CSC232 - Lecture 1

Introduction to Parametric Equations

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Parametric Equations

Recall the following from high school and first-year calculus:

Equation Type	**Example**
Cartesian Equation	$y = x^2$
Function in Cartesian Form	$y = f(x) = x^2$
Parametric Equation	$\begin{cases} x = t \\ y = t^2 \end{cases}$

Table 1: Comparison of equation representations

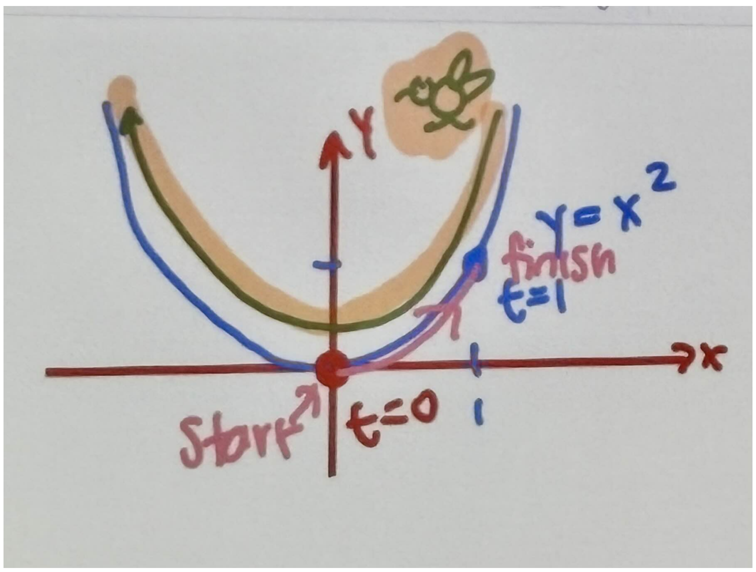

Figure 1: Graph of a parabola: $y = x^2$

Parametric equations are of the form:

$$x = f(t), \quad y = g(t), \quad t \in \mathbb{R}$$

For example:

$$x = t, \quad y = t^2, \quad t \in \mathbb{R}$$

This yields points such as:

$$(1, 1), \quad (0, 0), \quad (-1, 1)$$

Alternatively:

$$x = -t, \quad y = t^2, \quad t \in \mathbb{R}$$

This yields points such as:

$$(-1, 1), \quad (0, 0)$$

Drawing Parametric Equations

Two methods are commonly used to sketch parametric equations:

- Use a table of values for manual computation.
- Convert to a Cartesian equation (eliminate t) and sketch the graph, if possible.

Example: Sketch $x = t^2, y = t^3$ for $-\infty < t < \infty$.
Table of Values:

t	$x = t^2$	$y = t^3$	(x, y)
2	4	8	$(4, 8)$
1	1	1	$(1, 1)$
0	0	0	$(0, 0)$
-1	1	-1	$(1, -1)$
-2	4	-8	$(4, -8)$

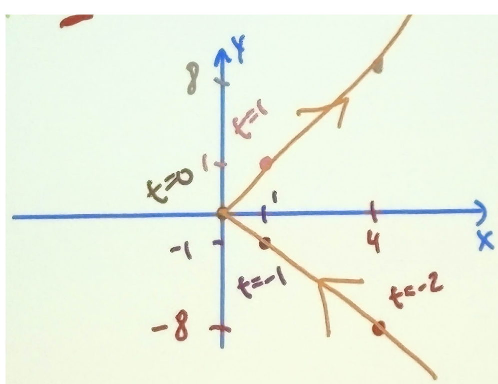

Figure 2: Sketch of $x = t^2, y = t^3$

Other Examples

A cartesian equation is given by:

$$x = t^2,$$
$$y = t^3.$$

From these, we derive:

$$x^3 = (t^2)^3 = t^6,$$
$$y^2 = (t^3)^2 = t^6.$$

Thus, we have:

$$x^3 = t^6 = y^2 \implies x^3 = y^2.$$

Rewriting $y^2 = x^3$, we solve for y:

$$y = \pm x^{\frac{3}{2}},$$

which gives the two solutions:

$$y = x^{\frac{3}{2}} \quad \text{and} \quad y = -x^{\frac{3}{2}}.$$