5 RBC model with leisure and habit formation

Consider the basic Real Business Cycle (RBC) model with leisure and habit formation. The representative household maximizes present as well as expected future utility

$$\max_{\{C_t, I_t, L_t, K_t\}} E_t \sum_{j=0}^{\infty} \beta^j U_{t+j}$$

with $\beta < 1$ denoting the discount factor and E_t is expectation given information at time t. The contemporaneous utility function

$$U_t = \gamma \ln(C_t - \phi C_{t-1}) + (1-\gamma) \ln(1 - L_t)$$

has three arguments: consumption C_t and labor L_t of the current period and consumption C_{t-1} of the previous period. The marginal utility of consumption is positive, whereas more labor reduces utility. Accordingly, γ is the elasticity of substitution between consumption and labor and ϕ a coefficient of persistence in habits. In each period the household takes the real wage W_t as given and supplies perfectly elastic labor service to the representative firm. In return, she receives real labor income in the amount of $W_t L_t$ and, additionally, profits Π_t from the firm as well as revenue from lending capital in the previous period K_{t-1} at interest rate R_t to the firms, as it is assumed that the firm and capital stock are owned by the household. Income and wealth are used to finance consumption C_t and investment I_t. In total, this defines the (real) budget constraint of the household:

$$C_t + I_t = W_t L_t + R_t K_{t-1} + \Pi_t$$

The law of motion for capital K_t at the end of period t is given by

$$K_t = (1-\delta) K_{t-1} + I_t$$

where δ is the depreciation rate. Assume that the transversality condition is full-filled.

Productivity A_t is the driving force of the economy and evolves according to

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_t^A$$

where ρ_A denotes the persistence parameter and ε_t^A is assumed to be normally distributed with mean zero and variance σ_A^2.

Real profits Π_t of the representative firm are revenues from selling output Y_t minus costs from labor $W_t L_t$ and renting capital $R_t K_{t-1}$:

$$\Pi_t = Y_t - W_t L_t - R_t K_{t-1}$$

The representative firm maximizes expected profits

$$\max_{\{L_t, K_{t-1}\}} E_t \sum_{j=0}^{\infty} \beta^j Q_{t+j} \Pi_{t+j}$$

subject to a Cobb-Douglas production function

$$Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha}$$

The discount factor takes into account that firms are owned by the household, i.e. $\beta^j Q_{t+j}$ is the present value of a unit of consumption in period $t+j$ or, respectively, the marginal utility of an additional unit of profit; therefore $Q_{t+j} = \frac{\partial U_{t+j}/\partial C_{t+j}}{\partial U_t / \partial C_t}$.

Finally, we have the non-negativity constraints $K_t \geq 0$, $C_t \geq 0$ and $0 \leq L_t \leq 1$ and clearing of the labor as well as goods market in equilibrium, i.e.

$$Y_t = C_t + I_t$$

1. Briefly provide intuition behind the consumption habit formation behavior.

2. Show that the first-order conditions of the representative household are given by

$$E_t\left[\frac{\frac{1}{C_t-\phi C_{t-1}}-\beta\phi\frac{1}{C_{t+1}-\phi C_t}}{\frac{1}{C_{t+1}-\phi C_t}-\beta\phi\frac{1}{C_{t+2}-\phi C_{t+1}}}\right]=E_t\left[\beta(R_{t+1}+1-\delta)\right]$$

$$E_t\left[\gamma\frac{1}{C_t-\phi C_{t-1}}-\beta\gamma\phi\frac{1}{C_{t+1}-\phi C_t}\right]W_t=(1-\gamma)\frac{1}{1-L_t}$$

   Interpret these equations in economic terms.

3. Show that the first-order conditions of the representative firm are given by

$$W_t=(1-\alpha)A_t\left(\frac{K_{t-1}}{L_t}\right)^{\alpha}$$

$$R_t=\alpha A_t\left(\frac{L_t}{K_{t-1}}\right)^{1-\alpha}$$

   Interpret these equations in economic terms.

4. Discuss how to calibrate the parameter ϕ.

5. Write a DYNARE mod file for this model with a feasible calibration and compute the steady state of the model either analytically or numerically.

6. Study the effects of a positive aggregate productivity shock using an impulse response analysis. What are the main differences relative to the same shock in the basic RBC model without consumption habit, i.e. $\phi = 0$?

7. Simulate data for consumption growth for 200 periods. Estimate three parameters (of your choosing) with

   (i) maximum likelihood methods

   (ii) Bayesian methods

   Provide feasible upper and lower bounds and discuss the intuition behind your priors.

8. Explain whether or not you are satisfied with your estimation results?