7 RBC model with leisure and investment-specific technological change

Consider the basic Real Business Cycle (RBC) model with leisure and investment-specific technological change. The representative household maximizes present as well as expected future utility

$$\max_{\{C_t, I_t, L_t, K_t\}} E_t \sum_{j=0}^{\infty} \beta^j U_{t+j}$$

with $\beta < 1$ denoting the discount factor and E_t is expectation given information at time t. The contemporaneous utility function

$$U_t = \gamma \ln(C_t) + (1-\gamma)\ln(1-L_t)$$

has two arguments: consumption C_t and labor L_t. The marginal utility of consumption is positive, whereas more labor reduces utility. Accordingly, γ is the elasticity of substitution between consumption and labor. In each period the household takes the real wage W_t as given and supplies perfectly elastic labor service to the representative firm. In return, she receives real labor income in the amount of $W_t L_t$ and, additionally, profits Π_t from the firm as well as revenue from lending capital K_{t-1} in the previous period at interest rate R_t to the firms, as it is assumed that the firm and capital stock are owned by the household. Income and wealth are used to finance consumption C_t and investment I_t. In total, this defines the (real) budget constraint of the household:

$$C_t + I_t = W_t L_t + R_t K_{t-1} + \Pi_t$$

The law of motion for capital K_t at the end of period t is given by

$$K_t = (1-\delta)K_{t-1} + Z_t I_t$$

where δ is the depreciation rate and Z_t investment-specific technological change. Assume that the transversality condition is full-filled.

The model includes two driving forces of the economy, a neutral technological progress A_t and a technological progress specific to investment Z_t. The laws of motion for these processes are given by:

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_t^A$$
$$\ln Z_t = \rho_Z \ln Z_{t-1} + \varepsilon_t^Z$$

where ρ_A and ρ_Z denote the persistence parameters and ε_t^A and ε_t^Z are assumed to be independently normally distributed with zero means and variances equal to σ_A^2 and σ_Z^2, respectively.

Real profits Π_t of the representative firm are revenues from selling output Y_t minus costs from labor $W_t L_t$ and renting capital $R_t K_{t-1}$:

$$\Pi_t = Y_t - W_t L_t - R_t K_{t-1}$$

The representative firm maximizes expected profits

$$\max_{\{L_t, K_{t-1}\}} E_t \sum_{j=0}^{\infty} \beta^j Q_{t+j} \Pi_{t+j}$$

subject to a Cobb-Douglas production function

$$Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha}$$

The discount factor takes into account that firms are owned by the household, i.e. $\beta^j Q_{t+j}$ is the present value of a unit of consumption in period $t+j$ or, respectively, the marginal utility of an additional unit of profit; therefore $Q_{t+j} = \frac{\partial U_{t+j}/\partial C_{t+j}}{\partial U_t/\partial C_t}$.

Finally, we have the non-negativity constraints $K_t \geq 0$, $C_t \geq 0$ and $0 \leq L_t \leq 1$ and clearing of the labor as well as goods market in equilibrium, i.e.

$$Y_t = C_t + I_t$$

1. Briefly provide intuition behind the introduction of investment-specific technological change.
2. Show that the first-order conditions of the agents are given by

$$\begin{aligned} E_t\left[\frac{C_{t+1}}{C_t}\right] &= \beta E_t\left[\frac{Z_t}{Z_{t+1}}\left(1-\delta+Z_{t+1}R_{t+1}\right)\right], \\ W_t &= \frac{1-\gamma}{\gamma}\frac{C_t}{1-L_t}, \end{aligned}$$

   Interpret these equations in economic terms.
3. Show that the first-order conditions of the representative firm are given by

$$\begin{aligned} W_t &= (1-\alpha)A_t\left(\frac{K_{t-1}}{L_t}\right)^{\alpha}, \\ R_t &= \alpha A_t\left(\frac{L_t}{K_{t-1}}\right)^{1-\alpha} \end{aligned}$$

   Interpret these equations in economic terms.
4. Discuss how to calibrate the parameters ρ_Z and σ_Z^2.
5. Write a DYNARE mod file for this model with a feasible calibration and compute the steady state of the model either analytically or numerically.
6. Study the effects of both a positive neutral productivity shock and a positive investment-specific productivity shock using an impulse response analysis. How would you design a short-run identification scheme for a SVAR model based on your DSGE model to disentangle both technological shocks? In other words, which variable(s) behave differently in the short-run?
7. Simulate data for investment and consumption growth for 200 periods. Estimate three parameters (of your choosing) with
   (i) maximum likelihood methods
   (ii) Bayesian methods

   Provide feasible upper and lower bounds and discuss the intuition behind your priors.
8. Explain whether or not you are satisfied with your estimation results?