n^2}$ converge.

EJ. 2 $\sum_{n=2}^{\infty} \frac{1}{n \cdot \ln(n)}$

$$\rightarrow \lim_{a\to\infty} \int^a \frac{dx}{x \ln(x)}$$

$$\to \lim_{a\to\infty} \int_2^a \frac{dx}{x\ln(x)}$$

$u = \ln(x)$

$du = \frac{1}{x} \cdot dx$

$$\int \frac{du}{u} = \ln(u)$$

$$= \ln(\ln(x)) \Big|_2^a$$

$$\lim_{a\to\infty} \ln(\ln(x)) \Big|_2^a$$

∞

$$\lim_{a\to\infty} \ln(\cancel{\ln(a)}) - \ln(\ln(2))$$

∞

$$\infty - \ln(\ln(2)) = \infty.$$

$$\therefore \sum_{n=2}^{\infty} \frac{1}{n(\ln(n))} \text{ diverge.}$$

EJERCICIOS ASIGNADOS

1. **Encuentre el término general de la siguiente sucesión y determine si converge o no.**

$$\left\{2, -\frac{2}{3}, \frac{2}{9}, -\frac{2}{27}, \frac{2}{81}, \ldots\right\}$$

2. **Demuestre de dos formas diferentes la convergencia o divergencia de esta serie:**

$$\sum_{n=0}^{\infty} \left(\frac{1}{e}\right)^n$$

a. Identificando que tipo de serie representa y concluyendo sobre su convergencia o divergencia.
b. Utilizando el criterio de la integral.

3. **Utilizando la prueba del cociente o la razón, determine si la siguiente serie converge o diverge:**

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$$