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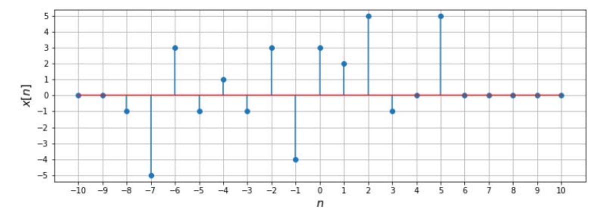
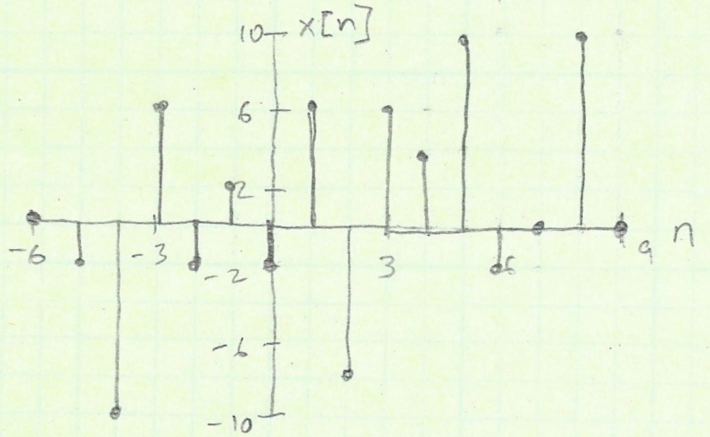
Given: For the signal $x[n]$, plot the following signals over a large enough time interval to show all nonzero samples by hand. (Note that $x[n] = 0$ for $n \leq -9$ and for $n \geq 6$.) (34 points total)



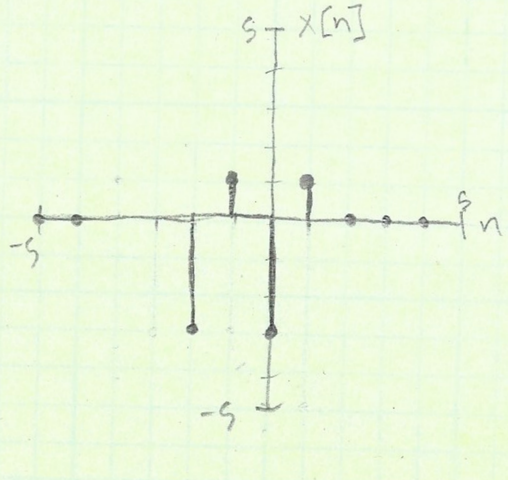
Find: a) $2x[n-3]$ b) $-x[3n]$ c) $x[-\frac{1}{4}n]$
d) even & odd parts of $x[n]$ e) smallest integer k for which $x[kn]$ contains only one nonzero sample

Solution:

a) $2x[n-3]$; The 2 scales amplitude by 2
$2x[n-(-3)]$; The -3 shifts to the right by 3
$n \leq -6$ & $n \geq 9$ are all zeros now



b) $(-1)x[3n]$; the -1 scales amplitude by -1
the 3 decimates by 3



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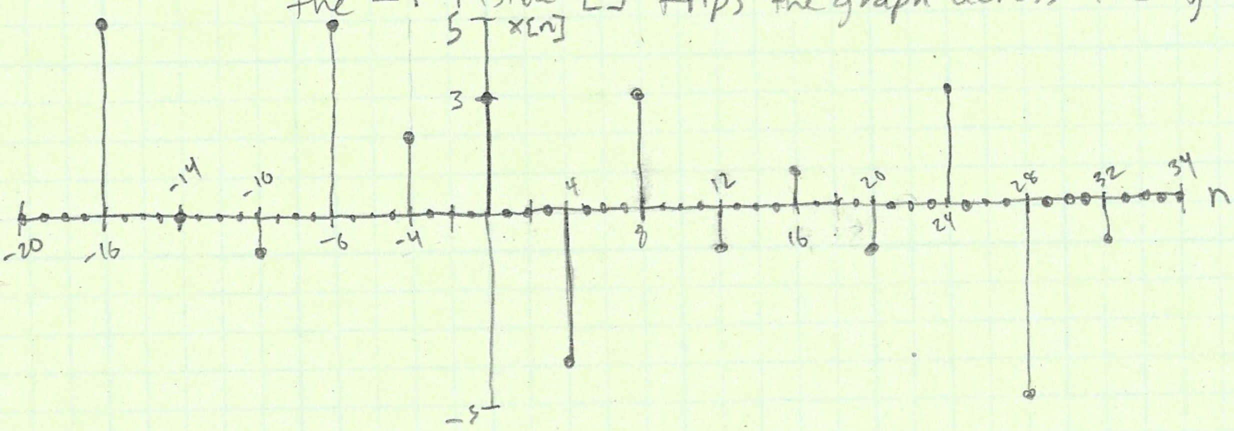
c) $x[-\frac{1}{4}n]$; $k = 4$, $k-1$ = # of zeros added between original signals ; # zeros = 3

the -1 inside [] flips the graph across the y axis



d) Even & odd parts of x[n]

$X_{even} = \frac{x[n]+x[-n]}{2}$; $X_{odd} = \frac{x[n]-x[-n]}{2}$

n=0

$x[n] = [-1, -5, 3, -1, 1, -1, 3, -4, 3, 2, 5, -1, 0, 5]\ 0, 0, 0$

$x[-n] = 0, 0, 0\ [5, 0, -1, 5, 2, 3, -4, 3, -1, 1, -1, 3, -5, -1]$

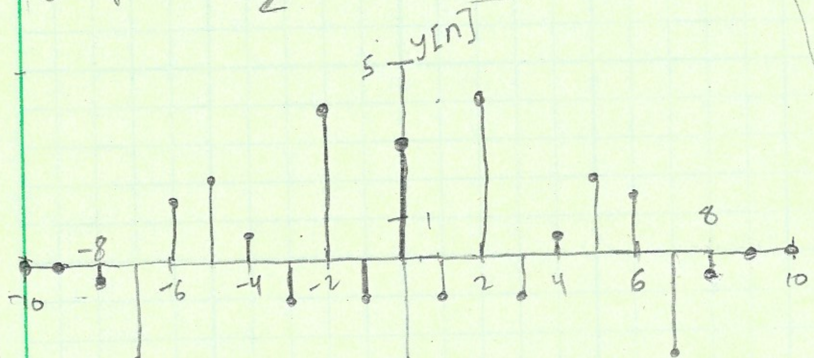
$X_{even} = \frac{x[n]+x[-n]}{2} = [-\frac{1}{2}, -2.5, 1.5, 2, \frac{1}{2}, -1, 4, -1, 3, -1, 4, -1, \frac{1}{2}, 2, 1.5, -2.5, -\frac{1}{2}]$

n=-8, n=-6, n=-4, n=-2, n=0, n=2, n=4, n=6, n=8

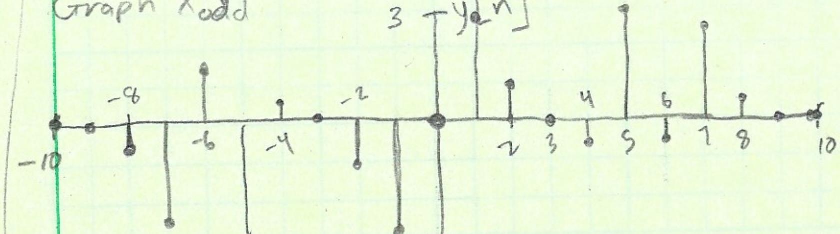
$X_{odd} = \frac{x[n]-x[-n]}{2} = [-\frac{1}{2}, -2.5, 1.5, -3, \frac{1}{2}, 0, -1, -3, 0, 3, 1, 0, -\frac{1}{2}, 3, -1.5, 2.5, \frac{1}{2}]$

* Note: make a table next time

Graph $\frac{x[n]+x[-n]}{2} = y[n] = X_{even}$



Graph X_{odd}



e) since $n=-8$ is the largest $|n|$ (8) value, to decimate all but the origin, $k=9$

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Given:

$$x_1[n] = \begin{cases} 5 - \frac{n}{2} ; & 0 \le n \le 20 \\ 0 ; & \text{otherwise} \end{cases}$$

$$x_2[n] = \begin{cases} 5 - \frac{n}{2} - 1 & 0 \le n \le 20 \\ -1 , & \text{otherwise} \end{cases}$$

Find: a) Determine forward differences $y_1[n]$ & $y_2[n]$

How do $y_1[n]$ & $y_2[n]$ compare?

*b) & c) in Solutions

Solution:

a) $y_1[n] = x_1[n+1] - x_1[n]$

$y_1[-2] = 0 - 0 = 0$

$y_1[-1] = 5 - 0 = 5$

$y_1[0] = 4.5 - 5 = -0.5$

$y_1[20] = 0 - -5 = 5$

$y_1[21] = 0 - 0 = 0$

$y_2[n] = x_2[n+1] - x_2[n]$

$y_2[-2] = -1 - -1 = 0$

$y_2[-1] = 4 + 1 = 5$

$y_2[0] = 3.5 - 4 = -0.5$

$y_2[20] = -1 - -6 = 5$

$y_2[21] = -1 - -1 = 0$

$$y_1[n] = \begin{cases} 5 ; & n = -1 \text{ \& } n = 20 \\ -1/2 ; & 0 \le n \le 19 \\ 0 ; & \text{otherwise} \end{cases} \qquad y_2[n] = \begin{cases} 5 ; & n = -1 \text{ \& } n = 20 \\ -1/2 ; & 0 \le n \le 19 \\ 0 ; & \text{otherwise} \end{cases}$$

How do they compare?

$y_1[n] = y_2[n]$ ∴ They are the same

b) Determine forward difference $y_3[n]$ of the signal

$$x_3[n] = \begin{cases} 5 - \frac{n}{2} - 1 ; & 0 \le n \le 20 \\ 0 ; & \text{otherwise} \end{cases}$$

$y_3[n] = x_3[n+1] - x_3[n]$

$y_3[-2] = 0 - 0 = 0$

$y_3[-1] = 5 - 0 - 1 =$

$y_3[0] = 3.5 - 4 = -1/2$

$y_3[20] = 0 - -6 = 6$

$y_3[21] = 0 - 0 = 0$

$$y_3[n] = \begin{cases} 4 ; & n = -1 \\ 6 ; & n = 20 \\ -1/2 ; & 0 \le n \le 19 \\ 0 ; & \text{otherwise} \end{cases}$$

How does $y_3[n]$ differ from $y_1[n]$

The only difference is at $n = 20$, $n = -1$

$y_3[n]$ is 6, 4 and $y_1[n]$ is 5, 5 respectivly.

This is because of the "-1" in $x_n[3]$, $0 \le n \le 20$

c)

$$\check{x}[n] = \sum_{m=-\infty}^{n-1} y_2[m] \quad ; \quad y_2[n] = \begin{cases} 5 & ; n=-1 \; \& \; n=20 \\ -1/2 & ; 0 \le n \le 19 \\ 0 & ; \text{otherwise} \end{cases}$$

$$\check{x}[n] = \begin{cases} 5, & n=-1, \\ 4-\frac{1}{2}n; & 0 \le n \le 19 \\ 0 & ; \text{otherwise} \end{cases}$$

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Given: Determine if following signals are periodic. Plot in python. If complex, determine magnitude & phase. Find and plot its real & imaginary parts. For each periodic signal, determine the fundamental period, frequency, and angular frequency, & calculate average power in python. Assume $x[n]$ units are furlongs per fortnight. units^2

Find: a) $x[n] = 2\sin\left(\frac{9n}{163} - \frac{3\pi}{4}\right)$ b) $x[n] = 2\cos\left(\frac{3\pi n}{163} - \frac{\pi}{4}\right)$

c) $x[n] = 3e^{\left(-\frac{\pi}{48} + j\frac{2\pi}{253}\right)n}$ d) $x[n] = 3e^{-j\left(\frac{8\pi n}{287} - \frac{3\pi}{2}\right)}$

e) $x[n] = 3e^{-j\left(-\frac{5n}{96} - \pi/3\right)}$

Solution:

a) $x[n] = 2\sin\left(2\pi\left(\frac{n}{\left(\frac{326\pi}{9}\right)}\right) - \frac{3\pi}{4}\right)$

$T_{01} = \frac{326\pi}{9}$ $\boxed{\text{since } T_{01} \text{ is irrational, } x[n] \text{ is aperiodic}}$

b) $x[n] = 2\cos\left(2\pi\left(\frac{n}{326/3}\right) - \pi/4\right)$

$T_{01} = \frac{326}{3}$ $\boxed{\text{since } T_{01} \text{ is rational, } x[n] \text{ is periodic}}$

Fundamental Period $T_0 = \frac{326}{3} = 108.67$ time unit

Fundamental Frequency $f_0 = \frac{1}{T_0} = \frac{3}{326} = 9.2$ mHz

Fundamental Angular Frequency $\omega_0 = 2\pi f_0 = \frac{3\pi}{163}$ rad/time unit

c) $x[n] = 3e^{\left(-\frac{\pi}{48} + j\frac{2\pi}{253}\right)n}$

$= 3e^{-\pi/48}\left(\cos\left(\frac{2\pi}{253}n\right) + j\sin\left(\frac{2\pi}{253}n\right)\right)$

$T_{01} = \frac{2\pi}{\omega_0} = \frac{2\pi}{\left(2\pi/253\right)} = 253$

T_{01} is rational, but since $|x[n]|$ decreases exponentially, $x[n]$ is aperiodic

$T_0 = 253$ time unit

~~$f_0 = \frac{1}{T_0} = \frac{1}{253} = 3.953$ mHz~~

~~$\omega_0 = 2\pi\frac{1}{T_0} = \frac{2\pi}{253}$ radians/time unit~~

Re: $3e^{-\pi n/48} \cdot \cos\left(\frac{2\pi}{253}n\right)$

Im: $3e^{-\pi n/48} j\sin\left(\frac{2\pi}{253}n\right)$

Magnitude $= 3e^{-\pi/48}$ $\theta = \frac{2\pi}{253}n$

~~Re + Im = 2.8091 + 0.06977j~~

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d) $x[n] = 3e^{j\left(\frac{8\pi n}{287} - \frac{3\pi}{2}\right)}$ Using Eulers Formula, we get

$$x[n] = 3\left(\cos\left(\frac{8\pi n}{287} - \frac{3\pi}{2}\right) + j\sin\left(\frac{8\pi n}{287} - \frac{3\pi}{2}\right)\right)$$

$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\underbrace{(8\pi/287)}_{2\pi(4/287)}} = \frac{287}{4}$$

Since T_0 is rational, $x[n]$ is periodic

$T_0 = \frac{287}{4}$ time units

$f_0 = \frac{1}{T_0} = \frac{4}{287} = 13.9\,\text{mHz}$

$\omega_0 = 2\pi f_0 = 2\pi(13.9e\text{-}3) = 87.34e\text{-}3$ radians/time unit

mag: $|x[n]| = 3$ $\theta = \frac{8\pi n}{287} - \frac{3\pi}{2}$

$Re + Im = 3\left(\cos\left(\frac{8\pi n}{287} - \frac{3\pi}{2}\right) + j\sin\left(\frac{8\pi n}{287} - \frac{3\pi}{2}\right)\right)$

e) $x[n] = 3e^{j\left(-\frac{5n}{96} - \pi/3\right)}$

$$x[n] = 3\left(\cos\left(\underbrace{-\frac{5n}{96}}_{-2\pi\left(\frac{5n}{192\pi}\right)} - \frac{\pi}{3}\right) + j\sin\left(-\frac{5n}{96} - \frac{\pi}{3}\right)\right)$$

$T_0 = \frac{192\pi}{5}$ since T_0 is irrational, $x[n]$ is aperiodic

$\theta = -\frac{5n}{96} - \frac{\pi}{3}$ mag: 3

$$Re + Im = 3\cos\left(-\frac{5n}{96} - \frac{\pi}{3}\right) + 3j\sin\left(-\frac{5n}{96} - \frac{\pi}{3}\right)$$

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Given: Determine whether the following systems have the properties linearity, time invariance, BIBO stability, memory, causality, or invertibility. If
Find: the system is invertible, find its inverse. (56 points total)

(a) $y(t) = (t^2 - 1)x(t-1)$ (14 points)
(b) $y[n] = e^{-x[n]}$ (14 points)
(c) $y(t) = 1 - x(t)$ (14 points)
(d) $y[n] = \sum_{n_0}^{n} x[n]$ where n_0 is a fixed, finite time (14 points)

Solution:

a) $y(t) = (t^2-1)\,x(t-1)$

$y_1(t) + y_2(t) = (t^2-1)(x_1(t-1) + x_2(t-1))$

Let $x(t-1) = x_1(t-1) + x_2(t-1)$
Then $y(t) = (t^2-1)(x_1(t-1) + x_2(t-1))$
Since $y(t) = y_1(t) + y_2(t)$, it's additive

Let $x(t-1) = A x_1(t-1)$; A = constant
then $y(t) = (t^2-1) A x_1(t-1) = A(t^2-1)x(t-1)$
Therefore it's homogeneous $A \cdot y(t)$

Since $y(t)$ is both additive & homogeneous, it's linear

It has memory because y at t depends on x at $t-1$

$y(t-t_0) = ((t-t_0)^2-1)\,x(t-t_0-1)$
$y(t) = (t^2-1)\,x(t-t_0-1)$ since $y(t-t_0) \neq y(t) @ x(t-t_0)$
∴ It is time variant

Since we have $(t^2-1)\cdot x(t-1)$ it only depends on past times, therefore it's causal

For bounded input $-\infty < x(t-1) < \infty$, $y(t) \to \infty$
∴ NOT Stable

Let $t = 1$ & $t = -1$. This yields $y(t) = 0$ for both inputs, so it is not invertable

b) $y[n] = e^{-x[n]}$

$y_1[n] + y_2[n] = e^{-x_1[n]} + e^{-x_2[n]}$

Let $x[n] = x_1[n] + x_2[n]$

Then $y[n] = e^{-(x_1[n] + x_2[n])} = e^{-x_1[n]} e^{-x_2[n]}$

Since $y[n] \neq y_1[n] + y_2[n]$, $y[n]$ is not additive and therefore, it's not linear

$y[n]$ only depends on current input $x[n]$, so it's memoryless

$y[n-n_0] = e^{-x[n-n_0]}$

$y[n] @ x[n-n_0] = e^{-x[n-n_0]}$

Since $y[n-n_0] = y[n] @ x[n-n_0]$, it's time invariant

Since $y[n]$ only depends on current times, so it's causal

Since it's $e^{-x[n]}$, bounded inputs will have bounded outputs, so it's stable, $e^{-\infty} = 0$

Since $y[n] = e^{-x[n]}$, different $x[n]$ will always produce different $y[n]$, & $y[n] \neq 0$ for nonzero $x[n]$, so the inverse of $y[n]$ will exist

$$\ln(y[n]) = \ln\left(e^{-x[n]}\right)$$
$$= -x[n]$$
$$x[n] = -\ln(y[n])$$
$$x[n] = \ln(1/y[n])$$

$\therefore$ inverse of $y[n] = \ln(1/x[n])$

c) $y(t) = 1 - x(t)$

Due to the 1, it's <u>NOT linear</u>

$y(t-t_0) = 1 - x(t-t_0)$

$y(t) @ x(t-t_0) = 1 - x(t-t_0)$

Since $y(t-t_0) = y(t) @ x(t-t_0)$, $y(t)$ is <u>time invariant</u>

We know that $x(t)$ is <u>stable</u>, so $1-x(t)$ is also <u>stable</u>

$y(t)$ only depends on current times, so it's <u>causal</u>

Since $y(t)$ does not depend on future or past times, it's <u>memoryless</u>

$-y(t) = -(1-x(t)) \Rightarrow \underline{y(t) = x(t)+1}$

It <u>is invertible</u>

d) $y[n] = \sum_{n_0}^{n} x[n]$ where n_0 is a fixed, finite time

$\check{x}[n] = a x_1[n] + b x_2[n]$

$\check{y}[n] = \sum_{n_0}^{n} (a x_1[n] + b x_2[n])$

$= a \sum_{n_0}^{n} x_1[n] + b \sum_{n_0}^{n} x_2[n]$

$= a y_1[n] + b y_2[n]$ $\therefore$ <u>It's linear</u>

$y[n-n_0] = \sum_{n_0}^{n} x[n-n_0]$

$y[n] @ x[n-n_0] = \sum_{n_0}^{n-n_0} x[n]$

Since $y[n-n_0] \neq y[n] @ x[n-n_0]$ it's <u>time variant</u>

Since it's a summation, $y[n]$ depends on time from $n \to n_0$ so the system has <u>memory</u>

Since $y[n]$ depends on time from $n \to n_0$, when $n < n_0$ the system is <u>acausal</u>

Since it's a summation, if the input is bounded the output will be bounded too, $\therefore$ <u>stable</u>

Since it's a summation, you can have 2 distinct input signals that result in the same output by varying the bounds $\therefore$ $\boxed{\text{It's NOT INVERTIBLE}}$