ECE 315 | HW#3 | Problem 1 | Phil Nevins

#1 Given:

$h[-m]$ — $x[n]$

$h[-1] = 1$ — $x[-1] = 1$

$h[0] = 2$ — $x[0] = -2$

$h[1] = 3$ — $x[1] = 2$

$x[2] = -1$

Find: a) Find & plot $h[-m]$ as a funct. of m.

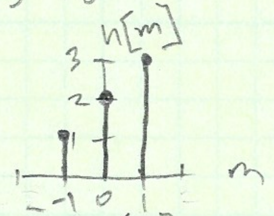
b) Find & plot $h[-3-m] = h[-(m+3)]$ as a funct. of m.

c) Calculate all nonzero for $y[n]$ w/ input $x[n]$ via convolution

d) verify using python

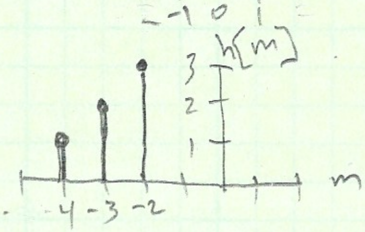
Solution:

a)



note: [-m] flips across h[m] axis

b)



note: [-3-m] flips across h[m] and shifts left 3

c) $h[n] * x[n] = y[n]$

$y[n] = 0 \; ; \; n \le -2$

$y[-1] = x[-1]\,h[1] = (1)\cdot(3) = 3$

$y[0] = x[-1]\,h[0] + x[0]\,h[1] = (1)(2) + (-2)(3) = -4$

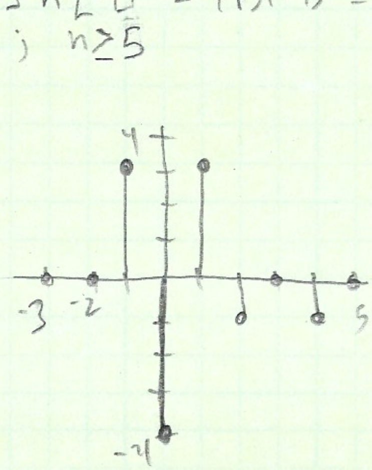
$y[1] = x[-1]\,h[-1] + x[0]\,h[0] + x[1]\,h[1] = (1)(1) + (-2)(2) + (2)(3) = 3$

$y[2] = x[0]\,h[-1] + x[1]\,h[0] + x[2]\,h[1] = (1)(-2) + (2)(2) + (3)(-1) = -1$

$y[3] = x[1]\,h[-1] + x[2]\,h[0] = (1)(2) + (2)(-1) = 0$

$y[4] = x[2]\,h[-1] = (1)(-1) = -1$

$y[n] = 0 \; ; \; n \ge 5$



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#2 $-8y[n] + 2y[n-1] + y[n-2] = -2x[n] + 3x[n-1]$

a) Find $y_h[n]$ to homogeneous eq

$$-8y_h[n] + 2y_h[n-1] + y_h[n-2] = 0$$

$$y_h[n] = c_1 r_1^n + c_2 r_2^n \quad ; \quad -8r^2 + 2r + 1 = 0$$

$r_1 = -1/4, \; r_2 = 1/2$ $\quad \boxed{\therefore\; y_h[n] = c_1(-1/4)^n + c_2(1/2)^n}$

b) Find I.V. $\hat{h}[0]$ & $\hat{h}[1]$ for impulse response $\hat{h}[n]$ where

$$-8\hat{h}[n] + 2\hat{h}[n-1] + \hat{h}[n-2] = \delta[n], \quad \hat{h}[n] = 0 \text{ for } n<0$$

$$\hat{h}[n] = \left(-\tfrac{1}{8}\right)\left(\delta[n] + 2\hat{h}[n-1] + \hat{h}[n-2]\right)$$

note: $a_0 = -8$, $a_1 = 2$, $a_2 = 1$

$$\hat{h}[0] = \left(-\tfrac{1}{8}\right)\left(1 + 2\hat{h}[-1] + \hat{h}[-2]\right) = \left(-\tfrac{1}{8}\right)(1+0+0) = \frac{1}{a_0} = -\frac{1}{8}$$

$$\hat{h}[1] = \left(-\tfrac{1}{8}\right)\left(0 + 2\hat{h}[0] + \hat{h}[-1]\right) = -\frac{a_1}{a_0^2} = -\frac{1}{32}$$

$$\boxed{\hat{h}[0] = -1/8 \;\;\&\;\; \hat{h}[1] = -1/32}$$

c) $\hat{h}[n] = y_h[n]u[n] = \left(c_1(-1/4)^n + c_2(1/2)^n\right)u[n]$

Note: solver TI 89, $c_1 = x$, $c_2 = y$

$\hat{h}[0] \Rightarrow c_1 + c_2 = -1/8$

$\hat{h}[1] \Rightarrow -0.25c_1 + 0.5c_2 = -1/32$

$c_1 = -41.667e{-3} = -1/24; \quad c_2 = -83.334e{-3} = -1/12$

$$\boxed{\therefore\; \hat{h}[n] = \left[(-41.667e{-3})\left(-\tfrac{1}{4}\right)^n + (-83.334e{-3})\left(\tfrac{1}{2}\right)^n\right]u[n]}$$

d) $h[n] = -2\hat{h}[n] + 3\hat{h}[n-1]$

$$\boxed{\begin{aligned} h[n] = &-2\left(-41.667e{-3}\left(-\tfrac{1}{4}\right)^n + \left(-83.334e{-3}\left(\tfrac{1}{2}\right)^n\right)\right)u[n] \\ &+ 3\left(-41.667e{-3}\left(-\tfrac{1}{4}\right)^{n-1} + \left(-83.334e{-3}\left(\tfrac{1}{2}\right)^{n-1}\right)\right)u[n-1] \end{aligned}}$$

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#3

$$-8y[n] + 2y[n-1] + y[n-2] = -2x[n] + 3x[n-1]$$

a) Find the transfer function

$a_0 = -8 \quad a_1 = 2 \quad a_2 = 1$ & $b_0 = -2, \quad b_1 = 3$

$$\therefore \quad H(z) = \frac{\sum_{k=0}^{1} b_k z^{-k}}{\sum_{k=0}^{2} a_k z^{-k}} = \frac{-2 + 3z^{-1}}{-8 + 2z^{-1} + z^{-2}} \cdot \frac{z^2}{z^2}$$

$$\frac{-2z^2 + 3z}{-8z^2 + 2z + 1}$$

$$\boxed{H(z) = \frac{z(-2z+3)}{-8z^2 + 2z + 1}}$$

b) Find frequency response

$$\boxed{H(e^{j\Omega}) = \frac{e^{j\Omega}(-2e^{j\Omega} + 3)}{-8(e^{j2\Omega}) + 2(e^{j\Omega}) + 1}}$$

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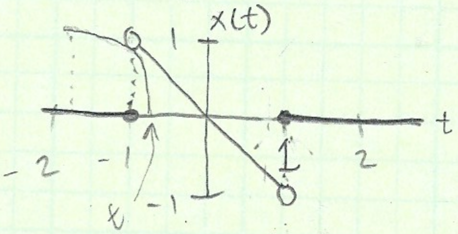
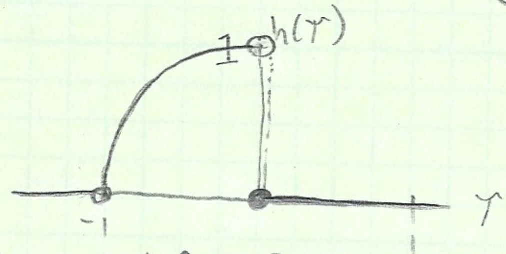
$\boxed{\#4}$

a) Plot $x(t) = \begin{cases} -t & ; -1 < t < 1 \\ 0 & ; \text{otherwise} \end{cases}$



b) $h(t) = \begin{cases} 1-t^2 & ; 0 < t < 1 \\ 0 & ; \text{otherwise} \end{cases} \Rightarrow h(-\tau) = \begin{cases} 1-t^2 & ; 0 < \tau < -1 \\ 0 & ; \text{otherwise} \end{cases}$

Plot $h(-\tau)$



c) $h(-2-\tau) = \begin{cases} 1-t^2 & ; -2 < \tau < -3 \\ 0 & ; \text{otherwise} \end{cases}$

Plot $h(-2-\tau)$

d) Calculate $y(t) = x(t) * h(t)$

$h(t-\tau) = \begin{cases} 1-(t-\tau)^2 & ; t-1 < \tau < t \\ 0 & ; \text{otherwise} \end{cases}$

$-1 < t \leq 0$

$$y(t) = \int_{-1}^{t} (-\tau)(1-(t-\tau)^2)\, d\tau$$

$$= \frac{t^4}{12} - t^2 - \frac{3t}{4} + \frac{1}{4}$$

#4

d continued)

$0 < t \le 1$

$$y(t) = \int_{t-1}^{t} (-\tau)(1-(t-\tau)^2)\,d\tau = \frac{1}{4} - \frac{2t}{3}$$

$1 < t < 2$

$$y(t) = \int_{t-1}^{1} (-\tau)(1-(t-\tau)^2)\,d\tau = -\frac{t^4}{12} + t^2 - \frac{4t}{3}$$

$$\boxed{y(t) = \begin{cases} \frac{t^4}{12} - t^2 - \frac{2t}{3} + \frac{1}{4}; & -1 < t \le 0 \\ \frac{1}{4} - \frac{2t}{3}; & 0 < t \le 1 \\ -\frac{t^4}{12} + t^2 - \frac{4t}{3}; & 1 < t \le 2 \end{cases}}$$

$y(0^-) = 1/4$ ✓

$y(0^+) = 1/4$

$y(1^-) = 1/4 - \frac{2(1)}{3} = -5/12$ ✓

$y(1^+) = -\frac{(1)^4}{12} + (1)^2 - \frac{4(1)}{3} = -5/12$

$y(-1^+) = \frac{(-1)^4}{12} - (-1)^2 - \frac{2(-1)}{3} + 1/4 = 0$ ✓

$y(2^-) = -\frac{2^4}{12} + 2^2 - \frac{4(2)}{3} = 0$

checking continuality across all bounds shows the answer is reasonable

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$\boxed{\#5}$ $2y''(t) - 5y'(t) - 7y(t) = -3x''(t) + 11x(t)$

a) Find transfer function

$a_2 = 2$, $a_1 = -5$, $a_0 = -7$ $ $b_2 = -3$, $b_1 = 0$, $b_0 = 11$

$$\boxed{H(s) = \frac{\sum_{k=0}^{2} b_k s^k}{\sum_{k=0}^{2} a_k s^k} = \frac{-3s^2 + 11}{2s^2 - 5s - 7}}$$

b) Find frequency response

$$\boxed{H(j\omega) = \frac{-3(j\omega)^2 + 11}{2(j\omega)^2 - 5(j\omega) - 7}}$$