Final Exam

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PRE-PROCESSING

We start by setting the random seed and loading useful libraries.

```
set.seed(02022018)
require(RCurl)
require(gdata)
library(dplyr)
library(magrittr)
library(ggplot2)
library(MASS)
#library(MVN) #multivariate normality test
#library(MASS)
#library(caret)
```

Now we load our data frames for the exercises.

```
urls <- c("https://www.mimuw.edu.pl/~noble/courses/SDA/data/ushighways.txt",
          "https://www.mimuw.edu.pl/~noble/courses/SDA/data/PET.txt",
          "https://www.mimuw.edu.pl/~noble/courses/SDA/data/ozone.csv",
          "https://www.mimuw.edu.pl/~noble/courses/SDA/data/pendigits.txt",
          "https://www.mimuw.edu.pl/~noble/courses/SDA/data/carmarks.txt",
          "https://www.mimuw.edu.pl/~noble/courses/SDA/data/primate.scapulae.txt")
ushighways <- read.table(urls[1], header=T)
#download.file("https://www.mimuw.edu.pl/~noble/courses/SDA/data/bodyfat2.XLS", destfile="bodyfat2.xls"
bodyfat <- read.xls("bodyfat2.xlsx")
#data(yarn, package="pls")
yarn <- read.table(urls[2], header=T)
ozone <- read.csv(urls[3])
pendigits <- read.table(urls[4])
carmarks <- read.table(urls[5], sep=";", header=T)
scapular <- read.table(urls[6], sep=" ", header=T)
```

EXERCISE 1

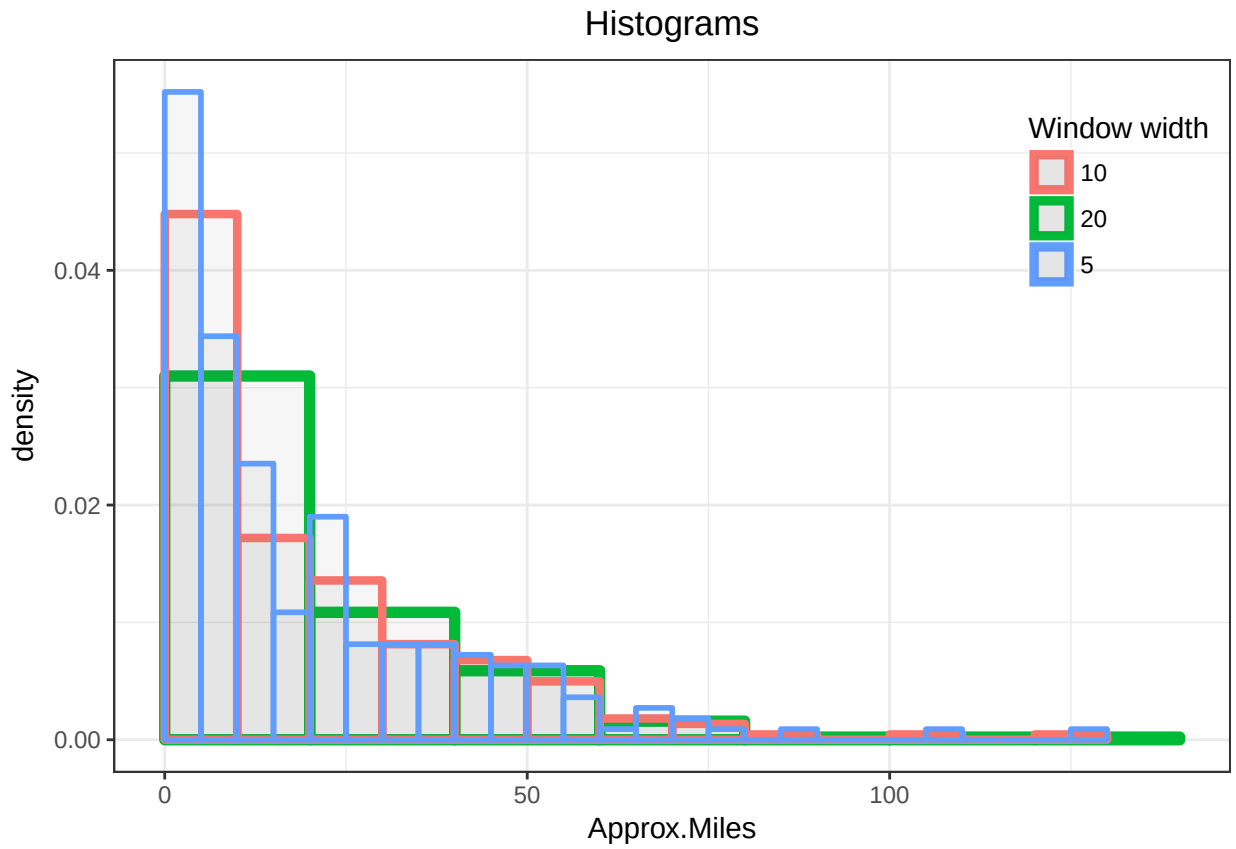
Let us take a look at data.

```
head(ushighways)
```

```
##   Interstate.. State Approx.Miles    Location
## 1          165    AL            4      MOBILE
## 2          359    AL            4  TUSCALOOSA
## 3          459    AL           32  BIRMINGHAM
## 4          759    AL            4     GADSDEN
## 5          565    AL           21  HUNTSVILLE
## 6          430    AR           13 LITTLE ROCK
```

We first draw some histograms with different window widths.

```
ggplot(ushighways, aes(x=Approx.Miles)) +
    geom_histogram(size=2, alpha=0.05, binwidth=20, boundary=0,
                   aes(color="20", y=..density..)) +
    geom_histogram(size=1.5, alpha=0.05, binwidth=10, boundary=0,
                   aes(color="10", y=..density..)) +
    geom_histogram(size=1, alpha=0.05, binwidth=5, boundary=0,
                   aes(color="5", y=..density..)) +
    theme_bw() + theme(plot.title = element_text(hjust = 0.5)) + ggtitle("Histograms") +
    guides(color=guide_legend(title="Window width")) +
    theme(legend.position=c(0.9,0.8), legend.background=element_rect(fill=alpha('blue',0)))
```

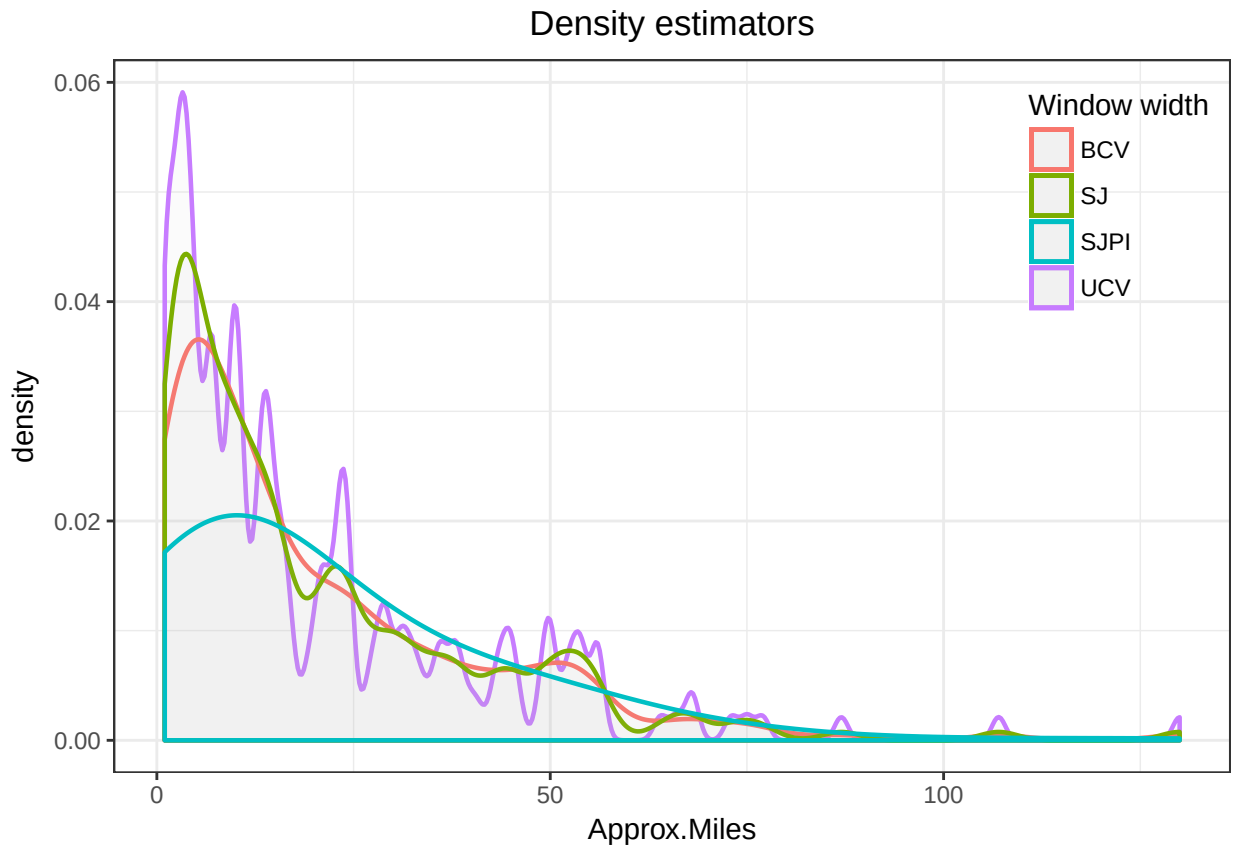


The second picture with window width of 10 miles looks quite good. It is well-splitted and shows the monotonicity of the distribution.

Now we will use UCV, BCV and SJPI estimators for window width to plot densities with gaussian kernel.

```
sjpi <- width.SJ(ushighways$Approx.Miles, method="dpi")
ggplot(ushighways, aes(x=Approx.Miles)) +
    geom_density(fill="grey", size=0.8, alpha=0.05, bw="ucv", aes(color="UCV")) +
    geom_density(fill="grey", size=0.8, alpha=0.05, bw="bcv", aes(color="BCV")) +
    geom_density(fill="grey", size=0.8, alpha=0.05, bw="sj", aes(color="SJ")) +
    geom_density(fill="grey", size=0.8, alpha=0.05, bw=sjpi, aes(color="SJPI")) +
    theme_bw() + theme(plot.title = element_text(hjust = 0.5)) +
    ggtitle("Density estimators") + guides(color=guide_legend(title="Window width")) +
    theme(legend.position=c(0.9,0.8), legend.background=element_rect(fill=alpha('blue',0)))
```

```
## Warning in bw.ucv(x): minimum occurred at one end of the range
```



The BCV estimator (red one) looks quite well-fitted and monotonic. The other kernel choices gave very similar results.

EXERCISE 2

Let us take a look at data.

```
head(bodyfat)
```

```
##   density bodyfat age weight height neck chest abdomen   hip thigh knee
## 1  1.0708    12.3  23 154.25  67.75 36.2  93.1    85.2  94.5  59.0 37.3
## 2  1.0853     6.1  22 173.25  72.25 38.5  93.6    83.0  98.7  58.7 37.3
## 3  1.0414    25.3  22 154.00  66.25 34.0  95.8    87.9  99.2  59.6 38.9
## 4  1.0751    10.4  26 184.75  72.25 37.4 101.8    86.4 101.2  60.1 37.3
## 5  1.0340    28.7  24 184.25  71.25 34.4  97.3   100.0 101.9  63.2 42.2
## 6  1.0502    20.9  24 210.25  74.75 39.0 104.5    94.4 107.8  66.0 42.0
##   ankle biceps forearm wrist
## 1  21.9   32.0    27.4  17.1
## 2  23.4   30.5    28.9  18.2
## 3  24.0   28.8    25.2  16.6
## 4  22.8   32.4    29.4  18.2
## 5  24.0   32.2    27.7  17.7
## 6  25.6   35.7    30.6  18.8
```

EXERCISE 3

Let us take a look at data.

```
head(yarn)
```

```
##      X.1    X.2    X.3    X.4    X.5    X.6    X.7    X.8    X.9   X.10
## 1 3.0663 3.0861 3.1079 3.0972 2.9979 2.8273 2.6233 2.4039 2.1931 2.0058
## 2 3.0675 3.0857 3.0958 3.0692 2.9818 2.8408 2.6760 2.5059 2.3506 2.2230
## 3 3.0750 3.0966 3.0916 3.0288 2.8849 2.6885 2.4764 2.2694 2.0824 1.9195
## 4 3.0828 3.0973 3.1010 3.0735 2.9913 2.8709 2.7392 2.6102 2.5000 2.4237
## 5 3.1029 3.1034 3.0848 3.0228 2.8927 2.7159 2.5384 2.3764 2.2397 2.1346
## 6 3.0815 3.0849 3.0487 2.9305 2.7323 2.5089 2.2944 2.0995 1.9328 1.7925
##     X.11   X.12   X.13   X.14   X.15   X.16   X.17   X.18   X.19   X.20
## 1 1.8379 1.6907 1.5777 1.5033 1.4381 1.3373 1.2206 1.1410 1.1161 1.1329
## 2 2.1192 2.0352 1.9795 1.9554 1.9396 1.8931 1.8198 1.7550 1.7122 1.6832
## 3 1.7747 1.6490 1.5533 1.4907 1.4363 1.3540 1.2603 1.1979 1.1788 1.1900
## 4 2.3774 2.3526 2.3506 2.3721 2.4009 2.4042 2.3756 2.3321 2.2768 2.2053
## 5 2.0534 1.9909 1.9522 1.9393 1.9337 1.9047 1.8535 1.8047 1.7650 1.7266
## 6 1.6693 1.5633 1.4836 1.4310 1.3851 1.3193 1.2468 1.1999 1.1851 1.1896
##     X.21   X.22   X.23   X.24   X.25   X.26   X.27   X.28   X.29   X.30
## 1 1.1604 1.1638 1.1527 1.1515 1.1277 1.0681 1.0197 1.0084 1.0321 1.0828
## 2 1.6461 1.5810 1.5091 1.4521 1.3802 1.2865 1.2045 1.1508 1.1308 1.1438
## 3 1.2053 1.1985 1.1804 1.1727 1.1513 1.1042 1.0665 1.0593 1.0807 1.1245
## 4 2.1085 1.9815 1.8544 1.7459 1.6310 1.5083 1.3954 1.2976 1.2297 1.2022
## 5 1.6753 1.5999 1.5207 1.4575 1.3898 1.3114 1.2421 1.1904 1.1630 1.1655
## 6 1.1931 1.1778 1.1538 1.1405 1.1220 1.0875 1.0604 1.0562 1.0742 1.1106
##     X.31   X.32   X.33   X.34   X.35   X.36   X.37   X.38   X.39   X.40
## 1 1.1408 1.1788 1.2018 1.2349 1.2776 1.3125 1.3431 1.3766 1.4145 1.4571
## 2 1.1761 1.2041 1.2292 1.2690 1.3180 1.3574 1.3924 1.4335 1.4792 1.5239
## 3 1.1751 1.2105 1.2357 1.2724 1.3204 1.3659 1.4096 1.4529 1.4976 1.5460
## 4 1.2088 1.2278 1.2553 1.3016 1.3567 1.4006 1.4399 1.4881 1.5412 1.5877
## 5 1.1901 1.2163 1.2443 1.2880 1.3421 1.3918 1.4400 1.4908 1.5430 1.5931
## 6 1.1542 1.1873 1.2147 1.2547 1.3070 1.3618 1.4175 1.4692 1.5190 1.5724
##     X.41   X.42   X.43   X.44   X.45   X.46   X.47   X.48   X.49   X.50
## 1 1.5031 1.5457 1.5762 1.6084 1.6605 1.7264 1.7977 1.8791 1.9835 2.1216
## 2 1.5657 1.6045 1.6366 1.6725 1.7260 1.7932 1.8686 1.9561 2.0683 2.2159
## 3 1.5938 1.6320 1.6575 1.6918 1.7518 1.8215 1.8862 1.9618 2.0727 2.2230
## 4 1.6249 1.6592 1.6928 1.7325 1.7870 1.8548 1.9343 2.0282 2.1481 2.3036
## 5 1.6366 1.6709 1.6982 1.7365 1.7976 1.8684 1.9377 2.0199 2.1391 2.2995
## 6 1.6216 1.6547 1.6754 1.7124 1.7793 1.8510 1.9086 1.9792 2.0954 2.2553
##     X.51   X.52   X.53   X.54   X.55   X.56   X.57   X.58   X.59   X.60
## 1 2.2734 2.4047 2.5122 2.6176 2.7415 2.8889 3.0358 3.1401 3.1976 3.2279
## 2 2.3712 2.4851 2.5590 2.6399 2.7597 2.9196 3.0851 3.1855 3.2077 3.2086
## 3 2.3876 2.5381 2.6614 2.7637 2.8644 2.9774 3.0972 3.1887 3.2271 3.2448
## 4 2.4589 2.5513 2.5923 2.6528 2.7686 2.9346 3.1110 3.2062 3.2019 3.1915
## 5 2.4665 2.5950 2.6826 2.7628 2.8580 2.9830 3.1250 3.2115 3.2239 3.2267
## 6 2.4310 2.5975 2.7325 2.8296 2.9058 2.9825 3.0743 3.1621 3.2042 3.2293
##     X.61   X.62   X.63   X.64   X.65   X.66   X.67   X.68   X.69   X.70
## 1 3.2512 3.2835 3.3212 3.3514 3.3670 3.3590 3.3159 3.2437 3.1262 2.9523
## 2 3.2257 3.2590 3.2948 3.3264 3.3504 3.3609 3.3297 3.2577 3.1240 2.8986
## 3 3.2838 3.3285 3.3538 3.3764 3.3896 3.3667 3.3145 3.2266 3.0409 2.8017
## 4 3.2100 3.2352 3.2603 3.2975 3.3337 3.3377 3.2949 3.2351 3.0970 2.8169
## 5 3.2484 3.2761 3.2982 3.3229 3.3476 3.3372 3.2867 3.2119 3.0177 2.7192
## 6 3.2794 3.3154 3.3222 3.3380 3.3533 3.3176 3.2418 3.1224 2.8633 2.5717
```

```
##     X.71   X.72   X.73   X.74   X.75   X.76   X.77   X.78   X.79   X.80
## 1 2.7794 2.6562 2.5837 2.5444 2.5277 2.5333 2.5613 2.6092 2.6722 2.7478
## 2 2.6815 2.5399 2.4638 2.4274 2.4160 2.4249 2.4540 2.5024 2.5677 2.6458
## 3 2.6094 2.4879 2.4199 2.3875 2.3800 2.3926 2.4244 2.4747 2.5401 2.6181
## 4 2.5605 2.4060 2.3280 2.2933 2.2850 2.2969 2.3275 2.3747 2.4371 2.5129
## 5 2.4807 2.3432 2.2758 2.2470 2.2416 2.2552 2.2888 2.3401 2.4053 2.4832
## 6 2.3675 2.2514 2.1925 2.1672 2.1641 2.1799 2.2140 2.2654 2.3321 2.4104
##     X.81   X.82   X.83   X.84   X.85   X.86   X.87   X.88   X.89   X.90
## 1 2.8329 2.9196 3.0015 3.0733 3.1329 3.1796 3.2105 3.2331 3.2502 3.2586
## 2 2.7312 2.8192 2.9053 2.9842 3.0542 3.1125 3.1623 3.2021 3.2261 3.2396
## 3 2.7061 2.7947 2.8782 2.9610 3.0397 3.1053 3.1609 3.2047 3.2294 3.2436
## 4 2.5980 2.6879 2.7764 2.8610 2.9408 3.0118 3.0706 3.1137 3.1476 3.1750
## 5 2.5701 2.6603 2.7483 2.8323 2.9123 2.9827 3.0424 3.0970 3.1408 3.1664
## 6 2.4958 2.5848 2.6741 2.7634 2.8494 2.9249 2.9928 3.0519 3.0922 3.1194
##     X.91   X.92   X.93   X.94   X.95   X.96   X.97   X.98   X.99  X.100
## 1 3.2692 3.2772 3.2773 3.2683 3.2452 3.2278 3.2205 3.2091 3.1986 3.1937
## 2 3.2471 3.2524 3.2528 3.2509 3.2488 3.2413 3.2300 3.2125 3.1908 3.1746
## 3 3.2559 3.2638 3.2606 3.2489 3.2372 3.2255 3.2127 3.1988 3.1850 3.1696
## 4 3.1908 3.1965 3.1910 3.1808 3.1767 3.1746 3.1650 3.1493 3.1331 3.1134
## 5 3.1820 3.1919 3.1930 3.1874 3.1768 3.1660 3.1556 3.1399 3.1204 3.0996
## 6 3.1398 3.1507 3.1530 3.1497 3.1416 3.1266 3.1072 3.0852 3.0615 3.0416
##    X.101  X.102  X.103  X.104  X.105  X.106  X.107  X.108  X.109  X.110
## 1 3.1872 3.1732 3.1540 3.1392 3.1267 3.1084 3.0922 3.0802 3.0668 3.0553
## 2 3.1665 3.1568 3.1366 3.1168 3.1036 3.0870 3.0661 3.0470 3.0316 3.0190
## 3 3.1501 3.1299 3.1112 3.0920 3.0760 3.0619 3.0432 3.0224 3.0055 2.9947
## 4 3.0893 3.0717 3.0611 3.0451 3.0224 3.0018 2.9854 2.9685 2.9541 2.9456
## 5 3.0812 3.0595 3.0324 3.0106 2.9907 2.9682 2.9469 2.9272 2.9113 2.8994
## 6 3.0204 2.9934 2.9691 2.9451 2.9194 2.8962 2.8758 2.8590 2.8415 2.8221
##    X.111  X.112  X.113  X.114  X.115  X.116  X.117  X.118  X.119  X.120
## 1 3.0462 3.0344 3.0206 3.0060 2.9881 2.9677 2.9488 2.9304 2.9088 2.8820
## 2 3.0068 2.9941 2.9808 2.9678 2.9513 2.9296 2.9083 2.8884 2.8676 2.8408
## 3 2.9810 2.9622 2.9463 2.9300 2.9091 2.8866 2.8636 2.8374 2.8058 2.7747
## 4 2.9372 2.9255 2.9097 2.8933 2.8791 2.8601 2.8397 2.8201 2.7973 2.7751
## 5 2.8890 2.8770 2.8599 2.8403 2.8206 2.7990 2.7748 2.7497 2.7230 2.6925
## 6 2.8058 2.7915 2.7768 2.7584 2.7324 2.7034 2.6764 2.6493 2.6189 2.5847
##    X.121  X.122  X.123  X.124  X.125  X.126  X.127  X.128  X.129  X.130
## 1 2.8537 2.8241 2.7893 2.7557 2.7260 2.6924 2.6536 2.6072 2.5405 2.4500
## 2 2.8082 2.7738 2.7401 2.7084 2.6762 2.6422 2.6003 2.5485 2.4831 2.3965
## 3 2.7436 2.7049 2.6659 2.6288 2.5885 2.5453 2.4986 2.4435 2.3701 2.2787
## 4 2.7500 2.7165 2.6809 2.6493 2.6203 2.5863 2.5449 2.4958 2.4319 2.3520
## 5 2.6586 2.6206 2.5821 2.5459 2.5079 2.4664 2.4211 2.3667 2.2955 2.2098
## 6 2.5470 2.5057 2.4638 2.4226 2.3782 2.3305 2.2813 2.2237 2.1507 2.0656
##    X.131  X.132  X.133  X.134  X.135  X.136  X.137  X.138  X.139  X.140
## 1 2.3520 2.2615 2.1826 2.1145 2.0522 1.9894 1.9224 1.8526 1.7848 1.7248
## 2 2.3020 2.2142 2.1365 2.0681 2.0062 1.9456 1.8817 1.8135 1.7458 1.6854
## 3 2.1837 2.0978 2.0253 1.9637 1.9081 1.8537 1.7964 1.7350 1.6739 1.6201
## 4 2.2644 2.1799 2.1048 2.0383 1.9779 1.9197 1.8590 1.7927 1.7250 1.6643
## 5 2.1224 2.0418 1.9710 1.9099 1.8552 1.8022 1.7473 1.6876 1.6259 1.5709
## 6 1.9802 1.9046 1.8417 1.7885 1.7406 1.6948 1.6474 1.5950 1.5400 1.4918
##    X.141  X.142  X.143  X.144  X.145  X.146  X.147  X.148  X.149   X.150
## 1 1.6786 1.6477 1.6238 1.5915 1.5351 1.4468 1.3313 1.2072 1.0979 1.02230
## 2 1.6382 1.6045 1.5763 1.5392 1.4800 1.3956 1.2905 1.1817 1.0890 1.02460
## 3 1.5811 1.5590 1.5449 1.5200 1.4682 1.3854 1.2793 1.1715 1.0839 1.02640
## 4 1.6158 1.5796 1.5477 1.5049 1.4414 1.3581 1.2609 1.1660 1.0887 1.03470
```

```
## 5 1.5303 1.5058 1.4880 1.4576 1.4014 1.3203 1.2235 1.1313 1.0605 1.01370
## 6 1.4601 1.4474 1.4428 1.4239 1.3743 1.2934 1.1933 1.0994 1.0309 0.98797
##     X.151   X.152   X.153   X.154   X.155   X.156   X.157   X.158   X.159
## 1 0.98235 0.96873 0.96943 0.97208 0.96891 0.95732 0.93511 0.90178 0.86321
## 2 0.98747 0.96987 0.96509 0.96567 0.96122 0.94465 0.91598 0.87981 0.84280
## 3 0.99373 0.97850 0.97561 0.97556 0.96874 0.95167 0.92322 0.88488 0.84445
## 4 0.99942 0.97663 0.96524 0.96287 0.95665 0.93488 0.89989 0.86141 0.82631
## 5 0.98274 0.96252 0.95336 0.95079 0.94237 0.91993 0.88499 0.84439 0.80603
## 6 0.95948 0.94130 0.93451 0.93157 0.92117 0.89906 0.86508 0.82293 0.78174
##     X.160   X.161   X.162   X.163   X.164   X.165   X.166   X.167   X.168
## 1 0.82637 0.79551 0.76532 0.72361 0.66715 0.60730 0.55709 0.51899 0.48948
## 2 0.80988 0.78279 0.75227 0.70640 0.64862 0.59210 0.54623 0.51148 0.48445
## 3 0.80914 0.78145 0.75225 0.70873 0.65284 0.59801 0.55402 0.52105 0.49547
## 4 0.79731 0.77390 0.74286 0.69303 0.63437 0.58120 0.53939 0.50764 0.48281
## 5 0.77479 0.75075 0.72096 0.67334 0.61650 0.56502 0.52510 0.49511 0.47173
## 6 0.74842 0.72380 0.69576 0.65136 0.59709 0.54753 0.50948 0.48122 0.45923
##     X.169   X.170   X.171   X.172   X.173   X.174   X.175   X.176   X.177
## 1 0.46553 0.44558 0.42895 0.41522 0.40396 0.39468 0.38694 0.38034 0.37455
## 2 0.46251 0.44425 0.42903 0.41642 0.40605 0.39750 0.39039 0.38431 0.37903
## 3 0.47468 0.45751 0.44333 0.43155 0.42185 0.41395 0.40746 0.40193 0.39712
## 4 0.46266 0.44592 0.43195 0.42036 0.41079 0.40287 0.39628 0.39071 0.38591
## 5 0.45281 0.43720 0.42423 0.41349 0.40468 0.39742 0.39141 0.38635 0.38202
## 6 0.44142 0.42680 0.41478 0.40487 0.39673 0.39006 0.38461 0.38008 0.37623
##     X.178   X.179   X.180   X.181   X.182   X.183   X.184   X.185   X.186
## 1 0.36945 0.36513 0.36175 0.35944 0.35826 0.35811 0.35872 0.35992 0.36171
## 2 0.37447 0.37064 0.36768 0.36577 0.36489 0.36488 0.36562 0.36691 0.36866
## 3 0.39299 0.38954 0.38688 0.38524 0.38457 0.38469 0.38553 0.38706 0.38911
## 4 0.38183 0.37850 0.37599 0.37442 0.37380 0.37399 0.37479 0.37609 0.37788
## 5 0.37833 0.37537 0.37320 0.37189 0.37146 0.37181 0.37278 0.37425 0.37624
## 6 0.37296 0.37035 0.36846 0.36737 0.36711 0.36756 0.36865 0.37033 0.37259
##     X.187   X.188   X.189   X.190   X.191   X.192   X.193   X.194   X.195
## 1 0.36414 0.36730 0.37142 0.37653 0.38241 0.38869 0.39506 0.40145 0.40788
## 2 0.37109 0.37435 0.37843 0.38336 0.38900 0.39496 0.40097 0.40699 0.41298
## 3 0.39170 0.39503 0.39923 0.40425 0.40998 0.41618 0.42245 0.42862 0.43474
## 4 0.38036 0.38365 0.38773 0.39255 0.39793 0.40360 0.40927 0.41487 0.42051
## 5 0.37888 0.38223 0.38637 0.39128 0.39680 0.40263 0.40853 0.41435 0.42003
## 6 0.37538 0.37875 0.38290 0.38784 0.39345 0.39948 0.40562 0.41163 0.41735
##     X.196   X.197   X.198   X.199   X.200   X.201   X.202   X.203   X.204
## 1 0.41431 0.42052 0.42601 0.42998 0.43154 0.43035 0.42697 0.42250 0.41825
## 2 0.41893 0.42476 0.43000 0.43390 0.43583 0.43557 0.43358 0.43088 0.42864
## 3 0.44069 0.44627 0.45129 0.45512 0.45696 0.45673 0.45504 0.45273 0.45082
## 4 0.42615 0.43160 0.43654 0.44044 0.44285 0.44362 0.44316 0.44230 0.44209
## 5 0.42554 0.43082 0.43559 0.43939 0.44176 0.44263 0.44242 0.44197 0.44220
## 6 0.42276 0.42787 0.43249 0.43615 0.43842 0.43932 0.43933 0.43920 0.43979
##     X.205   X.206   X.207   X.208   X.209   X.210   X.211   X.212   X.213
## 1 0.41552 0.41521 0.41748 0.42162 0.42685 0.43381 0.44468 0.46086 0.48147
## 2 0.42798 0.42972 0.43389 0.43978 0.44676 0.45544 0.46760 0.48429 0.50455
## 3 0.45056 0.45277 0.45739 0.46374 0.47114 0.48019 0.49281 0.51030 0.53191
## 4 0.44359 0.44740 0.45344 0.46113 0.46992 0.48031 0.49373 0.51083 0.53064
## 5 0.44408 0.44825 0.45475 0.46290 0.47207 0.48282 0.49665 0.51445 0.53545
## 6 0.44190 0.44624 0.45299 0.46139 0.47066 0.48146 0.49555 0.51388 0.53584
##     X.214   X.215   X.216   X.217   X.218   X.219   X.220   X.221   X.222
## 1 0.50364 0.52455 0.54347 0.56134 0.57942 0.60036 0.62888 0.66760 0.71161
## 2 0.52586 0.54594 0.56418 0.58120 0.59803 0.61709 0.64239 0.67568 0.71198
```

```
## 3 0.55492 0.57674 0.59689 0.61636 0.63667 0.66085 0.69249 0.73179 0.77257
## 4 0.55115 0.57042 0.58781 0.60388 0.61953 0.63663 0.65837 0.68569 0.71397
## 5 0.55755 0.57858 0.59783 0.61613 0.63512 0.65721 0.68492 0.71795 0.75057
## 6 0.55931 0.58178 0.60263 0.62315 0.64539 0.67225 0.70594 0.74466 0.78146
##     X.223   X.224   X.225   X.226   X.227   X.228   X.229   X.230   X.231
## 1 0.74849 0.76797 0.77298 0.77332 0.77099 0.76190 0.74646 0.73044 0.71725
## 2 0.74100 0.75583 0.76058 0.76421 0.76916 0.77233 0.77165 0.76655 0.75539
## 3 0.80412 0.81955 0.82342 0.82480 0.82465 0.81828 0.80530 0.78937 0.77181
## 4 0.73535 0.74604 0.75098 0.75804 0.77037 0.78596 0.79992 0.80539 0.79578
## 5 0.77430 0.78573 0.79025 0.79536 0.80285 0.80953 0.81181 0.80642 0.78996
## 6 0.80759 0.81979 0.82370 0.82656 0.82844 0.82490 0.81444 0.79810 0.77563
##     X.232   X.233   X.234   X.235   X.236   X.237   X.238   X.239   X.240
## 1 0.70428 0.68814 0.66857 0.64598 0.62081 0.59539 0.57245 0.55306 0.53702
## 2 0.73598 0.71012 0.68303 0.65673 0.63022 0.60396 0.57987 0.55894 0.54092
## 3 0.75052 0.72540 0.69906 0.67289 0.64675 0.62115 0.59753 0.57685 0.55948
## 4 0.76955 0.73416 0.70002 0.67048 0.64306 0.61620 0.59079 0.56798 0.54782
## 5 0.76176 0.72714 0.69369 0.66426 0.63709 0.61060 0.58543 0.56287 0.54336
## 6 0.74663 0.71376 0.68164 0.65282 0.62640 0.60074 0.57620 0.55427 0.53596
##     X.241   X.242   X.243   X.244   X.245   X.246   X.247   X.248   X.249
## 1 0.52345 0.51154 0.50104 0.49145 0.48152 0.47018 0.45690 0.44182 0.42590
## 2 0.52500 0.51086 0.49840 0.48715 0.47660 0.46636 0.45571 0.44413 0.43148
## 3 0.54519 0.53283 0.52160 0.51141 0.50170 0.49144 0.47986 0.46675 0.45222
## 4 0.52966 0.51330 0.49874 0.48584 0.47486 0.46588 0.45797 0.44993 0.44056
## 5 0.52657 0.51194 0.49879 0.48693 0.47670 0.46785 0.45916 0.44957 0.43827
## 6 0.52109 0.50842 0.49673 0.48596 0.47657 0.46788 0.45824 0.44694 0.43373
##     X.250   X.251   X.252   X.253   X.254   X.255   X.256   X.257   X.258
## 1 0.41059 0.39677 0.38416 0.37199 0.36018 0.34883 0.33808 0.32838 0.32001
## 2 0.41788 0.40389 0.39018 0.37691 0.36412 0.35194 0.34082 0.33122 0.32323
## 3 0.43679 0.42174 0.40788 0.39510 0.38335 0.37256 0.36253 0.35353 0.34613
## 4 0.42853 0.41426 0.39958 0.38519 0.37124 0.35833 0.34700 0.33749 0.32981
## 5 0.42443 0.40902 0.39406 0.38027 0.36746 0.35563 0.34515 0.33637 0.32935
## 6 0.41828 0.40202 0.38708 0.37409 0.36264 0.35224 0.34281 0.33475 0.32831
##     X.259   X.260   X.261   X.262   X.263   X.264   X.265   X.266   X.267
## 1 0.31308 0.30753 0.30313 0.29980 0.29748 0.29592 0.29489 0.29440 0.29432
## 2 0.31686 0.31195 0.30806 0.30499 0.30283 0.30149 0.30066 0.30031 0.30030
## 3 0.34026 0.33545 0.33164 0.32891 0.32703 0.32573 0.32493 0.32464 0.32460
## 4 0.32389 0.31947 0.31605 0.31332 0.31139 0.31023 0.30955 0.30928 0.30932
## 5 0.32394 0.31977 0.31649 0.31397 0.31218 0.31108 0.31046 0.31021 0.31026
## 6 0.32331 0.31934 0.31620 0.31389 0.31234 0.31140 0.31080 0.31052 0.31061
##     X.268      y train
## 1 0.29443 100.00     1
## 2 0.30041  80.22     1
## 3 0.32474  79.49     1
## 4 0.30947  60.80     1
## 5 0.31049  59.97     1
## 6 0.31093  60.48     1
```

EXERCISE 4

Let us take a look at data.

```
head(ozone)
```

```
##   Station   Av8top      Lat       Lon
```

```
## 1      60 7.225806 34.13583 -117.9236
## 2      69 5.899194 34.17611 -118.3153
## 3      72 4.052885 33.82361 -118.1875
## 4      74 7.181452 34.19944 -118.5347
## 5      75 6.076613 34.06694 -117.7514
## 6      84 3.157258 33.92917 -118.2097
```

EXERCISE 5

Let us take a look at data.

```
head(pendigits)
```

```
##    V1  V2 V3  V4  V5  V6  V7  V8 V9 V10 V11 V12 V13 V14 V15 V16 V17 V18
## 1  47 100 27  81  57  37  26   0  0  23  56  53 100  90  40  98   8   1
## 2   0  89 27 100  42  75  29  45 15  15  37   0  69   2 100   6   2   2
## 3   0  57 31  68  72  90 100 100 76  75  50  51  28  25  16   0   1   3
## 4   0 100  7  92   5  68  19  45 86  34 100  45  74  23  67   0   4   6
## 5   0  67 49  83 100 100  81  80 60  60  40  40  33  20  47   0   1   3
## 6 100 100 88  99  49  74  17  47  0  16  37   0  73  16  20  20   6   4
##   V19 V20 V21 V22 V23 V24 V25 V26 V27 V28 V29 V30 V31 V32 V33 V34 V35 V36
## 1  47 100  27  81  57  37  26   0   0  23  56  53 100  90  40  98   8   1
## 2   0  89  27 100  42  75  29  45  15  15  37   0  69   2 100   6   2   2
## 3   0  57  31  68  72  90 100 100  76  75  50  51  28  25  16   0   1   3
## 4   0 100   7  92   5  68  19  45  86  34 100  45  74  23  67   0   4   6
## 5   0  67  49  83 100 100  81  80  60  60  40  40  33  20  47   0   1   3
## 6 100 100  88  99  49  74  17  47   0  16  37   0  73  16  20  20   6   4
```

EXERCISE 6

Let us take a look at data.

```
head(carmarks)
```

```
##   CARMARK ECONOMY SERVICE VALUE PRICE DESIGN SPORT SAFETY EASYINESS
## 1    A100     3.9     2.8   2.2   4.2    3.0   3.1    2.4       2.8
## 2    BMW3     4.8     1.6   1.9   5.0    2.0   2.5    1.6       2.8
## 3    CiAX     3.0     3.8   3.8   2.7    4.0   4.4    4.0       2.6
## 4    Ferr     5.3     2.9   2.2   5.9    1.7   1.1    3.3       4.3
## 5    FiUn     2.1     3.9   4.0   2.6    4.5   4.4    4.4       2.2
## 6    FoFi     2.3     3.1   3.4   2.6    3.2   3.3    3.6       2.8
```

EXERCISE 7

Let us take a look at data.

```
head(scapular)
```

```
##   genus AD.BD AD.CD EA.CD Dx.CD SH.ACR EAD beta gamma     class classdigit
## 1    54 65.56 166.0 50.55 12.80   70.3 115   14    45 Hylobates          1
## 2    54 50.91  93.9 61.82 13.09   75.0 121   20    54 Hylobates          1
## 3    54 46.15  80.8 64.10 11.80   70.0 120   25    61 Hylobates          1
## 4    54 70.29 220.5 50.00 12.75   61.1 113   12    45 Hylobates          1
## 5    54 63.16 144.0 57.89 12.98   64.9 115   14    46 Hylobates          1
```

```
## 6    54 50.72 134.6 56.23 11.88   52.6 136   14    46 Hylobates          1
```

EXERCISE 8

Let us take a look at data.

```
head(scapular)
```

```
##   genus AD.BD AD.CD EA.CD Dx.CD SH.ACR EAD beta gamma     class classdigit
## 1    54 65.56 166.0 50.55 12.80   70.3 115   14    45 Hylobates          1
## 2    54 50.91  93.9 61.82 13.09   75.0 121   20    54 Hylobates          1
## 3    54 46.15  80.8 64.10 11.80   70.0 120   25    61 Hylobates          1
## 4    54 70.29 220.5 50.00 12.75   61.1 113   12    45 Hylobates          1
## 5    54 63.16 144.0 57.89 12.98   64.9 115   14    46 Hylobates          1
## 6    54 50.72 134.6 56.23 11.88   52.6 136   14    46 Hylobates          1
```