Interpreting Deep Learning Models with Marginal Attribution by Conditioning on Quantiles

Anna Kozak

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Interpreting Deep Learning Models with Marginal Attribution by Conditioning on Quantiles

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- PDP
- ICE
- ALE
- LIME
- Shapley values
- **Marginal Attribution by Conditioning on Quantiles (MACQ)**

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Mario Wüthrich

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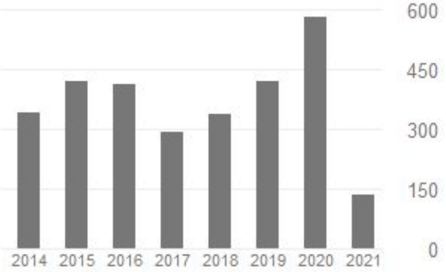
TITEL	ZITIERT VON	JAHR
Stochastic claims reserving methods in insurance MV Wüthrich, M Merz John Wiley & Sons	462	2008
Stochastic mortality in life insurance: market reserves and mortality-linked insurance contracts M Dahl Insurance: mathematics and economics 35 (1), 113-136	452	2004
Copula convergence theorems for tail events A Juri, MV Wüthrich Insurance: Mathematics and Economics 30 (3), 405-420	178	2002
Modelling the claims development result for solvency purposes MV Wüthrich, M Merz, H Bühlmann, M De Felice, A Gisler, F Moriconi Casualty Actuarial Society E-Forum, 542-568	153	2008

Zitiert von — ALLE ANZEIGEN

	Alle	Seit 2016
Zitate	5153	2228
h-index	37	26
i10-index	104	70



$$\mu : \mathbb{R}^q \to \mathbb{R}, \qquad \boldsymbol{x} \mapsto \mu(\boldsymbol{x}), \qquad\qquad \mathbb{E}[Y|\boldsymbol{x}] = \mu(\boldsymbol{x})$$

Select a quantile level $\alpha \in (0,1)$, the α-quantile of $\mu(\boldsymbol{X})$ is given by

$$F^{-1}_{\mu(\mathbf{X})}(\alpha) = \inf\left\{y \in \mathbb{R};\ F_{\mu(\mathbf{X})}(y) \geq \alpha\right\},$$

where $F_{\mu(\mathbf{X})}(y) = P[\mu(\boldsymbol{X}) \leq y]$ describes the distribution function of $\mu(\boldsymbol{X})$.

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The *1st order attributions* to components $1 \leq j \leq q$ on quantile level α are defined by

$$S_j(\mu;\alpha) \ = \ \mathbb{E}_P\left[X_j \mu_j(\boldsymbol{X}) \middle| \mu(\boldsymbol{X}) = F^{-1}_{\mu(\boldsymbol{X})}(\alpha)\right].$$

These are the marginal attributions by conditioning on quantiles (MACQ).

Taylor expansion

$$\mu(\mathbf{0}) \approx \mu(\boldsymbol{x}) - (\nabla_{\boldsymbol{x}}\mu(\boldsymbol{x}))^\top \boldsymbol{x}.$$

$$F^{-1}_{\mu(\boldsymbol{X})}(\alpha) = \mathbb{E}_P\left[\mu(\boldsymbol{X}) \,\middle|\, \mu(\boldsymbol{X}) \models F^{-1}_{\mu(\boldsymbol{X})}(\alpha)\right] \approx \mu(\mathbf{0}) + \sum_{j=1}^{q} S_j(\mu;\alpha).$$

2nd order Taylor expansion

$$\mu(\boldsymbol{x}+\boldsymbol{\epsilon}) = \mu(\boldsymbol{x}) + (\nabla_{\boldsymbol{x}}\mu(\boldsymbol{x}))^\top \boldsymbol{\epsilon} + \frac{1}{2}\boldsymbol{\epsilon}^\top (\nabla^2_{\boldsymbol{x}}\mu(\boldsymbol{x}))\boldsymbol{\epsilon} + o(\|\boldsymbol{\epsilon}\|^2),$$

$$F^{-1}_{\mu(\mathbf{X})}(\alpha) \;\approx\; \mu\left(\mathbf{0}\right) + \sum_{j=1}^{q} S_j(\mu;\alpha) - \frac{1}{2}\sum_{j,k=1}^{q} T_{j,k}(\mu;\alpha),$$

2nd order Taylor expansion

$$\mu(\boldsymbol{x}+\boldsymbol{\epsilon}) = \mu(\boldsymbol{x}) + (\nabla_{\boldsymbol{x}}\mu(\boldsymbol{x}))^\top \boldsymbol{\epsilon} + \frac{1}{2}\boldsymbol{\epsilon}^\top (\nabla^2_{\boldsymbol{x}}\mu(\boldsymbol{x}))\boldsymbol{\epsilon} + o(\|\boldsymbol{\epsilon}\|^2),$$

$$F^{-1}_{\mu(\mathbf{X})}(\alpha) \;\approx\; \mu\left(\mathbf{0}\right) + \sum_{j=1}^{q} S_j(\mu;\alpha) - \frac{1}{2}\sum_{j,k=1}^{q} T_{j,k}(\mu;\alpha),$$

$$1 \le j,k \le q, \qquad T_{j,k}(\mu;\alpha) = \mathbb{E}_P\left[X_j X_k \mu_{j,k}(\boldsymbol{X}) \,\middle|\, \mu(\boldsymbol{X}) = F^{-1}_{\mu(\mathbf{X})}(\alpha)\right]$$

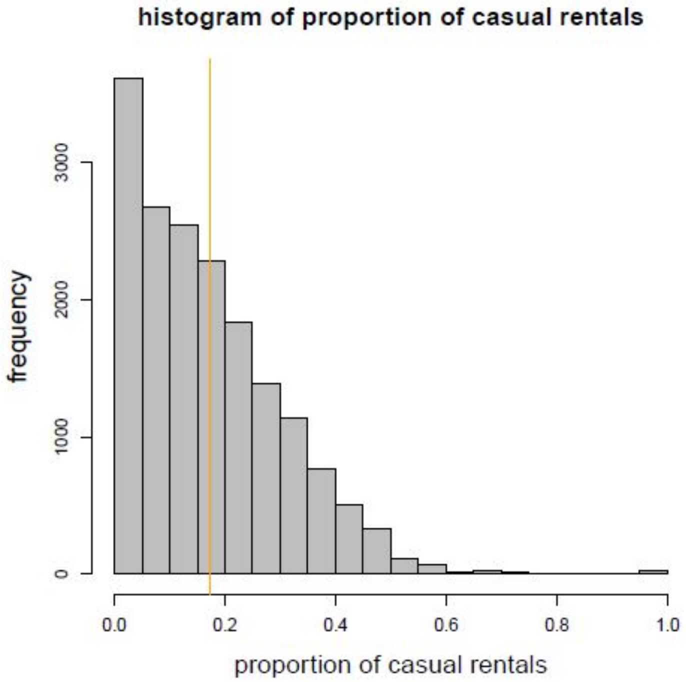
$$F^{-1}_{\mu(\mathbf{X})}(\alpha) \;\approx\; \mu\left(\mathbf{0}\right) + \sum_{j=1}^{q}\left(S_j(\mu;\alpha) - \frac{1}{2}T_{j,j}(\mu;\alpha)\right) - \sum_{1\le j<k\le q} T_{j,k}(\mu;\alpha)$$

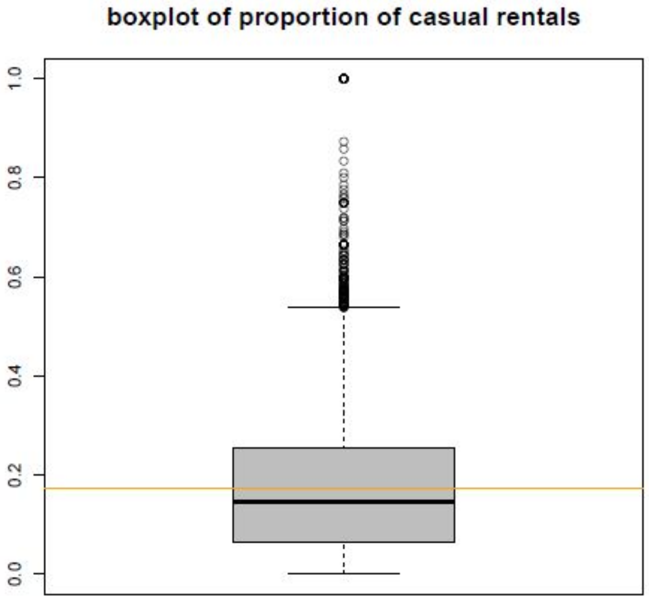
Example - bike rental data

Listing 1: Excerpt of bike rental data.

```
'data.frame':   17379 obs. of  13 variables:
 $ date       : Date, format: "2011-01-01" "2011-01-01" "2011-01-01" ...
 $ year       : num  2011 2011 2011 2011 2011 ...
 $ month      : int  1 1 1 1 1 1 1 1 1 1 ...
 $ hour       : int  0 1 2 3 4 5 6 7 8 9 ...
 $ weekday    : int  6 6 6 6 6 6 6 6 6 6 ...
 $ holiday    : Factor w/ 2 levels "holiday","no-holiday": 2 2 2 2 2 2 2 2 2 2 ...
 $ workingday: Factor w/ 2 levels "no-working","workingday": 1 1 1 1 1 1 1 1 1 1 ...
 $ weather    : num  1 1 1 1 1 2 1 1 1 1 ...
 $ temp       : num  0.24 0.22 0.22 0.24 0.24 0.24 0.22 0.2 0.24 0.32 ...
 $ temp_feel : num  0.288 0.273 0.273 0.288 0.288 ...
 $ humidity  : num  0.81 0.8 0.8 0.75 0.75 0.75 0.8 0.86 0.75 0.76 ...
 $ windspeed : num  0 0 0 0 0 0.0896 0 0 0 0 ...
 $ casual     : int  3 8 5 3 0 0 2 1 1 8 ...
 $ registered: int  13 32 27 10 1 1 0 2 7 6 ...
 $ count      : int  16 40 32 13 1 1 2 3 8 14 ...
```

Example - bike rental data

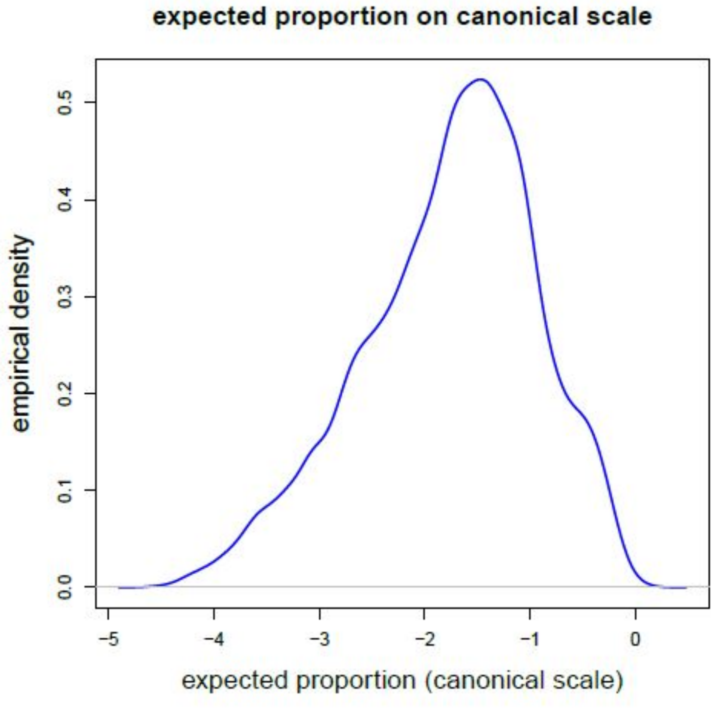


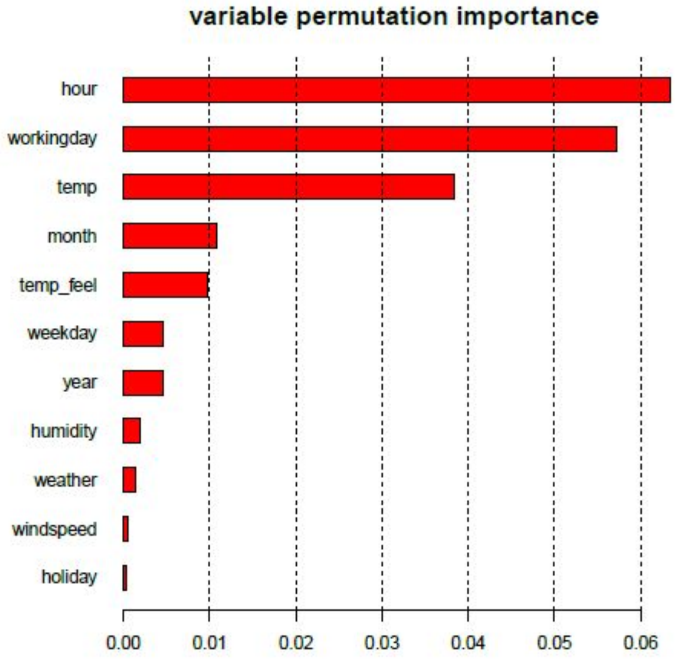


Model

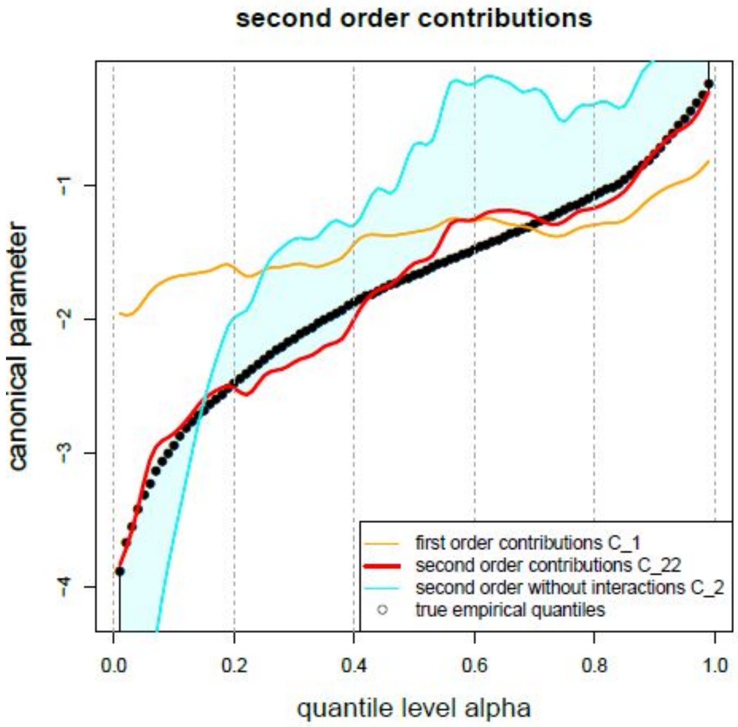
- fully-connected feed-forward neural network
- three hidden layers (20, 15, 10)
- sigmoid output activation
- hyperbolic tangent as activation function in 3 hidden layers

Example - bike rental data

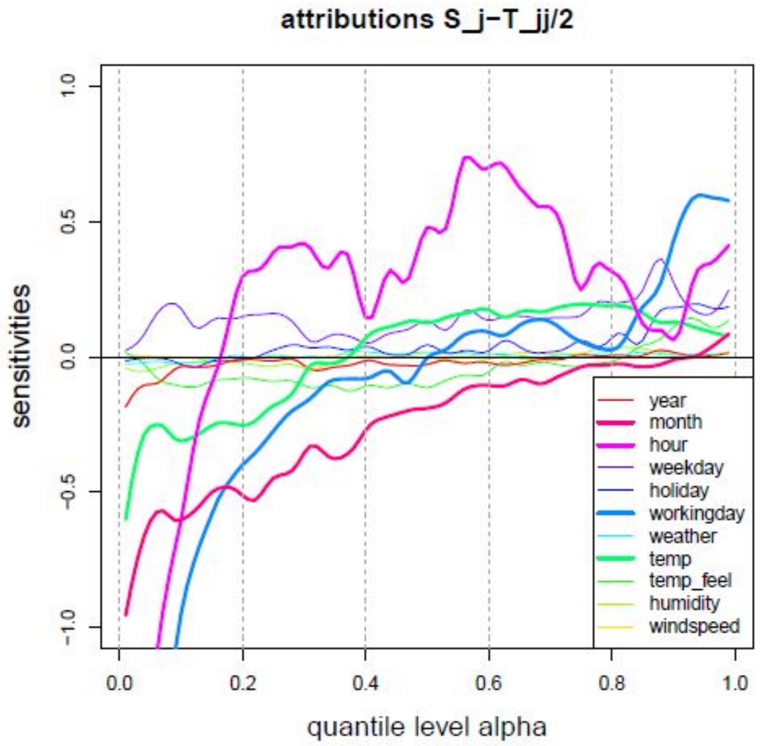


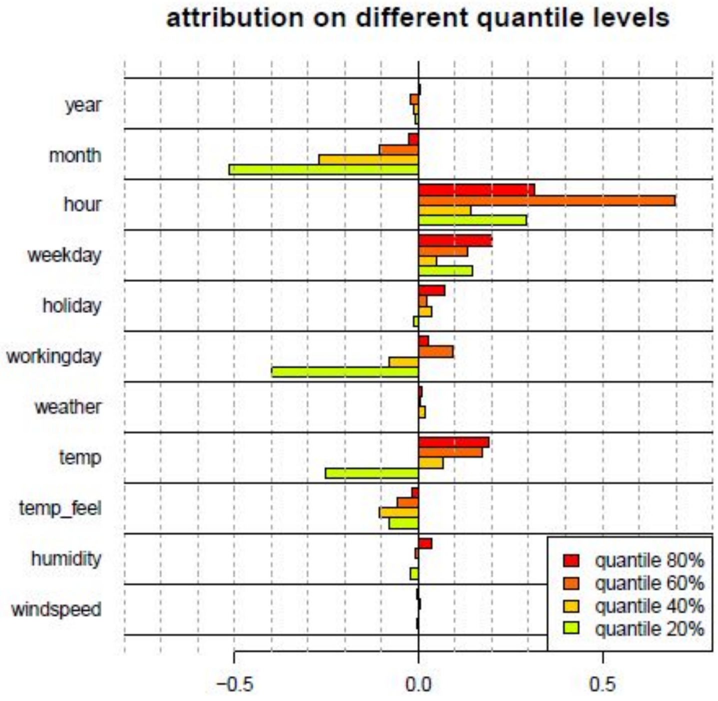


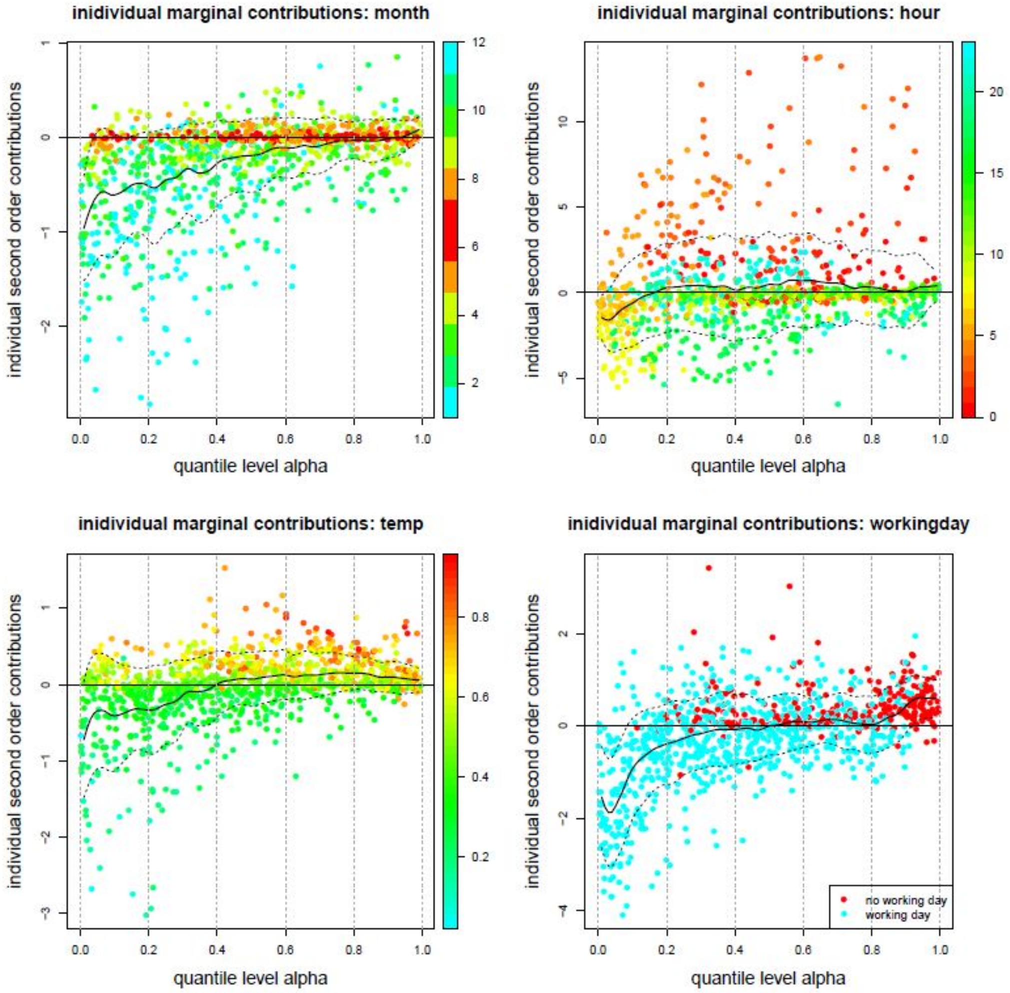
Example - bike rental data



Example - bike rental data






inidividual marginal contributions: month
inidividual marginal contributions: hour
inidividual marginal contributions: temp
inidividual marginal contributions: workingday
individual second order contributions
quantile level alpha
no working day
working day