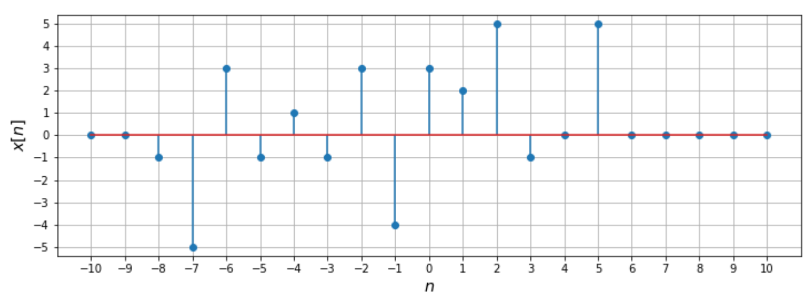
Signals & Systems
Homework #2

ECE 315 – Fall 2022
190 points total

Due Monday, October 24, 2022

1. For the signal $x[n]$, plot the following signals over a large enough time interval to show all nonzero samples by hand. (Note that $x[n] = 0$ for $n \le -9$ and for $n \ge 6$.) (34 points total)



   (a) $2x[n-3]$ (6 points)
   (b) $-x[3n]$ (6 points)
   (c) $x[-\frac{n}{4}]$ (6 points)
   (d) The even and odd parts of $x[n]$ (12 points)
   (e) What is the smallest integer k for which $x[kn]$ contains only one nonzero sample from the original signal $x[n]$? (4 points)

2. (32 points total)

   (a) Determine the forward differences $y_1[n]$ and $y_2[n]$, respectively, of the signals

   $$x_1[n] = \begin{cases} 5 - \frac{n}{2}, & 0 \leq n \leq 20 \\ 0, & \text{otherwise} \end{cases}$$

   and

   $$x_2[n] = \begin{cases} 5 - \frac{n}{2} - 1, & 0 \leq n \leq 20 \\ -1, & \text{otherwise} \end{cases}$$

   by hand. How do $y_1[n]$ and $y_2[n]$ compare to each other? Use MATLAB or python to plot the signals and their forward differences. (Note: MATLAB and matplotlib have functions called "stem" for making stem plots.) (16 points)

   (b) Determine the forward difference $y_3[n]$ of the signal

   $$x_3[n] = \begin{cases} 5 - \frac{n}{2} - 1, & 0 \leq n \leq 20 \\ 0, & \text{otherwise.} \end{cases}$$

   by hand. Use MATLAB or python to plot $x_3[n]$ and $y_3[n]$. How does $y_3[n]$ differ from $y_1[n]$? (8 points)

   (c) Calculate the accumulation

   $$\tilde{x}[n] = \sum_{m=-\infty}^{n-1} y_2[m]$$

   and plot it using MATLAB or python. Which of the three signals $x_1[n]$, $x_2[n]$, or $x_3[n]$ does it match? (8 points)

3. Determine whether the following signals are periodic and plot them. Create all plots in this problem using MATLAB or python. If a signal is complex, determine its magnitude and phase and find and plot its real and imaginary parts. For each periodic signal, determine its

   - fundamental period,
   - fundamental frequency,
   - fundamental angular frequency,
   - and average power.

   Use MATLAB or python to calculate the average power. Include the units for these quantities. Assume that the unit of $x[n]$ is furlongs per fortnight. (56 points total)

   (a) $x[n] = 2\sin\left(\frac{9n}{163} - \frac{3\pi}{4}\right)$ (10 points)

   (b) $x[n] = 2\cos\left(\frac{3\pi n}{163} - \frac{\pi}{4}\right)$ (10 points)

   (c) $x[n] = 3e^{(-\pi/48+j2\pi/253)n}$ (12 points)

   (d) $x[n] = 3e^{j(8\pi n/287-3\pi/2)}$ (12 points)

   (e) $x[n] = 3e^{j(-5n/96-\pi/3)}$ (12 points)

4. Plot the real and imaginary parts of the two signals

$$x_1[n] = 6\exp\left(j\left(\frac{3\pi n}{51} - \frac{\pi}{12}\right)\right)$$
$$x_2[n] = 6\exp\left(j\left(\frac{207\pi n}{51} - \frac{\pi}{12}\right)\right)$$

   using MATLAB or python. Describe how the two signals are related to each other. (12 points)

5. Determine whether the following systems have the properties linearity, time invariance, BIBO stability, memory, causality, or invertibility. If the system is invertible, find its inverse. (56 points total)

   (a) $y(t) = (t^2 - 1)x(t-1)$ (14 points)

   (b) $y[n] = e^{-x[n]}$ (14 points)

   (c) $y(t) = 1 - x(t)$ (14 points)

   (d) $y[n] = \sum_{n_0}^{n} x[n]$ where n_0 is a fixed, finite time (14 points)