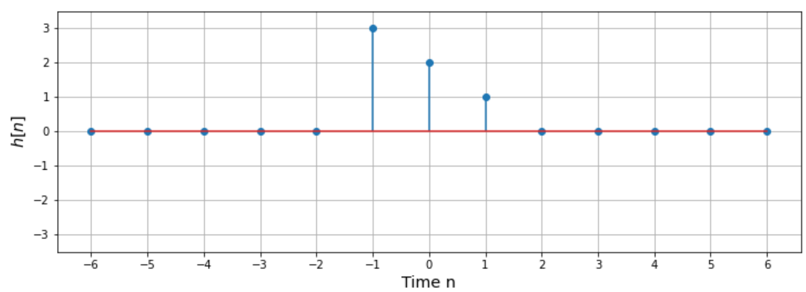
Signals & Systems
Homework #3

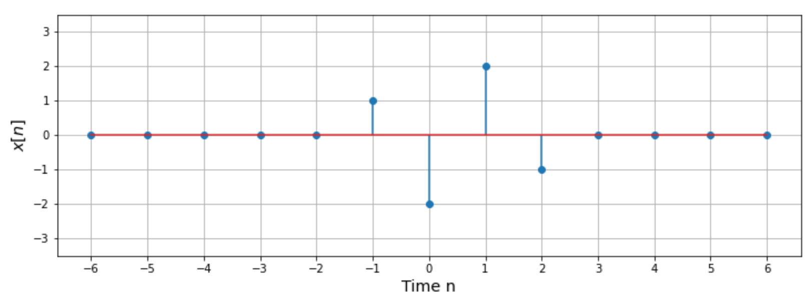
ECE 315 – Fall 2021
153 points total

Due Wednesday, November 9, 2022

1. Suppose a discrete-time LTI system has the impulse response $h[n]$



and that the system is stimulated with the signal $x[n]$



where $h[n]$ and $x[n]$ are 0 at all values of n not shown.

(a) Find and plot the signal $h[-m]$ as a function of the integer m by hand. (5 points)

(b) Find and plot the signal $h[-3-m] = h[-(m+3)]$ as a function of the integer m by hand. (5 points)

(c) Calculate all nonzero values of the output $y[n]$ of the system in response to the input signal $x[n]$ using convolution by hand. (15 points)

(d) Verify your solution using the MATLAB function `conv` or the NumPy function `convolve`. Include your code and its output in your answer. (7 points)

2. Consider the discrete-time LTI system

$$-8y[n] + 2y[n-1] + y[n-2] = -2x[n] + 3x[n-1].$$

(a) Find the general solution $y_h[n]$ to the homogeneous equation

$$-8y_h[n] + 2y_h[n-1] + y_h[n-2] = 0. \text{ (5 points)}$$

(b) Find initial values $\hat{h}[0]$ and $\hat{h}[1]$ for the impulse response $\hat{h}[n]$, where

$$-8\hat{h}[n] + 2\hat{h}[n-1] + \hat{h}[n-2] = \delta[n]$$

and $\hat{h}[n] = 0$ for $n < 0$. (5 points)

(c) Use the initial values calculated in the previous part to determine the values of the undetermined coefficients in the solution $\hat{h}[n] = y_h[n]u[n]$, which is the impulse response for the problem

$$-8y[n] + 2y[n-1] + y[n-2] = x[n]. \text{ (10 points)}$$

(d) Use the solution calculated in the previous part to construct the impulse response for the original difference equation

$$-8y[n] + 2y[n-1] + y[n-2] = -2x[n] + 3x[n-1]. \text{ (5 points)}$$

(e) Use MATLAB or python to plot the impulse response of the original system. If the impulse response is complex, plot its magnitude and phase and plot it as a stem plot in the 3 dimensions of its real and imaginary parts and time n. (10 points)

3. Consider the discrete-time LTI system

$$-8y[n] + 2y[n-1] + y[n-2] = -2x[n] + 3x[n-1].$$

(a) Find the transfer function for the system analytically. (5 points)

(b) Find the frequency response for the system analytically and use MATLAB to plot the magnitude and phase of your result as functions of angular frequency Ω for $0 \leq \Omega \leq \pi$. (10 points)

(c) Use the MATLAB function `freqz` or the SciPy function `freqz` to determine and plot the magnitude and phase of the frequency response for $0 \leq \Omega \leq \pi$. (7 points)

4. Suppose that

$$h(t) = \begin{cases} 1 - t^2, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$$

is the impulse response for a continuous-time LTI system and that the system is stimulated with the signal

$$x(t) = \begin{cases} -t, & -1 < t < 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Plot the signal $x(t)$ by hand. (5 points)

(b) Find and plot the signal $h(-\tau)$ as a function of τ by hand. (5 points)

(c) Find and plot the signal $h(-2-\tau) = h(-(\tau+2))$ as a function of τ by hand. (5 points)

(d) Calculate the output $y(t)$ of the system in response to the input signal $x(t)$ using convolution by hand. (20 points)

(e) Use MATLAB or python to plot the convolution $y(t)$. (7 points)

5. Consider the continuous-time LTI system

$$2\,y''(t) - 5\,y'(t) - 7\,y(t) = -3x''(t) + 11\,x(t)$$

(a) Find the transfer function for the system analytically. (5 points)

(b) Find the frequency response for the system analytically and use MATLAB or python to plot the magnitude and phase of your result as functions of angular frequency ω for $-10 \leq \omega \leq 10$. (10 points)

(c) Use the MATLAB function `freqs` or the SciPy function `freqs` to determine and plot the magnitude and phase of the frequency response for $-10 \leq \omega \leq 10$. (7 points)