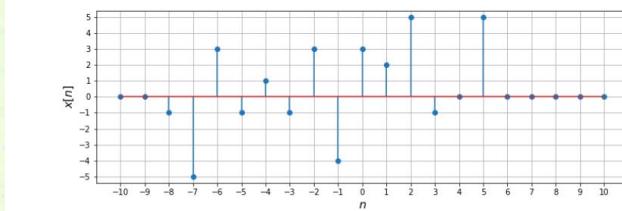


ven:

For the signal $x[n]$, plot the following signals over a large enough time interval to show all nonzero samples by hand. (Note that $x[n] = 0$ for $n \leq -9$ and for $n \geq 6$.) (34 points total)

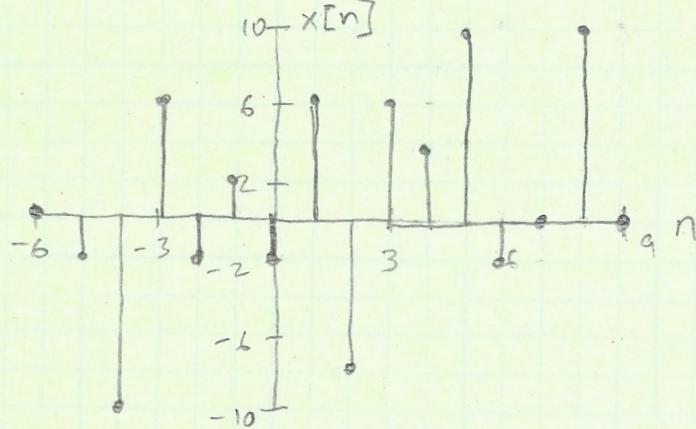


- Find: a) $2x[n-3]$ b) $-x[3n]$ c) $x[-\frac{1}{4}n]$
 d) even & odd parts of $x[n]$ e) smallest integer k for which $x[kn]$ contains only one nonzero sample

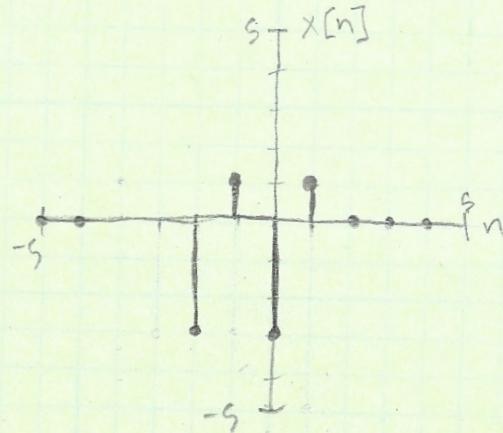
Solution:

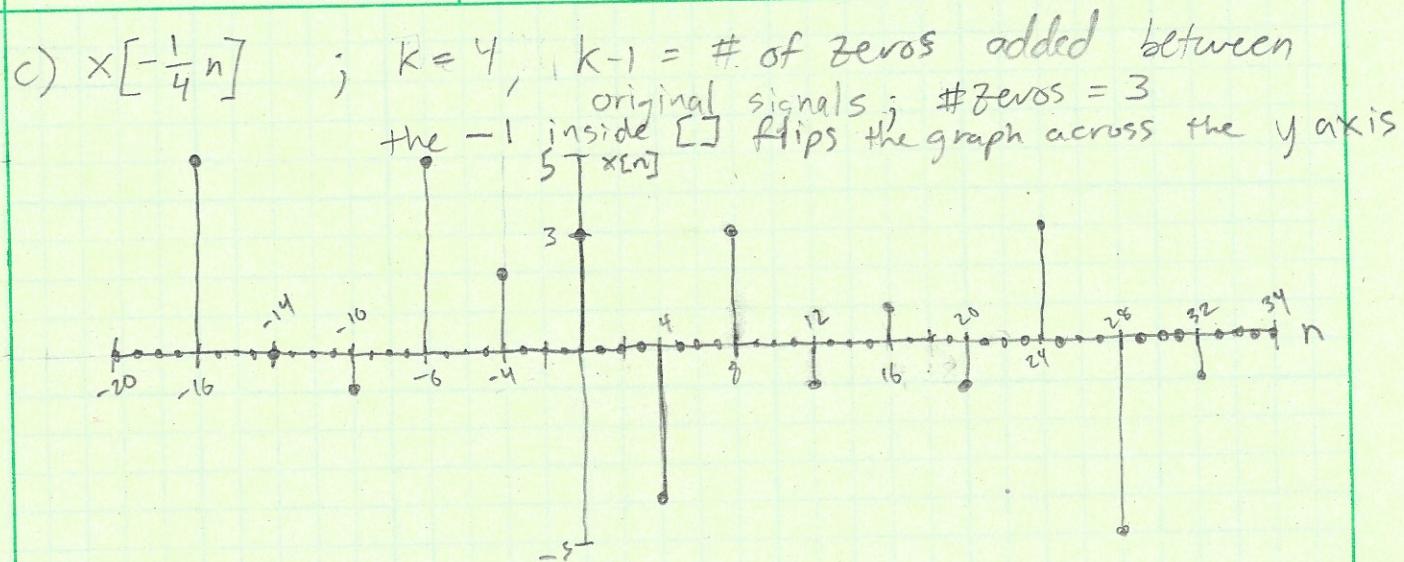
a) $2x[n-3]$; The 2 scales amplitude by 2

$2x[n-(-3)]$; The -3 shifts to the right by 3
 $n \leq -6$ & $n \geq 9$ are all zeros now



b) $(-1)x[3n]$; the -1 scales amplitude by -1
 the 3 decimates by 3





d) Even & odd parts of $x[n]$

$$x_{\text{even}} = \frac{x[n] + x[-n]}{2}; x_{\text{odd}} = \frac{x[n] - x[-n]}{2}$$

$$x[n] = [-1, -5, 3, -1, 1, -1, 3, -4, 3, 2, 5, -1, 0, 5]_0, 0, 0$$

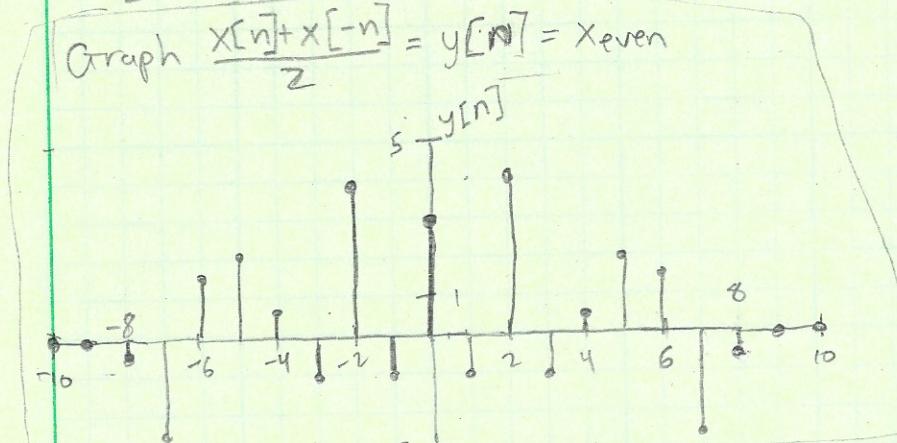
$$x[-n] = 0, 0, 0 [5, 0, -1, 5, 2, 3, -4, 3, -1, 1, -1, 3, -5, -1]$$

$$x_{\text{even}} = \frac{x[n] + x[-n]}{2} = \left[\begin{array}{c} \frac{1}{2}, -2.5, 1.5, 2, \frac{1}{2}, -1, 4, -1, 3, -1, 4, -1, \frac{1}{2}, 2, 1.5, -2.5, -\frac{1}{2} \\ \uparrow \quad \uparrow \end{array} \right]_{n=-8}^8$$

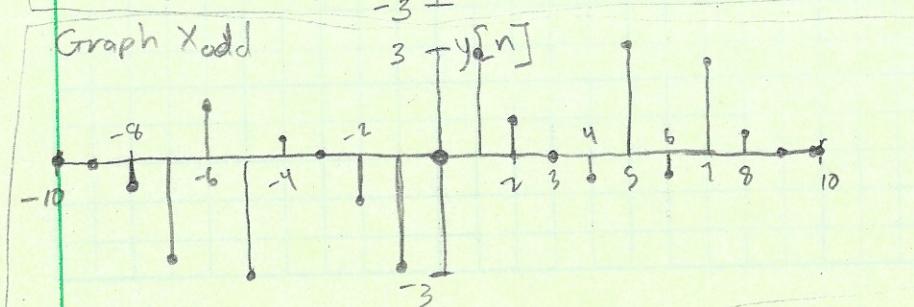
$$x_{\text{odd}} = \frac{x[n] - x[-n]}{2} = \left[\begin{array}{c} -\frac{1}{2}, -2.5, 1.5, -3, \frac{1}{2}, 0, -1, -3, 0, 3, 1, 0, -\frac{1}{2}, 3, -1.5, 2.5, \frac{1}{2} \\ \downarrow \quad \downarrow \end{array} \right]_{n=-8}^8$$

Graph $\frac{x[n] + x[-n]}{2} = y[n] = x_{\text{even}}$

* Note: make a table next time



Graph x_{odd}



e) Since $n = -8$ is the largest $|n|/(\theta)$ value, to decimate all but the origin, $K=9$

Given:

$$x_1[n] = \begin{cases} 5 - \frac{n}{2}; & 0 \leq n \leq 20 \\ 0; & \text{otherwise} \end{cases}$$

$$x_2[n] = \begin{cases} 5 - \frac{n}{2} - 1 & 0 \leq n \leq 20 \\ -1, & \text{otherwise} \end{cases}$$

Find a) Determine forward differences $y_1[n]$ & $y_2[n]$ How do $y_1[n]$ & $y_2[n]$ compare?* b) & c) in solutionsSolution:

$$\text{a) } y_1[n] = x_1[n+1] - x_1[n]$$

$$y_1[-2] = 0 - 0 = 0$$

$$y_1[-1] = 5 - 0 = 5$$

$$y_1[0] = 4.5 - 5 = -0.5$$

$$y_1[20] = 0 - 5 = 5$$

$$y_1[21] = 0 - 0 = 0$$

$$y_1[n] = \begin{cases} 5; & n=-1 \text{ or } n=20 \\ -\frac{1}{2}; & 0 \leq n \leq 19 \\ 0; & \text{otherwise} \end{cases}$$

$$y_2[n] = x_2[n+1] - x_2[n]$$

$$y_2[-2] = -1 - -1 = 0$$

$$y_2[-1] = 4 + 1 = 5$$

$$y_2[0] = 3.5 - 4 = -0.5$$

$$y_2[20] = -1 - -6 = 5$$

$$y_2[21] = -1 - -1 = 0$$

$$y_2[n] = \begin{cases} 5; & n=-1 \text{ or } n=20 \\ -\frac{1}{2}; & 0 \leq n \leq 19 \\ 0; & \text{otherwise} \end{cases}$$

How do they compare?

 $y_1[n] = y_2[n] \therefore$ They are the sameb) Determine forward difference $y_3[n]$ of the signal

$$x_3[n] = \begin{cases} 5 - \frac{n}{2} - 1; & 0 \leq n \leq 20 \\ 0; & \text{otherwise} \end{cases}$$

$$y_3[n] = x_3[n+1] - x_3[n]$$

$$y_3[-2] = 0 - 0 = 0$$

$$y_3[-1] = 5 - 0 - 1 = 4$$

$$y_3[0] = 3.5 - 4 = -\frac{1}{2}$$

$$y_3[20] = 0 - -6 = 6$$

$$y_3[21] = 0 - 0 = 0$$

$$y_3[n] = \begin{cases} 4; & n=-1 \\ 6; & n=20 \\ -\frac{1}{2}; & 0 \leq n \leq 19 \\ 0; & \text{otherwise} \end{cases}$$

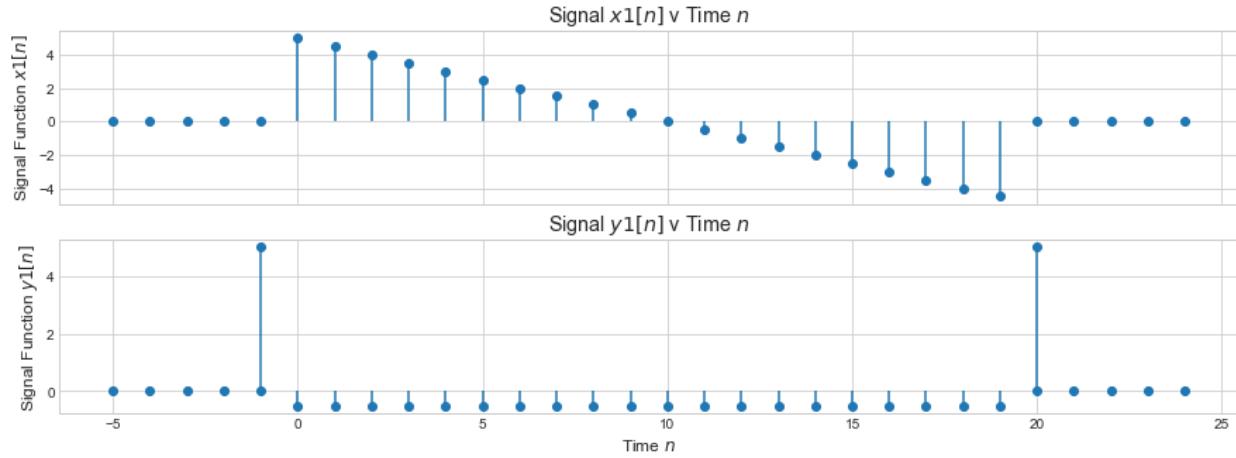
How does $y_3[n]$ differ from $y_1[n]$?
The only difference is at $n=20, n=-1$ $y_3[n]$ is 6, 4 and $y_1[n]$ is 5, 5 respectively.This is because of the " -1 "in $x_3[n]$, $0 \leq n \leq 20$

c) $\tilde{x}[n] = \sum_{m=-\infty}^{n-1} y_2[m] ; y_2[n] = \begin{cases} 5 & ; n = -1 \text{ &} n = 20 \\ -1/2 & ; 0 \leq n \leq 19 \\ 0 & ; \text{otherwise} \end{cases}$

$$\tilde{x}[n] = \begin{cases} 5, & n = -1, \\ 4 - \frac{1}{2}n; & 0 \leq n \leq 19 \\ 0 & ; \text{otherwise} \end{cases}$$

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 HW#2 Problem 2

PART A
x1[n] and y1[n]



```
# -*- coding: utf-8 -*-
"""

```

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 10/21/2022
 ECE 315 Signals & Systems
 HW#2, Problem 2a

```
"""
import numpy as np
import matplotlib.pyplot as plt
```

```
#Setup Plot
plt.style.use('seaborn-whitegrid')
fig, (ax1, ax2) = plt.subplots(2, figsize=(15, 5), sharex=True)
```

```
#Plot Points y1[n]
for i in range(0, 20):
    ax2.stem(i, -0.5)
    ax2.stem(-1, 5)
    ax2.stem(20, 5)
```

```
for i in range(-5, 0):
    ax2.stem(i, 0)
```

```
for i in range (20, 25):
    ax2.stem(i, 0)
```

```
#Plot Points x1[n]
for i in range(0, 20):
```

```

ax1.stem(i, (5 - i/2))

for i in range(-5, 0):
    ax1.stem(i, 0)

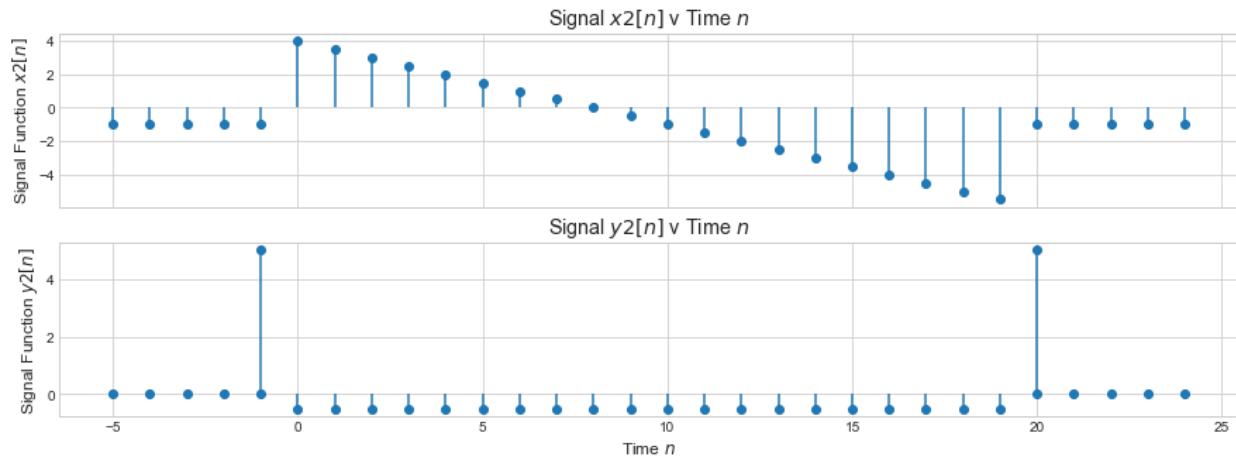
for i in range (20, 25):
    ax1.stem(i, 0)

#Plot Settings
ax2.set_xlabel("Time $n$", fontsize='large')
ax2.set_ylabel("Signal Function $y1[n]$", fontsize='large')
ax2.set_title("Signal $y1[n]$ v Time $n$", fontsize='x-large')

ax1.set_ylabel("Signal Function $x1[n]$", fontsize='large')
ax1.set_title("Signal $x1[n]$ v Time $n$", fontsize='x-large')

```

x2[n] and y2[n]



```

# -*- coding: utf-8 -*-
"""


```

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ECE 315 Signals & Systems
HW#2, Problem 2a

```

import numpy as np
import matplotlib.pyplot as plt
```

```

#Setup Plot
plt.style.use('seaborn-whitegrid')
fig, (ax1, ax2) = plt.subplots(2, figsize=(15, 5), sharex=True)
```

```

#Plot Points y2[n]
for i in range(0, 20):
```

```

ax2.stem(i, -0.5)
ax2.stem(-1, 5)
ax2.stem(20, 5)

for i in range(-5, 0):
    ax2.stem(i, 0)

for i in range (20, 25):
    ax2.stem(i, 0)

#Plot Points x2[n]
for i in range(0, 20):
    ax1.stem(i, (5 - i/2 - 1))

for i in range(-5, 0):
    ax1.stem(i, -1)

for i in range (20, 25):
    ax1.stem(i, -1)

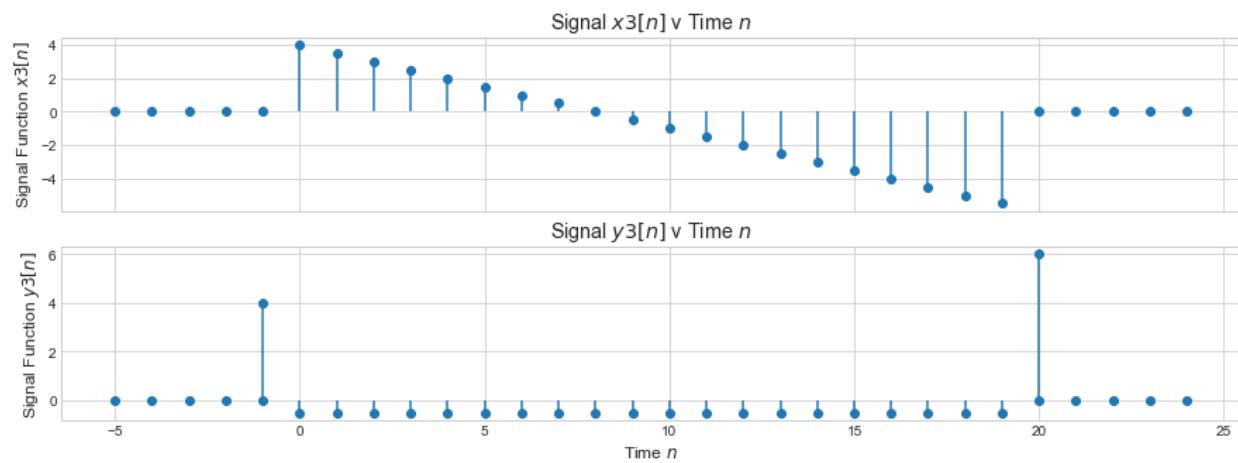
#Plot Settings
ax2.set_xlabel("Time $n$", fontsize='large')
ax2.set_ylabel("Signal Function $y2[n]$", fontsize='large')
ax2.set_title("Signal $y2[n]$ v Time $n$", fontsize='x-large')

ax1.set_ylabel("Signal Function $x2[n]$", fontsize='large')
ax1.set_title("Signal $x2[n]$ v Time $n$", fontsize='x-large')

```

PART B

x3[n] and y3[n]



```

# -*- coding: utf-8 -*-
"""

```

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10/21/2022
ECE 315 Signals & Systems
HW#2, Problem 2b
"""

```
import numpy as np
import matplotlib.pyplot as plt

#Setup Plot
plt.style.use('seaborn-whitegrid')
fig, (ax1, ax2) = plt.subplots(2, figsize=(15, 5), sharex=True)

#Plot Points y3[n]
for i in range(0, 20):
    ax2.stem(i, -0.5)
ax2.stem(-1, 4)
ax2.stem(20, 6)

for i in range(-5, 0):
    ax2.stem(i, 0)

for i in range (20, 25):
    ax2.stem(i, 0)

#Plot Points x3[n]
for i in range(0, 20):
    ax1.stem(i, (5 - i/2 - 1))

for i in range(-5, 0):
    ax1.stem(i, 0)

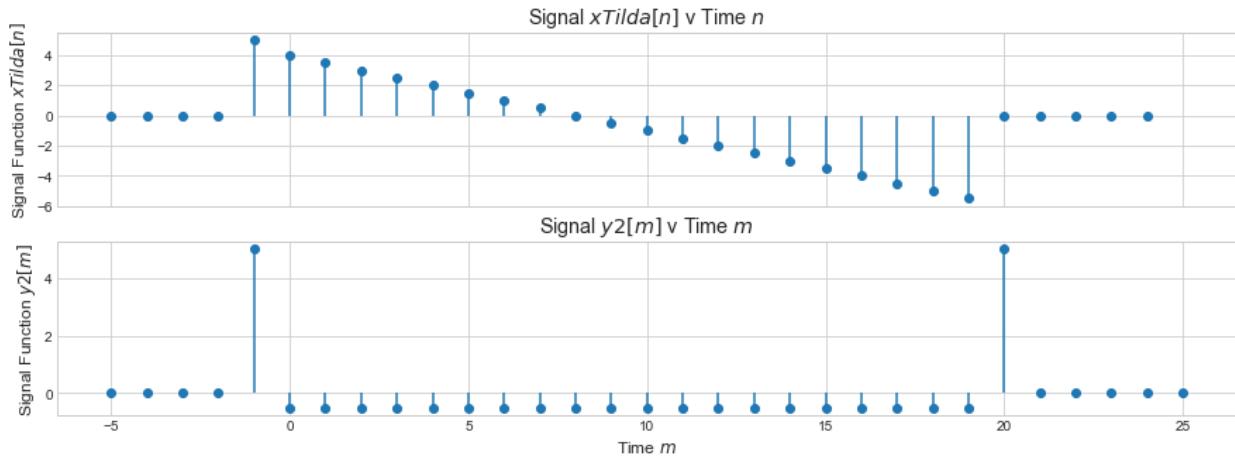
for i in range (20, 25):
    ax1.stem(i, 0)

#Plot Settings
ax2.set_xlabel("Time $n$", fontsize='large')
ax2.set_ylabel("Signal Function $y3[n]$", fontsize='large')
ax2.set_title("Signal $y3[n]$ v Time $n$", fontsize='x-large')

ax1.set_ylabel("Signal Function $x3[n]$", fontsize='large')
ax1.set_title("Signal $x3[n]$ v Time $n$", fontsize='x-large')
```

PART C

Accumulation. $\sim x[n]$ and $y2[m]$



Calculated by visual inspection of $y2[m]$

```
# -*- coding: utf-8 -*-
"""
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10/21/2022
ECE 315 Signals & Systems
HW#2, Problem 2c
"""

import numpy as np
import matplotlib.pyplot as plt

#Setup Plot
plt.style.use('seaborn-whitegrid')
fig, (ax1, ax2) = plt.subplots(2, figsize=(15, 5), sharex=True)

#Plot Points y2[m]
for i in range(0, 20):
    ax2.stem(i, -0.5)
    ax2.stem(-1, 5)
    ax2.stem(20, 5)
for i in range(-5, -1):
    ax2.stem(i, 0)
for i in range(21, 26):
    ax2.stem(i, 0)

#Plot Points x[n]
for i in range(0, 20):
    ax1.stem(i, (4 - i/2))
    ax1.stem(-1, 5)
```

```
ax1.stem(20, 0)

for k in range(-5, -1):
    ax1.stem(k, 0)

for j in range (21, 25):
    ax1.stem(j, 0)

#Plot Settings
ax2.set_xlabel("Time $m$", fontsize='large')
ax2.set_ylabel("Signal Function $y2[m]$", fontsize='large')
ax2.set_title("Signal $y2[m]$ v Time $m$", fontsize='x-large')

ax1.set_ylabel("Signal Function $xTilda[n]$", fontsize='large')
ax1.set_title("Signal $xTilda[n]$ v Time $n$", fontsize='x-large')
```

Given: Determine if following signals are periodic. Plot in python. If complex, determine magnitude & phase. Find and plot its real & imaginary parts. For each periodic signal, determine the fundamental period, frequency, and angular frequency, & calculate average power in python. Assume $x[n]$ units are furlongs per fortnight. units^2

Find: a) $x[n] = 2 \sin\left(\frac{9n}{163} - \frac{3\pi}{4}\right)$ b) $x[n] = 2 \cos\left(\frac{3\pi n}{163} - \frac{\pi}{4}\right)$

c) $x[n] = 3e^{j(-\frac{\pi}{48} + j\frac{2\pi}{253})n}$ d) $x[n] = 3e^{j(\frac{9\pi n}{163} - \frac{3\pi}{2})}$

e) $x[n] = 3e^{j(-\frac{5n}{96} - \pi/3)}$

Solution:

a) $x[n] = 2 \sin\left(2\pi\left(\frac{n}{\frac{326\pi}{9}}\right) - \frac{3\pi}{4}\right)$

$$T_{01} = \frac{326\pi}{9}$$

since T_{01} is irrational, $x[n]$ is aperiodic

b) $x[n] = 2 \cos\left(2\pi\left(\frac{n}{\frac{326}{3}}\right) - \frac{\pi}{4}\right)$

$$T_{01} = \frac{326}{3}$$

since T_{01} is rational, $x[n]$ is periodic

Fundamental Period $T_0 = \frac{326}{3} = 108.67$ time unit

Fundamental Frequency $f_0 = \frac{1}{T_0} = \frac{3}{326} = 9.2$ mHz

Fundamental Angular Frequency $\omega_0 = 2\pi f_0 = \frac{3\pi}{163}$ rad/time unit

c) $x[n] = 3e^{(-\frac{\pi}{48} + j\frac{2\pi}{253})n}$

$$= 3e^{-\frac{\pi}{48}n} \left(\cos\left(\frac{2\pi}{253}n\right) + j \sin\left(\frac{2\pi}{253}n\right) \right)$$

$$T_{01} = \frac{2\pi}{\omega_0} = \frac{2\pi}{\left(\frac{2\pi}{253}\right)} = 253$$

T_{01} is rational, but since $|x[n]|$ decreases exponentially, $x[n]$ is aperiodic

~~$T_0 = 253$ time unit~~

~~$f_0 = \frac{1}{T_0} = \frac{1}{253} = 3.953$ mHz~~

~~$\omega_0 = 2\pi \frac{1}{T_0} = \frac{2\pi}{253}$ radians/time unit~~

~~Re: $3e^{-\frac{\pi}{48}n} \cdot \cos\left(\frac{2\pi}{253}n\right)$~~

~~Im: $3e^{-\frac{\pi}{48}n} j \sin\left(\frac{2\pi}{253}n\right)$~~

Magnitude = $3e^{-\frac{\pi}{48}n}$ $\theta = \frac{2\pi}{253}n$

~~Re + Im = $2.8091 + 0.06977j$~~

d) $x[n] = 3e^{j(\frac{8\pi n}{287} - \frac{3\pi}{2})}$ Using Eulers Formula, we get

$$x[n] = 3 \left(\cos \left(\frac{8\pi n}{287} - \frac{3\pi}{2} \right) + j \sin \left(\frac{8\pi n}{287} - \frac{3\pi}{2} \right) \right)$$

$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\frac{8\pi}{287}} = \frac{287}{4}$$

$\overbrace{\qquad\qquad\qquad}^{2\pi \left(\frac{4}{287} \right)}$

Since T_0 is rational,
 $x[n]$ is periodic

$$T_0 = \frac{287}{4} \text{ time units}$$

$$f_0 = \frac{1}{T_0} = \frac{4}{287} = 13.9 \text{ mHz}$$

$$\omega_0 = 2\pi f_0 = 2\pi (13.9 \times 10^{-3}) = 87.34 \times 10^{-3} \text{ radians/time unit}$$

$$\text{mag} : |x[n]| = 3 \quad \theta = \frac{8\pi n}{287} - \frac{3\pi}{2}$$

$$\text{Re} + \text{Im} = 3 \left(\cos \left(\frac{8\pi n}{287} - \frac{3\pi}{2} \right) + j \sin \left(\frac{8\pi n}{287} - \frac{3\pi}{2} \right) \right)$$

$$e) x[n] = 3e^{j(-\frac{5n}{96} - \frac{\pi}{3})}$$

$$x[n] = 3 \left(\cos \left(-\frac{5n}{96} - \frac{\pi}{3} \right) + j \sin \left(-\frac{5n}{96} - \frac{\pi}{3} \right) \right)$$

$\overbrace{\qquad\qquad\qquad}^{2\pi \left(\frac{5n}{192\pi} \right)}$

$$T_0 = \frac{192\pi}{5}$$

since T_0 is irrational, $x[n]$ is aperiodic

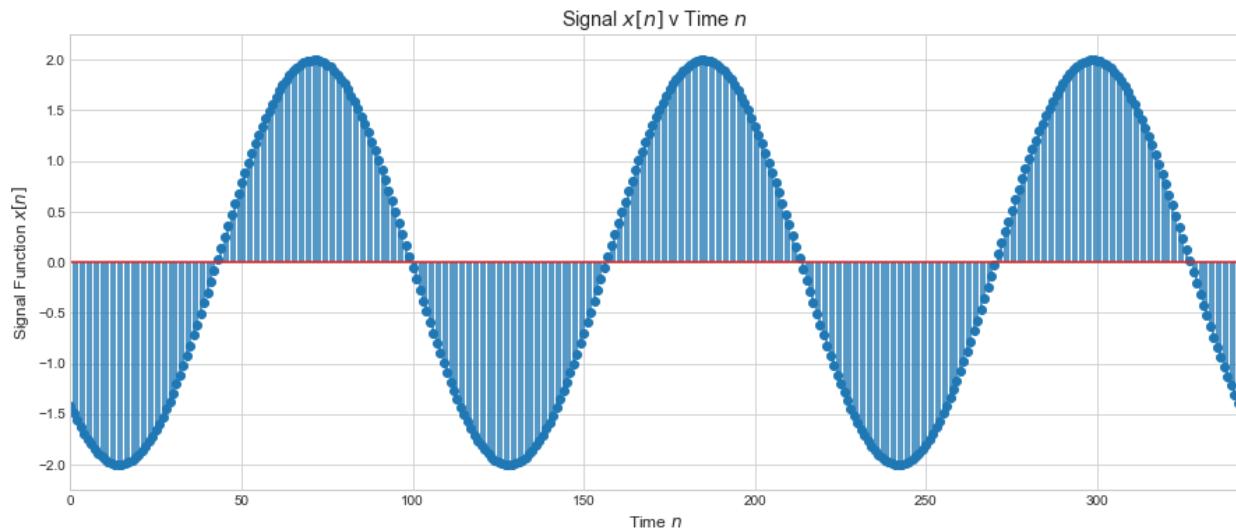
$$\theta = -\frac{5n}{96} - \frac{\pi}{3} \quad \text{mag: } 3$$

$$\text{Re} + \text{Im} = 3 \cos \left(-\frac{5n}{96} - \frac{\pi}{3} \right) + 3j \sin \left(-\frac{5n}{96} - \frac{\pi}{3} \right)$$

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10/21/2022
ECE 315 Signals and Systems
HW#2 Problem 3

Note: Some of the problems are plotted with 3 periods and some with 1. They vary to give a good graph visually.

PART A (Aperiodic)



```
# -*- coding: utf-8 -*-
"""
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10/21/2022
ECE 315 Signals & Systems
HW#2, Problem 3a
"""

import numpy as np
import matplotlib.pyplot as plt

#Period
N0 = ((326*np.pi) / 9)

#Number of periods to plot, starting from n = 0
numPeriods = 3
lastn = numPeriods*N0

#Time
n = np.arange(0, lastn + 1)

#Phase Shift
oneVec = np.ones(np.size(n))
```

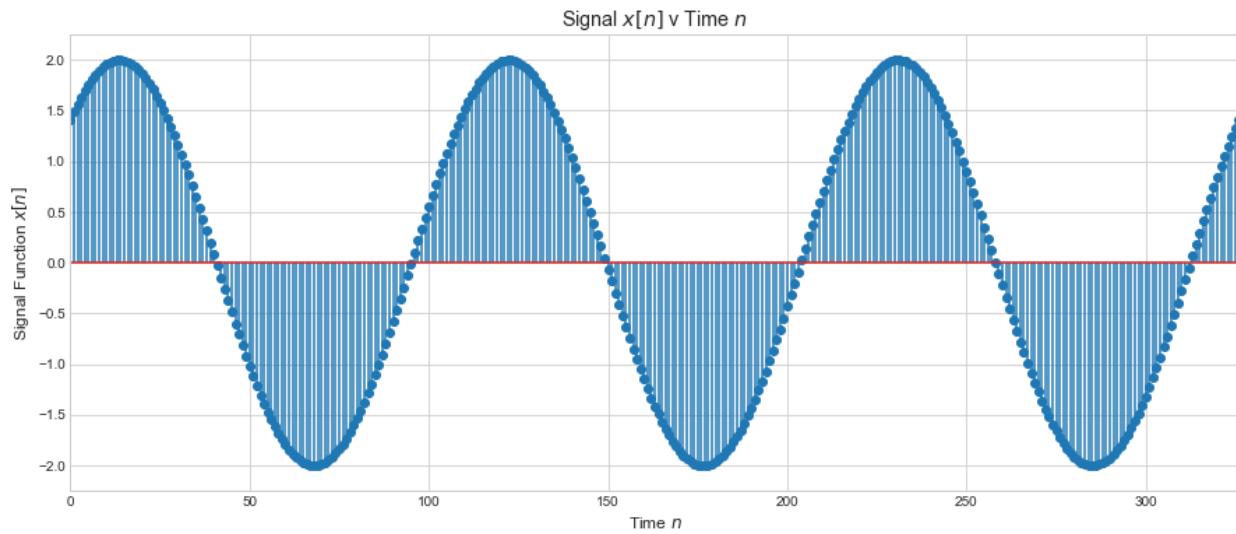
```

#Signal
x = 2.0*np.sin(((9.0)/163.0)*n - (0.75*np.pi)*oneVec)
fig, ax = plt.subplots(1, figsize=(15,6))
ax.stem(n, x)
ax.set_xlim([0, lastn])
ax.set_ylim([-2.25, 2.25])

#Plot Settings
ax.set_xlabel("Time $n$", fontsize='large')
ax.set_ylabel("Signal Function $x[n]$", fontsize='large')
ax.set_title("Signal $x[n]$ v Time $n$", fontsize='x-large')

```

PART B (periodic and average power)



The average power of $x[n]$ is 2.00 (furlongs 2 / fortnight).

```

# -*- coding: utf-8 -*-
"""
Philip Nevins
10/21/2022
ECE 315 Signals & Systems
HW#2, Problem 3b
"""

import numpy as np
import matplotlib.pyplot as plt

```

```

#Period
N0 = 326/3

```

```

#Number of periods to plot, starting from n = 0
numPeriods = 3

```

```

lastn = numPeriods*N0

#Time
n = np.arange(0, lastn + 1)

#Phase Shift
oneVec = np.ones(np.size(n))

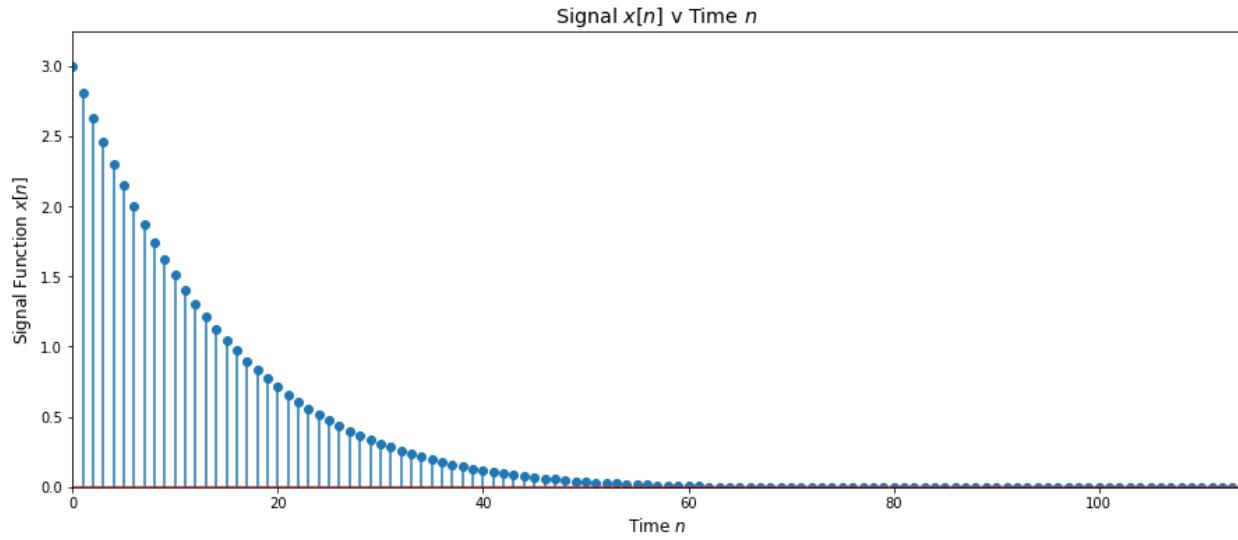
#Signal
x = 2.0*np.cos(((3.0*np.pi)/163.0)*n - (np.pi/4)*oneVec)
fig, ax = plt.subplots(1, figsize=(15,6))
ax.stem(n, x)
ax.set_xlim([0, lastn])
ax.set_ylim([-2.25, 2.25])

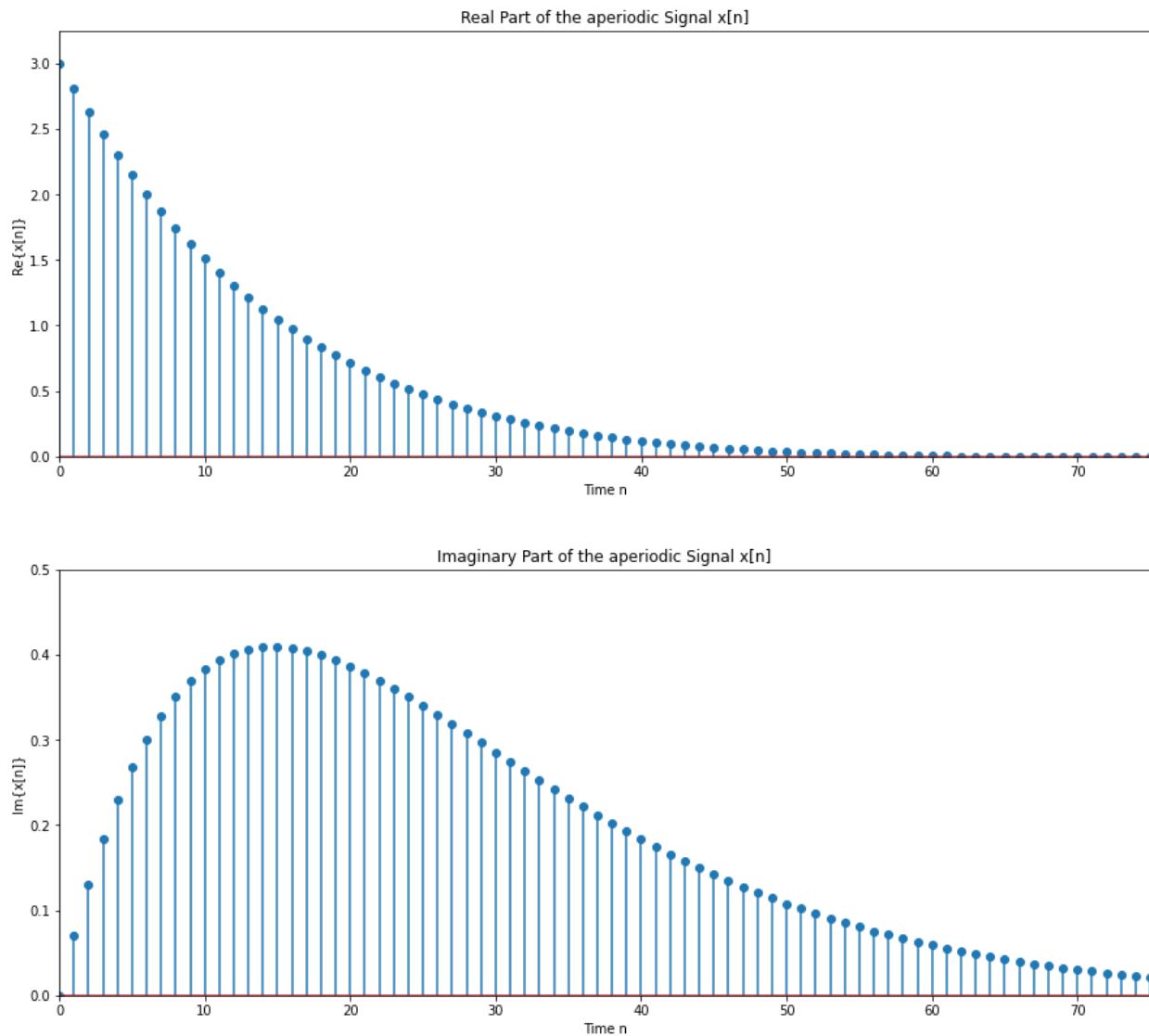
#Plot Settings
ax.set_xlabel("Time $n$", fontsize='large')
ax.set_ylabel("Signal Function $x[n]$", fontsize='large')
ax.set_title("Signal $x[n]$ v Time $n$", fontsize='x-large')

avgPower = np.dot(np.abs(x)[0:108],np.abs(x)[0:108])/108;
print("The average power of x[n] is {0} (furlongs^2 / fortnight).".format(avgPower))

```

PART C (aperiodic, imaginary)





```
# -*- coding: utf-8 -*-
"""

```

Philip Nevins

10/21/2022

ECE 315 Signals & Systems

HW#2, Problem 3c

"""

```
import numpy as np
import matplotlib.pyplot as plt
```

```
#Period
```

```
N0 = 253
```

```
#Number of periods to plot, starting from n = 0
```

```
numPeriods = 1
```

```
lastn = numPeriods*N0
```

```

#Time
n = np.arange(0, lastn + 1)

#Phase Shift
oneVec = np.ones(np.size(n))

#Signal
x = 3.0*np.exp(((2j*np.pi)/253.0) - (np.pi/48)*oneVec)*n
fig, ax = plt.subplots(1, figsize=(15,6))
ax.stem(n, x)
ax.set_xlim([0, lastn])
ax.set_ylim([0, 3.25])

#Plot Settings
ax.set_xlabel("Time $n$", fontsize='large')
ax.set_ylabel("Signal Function $x[n]$", fontsize='large')
ax.set_title("Signal $x[n]$ v Time $n$", fontsize='x-large')

# -*- coding: utf-8 -*-
"""
@author: phil nevins
"""

#x = 3.0*np.exp(((2j*np.pi)/253.0) - (np.pi/48)*oneVec)*n
import numpy as np
import matplotlib.pyplot as plt

lastn = 75
n = np.arange(0, lastn + 1) # time vector
oneVec = np.ones(np.size(n)) # vector of ones for phase shift
phase = (((2*n*np.pi)/253.0)*oneVec)
mag = np.exp(-n*np.pi/48)

xre = 3.0*mag*np.cos(phase) # real part of x[n]
xim = 3.0*mag*np.sin(phase) # imag part of x[n]

fig, ax = plt.subplots(1, figsize=(15,6))
ax.stem(n, xre)
ax.set_xlim([0, lastn])
ax.set_ylim([0, 3.25])
ax.set_title("Real Part of the aperiodic Signal x[n]")
ax.set_xlabel("Time n")
ax.set_ylabel("Re{x[n]}")

fig, ax = plt.subplots(1, figsize=(15,6))
ax.stem(n, xim)

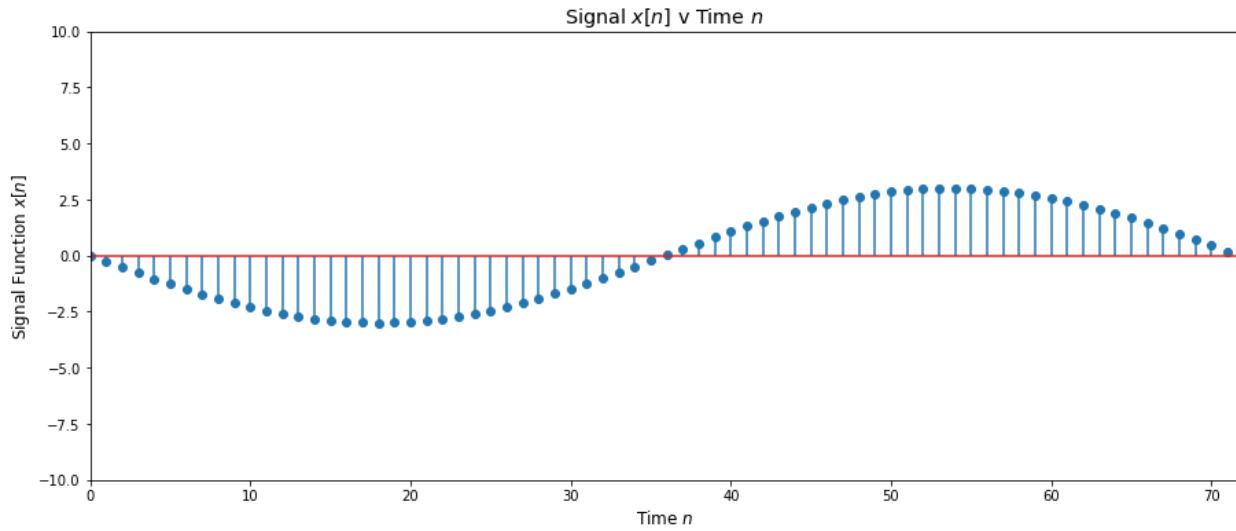
```

```

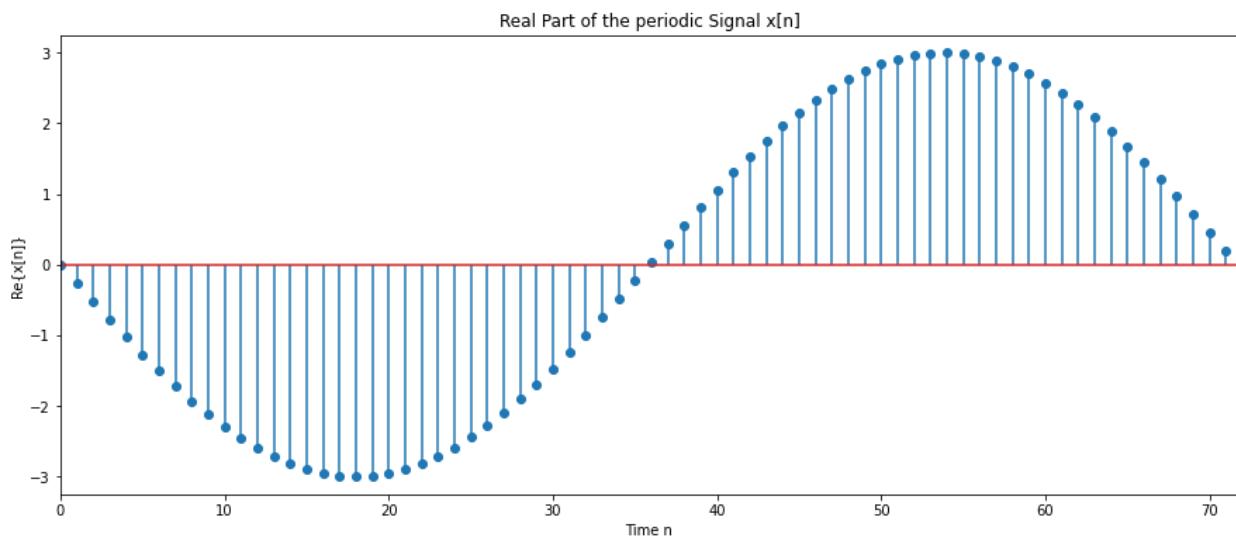
ax.set_xlim([0, lastn])
ax.set_ylim([0, 0.5])
ax.set_title("Imaginary Part of the aperiodic Signal x[n]")
ax.set_xlabel("Time n")
ax.set_ylabel("Im{x[n]}")

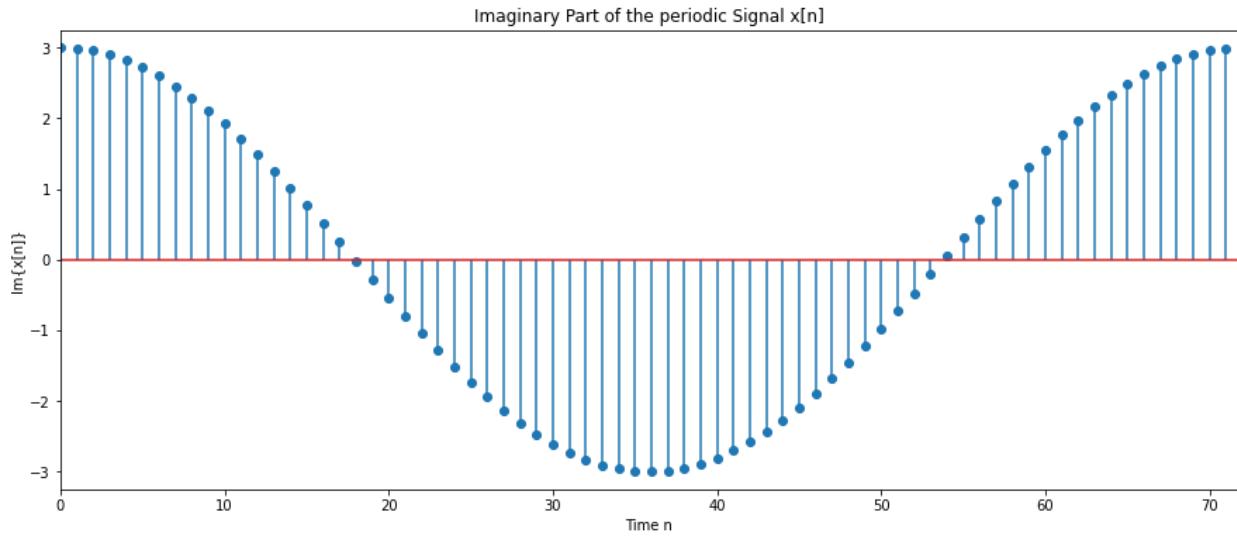
```

PART D (periodic, imaginary and average power)



The average power of $x[n]$ is 9.00 (furlongs^{^2} / fortnight).





```
# -*- coding: utf-8 -*-
"""

```

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 10/21/2022
 ECE 315 Signals & Systems
 HW#2, Problem 3d

```
"""

```

```
import numpy as np
import matplotlib.pyplot as plt
```

```
#Period
```

```
N0 = (287/4)
```

```
#Number of periods to plot, starting from n = 0
```

```
numPeriods = 1
```

```
lastn = numPeriods*N0
```

```
#Time
```

```
n = np.arange(0, lastn + 1)
```

```
#Phase Shift
```

```
oneVec = np.ones(np.size(n))
```

```
#Signal
```

```
x = 3.0*np.exp(((8j*np.pi*n) / 287) - (3j*np.pi/2)*oneVec)
```

```
fig, ax = plt.subplots(1, figsize=(15,6))
```

```
ax.stem(n, x)
```

```
ax.set_xlim([0, lastn])
```

```
ax.set_ylim([-10, 10])
```

```
#Plot Settings
```

```
ax.set_xlabel("Time $n$\", fontsize='large')
```

```

ax.set_ylabel("Signal Function $x[n]$", fontsize='large')
ax.set_title("Signal $x[n]$ v Time $n$", fontsize='x-large')

avgPower = np.dot(np.abs(x)[0:72],np.abs(x)[0:72])/72
print("The average power of x[n] is {0} (furlongs^2 / fortnight).".format(avgPower))

# -*- coding: utf-8 -*-
"""
@author: phil nevins
"""

#x = 3.0*np.exp(((8j*np.pi*n) / 287) - (3j*np.pi/2)*oneVec)
import numpy as np
import matplotlib.pyplot as plt

lastn = 287/4
n = np.arange(0, lastn + 1) # time vector
oneVec = np.ones(np.size(n)) # vector of ones for phase shift
phase = (((8*np.pi*n) / 287) - (3*np.pi/2)*oneVec)
mag = 3

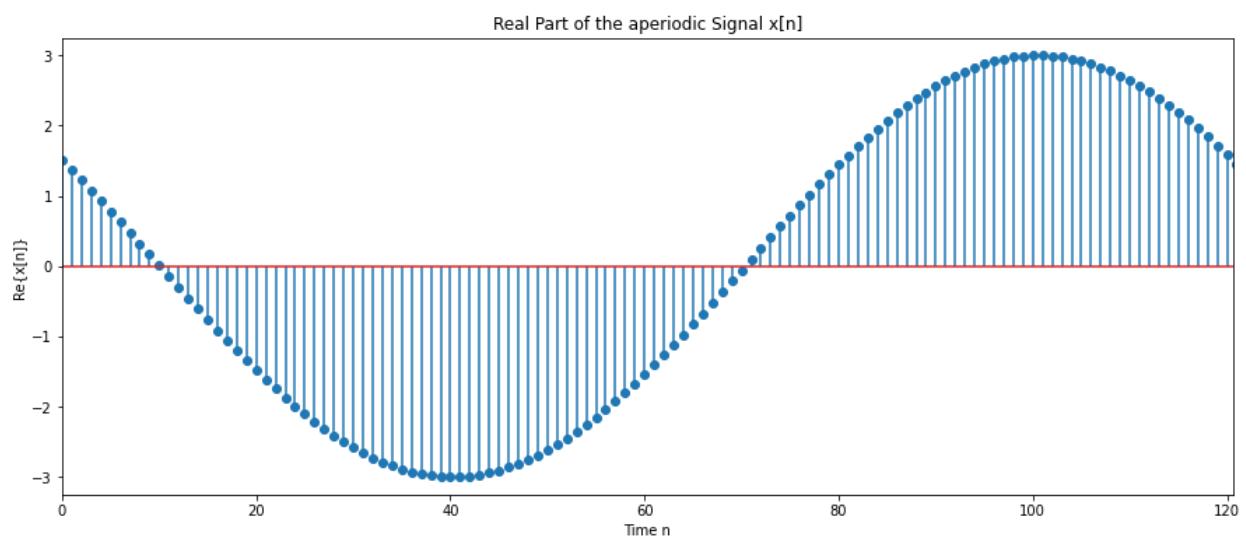
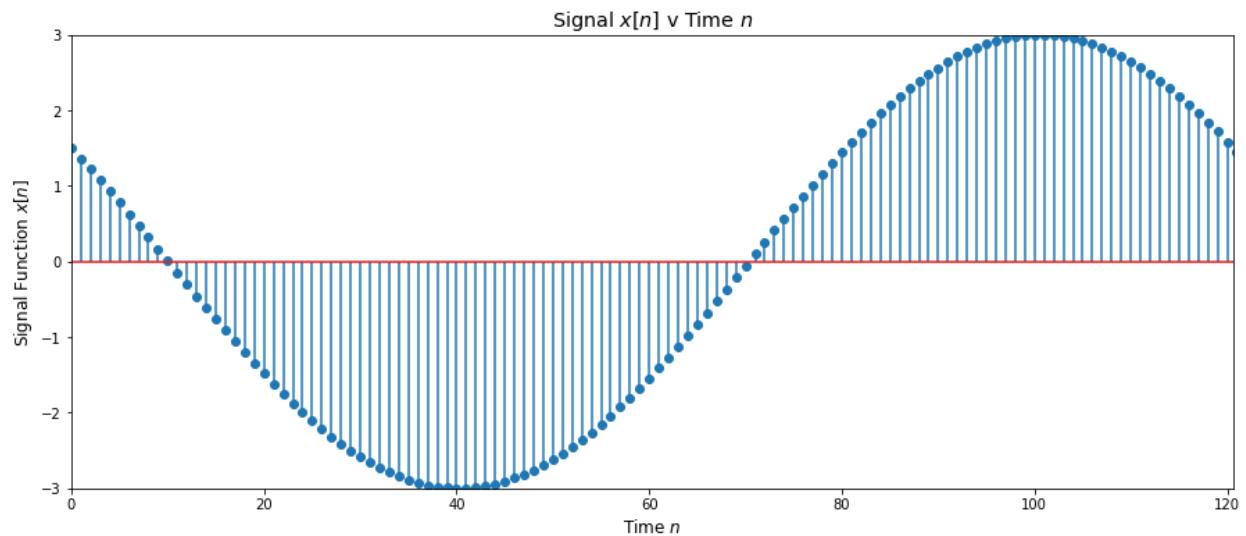
xre = mag*np.cos(phase) # real part of x[n]
xim = mag*np.sin(phase) # imag part of x[n]

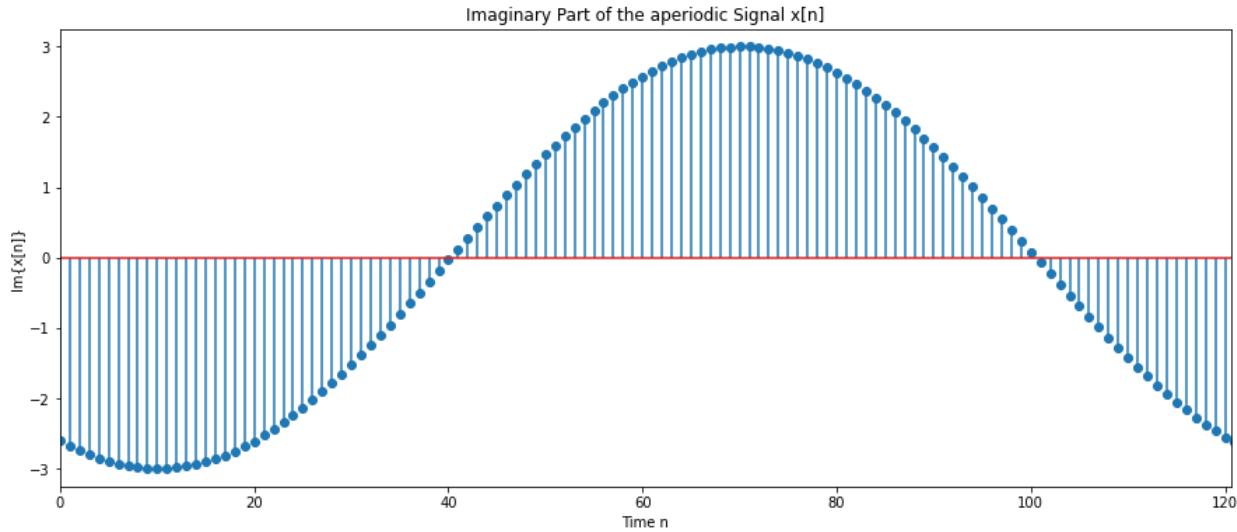
fig, ax = plt.subplots(1, figsize=(15,6))
ax.stem(n, xre)
ax.set_xlim([0, lastn])
ax.set_ylim([-3.25, 3.25])
ax.set_title("Real Part of the periodic Signal x[n]")
ax.set_xlabel("Time n")
ax.set_ylabel("Re{x[n]}")

fig, ax = plt.subplots(1, figsize=(15,6))
ax.stem(n, xim)
ax.set_xlim([0, lastn])
ax.set_ylim([-3.25, 3.25])
ax.set_title("Imaginary Part of the periodic Signal x[n]")
ax.set_xlabel("Time n")
ax.set_ylabel("Im{x[n]}")

```

PART E (aperiodic and imaginary)





```
# -*- coding: utf-8 -*-
"""
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ECE 315 Signals & Systems
HW#2, Problem 3e
"""

import numpy as np
import matplotlib.pyplot as plt

#Period
N0 = (192*np.pi/5)

#Number of periods to plot, starting from n = 0
numPeriods = 1
lastn = numPeriods*N0

#Time
n = np.arange(0, lastn + 1)

#Phase Shift
oneVec = np.ones(np.size(n))

#Signal
x = 3.0*np.exp((( -5j*n) / 96) - (1j*np.pi/3)*oneVec)
fig, ax = plt.subplots(1, figsize=(15,6))
ax.stem(n, x)
ax.set_xlim([0, lastn])
ax.set_ylim([-3, 3])

#Plot Settings
```

```

ax.set_xlabel("Time $n$", fontsize='large')
ax.set_ylabel("Signal Function $x[n]$", fontsize='large')
ax.set_title("Signal $x[n]$ v Time $n$", fontsize='x-large')

# -*- coding: utf-8 -*-
"""
@author: pnevi
"""

#x = 3.0*np.exp(((5j*n) / 96) - (1j*np.pi/3)*oneVec)
import numpy as np
import matplotlib.pyplot as plt

lastn = 192*np.pi/5
n = np.arange(0, lastn + 1) # time vector
oneVec = np.ones(np.size(n)) # vector of ones for phase shift
phase = ((-5*n) / 96) - (np.pi/3)*oneVec
mag = 3.0

xre = mag*np.cos(phase) # real part of x[n]
xim = mag*np.sin(phase) # imag part of x[n]

fig, ax = plt.subplots(1, figsize=(15,6))
ax.stem(n, xre)
ax.set_xlim([0, lastn])
ax.set_ylim([-3.25, 3.25])
ax.set_title("Real Part of the aperiodic Signal x[n]")
ax.set_xlabel("Time n")
ax.set_ylabel("Re{x[n]}")

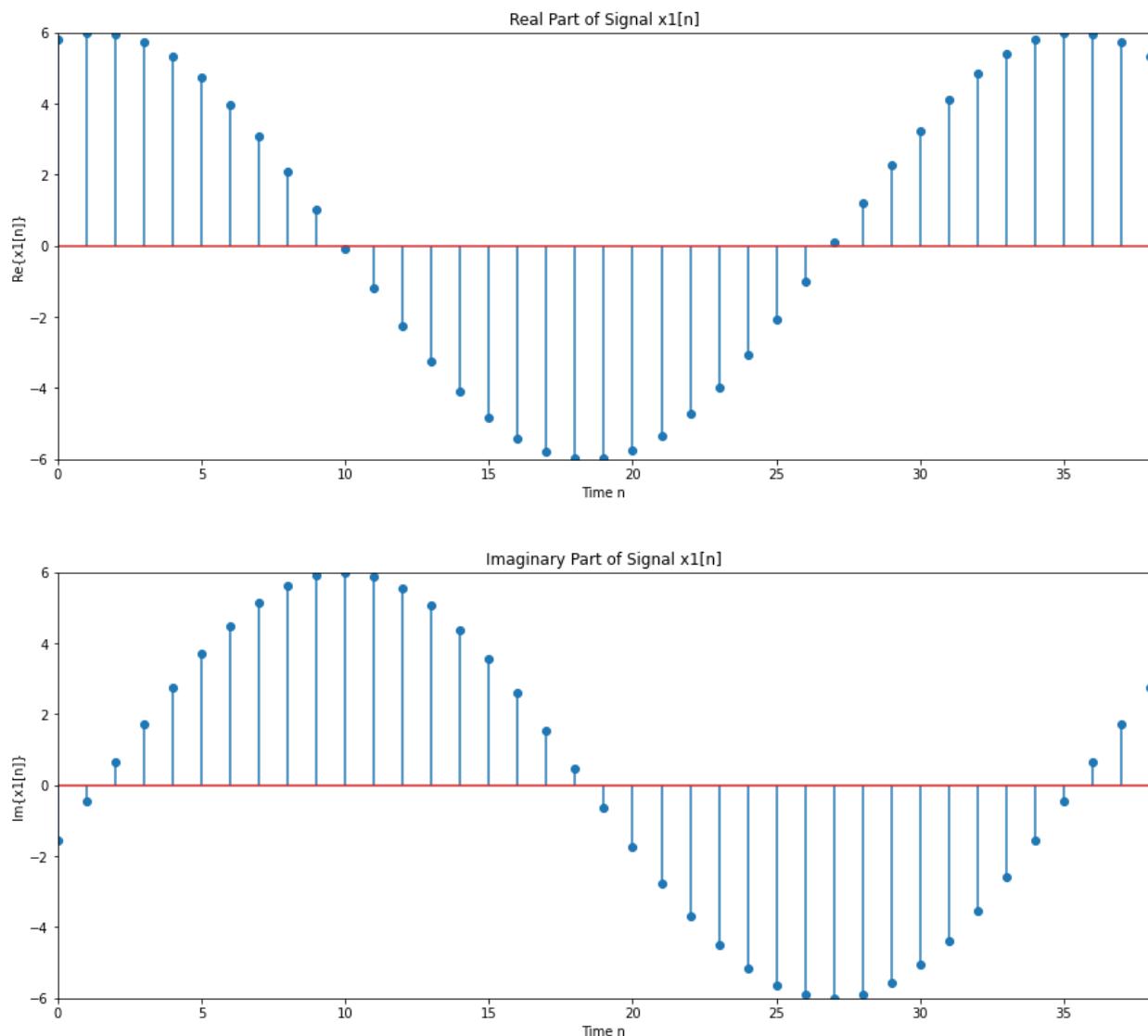
fig, ax = plt.subplots(1, figsize=(15,6))
ax.stem(n, xim)
ax.set_xlim([0, lastn])
ax.set_ylim([-3.25, 3.25])
ax.set_title("Imaginary Part of the aperiodic Signal x[n]")
ax.set_xlabel("Time n")
ax.set_ylabel("Im{x[n]}")

```

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 ECE 315 Signals and Systems
 HW#2 Problem 4

The two signals are identical. There is a very small shift due to the difference in the $3 \rightarrow 207$ in the numerator of the π^*n side of the phase. This makes sense, because $207/3 = 69$, so there only real difference is the phase angel.

Signal X1[n]



```
# -*- coding: utf-8 -*-
"""

```

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 10/21/2022

ECE 315 Signals & Systems

HW#2, Problem 4, $x_1[n]$

"""

```
import numpy as np
import matplotlib.pyplot as plt
```

```
lastn = 38
```

```
n = np.arange(0, lastn + 1) # time vector
```

```
oneVec = np.ones(np.size(n)) # vector of ones for phase shift
```

```
phase = ((3*np.pi*n) / 51) - (np.pi/12)*oneVec
```

```
mag = 6.0
```

```
xre = mag*np.cos(phase) # real part of  $x_1[n]$ 
xim = mag*np.sin(phase) # imag part of  $x_1[n]$ 
```

```
fig, ax = plt.subplots(1, figsize=(15,6))
```

```
ax.stem(n, xre)
```

```
ax.set_xlim([0, lastn])
```

```
ax.set_ylim([-6, 6])
```

```
ax.set_title("Real Part of Signal  $x_1[n]$ ")
```

```
ax.set_xlabel("Time n")
```

```
ax.set_ylabel("Re{ $x_1[n]$ }")
```

```
fig, ax = plt.subplots(1, figsize=(15,6))
```

```
ax.stem(n, xim)
```

```
ax.set_xlim([0, lastn])
```

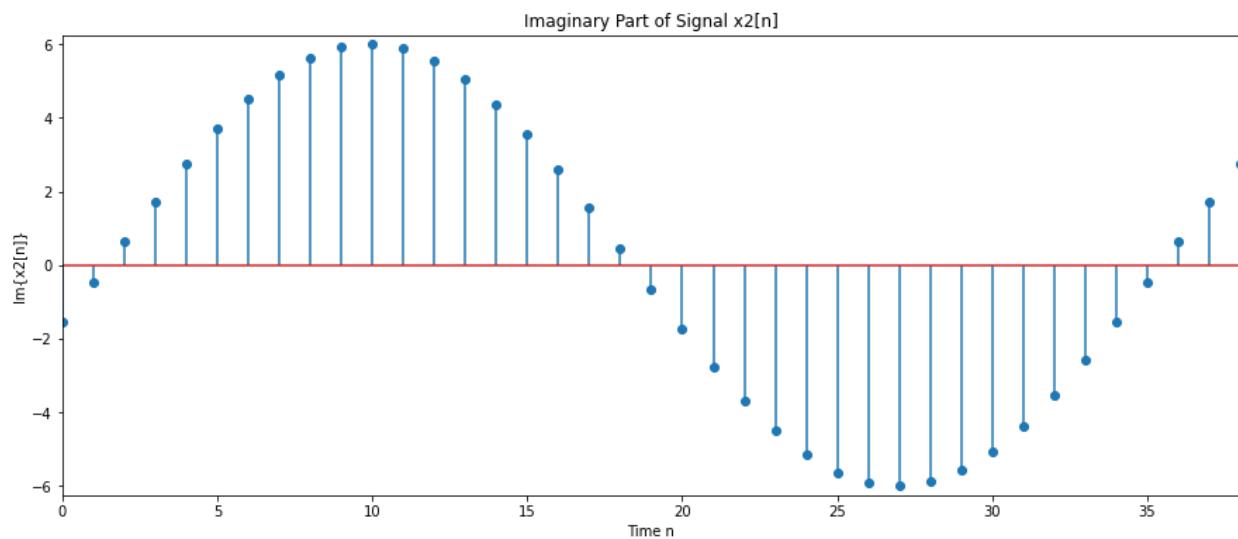
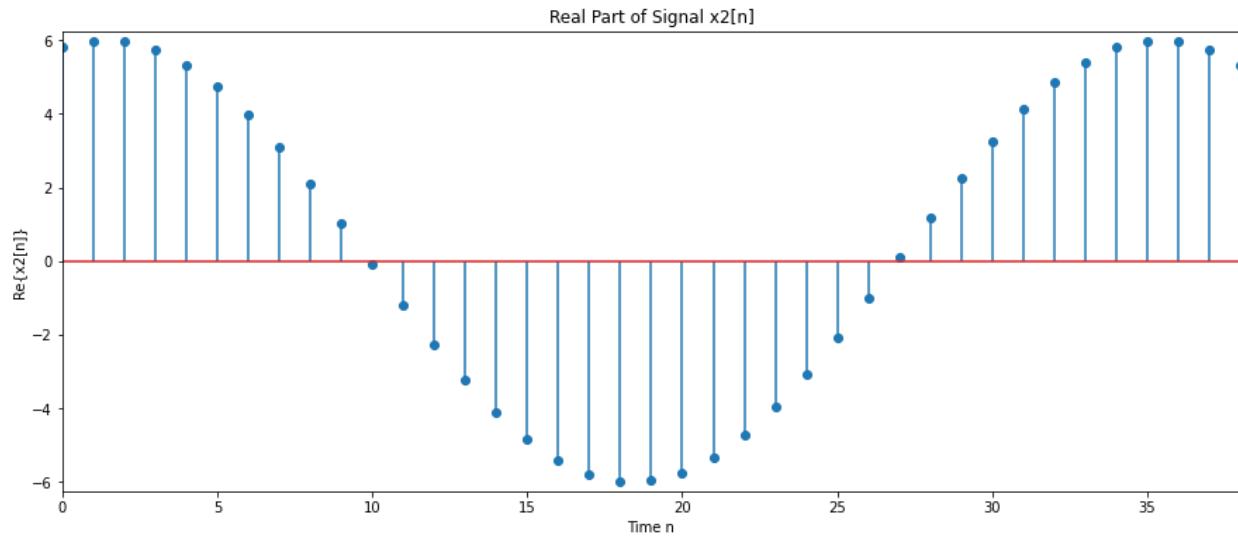
```
ax.set_ylim([-6, 6])
```

```
ax.set_title("Imaginary Part of Signal  $x_1[n]$ ")
```

```
ax.set_xlabel("Time n")
```

```
ax.set_ylabel("Im{ $x_1[n]$ }")
```

Signal X2[n]



```
# -*- coding: utf-8 -*-
```

```
""""
```

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10/21/2022

ECE 315 Signals & Systems

HW#2, Problem 4, x2[n]

```
""""
```

```
import numpy as np
import matplotlib.pyplot as plt
```

```
lastn = 38
```

```
n = np.arange(0, lastn + 1) # time vector
```

```

oneVec = np.ones(np.size(n)) # vector of ones for phase shift
phase = ((207*np.pi*n) / 51) - (np.pi/12)*oneVec
mag = 6.0

xre = mag*np.cos(phase) # real part of x2[n]
xim = mag*np.sin(phase) # imag part of x2[n]

fig, ax = plt.subplots(1, figsize=(15,6))
ax.stem(n, xre)
ax.set_xlim([0, lastn])
ax.set_ylim([-6.25, 6.25])
ax.set_title("Real Part of Signal x2[n]")
ax.set_xlabel("Time n")
ax.set_ylabel("Re{x2[n]}")

fig, ax = plt.subplots(1, figsize=(15,6))
ax.stem(n, xim)
ax.set_xlim([0, lastn])
ax.set_ylim([-6.25, 6.25])
ax.set_title("Imaginary Part of Signal x2[n]")
ax.set_xlabel("Time n")
ax.set_ylabel("Im{x2[n]}")

```

Given:

Determine whether the following systems have the properties linearity, time invariance, BIBO stability, memory, causality, or invertibility. If the system is invertible, find its inverse. (56 points total)

Find:

(a) $y(t) = (t^2 - 1)x(t - 1)$ (14 points)

(b) $y[n] = e^{-x[n]}$ (14 points)

(c) $y(t) = 1 - x(t)$ (14 points)

(d) $y[n] = \sum_{n_0}^n x[n]$ where n_0 is a fixed, finite time (14 points)

Solution:

a) $y(t) = (t^2 - 1)x(t - 1)$

$y_1(t) + y_2(t) = (t^2 - 1)(x_1(t-1) + x_2(t-1))$

Let $x(t-1) = x_1(t-1) + x_2(t-1)$

Then $y(t) = (t^2 - 1)(x_1(t-1) + x_2(t-1))$

Since $y(t) = y_1(t) + y_2(t)$, it's additive

Let $x(t-1) = Ax_1(t-1)$; $A = \text{constant}$

then $y(t) = (t^2 - 1)Ax_1(t-1) = A(t^2 - 1)x(t-1)$

Therefore it's homogeneous $A \cdot y(t)$ Since $y(t)$ is both additive & homogeneous, it's linearIt has memory because y at t depends on x at $t-1$

$y(t-t_0) = ((t-t_0)^2 - 1)x(t-t_0 - 1)$

$y(t) = (t^2 - 1)x(t - t_0 - 1)$ since $y(t-t_0) \neq y(t) @ x(t-t_0)$

∴ It is time variantsince we have $(t^2 - 1) \cdot x(t-1)$ it only depends on past times, therefore it's causalFor bounded input $-\infty < x(t-1) < \infty$, $y(t) \rightarrow \infty$
∴ NOT StableLet $t=1 \neq t=-1$. This yields $y(t)=0$ for both inputs, so it is not invertable

b) $y[n] = e^{-x[n]}$

$$y_1[n] + y_2[n] = e^{-x_1[n]} + e^{-x_2[n]}$$

$$\text{Let } x[n] = x_1[n] + x_2[n]$$

$$\text{Then } y[n] = e^{-(x_1[n] + x_2[n])} = e^{-x_1[n]} e^{-x_2[n]}$$

Since $y[n] \neq y_1[n] + y_2[n]$, $y[n]$ is not additive
and therefore, it's not linear

$y[n]$ only depends on current input $x[n]$, so it's memoryless

$$y[n-n_0] = e^{-x[n-n_0]}$$

$$y[n] @ x[n-n_0] = e^{-x[n-n_0]}$$

since $y[n-n_0] = y[n] @ x[n-n_0]$, it's time invariant

Since $y[n]$ only depends on current times, so it's causal

since it's $e^{-x[n]}$, bounded inputs will have bounded outputs,
so it's stable, $e^{-\infty} = 0$

Since $y[n] = e^{-x[n]}$, different $x[n]$ will always produce
different $y[n]$, & $y[n] \neq 0$ for nonzero $x[n]$, so
the inverse of $y[n]$ will exist

$$\ln(y[n]) = \ln(e^{-x[n]}) \\ = -x[n]$$

$$x[n] = -\ln(y[n])$$

$$x[n] = \ln(\frac{1}{y[n]})$$

$$\therefore \text{inverse of } y[n] = \ln(\frac{1}{x[n]})$$

c) $y(t) = 1 - x(t)$

Due to the 1, it's NOT linear

$$y(t-t_0) = 1 - x(t-t_0)$$

$$y(t) @ x(t-t_0) = 1 - x(t-t_0)$$

Since $y(t-t_0) = y(t) @ x(t-t_0)$, $y(t)$ is time invariant

We know that $x(t)$ is stable, so $1 - x(t)$ is also stable

$y(t)$ only depends on current times, so it's causal

Since $y(t)$ does not depend on future or past times, it's memoryless

$$-y(t) = -(1 - x(t)) \Rightarrow y(t) = x(t) + 1$$

I t is invertible

d) $y[n] = \sum_{n_0}^n x[n]$ where n_0 is a fixed, finite time

$$\tilde{x}[n] = ax_1[n] + bx_2[n]$$

$$\tilde{y}[n] = \sum_{n_0}^n (ax_1[n] + bx_2[n])$$

$$= a \sum_{n_0}^n x_1[n] + b \sum_{n_0}^n x_2[n]$$

$$= a y_1[n] + b y_2[n] \quad \therefore \text{It's linear}$$

$$y[n-n_0] = \sum_{n_0}^n x[n-n_0]$$

$$y[n] @ x[n-n_0] = \sum_{n_0}^n x[n]$$

Since $y[n-n_0] \neq y[n] @ x[n-n_0]$ it's time variant

Since it's a summation, $y[n]$ depends on time from $n \rightarrow n_0$ so the system has memory

Since $y[n]$ depends on time from $n \rightarrow n_0$, when

$n < n_0$ the system is acausal

since it's a summation, if the input is bounded the output will be bounded too, \therefore stable

Since it's a summation, you can have 2 distinct

input signals that result in the same output by varying the bounds \therefore IT'S NOT INVERTIBLE