MAT232 - Lecture 4

[Lesson Topic(s)]

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Prepared for January 16, 2025

Definitions and Theorems

Straight from the textbook — no fluff, just what we need.

Quick recap before diving into the lecture.

Let's Get Started

Time to dive into the lecture notes.

Grab your pen or pencil, and let's break this down step by step.

Recall the content from last lecture...

Note

Converting from cartesian coordniates (x, y) to polar coordinates r, θ.

$$x^2 + y^2 = r^2$$

$$\arctan\left(\frac{y}{x}\right) = \theta$$

Converting from polar coordinates (r, θ) to cartesian coordinates (x, y)

$$x = r\cos\theta$$

$$y = r\sin\theta$$

Remember how to between degrees and radians:

- Degrees to radians: [fill this in]
- Radians to degrees: [fill this in]

New

Concept

Note the convention for r in a polar-coordinate point:
PC is represented as (r, θ).

$$(-r, \theta) = (r, \theta + 180°)$$

Example of Plot Points

Example

Plot points: $(3, -45°), (3, 225°), (4, 330°), (1, -45°)$ **self-note: finish this part up**
self-note: add drawing from prof from camera roll here

Example of Converting from Polar Coordinates to Cartesian Coordinates

Example

Find the **rectangular coordinates** of the point p whose polar coordniates are $6, \dfrac{\pi}{3}$.

Solution

$$x = r\cos\theta = c\cos\left(\frac{\pi}{3}\right) = 6(\frac{1}{2}) = 3$$

$$y = r\sin\theta = 6\sin\left(\frac{\pi}{3}\right) = 6(\frac{\sqrt{3}}{2}) = 3\sqrt{3}$$

$$\frac{\pi}{3} = 60^\circ$$

Therefore, the cartesian coordinate is $x, y = (3, 3\sqrt{3})$.

Converting from Cartesian Coordinates to Polar Coordinates

Example

Find the polar coordinate of the point p whose rectangular coordinates are $-2, 2\sqrt{3}$.

Solution

Recall that (the circle equation):

$$x^2 + y^2 = r^2$$

It follows that:

$$(-2)^2 + (2\sqrt{3})^2 = r^2$$

$$4 + 4 \cdot 3 = r^2$$

$$16 = r^2$$

$$\pm\sqrt{16} = r$$

$$r = \pm 4$$

Note that the radius is positive. Thus:

$$r = 4.$$

Note that

Key Concepts

Definition

A **parametric equation** is a set of equations that express the coordinates of the points of a curve as functions of a variable, called a parameter.

Examples

Example

Example 1: Consider the parametric equations:

$$x = t, \quad y = t^2, \quad t \in \mathbb{R}.$$

- At $t = 0$, $(x, y) = (0, 0)$.
- At $t = 1$, $(x, y) = (1, 1)$.

This describes a parabola.

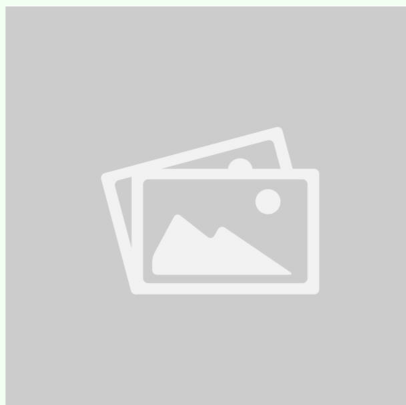

Figure 1: Sample image illustrating the concept.

Theorems and Proofs

Theorem

Theorem: If $x(t)$ and $y(t)$ are differentiable functions, the slope of the curve is given by:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \quad \text{provided } \frac{dx}{dt} \neq 0.$$

Figure 2: Graphical representation of the theorem.

Additional Notes

Note

Always check the domain of the parameter t when solving problems involving parametric equations.