Solving PDEs Associated with Economic Models

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- Used in EconPDEs
  - The Jacobian corresponding to a second-order PDE $F(f, f_x, f_{xx})$ with first-order finite difference scheme a Triangular Matrix of size n_x
  - The Jacboain corresponding to a system of second-order PDE $F(f^1, f^1_x, f^1_{xx}, \dots, f^J, f^J_x, f^J_{xx})$ with first-order finite difference scheme is a Sparse Matrix of size $n_x \times J$ with non zeros subdiagonal at $i \times n_x - 1, i \times n_x, i \times n_x + 1$ for $i \in [[-J, J]]$.

    Alternatively, it can be written as a Block Triangular Matrix where blocks are dense of size $J \times J$ by writing the system as $f^1(x_1), f^1(x_2), \dots f^J(x_1)$, etc
  - The Jacobian corresponding to the system $F(f, \nabla f, f_{x_2}, f_{x_1 x_1}, f_{x)y}, f_{yy})$ with first-order finite difference scheme is a Block Triangular Matrix of size $n_x \times n_y$ where each block is itself a Triangular Matrix.

    Alternatively, it can be written as a Block Triangular Matrix where blocks are dense of size 2×2

  In summary, EconPDEs would benefit from specialization on Triangular Matrix and Block Triangular Matrix (with blocks either dense or Triangular themselves).

  A package that wants to solve PDEs using high-order finite difference scheme would benefit from specialization on BandedMatrix / Block Banded Matrix

- Used by Jesse Perla
  - Some problems in econmics have the form of a system $F(f, f_x, f_{xx})$, and $\int f(x)g(x) = 1.$ where g is known. IN this case, the Jacobian is a Triangular matrix + dense row.

- Others
  - $F(f, f_x, f_{xx})$ with $n - th$ order finite difference scheme a Banded Matrix with band size n.

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