Setting quantum state Exercises

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1 Introduction

In this file you will have exercises for setting some basic quantum states working only on real numbers.

2 Reminder

Here are some reminders for basic gates:

PauliX gate (NOT or CX):

$|0\rangle = |1\rangle$
$|1\rangle = |0\rangle$

Hadamard gate:

$|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
$|1\rangle = |0\rangle - |1\rangle$

Controlled PauliX gate (CNOT or CX):

$|00\rangle = |00\rangle$
$|01\rangle = |11\rangle$
$|10\rangle = |10\rangle$
$|11\rangle = |01\rangle$

3 Easy

Find optimal way to make following states:

- $\frac{1}{\sqrt{2}}(|01> + |10>)$
- $\frac{1}{\sqrt{2}}(|001> - |110>)$
- $\frac{1}{2}(|000> - |001> + |110> - |111>)$

4 Medium

Advice: If we have qubits in state $\frac{1}{\sqrt{2}}$ ($|00>+ |01>$) and first qubit (this on right) is controlled qubit and the second (this on left) is target qubit, after applying controlled hadamard (CH) we get $\frac{1}{2}$ ($\sqrt{2}$ $|00>+ |01>+ |11>$) the part that had the controlled qubit set at 0 didn't change, but part that had controlled qubit set at 1 get hadamard gate at target qubit.

Find optimal way to make following states:

- example 011
  |010> - 50%
  |110> - 25%
  |111> - 25%
- example 011
  |000> - 25%
  |011> - 25%
  |110> - 25%
  |111> - 25%
- example 100
  |000> - 12.5%
  |001> - 12.5%
  |010> - 25%
  |110> - 25%
  |111> - 25%

5 Hard

Given that: $\theta = 2arcsin(\sqrt{P})$,
probability outcomes for RX(θ) are as following:

- P for |1>,
- 1-P for |0>.

  - example fun
    |000> - 33.(3)%
    |010> - 33.(3)%
    |100> - 33.(3)%
  - example f
    |000> - 30%
    |001> - 40%
    |101> - 30%
  - example very fun fun
    |001> - 37%
    |010> - 25%
    |101> - 12%
    |110> - 26%