Signals & Systems
Homework #4

ECE 315 – Fall 2022
195 points total

Due Wednesday, November 30

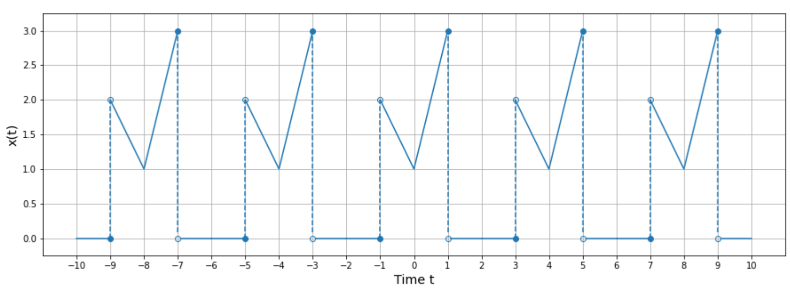
1. Consider the periodic continuous-time signal

$$x(t) = g(t) * \delta_4(t)$$

where

$$g(t) = \begin{cases} 1 - t, & -1 < t \leq 0 \\ 2t + 1, & 0 < t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

The signal $x(t)$ is shown below.



(a) Determine the continuous-time Fourier series (CTFS) coefficients for $x(t)$ by hand. Hints: This will require integration by parts. The final result will be complicated. (16 points)

(b) Use MATLAB or python to plot the real and imaginary parts and the magnitude and phase of the CTFS coefficients for harmonic numbers $k = -20, -19, \cdots, 20$. Include your code as part of your solution. (8 points)

(c) Use MATLAB or python to form the partial sums $x_N(t)$ of the CTFS and plot the partial sums with the signal $x(t)$ for $N = 1, 2, 5, 10, 50$ and for $-2 \leq t \leq 2$. Include your MATLAB or python code as part of your solution. (16 points)

2. Consider the system

$$y''(t) - 2y'(t) - 3y(t) = x(t).$$

(a) Use the Fourier properties to determine the harmonic response for the system. (The "harmonic response" is defined in Example 6.2 on p. 246 of the textbook.) (6 points)

(b) Determine the frequency response of the system. How is the harmonic response related to the frequency response? (6 points)

(c) Use the result from part (a) and the Fourier series pairs to determine the response of the system to the signal

$$x(t) = \text{tri}(t/2) * \delta_5(t). \text{ (6 points)}$$

3. Consider the continuous-time signal

$$x(t) = u(t)t^3 e^{-t}.$$

(a) Derive the continuous-time Fourier transform (CTFT) of $x(t)$ as a function of the angular frequency ω, as listed in Table 6.3 on p. 260 in the textbook. (10 points)

(b) Use MATLAB or python to determine the real and imaginary parts and the magnitude and phase of the CTFT and plot them as functions of the angular frequency ω for $-8 \leq \omega \leq 8$. Include your code as part of your solution. (20 points)

(c) Use the MATLAB function `fft` or the NumPy function `fft`, which calculate the discrete Fourier transform, to approximate the CTFT of $x(t)$. Create the approximation using $N = 262,144$ samples of $x(t)$ over the time interval $0 \leq t \leq 100$. Use MATLAB or python to plot the real and imaginary parts and the magnitude and phase of the approximation as a function of angular frequency ω for $-8 \leq \omega \leq 8$. Include your code in your solution. (10 points)

4. Consider the system

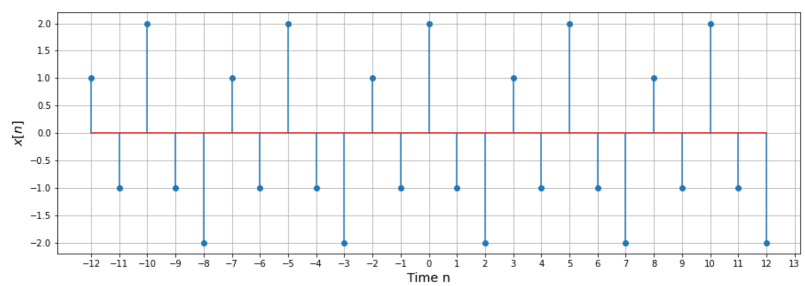
$$y''(t) - 5y' + 6y(t) = 2x'(t) - 3x(t).$$

(a) Find the frequency response for the system. (6 points)

(b) Use a partial fractions expansion and a table of CTFT pairs from the textbook to determine the response $y(t)$ of the system when the input is

$$x(t) = u(t)e^{-\frac{t}{5}} \sin(2\pi t). \text{ (16 points)}$$

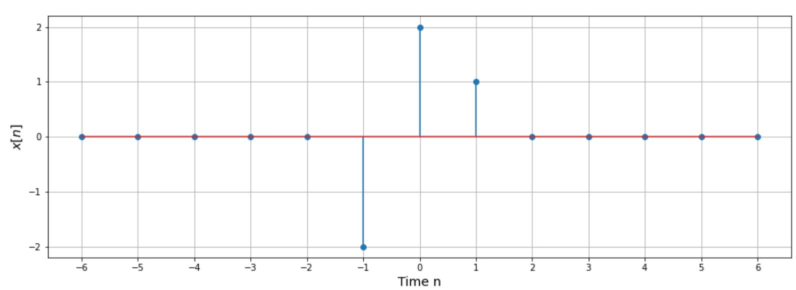
5. Consider the periodic discrete-time signal $x[n]$ shown below.



(a) Calculate the DTFS coefficients of $x[n]$ by hand and plot them using MATLAB or python. If they're complex-valued, plot the real and imaginary parts and the magnitude and phase. Include your MATLAB or python code in your answer. (16 points)

(b) Use MATLAB or python to show that the Fourier series representation of $x[n]$ has the expected values for one period of $x[n]$. Include your MATLAB or python code in your answer. (14 points)

6. Consider the discrete-time signal $x[n]$ shown below, where $x[n] = 0$ for $n \leq -2$ and for $n \geq 2$.



(a) Find the DTFT for the signal $x[n]$ by hand. (8 points)

(b) In this part, you'll be comparing the DTFT that you calculated and the DFT calculated using MATLAB or python.

   i. Use MATLAB or python to plot the magnitude and phase of the DTFT of $x[n]$ that you calculated in Part (a) as functions of angular frequency Ω. (6 points)

   ii. Next, create a MATLAB or python/NumPy vector of length 1024 that starts with the nonzero values of $x[n]$ and is otherwise 0. (This is called zero-padding and is often done when working with short signals. In this case, you're just adding values of $x[n]$ at times n when $x[n] = 0$.) Use the MATLAB or NumPy function `fft` to calculate the DFT of this signal and plot the magnitude and phase of the DFT as functions of angular frequency Ω. (10 points)

   iii. How do the magnitudes and phases of the DTFT and the DFT compare to each other? If they are different, use the properties of the DTFT or the properties of the DFT to explain why. (5 points)

(c) Show that the inverse DTFT produces the expected values of $x[n]$ for all integers n. This calculation should be done by hand. (16 points)