

# CSC232 - Lecture 1

Introduction to Parametric Equations

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Prepared for January 6, 2025

# Parametric Equations

## Introduction

Recall the following from high school and first-year calculus:

Equation Type	Example
Cartesian Equation	$y = x^2$
Function in Cartesian Form	$y = f(x) = x^2$
Parametric Equation	$\begin{cases} x = t \\ y = t^2 \end{cases}$

Table 1: Comparison of equation representations

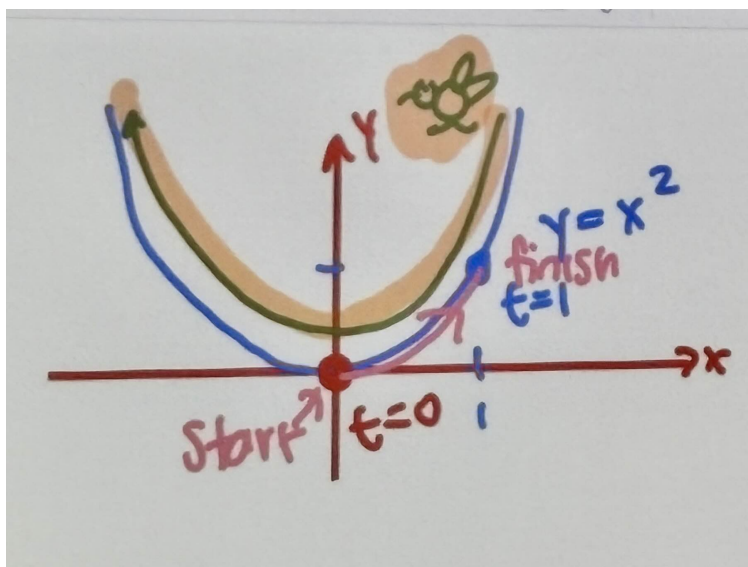


Figure 1: Graph of a parabola:  $y = x^2$

## Parametric Equation Form

Parametric equations are of the form:

$$x = f(t), \quad y = g(t), \quad t \in \mathbb{R}$$

For example:

$$x = t, \quad y = t^2, \quad t \in \mathbb{R}$$

This yields points such as:

$$(1, 1), \quad (0, 0), \quad (-1, 1)$$

Alternatively:

$$x = -t, \quad y = t^2, \quad t \in \mathbb{R}$$

This yields points such as:

$$(-1, 1), \quad (0, 0)$$

## Drawing Parametric Equations

### Methods to Sketch Parametric Equations

Two methods are commonly used to sketch parametric equations:

- Use a table of values for manual computation.
- Convert to a Cartesian equation (eliminate  $t$ ) and sketch the graph, if possible.

### Example

#### Example Problem

Sketch  $x = t^2, y = t^3$  for  $-\infty < t < \infty$ .

$t$	$x = t^2$	$y = t^3$	$(x, y)$
2	4	8	(4, 8)
1	1	1	(1, 1)
0	0	0	(0, 0)
-1	1	-1	(1, -1)
-2	4	-8	(4, -8)

Table 2: Table of values for  $x = t^2, y = t^3$

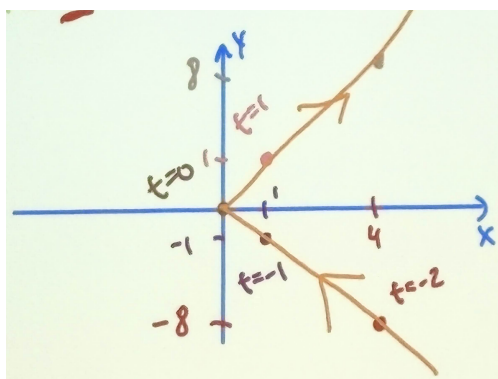


Figure 2: Sketch of  $x = t^2, y = t^3$

## Other Examples

### Advanced Example

A Cartesian equation is given by:

$$x = t^2,$$

$$y = t^3.$$

From these, we derive:

$$x^3 = (t^2)^3 = t^6,$$

$$y^2 = (t^3)^2 = t^6.$$

Thus, we have:

$$x^3 = t^6 = y^2 \implies x^3 = y^2.$$

Rewriting  $y^2 = x^3$ , we solve for  $y$ :

$$y = \pm x^{\frac{3}{2}},$$

which gives the two solutions:

$$y = x^{\frac{3}{2}} \quad \text{and} \quad y = -x^{\frac{3}{2}}.$$