ECE 315 | HW 4 | Problem 1 | Phil Nevins

a) $C_x[k] = \frac{1}{4}\int_{-1}^{1} (g(t) * \delta_4(t)) e^{-jk\frac{\pi}{2}t}$ $\omega_0 = \frac{2\pi}{4}$

$= \frac{1}{4}\left[\int_{-1}^{0}(1-t)e^{-jk\frac{\pi}{2}t}dt + \int_{0}^{1}(2t+1)e^{-jk\frac{\pi}{2}t}dt\right]$ $T_0 = 4$

$C_x[0] = \frac{1}{4}\left[\int_{-1}^{0}(1-t)dt + \int_{0}^{1}(2t+1)dt\right]$

$= \frac{1}{4}\left[\frac{3}{2} + 2\right]$

$C_x[0] = 7/8$

For $k \neq 0$: $\int_{-1}^{0}(1-t)e^{-jk\frac{\pi}{2}t}dt$

$u = 1-t$; $du = -dt$; $dv = e^{-jk\frac{\pi}{2}t}dt$; $v = -\frac{1}{j\frac{\pi}{2}k}e^{-j\frac{\pi}{2}kt}$

$\int_{-1}^{0}(1-t)e^{-jk\frac{\pi}{2}t}dt = u(t)v(t)\Big|_{-1}^{0} - \int_{-1}^{0}v(t)du$

$= (1-t)\left(-\frac{1}{j\frac{\pi}{2}k}e^{-jk\frac{\pi}{2}t}\right)\Big|_{-1}^{0} - \frac{1}{j\frac{\pi}{2}k}\int_{-1}^{0}e^{-j\frac{\pi}{2}kt}dt$

$= (1)\left(-\frac{1}{j\frac{\pi}{2}k}\right) + (2)\left(\frac{1}{j\frac{\pi}{2}k}e^{jk\pi/2}\right) - \left(\frac{1}{j\frac{\pi}{2}k}\right)^2\left(1 - e^{j\pi/2 k}\right)$

$= -\frac{1}{j\frac{\pi}{2}k} + \frac{2}{j\frac{\pi}{2}k}e^{jk\pi/2} - \left(\frac{1}{j\frac{\pi}{2}k}\right)^2\left(1 - e^{j\pi/2 k}\right)$

$\int_{0}^{1}(2t+1)e^{-jk\pi/2 t}dt$

$u = 2t+1$; $du = 2dt$; $dv = e^{-jk\frac{\pi}{2}t}dt$; $v = -\frac{1}{j\frac{\pi}{2}k}e^{-j\frac{\pi}{2}kt}$

$\int_{0}^{1}(2t+1)e^{-jk\pi/2 t}dt = u(t)v(t)\Big|_{0}^{1} - \int_{0}^{1}v(t)du$

$= (2t+1)\left(-\frac{1}{j\frac{\pi}{2}k}e^{-j\frac{\pi}{2}kt}\right)\Big|_{0}^{1} - \frac{2}{j\frac{\pi}{2}k}\int_{0}^{1}e^{-j\pi/2 kt}dt$

$= (\cancel{3})\left(-\frac{3\cdot 1}{j\frac{\pi}{2}k}e^{-j\frac{\pi}{2}k} - \frac{3\cdot 1}{j\frac{\pi}{2}k}\right) - \frac{2}{j\frac{\pi}{2}k}e^{-j\pi/2 k} + \frac{2}{j\frac{\pi}{2}k}$

$$C_x[k \neq 0] = e^{-j\frac{\pi}{2}k}\frac{2}{k^2\pi^2} + e^{j\frac{\pi}{2}k}\cdot\frac{1}{k^2\pi^2} - \frac{3}{k^2\pi^2} + je^{-j\frac{\pi}{2}k}\cdot\frac{3}{2k\pi} - je^{j\frac{\pi}{2}k}\cdot\frac{1}{\pi k}$$

$$C_x[k=0] = 7/8$$

a) $y''(t) - 2y'(t) - 3y(t) = x(t)$

$$c_x[k] = c_{y''}[k] - 2c_{y'}[k] - 3c_y[k]$$

$$c_{y''}[k] = \left(\frac{j2\pi k}{T}\right)^2 c_y[k]$$

$$c_{y'}[k] = \frac{j2\pi k}{T} c_y[k]$$

$$c_x[k] = \left[\left(\frac{j2\pi k}{T}\right)^2 - 2\left(\frac{j2\pi k}{T}\right) - 3\right] c_y[k]$$

$$H[k] = \frac{c_y[k]}{c_x[k]} = \frac{1}{(j2\pi k/T)^2 - 2(j2\pi k/T) - 3}$$

b) $N=2,\ M=0\ ;\ a_2 = 1,\ a_1 = -2,\ a_0 = -3\ ;\ b_0 = 1$

$$H(s) = \frac{1}{s^2 - 2s - 3}$$

Comparing a & b, the harmonic response is the frequency response with

$$\omega = \frac{2\pi k}{T}$$

$$H(j\omega) = \frac{1}{(j\omega)^2 - 2j\omega - 3}$$

c) $\omega = 2,\ T_0 = 5,\ m = 1$

$$c_x[k] = \frac{2}{5}\operatorname{sinc}^2(2k/5)\,\delta_1[k] = \frac{2}{5}\operatorname{sinc}^2\left(\frac{2k}{5}\right)$$

$$c_y[k] = H[k]\,c_x[k] = \frac{\frac{2}{5}\operatorname{sinc}^2\left(\frac{2k}{5}\right)}{(j2\pi k/T)^2 - 2(j2\pi k/T) - 3}$$

So the output of the system is

$$y(t) = \sum_{k=-\infty}^{\infty} c_y[k]\, e^{j2\pi k t/T_0}$$

$$y(t) = \frac{2}{5}\sum_{k=-\infty}^{\infty}\left(\frac{\operatorname{sinc}^2\left(\frac{2k}{5}\right)}{(j2\pi k/5)^2 - 2(j2\pi k/5) - 3}\right) e^{j2\pi k t/5}$$

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$x(t) = u(t)\, t^3 e^{-t}$

a) $X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{\infty} u(t)\, t^3 e^{-t} e^{-j\omega t}\, dt$

$X(j\omega) = \int_0^{\infty} t^3 e^{-(1+j\omega)t}\, dt$

$u = t^3 \qquad dv = e^{-(1+j\omega)t}$

$du = 3t^2\, dt \qquad v = -\frac{1}{1+j\omega} e^{-(1+j\omega)t}$

$$X(j\omega) = \underbrace{u(t)\, v(t)\Big|_0^{\infty}}_{0-0} - \int_0^{\infty} v(t)\, du$$

$$= 0 - 0 - \int_0^{\infty} \left(-\frac{1}{1+j\omega}\right) e^{-(1+j\omega)t} (3t^2)\, dt$$

$$= \frac{3}{1+j\omega} \int_0^{\infty} e^{-(1+j\omega)t} \cdot t^2\, dt$$

$u = t^2 \qquad dv = e^{-(1+j\omega)t}$

$du = 2t\, dt \qquad v = -\frac{1}{1+j\omega} e^{-(1+j\omega)t}$

$$X(j\omega) = \left(\frac{3}{1+j\omega}\right) u(t)\, v(t)\Big|_0^{\infty} - \int_0^{\infty} v(t)\, dt$$

$$= \left(\frac{3}{1+j\omega}\right)\left[-\frac{1}{1+j\omega} t^2 e^{-(1+j\omega)t}\Big|_0^{\infty} - \int_0^{\infty} \left(-\frac{1}{1+j\omega} e^{-(1+j\omega)t}\right) 2t\, dt \right]$$

$$= \left(\frac{3}{1+j\omega}\right)\left(\frac{2}{1+j\omega}\right) \underbrace{\int_0^{\infty} t\, e^{-(1+j\omega)t}\, dt}$$

$u = t \qquad dv = e^{-(1+j\omega)t}\, dt$

$du = dt \qquad v = -\frac{1}{1+j\omega} e^{-(1+j\omega)t}$

$$= u(t)\, v(t)\Big|_0^{\infty} - \int_0^{\infty} v(t)\, dt = \left(-\frac{1}{1+j\omega} t e^{-(1+j\omega)t}\Big|_0^{\infty} - \int_0^{\infty} \left(-\frac{1}{1+j\omega} e^{-(1+j\omega)t}\, dt\right.\right.$$

$$X(j\omega) = \left(\frac{6}{(1+j\omega)^3}\right) \int_0^{\infty} e^{-(1+j\omega)t}\, dt$$

$$= \left(\frac{6}{(1+j\omega)^3}\right)\left(-\frac{1}{1+j\omega} e^{-(1+j\omega)t}\Big|_0^{\infty}\right)$$

$$X(j\omega) = \frac{6}{(1+j\omega)^4}$$

From table 6.3, we should have,

$$\frac{n!}{(j\omega+\alpha)^{n+1}} \Leftrightarrow t^n e^{-\alpha t} u(t)$$

we have $n = 3$, $\alpha = 1$

$$\frac{3!}{(1+j\omega)^{3+1}} = \frac{6}{(1+j\omega)^4} \quad \checkmark \text{ correct}$$

$y''(t) - 5y'(t) + 6y(t) = 2x'(t) - 3x(t)$

a) $N = 2,\ M = 1$; $a_2 = 1,\ a_1 = -5,\ a_0 = 6$
$b_1 = 2,\ b_0 = -3$

$$H(s) = \frac{2s-3}{s^2-5s+6} \Rightarrow \frac{2s-3}{(s-2)(s-3)}$$

$$H(s) = \frac{2s-3}{(s-2)(s-3)}$$

b) $x(t) = u(t)e^{-t/5}\sin(2\pi t) \overset{F}{\longleftrightarrow} \dfrac{2\pi}{(j\omega+\frac{1}{5})^2+4\pi^2}$

$\alpha = 1/5,\ \omega_0 = 2\pi$

$$X(j\omega) = \frac{2\pi}{(j\omega+\frac{1}{5})^2+4\pi^2}$$

(2)

$$Y(j\omega) = H(j\omega)X(j\omega) = \frac{2j\omega-3}{(j\omega-2)(j\omega-3)}\left(\frac{2\pi}{(j\omega+\frac{1}{5})^2+(2\pi)^2}\right)$$

using s, we have

$$Y(s) = \frac{2s-3}{(s-2)(s-3)}\left(\frac{2\pi}{(s+\frac{1}{5})^2+4\pi^2}\right) = \frac{50\pi(2s-3)}{(25s+100\pi^2+1)(s^2-5s+6)}$$

Doing partial fraction expansion & slight manipulation:

$$Y(s) = \frac{-0.151335}{s-2} - \frac{0.291992}{s+39.5184} + \frac{0.443327}{s-3}$$

$\alpha = -2$ $\quad$ $\alpha = 39.5184$ $\quad$ $\alpha = -3$

$$Y(s) \overset{F}{\longleftrightarrow} y(t) = -0.291992e^{-39.5184t}u(t) + \left[0.151335e^{2t} - 0.443327e^{3t}\right]u(-t)$$

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a)

$$x[n] = \begin{cases} 1 & ; n=-2 \\ -1 & ; n=-1 \\ 2 & ; n=0 \\ -1 & ; n=1 \\ -2 & ; n=2 \end{cases}$$

$N_0 = 5$

$\Omega_0 = \frac{2\pi}{N_0} = \frac{2}{5}\pi$

$$c_x[k] = \frac{1}{N_0} \sum_{n=\langle N_0 \rangle} x[n] e^{-j\Omega_0 k n}$$

$$= \frac{1}{5} \sum_{0}^{5} x[n] e^{-j\Omega_0 k n}$$

$$= \frac{1}{5}\left[e^{j2\Omega_0 k} - e^{j\Omega_0 k} + 2 - e^{-j\Omega_0 k} - 2e^{j2\Omega_0 k} \right]$$

$$c_x[k] = \frac{-\left[\cos(2\Omega_0 k) + 2(\cos(\Omega_0 k) - 1)\right]}{5} - \frac{\sin(2\Omega_0 k)}{5} j$$

* simplified using TI 89

ECE 315 | HW#4 | Problem 6 | Phil Nevins

a)
$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$$
$$= \sum_{n=-1}^{1} x[n] e^{-j\Omega n}$$
$$= -2e^{j\Omega} + 2 + e^{-j\Omega}$$
$$X(e^{j\Omega}) = 2 - \cos(\Omega) - 3\sin(\Omega)j$$

b.iii) The magnitudes are the same, but the phase is off by a factor of ~3

c) $$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\Omega}) e^{j\Omega n} d\Omega$$
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} (2 - 2e^{j\Omega} + e^{-j\Omega}) e^{j\Omega n} d\Omega$$
$$= -\frac{1}{\pi}\int_{-\pi}^{\pi} e^{j\Omega(n+1)} d\Omega + \frac{1}{\pi}\int_{-\pi}^{\pi} e^{j\Omega n} d\Omega + \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\Omega(n-1)} d\Omega$$

$n=-1$
$$-\frac{1}{\pi}\int_{-\pi}^{\pi} e^{j\Omega(n+1)} d\Omega = -\frac{1}{\pi}\int_{-\pi}^{\pi} d\Omega = -\frac{1}{\pi}[\Omega]_{-\pi}^{\pi} = -2$$

$n \neq -1$
$$-\frac{1}{\pi}\int_{-\pi}^{\pi} e^{j\Omega(n+1)} d\Omega = -\frac{1}{\pi}\left[\frac{e^{j\Omega(n+1)}}{j(n+1)}\right]\Bigg|_{-\pi}^{\pi} = 0$$

$n=0$
$$\frac{1}{\pi}\int_{-\pi}^{\pi} e^{j\Omega n} d\Omega = \frac{1}{\pi}[\Omega]_{-\pi}^{\pi} = 2$$

$n \neq 0$
$$\frac{1}{\pi}\int_{-\pi}^{\pi} e^{j\Omega n} d\Omega = \frac{1}{\pi}\left[\frac{e^{j\Omega n}}{jn}\right]\Bigg|_{-\pi}^{\pi} = 0$$

$n=1$
$$\frac{1}{\pi}\int_{-\pi}^{\pi} e^{j\Omega(n-1)} d\Omega = \frac{1}{\pi}[\Omega]\Big|_{-\pi}^{\pi} = 1$$

$n \neq 1$
$$\frac{1}{\pi}\int_{-\pi}^{\pi} e^{j\Omega(n-1)} d\Omega = \frac{1}{\pi}\left[\frac{e^{j\Omega(n-1)}}{j(n-1)}\right]\Bigg|_{-\pi}^{\pi} = 0$$

this shows
$$x[n] = \begin{cases} -2 & ; n = 1 \\ 2 & ; n = 0 \\ 1 & ; n = 1 \\ 0 & ; \text{otherwise} \end{cases}$$

✓ correct