# MAT232 - Lecture 15

[Lesson Topic(s)]

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# Contents

# Definitions and Theorems

*Straight from the textbook — no fluff, just what we need.*

**Quick recap before diving into the lecture.**

# Let's Get Started

*Time to dive into the lecture notes.*

**Grab your pen or pencil, and let's break this down step by step.**

**self-note: complete this according to the posted lecture notes**

## SDT

- rel max
- rel min
- saddle point
- 

# Section 14.8: Lagrange Multipliers

**Illustration**

Absolute maximum? Find it using the gradient!

$$z = f(x, y)$$

The gradient would be pointing upwards.

$$g(x, y) = k$$

$k$ is a scalar/constant
Lagrange Multipliers

**Example**

Given $f(x, y)$ subject to a constant

$$g(x, y) = k$$

1: Solve (fix the stuff below later)
2: $\nabla g(x, y) \neq \bar{0}$; discard that $(x, y)$.
**self-note: finish this according to the posted lecture notes later** 3: Plug all $(x, y)$

Example

Find the minimum fencing required if the area is $800m^2$.

Illustration

**self-note: add the illustration of the fencing from the notes later on**

Solution

Perimeter:

$$P = 2y + x$$

Area:

$$800 = xy$$

Note that the $y$ is the constraint.

So...

$$y = \frac{800}{x}$$

So...

$$\begin{aligned} P(x) &= 2(\frac{800}{x}) + x \\ &= 1600x^{-1} + x \\ \implies P'(x) &= -1600x^{-2} + 1 \\ \implies P''(x) &= 3200x^{-3} \\ \implies P'(x) = 0 &= -\frac{1600}{x} + \frac{1}{1} \cdot \frac{x^2}{x^2} \\ \implies 0 &= \frac{-1600 + x^2}{x^2} \end{aligned}$$

So... Num:

$$0 = -1600 + x^2$$

This means

$$x = \pm 40$$

so only

$$x = 40.$$

Note that denom

$$x^2 = 0$$



$$P''(x) = 3200x^{-3} \quad \ldots \quad x = 40$$

$$\implies P''(40) = \frac{3200}{40^3} > 0$$

So, by SDT, rel min at $x = 40$. So...

$$800 = xy$$

Now, let's achieve the same result using Lagrange Multipliers!

## Application of Lagrange Multipliers

**Example**

Find the minimum fencing required if the area is $800m^2$.

**Solution**

Perimeter:

$$P = 2y + x \leftarrow f(x,y)$$

Area:

$$800 = xy \leftarrow \quad \text{constraint} \quad \leftarrow \quad g(x,y) = k$$

**Step No. 1**

Given $P(x,y) = 2y + x$ subject to the constraint $800 = xy$.

**Solution**

$$\begin{aligned} \nabla P(x,y) &= \lambda \nabla g(x,y) \\ \langle P_x, P_y \rangle &= \lambda \langle g_x, g_y \rangle \\ \langle 1, 2 \rangle &= \lambda \langle y, x \rangle \\ \textbf{self-note: fix this later} \implies \frac{1}{y} &= \lambda, \quad y \neq 0 \\ \wedge \frac{2}{x} &= \lambda \end{aligned}$$

Equate $\lambda$:

$$\begin{aligned} \frac{1}{y} &= \frac{2}{x} \\ x &= 2y \end{aligned}$$

**Constraint**

$$\begin{aligned} 800 &= xy \\ 800 &= 2y \cdot y \\ 400 &= y^2 \\ \pm 20 &= y \\ \text{So}\ldots \quad y &= 20 \\ \text{So}\ldots \quad x &= 2y \\ x &= 2 \cdot 20 \\ x &= 40 \end{aligned}$$



So $(40, 20)$.
In this case, because $x = \text{something}y$, we found the $y$-value.
**self-note: don't forget to add step no.2 and step no. 3 from the posted lecture notes as well!**

**Example**

Fidn the absolute max and absolute min of $f(x, y) = 5x - 3y$ subject to the constraint $x^2 + y^2 = 136$.

**Solution**

$$f(x, y) = 5x - 3y$$

$$g(x, y) = x^2 + y^2 = 136 = k$$

<u>Step no. 1</u>

$$\nabla f(x, y) = \lambda \nabla g(x, y)$$
$$\langle 5, -3 \rangle = \lambda \langle 2x, 2y \rangle$$
$$\text{Notice that} \quad 5 = 2\lambda x \wedge -3 = 2\lambda x.$$

- $\lambda \neq 0$:
  - $\frac{5}{2x} = \lambda \quad \frac{-3}{2y} = \lambda$
  - $5y = -3x$
  - $y = \frac{-3x}{5}$

  Time for the constraint:

$$x^2 + y^2 = 136$$
$$x^2 + \left(\frac{-3x}{5}\right)^2 = 136$$
$$x^2 + \frac{9x^2}{25} = 136$$
$$\frac{25x^2 + 9x^2}{25} = 136$$
$$\frac{34x^2}{25} = 136$$
$$34x^2 = 136 \cdot 25$$
$$34x^2 = 3400$$
$$x^2 = 100$$
$$x = \pm 10$$