MAT232 - Lecture 5

[Lesson Topic(s)]

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Definitions and Theorems

Straight from the textbook — no fluff, just what we need.

Quick recap before diving into the lecture.

Let's Get Started

Time to dive into the lecture notes.

Grab your pen or pencil, and let's break this down step by step.

Review from the Previous Lecture

Remark

Recall the following from last week's lecture:

- Given $r = f(\theta)$,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

- Circle:

$$(x-h)^2 + (y-k)^2 = r^2,$$

  where r is the radius and $(h, k)$is the centre.

Note

Term Test 1 is on Thursday January 30th, 2025 (week 4)!!!

Parabolas:

Example

$$y = ax^2 + bx + c, \quad a \neq 0$$

... complete the square

$$y = A(x - B)^2 + C$$

$$A > 0 \implies \text{up}$$

$$A < 0 \implies \text{down}$$

$$\text{Vertex: } (B, c)$$

Illustration

illustration goes here (see photo from camera roll for january 20th)

Figure 1: Sample image illustrating the concept.

Example: Sketching the Region of a Set

Example

Sketch the region of the set defined by

$$R = \{(x, y) \mid y \geqslant x^2 + 1\}$$

Solution

Consider the graph for the function $y = x^2 = 1$:

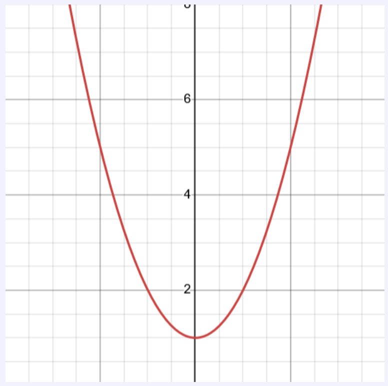

Figure 2: Graph of $y = x^2 + 1$.

Notice that

$$\begin{aligned} & y = x^2 + 1 \\ \Longrightarrow\ & 0 \geqslant (-2)^2 + 1 \\ \Longrightarrow\ & 0 \geqslant 5, \text{ which is not true.} \end{aligned}$$

Then, notice that

$$\begin{aligned} & 2 \geqslant 0^2 + 1 \\ \Longrightarrow\ & 2 \geqslant 1, \text{ which is true!} \end{aligned}$$

Here is the region being considered:

Figure 3: Sample image illustrating the concept.

Ellipse

Definition

The equation of an ellipse is defined by

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1.$$

Note

Recall the equation of the circle, which is based on the equation of the ellipse when $a = b = 1$:

$$\text{Circle:} \quad (x-h)^2 + (y-k)^2 = r^2,$$

where (h, k) is the centre, a represents the x-axis radius, and b represents the y-axis radius.

Example of Skecthing an Ellipse

Example

Sketch the region of the set defined by

$$A = \{(x, y) \mid x^2 + 4y^2 > 4\}.$$

Solution

Notice that

$$x^2 + 4y^2 = 4.$$

This means the centre is at $(0, 0)$. Also,

$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$

provides that the x-axis radius is $a = 2$ and the y-axis radius is $b = 1$.

Here is the corresponding illustration:

self-note: add the illustratoin from the lecture note from your camera roll

Figure 4: Illustration of ellipse.

Note

Note that dashed lines are used to denote that the edge of the ellipse is **not included** in the region A.

Check the point $(0, 0)$:

$$0^2 + 4 \cdot 0^2 > 4$$

$$\implies 0 > 4,$$

which is not true.

Therefore, the inside of the ellipse is **not** to be shaded in.

Check the point $(3, 0)$:

$$3^2 + 4 \cdot 0^2 > 4$$

Sketching a Region Involving a Hyperbola

Example

Lecture Title

Note

This template is designed for MAT232 lecture notes. Replace this content with your specific lecture details.

Key Concepts

Definition

A **parametric equation** is a set of equations that express the coordinates of the points of a curve as functions of a variable, called a parameter.

Examples

Example

Example 1: Consider the parametric equations:

$$x = t, \quad y = t^2, \quad t \in \mathbb{R}.$$

- At $t = 0$, $(x, y) = (0, 0)$.
- At $t = 1$, $(x, y) = (1, 1)$.

This describes a parabola.

Figure 5: Sample image illustrating the concept.

Theorems and Proofs

Theorem

Theorem: If $x(t)$ and $y(t)$ are differentiable functions, the slope of the curve is given by:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \quad \text{provided } \frac{dx}{dt} \neq 0.$$

Figure 6: Graphical representation of the theorem.

Additional Notes

Note

Always check the domain of the parameter t when solving problems involving parametric equations.