Macroeconometrics

Exam No. 2 of 3

Winter 2017/2018

- Answer **all** of the following four exercises in either German or English.
- Hand in your solutions before Monday January, 8 2018 at 12:00.
- Please e-mail the solutions files to willi@mutschler.eu.
- The solution files should contain your executable (and commented) Matlab functions and script files as well as an additional documentation preferably as `pdf`, not `doc` or `docx`.
- I will confirm the receipt of your work also by email.
- All students must work on their own, please also give your student ID number.
- It is advised to regularly check the learnweb and your emails in case of urgent updates.
- If there are any questions, do not hesitate to contact Willi Mutschler.

1 Properties Of Lag-Order Selection Criteria

Assume that the true Data-Generating-Process (DGP) follows the following VAR(4) model

$$y_t = \begin{pmatrix} 2.4 & 1.0 \\ 0 & 1.1 \end{pmatrix} y_{t-1} + \begin{pmatrix} -2.15 & -0.9 \\ 0 & -0.41 \end{pmatrix} y_{t-2} + \begin{pmatrix} 0.852 & 0.2 \\ 0 & 0.06 \end{pmatrix} y_{t-3} + \begin{pmatrix} -0.126 & 0 \\ 0 & 0.0003 \end{pmatrix} y_{t-4} + u_t$$

where u_t is a Gaussian white noise with contemporary covariance matrix $\Sigma_u = \begin{pmatrix} 0.9 & 0.2 \\ 0.2 & 0.5 \end{pmatrix}$.

Perform a Monte-Carlo analysis to study both the finite-sample as well as asymptotic properties of the Akaike Information Criterion (AIC) and the Schwarz Information Criterion (SIC):

$$AIC(m) = \log(det(\tilde{\Sigma}_u(m))) + \frac{2}{T}\varphi(m)$$
$$SIC(m) = \log(det(\tilde{\Sigma}_u(m))) + \frac{\log T}{T}\varphi(m)$$

where $\tilde{\Sigma}_u = T^{-1}\sum_{t=1}^{T} \hat{u}_t\hat{u}_t'$ is the residual covariance matrix estimator for a reduced-form VAR model of order m based on OLS residuals $\hat{u}_t$. The function $\varphi(m)$ corresponds to the total number of regressors in the system of VAR equations. The VAR order is chosen such that the respective criterion is minimized over the possible orders $m = 0, ..., p^{max}$. To this end, do the following.

- Set the number of Monte Carlo repetitions $R = 100$ and $p^{max} = 8$.
- Initialize output matrices aic and sic each of dimension $R \times 5$.
- For $r = 1, ..., R$ do the following:
  - Simulate 10100 observations for the DGP given above and discard the first 100 observations as burn-in phase. Save the remaining 10000 observations in a matrix Y.
  - Compute the lag criteria for 5 different sample sizes $T = \{80, 160, 240, 500, 10000\}$, i.e. use the last T observations of your simulated data matrix Y for computations.
  - Save the chosen lag order in the corresponding output object at position $[r, j]$ where $j = 1, ..., 5$ indicates the corresponding sample size.
- Look at the frequency tables of your output objects for the different subsamples. Hint: `tabulate(aic(:,1))` displays a frequency table for the AIC criterion with sample size equal to 80.

Given your results, do you agree with the following (general) findings?

1. AIC is not consistent for the true lag order, whereas SIC is consistent.
2. AIC never (asymptotically) selects a log order that is lower than the true lag order.
3. In finite samples, we usually have $\hat{p}_{SIC} \leq \hat{p}_{AIC}$.
4. In finite samples, AIC has a tendency to overestimate the lag order, SIC has a tendency to underestimate the lag order; hence, one should rely on AIC in finite samples.

2 Posterior distribution of sign-identified structural IRFs

Consider data for $y_t = (\Delta gnp_t, \Delta p_t, i_t)'$, where gnp_t denotes the log of U.S. real GNP, p_t the consumer price index in logs, and i_t the federal funds rate. The sample period consists of 1970Q1 to 2011Q1. Data is given in the Excel sheet "MonPolData" in the file `data.xlsx`.

1. Estimate the parameters of a VAR(4) model with constant by using Bayesian methods, i.e. a Gibbs sampling method. To this end:

   - Assume a Minnesota prior for the VAR coefficients, where the prior mean should reflect the view that the VAR follows a random walk. Set the hyperparameters for the prior covariance matrix V_0 such that the tightness parameter on lags of own and of other variables are both equal to 0.5, and the tightness parameter on the constant term is equal to 100.
   - Assume an inverse Wishart prior for the covariance matrix with degrees of freedoms equal to the number of variables and the identity matrix as prior scale matrix.
   - Initialize the first draw of the covariance matrix with OLS values.
   - Draw in total 30100 times from the conditional posteriors, where you discard draws of the coefficient matrix that are unstable.
   - Save the last $n_G = 100$, draws (A^r, Σ_u^r) $(r = 1, ..., n_G)$ for inference on the structural model.

2. Estimate the posterior of the structural impulse response function by considering the following sign restrictions pattern on the impact matrix

$$B_0^{-1} = \begin{bmatrix} > & > & < \\ > & < & < \\ > & * & > \end{bmatrix}$$

   where $*$ denotes unrestricted values. That is, for each draw (A^r, Σ_u^r) from the posterior of the reduced-form VAR Parameters:

   - Compute the lower-triangular Cholesky decomposition $P^r = chol(\Sigma_u^r)$.
   - Compute a random draw of the rotation matrix Q.
   - For each combination (A^r, P^r, Q) compute the set of implied structural impulse responses $\Theta^r(h)$ with horizons $h = 0, ..., 30$.
   - If $\Theta^r(0)$ satisfies the sign restrictions on impact, store the value of $\Theta^r(h)$. Otherwise discard it.
   - Repeat these steps until you have $n_Q = 200$ accepted draws for each (A^r, Σ_u^r).

   Note that in the end you should have $n_Q \cdot n_G = 20000$ accepted draws from the posterior of structural impulse response functions.

3. Display the vector of point-wise posterior medians of the structural impulse responses, i.e. the median response function.

4. Interpret your results for one structural shock (of your choice) on (i) the level of real gnp, (ii) the consumer price index and (iii) on the federal funds rate.

5. Name two shortcomings of using the median response function as a measure of central tendency in sign-identified SVARs.

3 How Well Does the IS-LM Model Fit Postwar US Data?

Consider a quarterly model for $y_t = (\Delta gnp_t, \Delta i_t, i_t - \Delta p_t, \Delta m_t - \Delta p_t)'$, where gnp_t denotes the log of GNP, i_t the yield on three-month Treasury Bills, m_t the growth in M1 and p_t the inflation rate in the CPI. There are four shocks in the system: an aggregate supply (AS), a money supply (MS), a money demand (MD) and an aggregate demand (IS) shock. Ignoring the lagged dependent variables for expository purposes, the unrestricted structural VAR model can be written as

$$\begin{aligned}
\Delta gnp_t &= -b_{12}\Delta i_t - b_{13}(i_t - \Delta p_t) - b_{14}(\Delta m_t - \Delta p_t) + \varepsilon_t^{AS} \\
\Delta i_t &= -b_{21}\Delta gnp_t - b_{23}(i_t - \Delta p_t) - b_{24}(\Delta m_t - \Delta p_t) + \varepsilon_t^{MS} \\
i_t - \Delta p_t &= -b_{31}\Delta gnp_t - b_{32}\Delta i_t - b_{34}(\Delta m_t - \Delta p_t) + \varepsilon_t^{MD} \\
\Delta m_t - \Delta p_t &= -b_{41}\Delta gnp_t - b_{42}\Delta i_t - b_{43}(i_t - \Delta p_t) + \varepsilon_t^{IS}
\end{aligned}$$

where b_{ij} denotes the ijth element of B_0^{-1}. Consider the following identification restrictions:

- Money supply shocks do not have contemporaneous effects on output growth, i.e.

$$\frac{\partial \Delta gnp_t}{\partial \varepsilon_t^{MS}} = 0$$

- Money demand shocks do not have contemporaneous effects on output growth, i.e.

$$\frac{\partial \Delta gnp_t}{\partial \varepsilon_t^{MD}} = 0$$

- Monetary authority does not react contemporaneously to changes in the price level, i.e.

$$\frac{\partial \Delta i_t}{\partial \Delta p_t} = 0$$

- Money supply shocks, money demand shocks and aggregate demand shocks do not have long-run effect on the log of real GNP, i.e. on the cumulated impulse response:

$$\frac{\partial gnp_t}{\partial \varepsilon_t^{MS}} = 0, \qquad \frac{\partial gnp_t}{\partial \varepsilon_t^{MD}} = 0, \qquad \frac{\partial gnp_t}{\partial \varepsilon_t^{IS}} = 0$$

- The structural shocks are assumed to have unit variance.

Solve the following exercises:

1. Derive the implied exclusion restrictions on the matrices B_0^{-1} and $\Theta(1)$.
2. Consider the data given in the excel file `gali1992.xlsx`. Estimate a VAR(4) model with constant term.
3. Estimate the structural impact matrix using a nonlinear equation solver, i.e. the objective is to find the unknown elements of B_0^{-1} such that

$$\begin{bmatrix} vech(B_0^{-1}B_0^{-1'} - \hat{\Sigma}_u) \\ \text{short-run restrictions on } B_0^{-1} \\ \text{long-run restrictions on } \Theta(1) \end{bmatrix}$$

   is minimized where the normalization $\Sigma_\varepsilon = I_3$ is imposed. Furthermore, normalize the shocks such that the diagonal elements of B_0^{-1} are positive.
4. Use the implied estimate of B_0^{-1} to plot the structural impulse response functions for (i) real GNP, (ii) the yield on Treasury Bills, (iii) the real interest rate and (iv) real money growth.

4 Inference In SVARs Identified By Exclusion Restrictions

Consider an exactly-identified structural VAR model subject to short- and/or long-run restrictions, where the structural impulse response of variable i to shock j at horizon h are simply denoted as $\theta \equiv \Theta_{ij,h}$. As an exact closed-form solution for the asymptotic standard errors of θ are only available under restrictive assumptions, we will rely on a numerical approximation.

1. Reconsider an exercise (of your choice) from the lecture on SVAR models identified with exclusion restrictions and estimate the structural impulse response function.

2. Write a Matlab function `bootstd(VAR,opt)` with the structure `VAR` from the reduced-form estimation step and corresponding options `opt` as inputs. The function computes $\widehat{std}(\hat{\theta}^*)$ via a bootstrap approximation, that is:

   - Set bootstrap repetitions B equal to 1000 (or higher) and initialize a $K \times K \times H \times B$ array `THETAstar`, where the first dimension corresponds to variable $i = 1, .., K$, the second dimension to shock $j = 1, ..., K$, the third dimension to the horizon of the IRFs $h = 0, .., H$ and the fourth dimension to the bootstrap repetition $b = 1., ..., B$.
   - For $b = 1, ..., B$ do the following (you may also try `parfor` instead of `for` in order to make use of Matlab's parallel computing toolbox):
     - Compute a bootstrap GDP y_t^b using the function `BootstrapGDP(VAR)` which implements a standard residual-based bootstrap approach using sampling with replacement techniques on the residuals. Furthermore, the initial values are randomly drawn in blocks.
     - Estimate the reduced-form and structural impulse response function on this artificial dataset with the same methodology, settings and identification restrictions as in the estimation on the original dataset.
     - Store the structural IRFs in `THETAstar` at position `(:,:,:,b)`.
   - Compute the standard deviation of the bootstrap structural IRFs using `std(THETAstart,0,4)` and use this as the output of your function `bootstd(VAR,opt)`.

3. Plot approximate 95% confidence intervals for the structural impulse response functions according to the delta method:

$$\hat{\theta} \pm z_{\gamma/2}\widehat{std}(\hat{\theta}^*)$$

   where $z_{\gamma/2}$ is the $\gamma/2$ quantile of the standard normal distribution.