

MAT232 - Lecture 15

[Lesson Topic(s)]

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Prepared for March 3, 2025

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Definitions and Theorems

Straight from the textbook — no fluff, just what we need.

Quick recap before diving into the lecture.

Let's Get Started

Time to dive into the lecture notes.

Grab your pen or pencil, and let's break this down step by step.

self-note: complete this according to the posted lecture notes

SDT

- rel max
- rel min
- saddle point
-

Section 14.8: Lagrange Multipliers

Illustration

Absolute maximum? Find it using the gradient!

$$z = f(x, y)$$

The gradient would be pointing upwards.

$$g(x, y) = k$$

k is a scalar/constant

Lagrange Multipliers

Example

Given $f(x, y)$ subject to a constant

$$g(x, y) = k$$

1: Solve (fix the stuff below later)

2: $\nabla g(x, y) \neq \bar{0}$; discard that (x, y) .

self-note: finish this according to the posted lecture notes later 3: Plug all (x, y)

Example

Find the minimum fencing required if the area is $800m^2$.

Illustration

self-note: add the illustration of the fencing from the notes later on

Solution

Perimeter:

$$P = 2y + x$$

Area:

$$800 = xy$$

Note that the y is the constraint.

So...

$$y = \frac{800}{x}$$

So...

$$\begin{aligned} P(x) &= 2\left(\frac{800}{x}\right) + x \\ &= 1600x^{-1} + x \\ \implies P'(x) &= -1600x^{-2} + 1 \\ \implies P''(x) &= 3200x^{-3} \\ \implies P'(x) = 0 &= -\frac{1600}{x} + \frac{1}{1} \cdot \frac{x^2}{x^2} \\ \implies 0 &= \frac{-1600 + x^2}{x^2} \end{aligned}$$

So... Num:

$$0 = -1600 + x^2$$

This means

$$x = \pm 40$$

so only

$$x = 40.$$

Note that denom

$$x^2 = 0$$

$$\begin{aligned} P''(x) &= 3200x^{-3} \quad \dots \quad x = 40 \\ \implies P''(40) &= \frac{3200}{40^3} > 0 \end{aligned}$$

So, by SDT, rel min at $x = 40$. So...

$$800 = xy$$

Now, let's achieve the same result using Lagrange Multipliers!

Application of Lagrange Multipliers

Example

Find the minimum fencing required if the area is $800m^2$.

Solution

Perimeter:

$$P = 2y + x \leftarrow f(x, y)$$

Area:

$$800 = xy \leftarrow \text{constraint} \quad \leftarrow \quad g(x, y) = k$$

Step No. 1

Given $P(x, y) = 2y + x$ subject to the constraint $800 = xy$.

Solution

$$\nabla P(x, y) = \lambda \nabla g(x, y)$$

$$\langle P_x, P_y \rangle = \lambda \langle g_x, g_y \rangle$$

$$\langle 1, 2 \rangle = \lambda \langle y, x \rangle$$

$$\text{self-note: fix this later} \implies \frac{1}{y} = \lambda, \quad y \neq 0$$

$$\wedge \frac{2}{x} = \lambda$$

Equate λ :

$$\frac{1}{y} = \frac{2}{x}$$

$$x = 2y$$

Constraint

$$800 = xy$$

$$800 = 2y \cdot y$$

$$400 = y^2$$

$$\pm 20 = y$$

$$\text{So... } y = 20$$

$$\text{So... } x = 2y$$

$$x = 2 \cdot 20$$

$$x = 40$$

So $(40, 20)$.

In this case, because $x = \text{something}y$, we found the y -value.