MAT232 - Lecture 6

vectors?

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Contents

Definitions and Theorems

Straight from the textbook — no fluff, just what we need.

Quick recap before diving into the lecture.

Introduction to Vectors

Definition

A **vector** is a quantity that has both magnitude and direction. Vectors can be optionally denoted in multiple ways:

- **Boldface Notation**: $\mathbf{v}$
- **Arrow Notation**: $\vec{v}$
- **Overline Notation**: $\overline{v}$

Note

In MAT232H5, the contents of a vector are typically written using angle bracket notation:

$$\mathbf{v} = \langle v_1, v_2, v_3 \rangle$$

For example, a 3D vector can be represented as:

$$\vec{v} = \langle 2, -1, 3 \rangle$$

Depending on the context, you might see $\mathbf{v} = \langle v_1, v_2 \rangle$ in 2D or $\mathbf{v} = \langle v_1, v_2, v_3, v_4 \rangle$ in higher dimensions.

Remark

Quantities such as velocity and force are examples of vectors because they require both magnitude and direction to be fully described.

Vector Representation

A **vector** in a plane is represented by a directed line segment (an arrow) with an **initial point** and a **terminal point**. The length of the segment represents its **magnitude**, denoted $\|\vec{v}\|$. A vector with the same initial and terminal point is called the **zero vector**, denoted $\vec{0}$.

Two vectors $\vec{v}$ and $\vec{w}$ are **equivalent** if they have the same magnitude and direction, written as $\vec{v} = \vec{w}$.

Exercise

Sketching Vectors

Sketch a vector in the plane from initial point $P(1, 1)$ to terminal point $Q(8, 5)$.

Basic Vector Operations

Scalar Multiplication

Multiplying a vector $\vec{v}$ by a scalar k results in a new vector $k\vec{v}$ with the following properties:

· Its magnitude is $|k|$ times the magnitude of $\vec{v}$.

· Its direction remains the same if $k > 0$.

· Its direction is reversed if $k < 0$.

· If $k = 0$ or $\vec{v} = \vec{0}$, then $k\vec{v} = \vec{0}$.

Note

The zero vector $\vec{0}$ is the vector with a magnitude of 0 and no direction (or any direction). It is the only vector that is orthogonal (perpendicular) to every vector, including itself.

Exercise

Scalar Multiplication
Given vector $\vec{v}$, sketch the vectors $3\vec{v}$, $\frac{1}{2}\vec{v}$, and $-\vec{v}$.

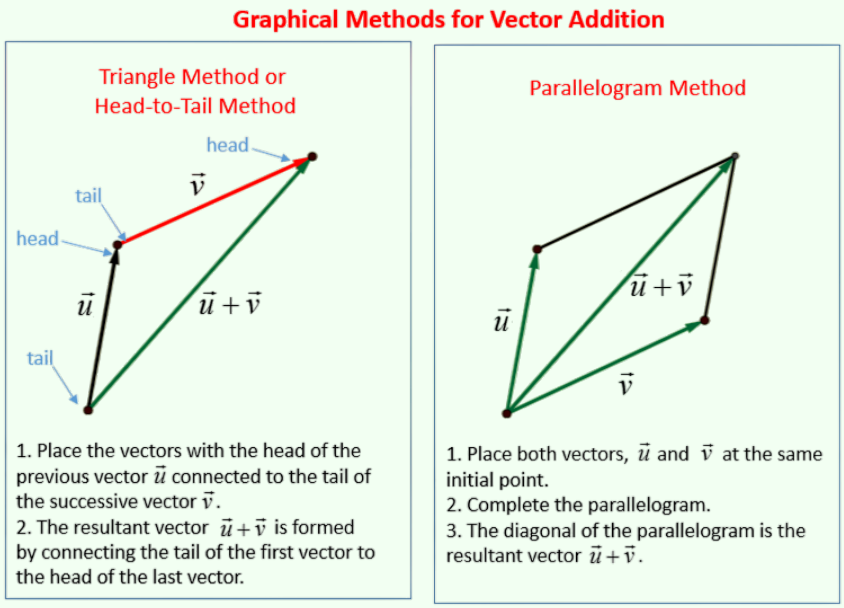
Vector Addition

The sum of two vectors $\vec{v}$ and $\vec{w}$ is constructed by placing the initial point of $\vec{w}$ at the terminal point of $\vec{v}$. The vector sum, $\vec{v} + \vec{w}$, is the vector from the initial point of $\vec{v}$ to the terminal point of $\vec{w}$.

Exercise

Vector Addition

Given vectors $\vec{v}$ and $\vec{w}$, sketch $\vec{v} + \vec{w}$ using both the triangle method and the parallelogram method.



Vector Subtraction

The difference $\vec{v} - \vec{w}$ is defined as $\vec{v} + (-\vec{w})$, where $-\vec{w}$ is the vector with the same magnitude as $\vec{w}$ but opposite direction.

Exercise

Vector Subtraction

Given vectors $\vec{v}$ and $\vec{w}$, sketch $\vec{v} - \vec{w}$.

Vector Components

A vector in standard position has its initial point at the origin $(0, 0)$. If the terminal point is (x, y), the vector is written in **component form** as $\vec{v} = \langle x, y \rangle$. The scalars x and y are called the **components** of $\vec{v}$.

Exercise

Expressing Vectors in Component Form

Express vector $\vec{v}$ with initial point $(-3, 4)$ and terminal point $(1, 2)$ in component form.

Magnitude of a Vector

Definition

The magnitude of a vector $\vec{v} = \langle x, y \rangle$ is its length, and is given by:

$$\|\vec{v}\| = \sqrt{x^2 + y^2}.$$

Exercise

Find the magnitude of the vector $\vec{v} = \langle 3, -4 \rangle$.

Properties of Vector Operations

Theorem

Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be vectors, and let k and c be scalars. Then:

1. $\vec{u} + \vec{v} = \vec{v} + \vec{u}$ (Commutative Property)
2. $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ (Associative Property)
3. $k(c\vec{v}) = (kc)\vec{v}$ (Associativity of Scalar Multiplication)
4. $k(\vec{u} + \vec{v}) = k\vec{u} + k\vec{v}$ (Distributive Property)

Proof

Proof of Commutative Property:
Let $\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$. Then:

$$\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2 \rangle = \langle v_1 + u_1, v_2 + u_2 \rangle = \vec{v} + \vec{u}.$$

Applications of Vectors

Example

Real-Life Applications

- A boat crossing a river experiences a force from its motor and a force from the river current. Both forces are vectors.
- A quarterback throwing a football applies a velocity vector to the ball, determining its speed and direction.

Let's Get Started

Time to dive into the lecture notes.

Grab your pen or pencil, and let's break this down step by step.