The deriving of a computational approach test for testing equality of several populations' means in the presence of nuisance parameter

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About Me

Live in Eskisehir, Turkey

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Using R since the lectures of Marek Gagolewski in 2011.

Founder of Eskisehir R User Group

Focused on deriving powerful statistical tests for ANOVA under non-normality, and implement them in doex, and visualizing tools in doexplot.

Introduction

The probability density function of the two-parameter exponential distribution:

$$f(x; a, b) = \frac{1}{a} \exp\left(-\frac{x-b}{a}\right),\ x > b,\ a > 0$$

where a is the scale and b is the location parameter. The interested hypothesis is in the following for testing the equality of means of the exponentially distributed populations.

$$H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$$

Candidate Methods

Test deriving methods in the presence of **nuisance**[1]parameters:

- Generalized p-value
- Parametric bootstrap
- Fiducial approach
- **Computational approach**

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In statistics, a nuisance parameter is any parameter which is not of immediate interest but which must be accounted for in the analysis of those parameters which are of interest. The classic example of a nuisance parameter is the variance, σ^2, of a normal distribution, when the mean, μ is of primary interest (Wikipedia).

Computational Approach Test

Pros:

- The CAT does not require knowledge of any sampling distribution.
- Better performance on the small sample size.

Cons:

- Long run time.

Computational Approach Test

Algorithm of the CAT

Algorithm 1 p-value calculation of the CAT

1: Calculate the observed values of the interested parameter θ_i and nuisance parameter δ_i for k samples.
2: Calculate the observed value of the test statistic $\hat{t}^*_{CAT}$ using the estimators in Step 1.
3: Find the restricted maximum likelihood (RML) estimator of the parameters for k samples under the true H_0.
4: **for** $j \leftarrow 1$ to M **do**
5:     **for** $i \leftarrow 1$ to n_i **do**
6:         **for** $j \leftarrow 1$ to k **do**
7:             Generate random samples from $X_{ij} \sim f(\hat{\theta}_{RML}, \hat{\delta}_{RML})$ for k samples.
8:         **end for**
9:     **end for**
10:     Calculate the observed values of the parameters for generated samples.
11:     Calculate the observed value of test statistic $\hat{t}^{(m)}_{CAT}$ using the estimators in Step 8.
12: **end for**
13: $p = \sum_{l=1}^{M} I(\hat{t}^{(m)}_{CAT} > t^*_{CAT})/M$ ▷ p-value of the CAT

Penalized Power

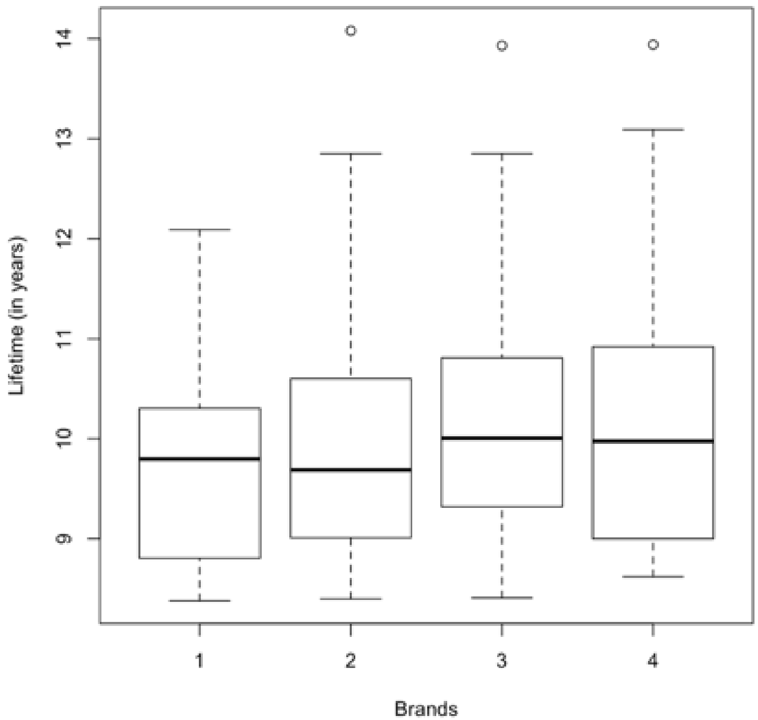
Any comparison of the powers is invalid when Type I error probabilities are different. Cavus et al. (2019) proposed the penalized power approach in the following formula to compare the power of the tests when Type I error probabilities are different.

$$\gamma_i = \frac{1 - \beta_i}{\sqrt{1 + \mid 1 - \frac{\alpha_i}{\alpha_0} \mid}}$$

where β_i is Type II error rate, α_i is Type I error of the test and α_0 is the nominal level.

Illustrative Example

Data consists the lifetimes of a component are supplied by different brands in a refrigerator which is collected from a local factory in Turkey and the sample size of the data are 15, 49, 54, 12, respectively.



Illustrative Example

Testing the mean lifetimes of the components under scale parameters, GP, PB, FA, and CAT tests are performed.

Method	**p-value**
GPV	0.6770
PB	0.7466
FA	0.7493
CAT	0.7510

There is no enough evidence to reject the null hypothesis at the 0.05 significance level.

References

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Thanks!

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