Macroeconometrics

Exam No. 1 of 3

Winter 2017/2018

- Answer **all** of the following four exercises in either German or English.
- Hand in your solutions before Tuesday November, 7 2017 at 14:00.
- Please e-mail the solutions file to willi@mutschler.eu.
- The solution files should contain your executable (and commented) Matlab functions and script files as well as an additional documentation preferably as `pdf`, not `doc` or `docx`.
- I will confirm the receipt of your work also by email.
- All students must work on their own, please also give your student ID number.
- It is advised to regularly check the learnweb in case of urgent updates.
- If there are any questions, do not hesitate to contact Willi Mutschler.

1 AR(1) with time trend

Consider the AR(1) model with constant and time trend

$$y_t = c + d \cdot t + \phi y_{t-1} + u_t$$

where u_t is $iid(0, \sigma^2)$, $|\phi| < 1$, $c \in \mathbb{R}$ and $d \in \mathbb{R}$.

1. Compute the unconditional first and second moments, i.e. the unconditional mean, variance, autocovariance and autocorrelation of y_t.

2. Why is this process not covariance-stationary? How could one proceed to make it covariance-stationary?

2 Portmanteau Test For Residual Autocorrelation

The portmanteau test checks the null hypothesis that there is no remaining residual autocorrelation at lags 1 to h against the alternative that at least one of the autocorrelations is nonzero. In other words, the pair of hypotheses:

$$H_0 : \rho_u(1) = \rho_u(2) = ... = \rho_u(h) = 0$$

versus:

$$H_1 : \rho_u(j) \neq 0 \text{ for at least one } j = 1, ..., h$$

is tested, where $\rho_u(j) = Corr(u_t, u_{t-j})$ denotes an autocorrelation coefficient of the residual series. Consider the Box-Pierce test statistic Q_h

$$Q_h = T \sum_{j=1}^{h} \hat{\rho}_u^2(j)$$

which has an approximate $\chi^2(h-p)$-distribution if the null hypothesis holds and T is the length of the residual series. The null hypothesis of no residual autocorrelation is rejected for large values of the test statistic.

1. Load Quarterly data for the price index of US Gross National Product given in gnpdeflator.txt by simply calling `load gnpdeflator.txt`. This is a chain-type price index with basis year 2005. The data is seasonally adjusted and spans from 1954.Q4 to 2007.Q4.
2. Compute the inflation series. That is, take the first difference of log(gnpdeflator).
3. Use the Akaike information criteria to determine the lag length $\hat{p}$.
4. Estimate two models: (i) an $AR(\hat{p})$ model and (ii) an $AR(1)$ model with OLS.
5. Set $h = \hat{p} + 10$ and compute Q_h as well as the corresponding p-value (`chi2pdf(`Q_h`,h-p)`) for both models.
6. Comment, based on your findings, whether the residuals are white noise.

3 Maximum Likelihood Estimation

Consider the AR(1) model with constant

$$y_t = c + \phi y_{t-1} + u_t$$

Assume that the error terms u_t are i.i.d. Laplace distributed with known density

$$f_{u_t}(u) = \frac{1}{2} \exp\left(-|u|\right)$$

Note that $E(u_t) = 0$ and $Var(u_t) = 2$.

1. Derive the log-likelihood function conditional on the first observation.
2. Write a MATLAB function that calculates the conditional log-likelihood of c and ϕ.
3. Load the dataset `LaPlace.txt` by running `load LaPlace.txt`.
4. Numerically find the maximum likelihood estimates of c and ϕ by minimizing the negative conditional log-likelihood function.
5. Compare your results with the maximum likelihood estimate under the assumption of Gaussianity. That is, redo the estimation by minimizing the negative Gaussian log-likelihood function.

4 Bootstrap Test Statistics

Consider the AR(1) model with constant

$$y_t = c + \phi y_{t-1} + u_t$$

for $t = 1, \ldots, T$ with i.i.d. error terms u_t and $E(u_t|y_{t-1}) = 0$. Usually, we construct a 95%-confidence interval for e.g. ϕ using the normal approximation

$$\left[\hat{\phi} - 1.96 \cdot SE(\hat{\phi});\ \hat{\phi} + 1.96 \cdot SE(\hat{\phi})\right]$$

with $\hat{\phi}$ denoting the OLS estimate and $SE(\hat{\phi})$ the estimated standard error of ϕ. If one does not know the asymptotic distribution of a test statistic (or it has a very complicated form), one often relies on a nonparametric approach. To this end, we are going to "bootstrap", i.e. recompute the t-statistics a large number of times on artificial data generated from resampled residuals. We will do this step-by-step, i.e. write a Matlab script for the following:

- Simulate $T = 100$ observations with $c = 1$, $\phi = 0.8$ and errors drawn from the exponential distribution using `u = exprnd(1,T,1)-1` such that $E(u_t) = 0$.
- Estimate the model with OLS and calculate the t-statistic $\tau = \frac{\hat{\phi}}{SE(\hat{\phi})}$.
- Store the OLS residuals in a vector $\hat{u} = (\hat{u}_2, \ldots, \hat{u}_T)'$.
- Set $B = 10000$ and initialize the output vector $\tau^* = (\tau_1^*, ..., \tau_B^*)$.
- For $b = 1, ..., B$:
  - Draw a sample **with replacement** from $\hat{u}$ and save it as $u^* = u_2^*, \ldots, u_T^*$.
    Hint: For sampling with replacement use either the `datasample` or `randi` function. If you don't have the necessary toolbox installed you can also use
    `ustar = uhat(ceil(size(uhat,1)*rand(T-1,1)),:)`.
  - Initialize an artificial time series y_t^* with T observations and set $y_1^* = y_1$.
  - For $t = 2, \ldots, T$ generate
    $$y_t^* = \hat{c} + \hat{\phi} y_{t-1}^* + u_t^*$$
  - Estimate an AR(1) model on this artificial dataset with OLS. Store the following t-statistic in your output vector at position b:
    $$\tau^\star = \frac{\phi^\star - \hat{\phi}}{SE(\phi^*)}$$
- Sort the output vector using `sort` such that $\tau_{(1)}^* \leq ... \leq \tau_{(B)}^*$.
- The bootstrap approximate confidence interval for ϕ is then
  $$\left[\hat{\phi} - \tau^*_{((1-\alpha/2)R)} \cdot SE(\hat{\phi});\ \hat{\phi} - \tau^*_{((\alpha/2)R)} \cdot SE(\hat{\phi})\right]$$
  Set $\alpha = 0.05$ and compare this with the normal approximation.
- Redo the exercise for $T = 30$ and $T = 10000$. Comment on your findings.