MAT232 - Lecture 3

[Lesson Topic(s)]

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Lecture Title

Note

This template is designed for MAT232 lecture notes. Replace this content with your specific lecture details.

Recall 1st Year Calculus

Definition

A definite integral...

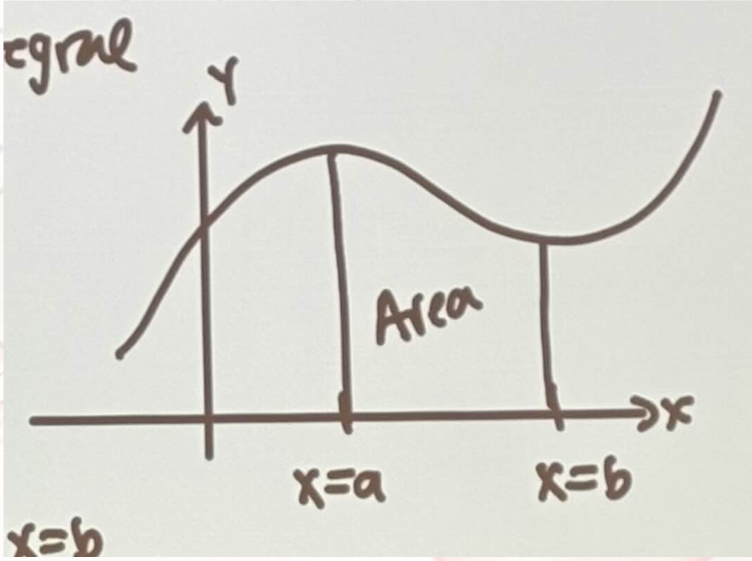


Figure 1: Sample image illustrating the concept.

Section 1.2 (...cont'd): Now, in MAT232...

Definition

self-note: make this definition proper A **parametric curve** has properties

$$x = f(t), \quad y = g(t), \quad \alpha \leqslant t \leqslant \beta$$

above the x-axis, and does not self-intersect. **self-note: show an image of self intersection and a cross to denote the "NO!"**

$$Area = \int_{x=a}^{x=b} f(x)dx = \int_{b}^{a} y(x)dx = \int_{c}^{d} x(y)dy$$

Aside:

$$Area = \int_{y=e}^{y=d} g(y)dy$$

Also...

$$Area = \int_{t=\alpha}^{t=\beta} g(t)f'(t)dt$$

$$Area = \int_{t=\alpha}^{t=\beta} f(t)g'(t)dt$$

Examples

Example

Example 1: Find the area under the curve of the cycloid defined by the equations

$$x = t - \sin(t), \quad y = 1 - \cos(t), \quad 0 \leqslant t \leqslant 2\pi.$$

- $x = f(t) = t - \sin(t)$
- $x' = f'(t) = 1 - \cos(t)$
- $y = g(t) = 1 - \cos(t)$

Recall the generic formula to find the area:

$$Area = \int_{t=\alpha}^{t=\beta} g(t)f'(t)dt$$

Applying $f(t), g(t)$ from this question:

$$\begin{aligned} Area &= \int_0^{2\pi} [1 - \cos(t)][1 - \cos(t)]dt \\ &= \int_0^{2\pi} [1 - 2\cos(t) + cos^2(t)]dt \end{aligned}$$

Recall the half-angles trigonometric identity:

$$\int \cos^2(x)dx = \int \frac{1 + cos(2x)}{2}dx$$

$$\int \cos(x)dx = \sin(x) + c$$

So... **self-note: finish off the work below on the ellipsis**

$$\begin{aligned} Area &= \dots \\ &= 3\pi \end{aligned}$$

Homework Practice Question

Example

Find the area under the curve defined by

$$x = 3\cos(t) + \cos(3t), \quad y = 3\sin(t) - \sin(3t), \quad 0 \leqslant t \leqslant \pi.$$

Hint: Recall that $\sin^2(x) + \cos^2(x) = 1$. Notice that...

$$mathgoeshere$$

answer: 3π

The Arc Length of a Parametric Curve

Theorem

Theorem: self-note: grab the actual theorem from the textbook lol

- (x_1, y_1) and (x_2, y_2) are points
- $\Delta x = x_1 - x_2$, $\Delta =$ Delta

The distance between two points is denoted by D as follows:

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Substitute Δx and Δy as follows:

$$D = \sqrt{\Delta x^2 + \Delta y^2}$$

It follows that...

$$D = \sqrt{(\frac{\Delta x}{\Delta t})^2 + (\frac{\Delta y}{\Delta t})^2}\Delta t$$

Now, notice the similarity to Riemann sums from MAT136. As $\Delta x \to 0$:

$$L = \int_{t=\alpha}^{t=\beta} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt$$

L is the (arc) length of a curve. This is confirmed to be included on term test 1, and will be on the formula sheet.

Figure 2: Graphical representation of the theorem.

Example

Example

Find the arc length of the curve defined by

$$x = 3\cos(t), \quad y = 3\sin(t), \quad t \in [0, 2\pi].$$

The arc length is denoted by L. Evaluate as follows:

$$\begin{aligned} L &= \int_0^{2\pi} \sqrt{(-3\sin(t))^2 + (3\cos(t))^2} dt \\ &= \textbf{self-note: finish this using the notes in the camera roll} \end{aligned}$$

Homework Practice Problem

Note

Find the arc length of the curve defined by

$$x = 3t^2, \quad y = 2t^3, \quad 1 \leqslant t \leqslant 3.$$

self-note: do the solution to this

Section 1.3: Polar Coordinates

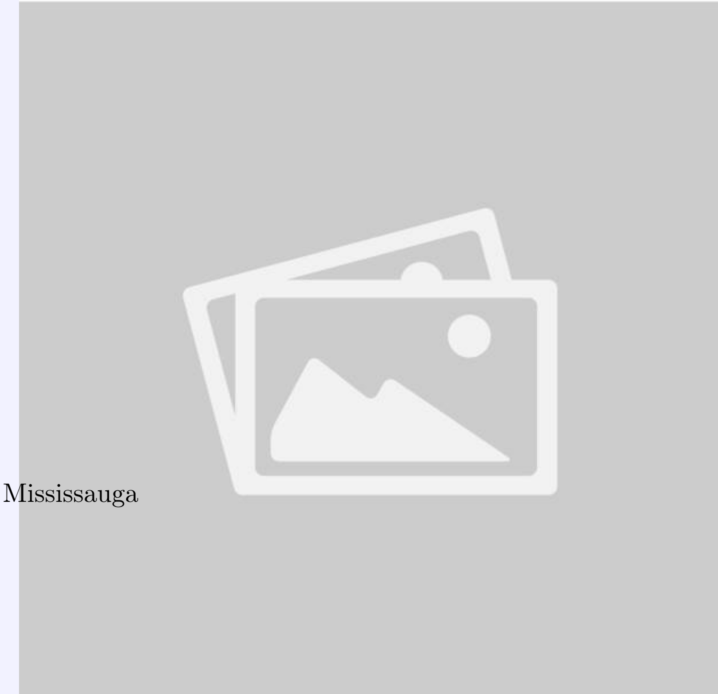
Definition

give the actual definition here from the textbook lol

Cartesian Coordniates:

Figure 3: Graphical representation of the theorem.

Polar Coordinates:



Additional Notes

Note

Always check the domain of the parameter t when solving problems involving parametric equations.

Further Visualization

Figure 5: Additional visualization for parametric curves.