书面作业2.3 参考解答或提示

第1部分 基础

无

第2部分 理论（T5，T6选做）

T1. 教材P97. 题17 (2)(4)（即证明由原推理的前提与结论构造的蕴含式重言）.

题17(2) $((P\vee\neg R)\wedge(Q\vee S)\wedge(R\rightarrow(S\wedge P))\rightarrow(S\rightarrow P)=\ldots=R\vee\neg S\vee P$, 显然，该蕴含式非重言式，故该推理无效.

题17(4) $((\neg P\rightarrow Q)\wedge(Q\rightarrow R)\wedge(R\rightarrow P))\rightarrow(P\vee Q\vee R)=\ldots=\neg(P\vee Q)\vee(P\vee Q)\vee R=1$, 故该推理有效.

T2. 教材P97. 题18 (1)(5).

题18(1) 证明:

(1) $C\vee D$ P
(2) $(C\vee D)\rightarrow\neg P$ P
(3) $\neg P$ T,I(1)(2)
(4) $\neg P\rightarrow(A\wedge\neg B)$ P
(5) $(A\wedge\neg B)$ T,I(3)(4)
(6) $(A\wedge\neg B)\rightarrow(R\vee S)$ P
(7) $R\vee S$ T,I(5)(6)

题18(5) 证明:

(1) $Q\rightarrow(R\rightarrow S)$ P (附加前提)
(2) $R\rightarrow(Q\rightarrow S)$ RE(1)
(3) $P\rightarrow(Q\rightarrow R)$ P
(4) P P(附加前提)
(5) $Q\rightarrow R$ T,I(3)(4)
(6) Q P(附加前提)
(7) R T,I(5)(6)
(8) $Q\rightarrow S$ T,I(2)(7)
(9) $P\rightarrow(Q\rightarrow S)$ CP(4)(8)
(10) $(Q\rightarrow(R\rightarrow S))\rightarrow(P\rightarrow(Q\rightarrow S))$ CP(1)(9)

T3. 用CP规则证明:

$A\rightarrow(B\rightarrow C)$，$(C\wedge D)\rightarrow E$，$\neg F\rightarrow(D\wedge\neg E)\Rightarrow A\rightarrow(B\rightarrow F)$.

证明:

1) A P（附加）
2) $A\rightarrow(B\rightarrow C)$ P
3) $B\rightarrow C$ T,I,(1),(2)
4) $(C\wedge D)\rightarrow E$ P
5) $C\rightarrow(D\rightarrow E)$ R,E,(4)
6) $B\rightarrow(D\rightarrow E)$ T,I,(3),(5)

7） $\neg F \to (D \wedge \neg E)$　P

8） $(D \to E) \to F$　R,E,(7)

9） $B \to F$　T,I,(6),(8)

10） $A \to (B \to F)$　CP

T4. 用反证法证明:

$(A \to B) \wedge (C \to D)$，$(B \to E) \wedge (D \to F)$，$\neg(E \wedge F)$，$A \to C \Rightarrow \neg A$.

证明：1） $\neg(\neg A)$　P（附加）

2） A　R,E,(1)

3） $A \to C$　P

4） C　T,I,(2),(3)

5） $(A \to B) \wedge (C \to D)$　P

6） $A \to B$　T,I,(5)

7） B　T,I,(2),(6)

8） $C \to D$　T,I,(5)

9） D　T,I,(4),(8)

10） $(B \to E) \wedge (D \to F)$　P

11） $B \to E$　T,I,(10)

12） E　T,I,(7),(11)

13） $D \to F$　T,I,(10)

14） F　T,I,(9),(13)

15） $\neg E \wedge F)$　P

16） $E \to \neg F$　R,E,(15)

17） $\neg F$　T,I,(12),(16)

18） $F \wedge \neg F$　T,I,(14),(17),矛盾

19) $\neg A$　反证法（IP）

T5. Prove that the following rule, called the Destructive Dilemma rule(破坏性二难), can be derived from the original and derived proof rules.

Premises: $\neg C \vee \neg D$，$A \to C$，$B \to D$

Conclusion: $\neg A \vee \neg B$.

Textbook: (假言易位) $A \to C \equiv \neg C \to \neg A$, $B \to D \equiv \neg D \to \neg B$, ...

(1) $A \to C$　P

(2) $\neg C \to \neg A$　RE(1)

(3) $B \to D$　P

(4) $\neg D \to \neg B$　RE(3)

(5) $\neg C \vee \neg D$　P

(6) $\neg A \vee \neg B$　TI(2)(4)(5) 构造性二难

QED（表示证明结束）.

或：结论转化为蕴含式，利用 CP 来证明，

(1) $A \to C$ P
(2) $B \to D$ P
(3) $\neg C \vee \neg D$ P
(4) A P(附加, for $A \to \neg B$)
(5) C TI(1)(4)
(6) $\neg\neg C$ RE(5)
(7) $\neg D$ TI(3)(6)
(8) $\neg B$ TI(2)(7)
(9) $A \to \neg B$ CP(4)(8)
(10) $\neg A \vee \neg B$ RE(9)
QED.

T6. Two students came up with the following different wffs to formalize the statement(命题) "If A then B else C."

$(A \wedge B) \vee (\neg A \wedge C)$.

$(A \to B) \wedge (\neg A \to C)$.

Prove that the two wffs are equivalent by finding formal proofs for the following two statements.

a. $((A \wedge B) \vee (\neg A \wedge C)) \to ((A \to B) \wedge (\neg A \to C))$.

b. $((A \to B) \wedge (\neg A \to C)) \to ((A \wedge B) \vee (\neg A \wedge C))$.

真值表方法（略）
推理方法：变换+CP+ IP（间接证明）？

a.（含两个蕴含式，两次 CP 规则）
(1) A P(附加, for $A \to B$)
(2) $\neg\neg A$ RE(1)
(3) $\neg\neg A \vee \neg C$ TI(2)
(4) $\neg(\neg A \wedge C)$ RE(3)
(5) $(A \wedge B) \vee (\neg A \wedge C)$ P(附加, for $((A \wedge B) \vee (\neg A \wedge C)) \to ((A \to B) \wedge (\neg A \to C))$)
(6) $A \wedge B$ TI(4)(5)
(7) B TI(6)
(8) $A \to B$ CP(1)-(7)
(9) $\neg A$ P(附加, for $\neg A \to C$)
(10) $\neg A \vee \neg B$ TI(9)
(11) $\neg(A \wedge B)$ RE(10)
(12) $\neg A \wedge C$ TI(5)(11)
(13) C TI(12)
(14) $\neg A \to C$ CP(9)-(13)
(15) $(A \to B) \wedge (\neg A \to C)$ TP(9)(14)
(16) $((A \wedge B) \vee (\neg A \wedge C)) \to ((A \to B) \wedge (\neg A \to C))$ CP(5)(15)
QED.

a.（等值变换，直接证明前件为真，后件亦为真）
(1) $(A \wedge B) \vee (\neg A \wedge C)$ P(附加, for $((A \wedge B) \vee (\neg A \wedge C)) \to ((A \to B) \wedge (\neg A \to C))$)

(2) $(A \vee \neg A) \wedge (A \vee C) \wedge (B \vee \neg A) \wedge (B \vee C)$ RE(1)
(3) $(\neg A \vee B) \wedge (A \vee C) \wedge (B \vee C)$ RE(2)
(4) $(\neg A \vee B) \wedge (A \vee C)$ TI(3)
(5) $(A \to B) \wedge (\neg A \to C)$ RE(4)
(6) $((A \wedge B) \vee (\neg A \wedge C)) \to ((A \to B) \wedge (\neg A \to C))$ CP(1)(5)
QED.

b.（直接证明方法）

(1) $(A \to B) \wedge (\neg A \to C)$ P(附加, for $((A \wedge B) \vee (\neg A \wedge C)) \to ((A \to B) \wedge (\neg A \to C))$)
(2) $(\neg B \to \neg A) \wedge (\neg A \to C)$ RE(1)
(3) $(\neg B \to C)$ TI(2)
(4) $(B \vee C)$ RE(3)
(5) $(\neg A \vee B) \wedge (A \vee C)$ RE(1)
(6) $(A \vee \neg A)$ TI(排中律)
(7) $(A \vee \neg A) \wedge (\neg A \vee B) \wedge (A \vee C) \wedge (B \vee C)$ TI(4)(5)(6)
(8) $(A \wedge B) \vee (\neg A \wedge C)$ RE(7)
(9) $(A \to B) \wedge (\neg A \to C) \to (A \wedge B) \vee (\neg A \wedge C)$ CP(1)(9)
QED.

也可以用构造性二难推得到（4）.
对于比较复杂的结论，就本题看，上述直接证明并非最佳方法. 结论为析取式时，还可以考虑反证法.

a.（IP 方法）

(1) $\neg((A \wedge B) \vee (\neg A \wedge C))$ P(附加,for $((A \to B) \wedge (\neg A \to C)) \to ((A \wedge B) \vee (\neg A \wedge C))$)
(2) $(\neg A \vee \neg B) \wedge (A \vee \neg C)$ RE(1)
(3) $(B \to \neg A) \wedge (C \to A)$ RE(2)
(4) $B \to \neg A$ TI(3)
(5) $C \to A$ TI(3)
(6) $(A \to B) \wedge (\neg A \to C)$ P
(7) $A \to B$ TI(6)
(8) $\neg A \to C$ TI(6)
(9) $A \to \neg A$ TI(4)(7)
(10) $\neg A$ RE(9)
(11) $\neg A \to A$ TI(5)(8)
(12) A RE(11)
(13) $\neg A \wedge A$ TI(10)(12) 矛盾
(14) $((A \to B) \wedge (\neg A \to C)) \to ((A \wedge B) \vee (\neg A \wedge C))$ IP(1)-(13)
QED.

第3部分 应用（T2选做）

T1. 教材P98. 题19 (2)(3)(5).

题19(2) 设P:红队第三;Q:黄队第二 R:蓝队第四;S:白队第一,则命题可符号化为:

$P \to (Q \to R), \neg S \vee P, Q \Rightarrow S \to R$

(1) S P (附加前提)
(2) $\neg S \vee P$ P
(3) P T,I(1)(2)

(4) $P \rightarrow (Q \rightarrow R)$ P
(5) $Q \rightarrow R$ T,I(3)(4)
(6) Q P
(7) R T,I(5)(6)
(8) $S \rightarrow R$ CP(1)(7)

题19(3) 设 P:6是偶数; Q:2整除7; R:5是素数, 则该命题可符号化为:

$P \rightarrow \neg Q$, $\neg R \vee Q$, $R \Rightarrow \neg P$

(1) $\neg(\neg P)$ P (附加前提)
(2) P T,I(1)
(3) $P \rightarrow \neg Q$ P
(4) $\neg Q$ T,I(2)(3)
(5) $\neg R \vee Q$ P
(6) $\neg R$ T,I(4)(5)
(7) R P
(8) $\neg R \wedge R$ T,I(6)(7)矛盾

题19(5) 设 P:今天是星期二; Q:要考计算机科学; R:我要考经济学; S:经济学教授病了,则命题可符号化为:

$P \rightarrow (Q \vee R)$, $S \rightarrow \neg R$, $P \wedge S \Rightarrow Q$

(1) $P \wedge S$ P
(2) S T,I(1)
(3) $S \rightarrow \neg R$ P
(4) $\neg R$ T,I(2)(3)
(5) $P \rightarrow (Q \vee R)$ P
(6) P T,I(1)
(7) $Q \vee R$ T,I(5)(6)
(8) Q T,I(4)(7)

T2. Consider the following argument that aims to prove that Superman does not exist.

If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil he would be impotent(虚弱无能); if he were unwilling to prevent evil he would be malevolent（邪恶）;Superman does not prevent evil; If superman exists he is neither malevolent nor impotent. Therefore Superman does not exist.

First, letters are employed to represent the propositions as follows:

a: Superman is able to prevent evil
w: Superman is willing to prevent evil
i: Superman is impotent
m: Superman is malevolent
p: Superman prevents evil
e: Superman exists

Then, the argument above is formalized in propositional logic as follows:

Premises	
P_1	$(a \wedge w) \to p$
P_2	$(\neg a \to i) \wedge (\neg w \to m)$
P_3	$\neg p$
P_4	$e \to \neg i \wedge \neg m$
Conclusion	$P_1 \wedge P_2 \wedge P_3 \wedge P_4 \Rightarrow \neg e$

直接证明（利用构造性二难推理）

(1) a∧w→p	P
(2) ¬p	P
(3) ¬(a ∧ w)	TI(1)(2)
(4) ¬a ∨ ¬w	RE(3)
(5) (¬a → i) ∧ (¬w → m)	P
(6) i ∨ m	TI(4)(5) 构造性二难
(7) ¬(¬i ∧ ¬m)	RE(7)
(8) e → ¬i ∧ ¬m	P
(9) ¬e	TI(10)

QED.

直接证明（利用二难推理）

(1) a∧w→p	P
(2) ¬p	P
(3) ¬(a ∧ w)	TI(1)(2)
(4) ¬a ∨ ¬w	RE(3)
(5) (¬a → i) ∧ (¬w → m)	P
(6) ¬a → i	TI(5)
(7) ¬a	P(附加，for ¬a → i∨m)
(8) i	TI(6)(7)
(9) i ∨ m	TI(8)
(10) ¬a → i∨m	CP(7)(9) 注：(7)-(10)为一个子证明
(11) ¬w → m	TI(5)
(12) ¬w	P(附加，for ¬w → i∨m)
(13) m	TI(11)(12)
(14) i ∨ m	TI(13)

(15) ¬w → i∨m	CP(12)(15) 注：(12)-(14)为一个子证明
(16) ¬a ∨ ¬w → i∨m	TI(10)(15) 二难推理
(17) i ∨ m	TI(4)(16)
(18) ¬(¬i ∧ ¬m)	RE(17)
(19) e → ¬i ∧ ¬m	P
(20) ¬e	TI(18)(19)

QED.

反证法

(1) e	P(附加)
(2) e →¬i ∧ ¬m	P
(3) ¬i ∧ ¬m	TI(1)(2)
(4) ¬i	TI(3)
(5) ¬m	TI(3)
(6) (¬a→i) ∧ (¬w→m)	P
(7) ¬a→i	TI(6)
(8) ¬w→m	TI(6)
(9) a	TI(4)(7)
(10) w	TI(5)(8)
(11) a∧w	TI(9)(10)
(12) a∧w→p	P
(13) p	TI(11)(12)
(14) ¬p	P
(15) p∧¬p	TI(13)(14), 矛盾
QED.	IP(1)-(15)

下面证明过程仅供参考，其给出依据中 P_n 指使用第 n 个条件，给出了具体的依据，利用了一些教材未给出的规则. 同学们可以尝试把证明依据按照教材方式简写或补充.

Proof that Superman does not exist

1.	$a \wedge w \to p$	*Premise* 1
2.	$(\neg a \to i) \wedge (\neg w \to m)$	*Premise* 2
3.	$\neg p$	*Premise* 3
4.	$e \to (\neg i \wedge \neg m)$	*Premise* 4
5.	$\neg p \to \neg(a \wedge w)$	1, *Contrapositive*
6.	$\neg(a \wedge w)$	*3, 5 Modus Ponens*
7.	$\neg a \vee \neg w$	*6, De Morgan's Law*
8.	$\neg(\neg i \wedge \neg m) \to \neg e$	*4, Contrapositive*
9.	$i \vee m \to \neg e$	*8, De Morgan's Law*
10.	$(\neg a \to i)$	2, $\wedge$ *Elimination*
11.	$(\neg w \to m)$	2, $\wedge$ *Elimination*
12.	$\neg\neg a \vee i$	10, $A \to B$ equivalent to $\neg A \vee B$
13.	$\neg\neg a \vee i \vee m$	11, $\vee$ Introduction
14.	$\neg\neg a \vee (i \vee m)$	
15.	$\neg a \to (i \vee m)$	14, $A \to B$ equivalent to $\neg A \vee B$
16.	$\neg\neg w \vee m$	11, $A \to B$ equivalent to $\neg A \vee B$
17.	$\neg\neg w \vee (i \vee m)$	
18.	$\neg w \to (i \vee m)$	17, $A \to B$ equivalent to $\neg A \vee B$
19.	$(i \vee m)$	7, 15, 18 $\vee$*Elimination*
20.	$\neg e$	*9, 19 Modus Ponens*

Second Proof

1.	$\neg p$	P_3
2.	$\neg(a \wedge w) \vee p$	P_1 $(A \to B \equiv \neg A \vee B)$
3.	$\neg(a \wedge w)$	1, 2 $A \vee B, \neg B \vdash A$
4.	$\neg a \vee \neg w$	3, De Morgan's Law
5.	$(\neg a \to i)$	P_2 ($\wedge$-Elimination)
6.	$\neg a \to i \vee m$	5, $x \to y \vdash x \to y \vee z$
7.	$(\neg w \to m)$	P_2 ($\wedge$-Elimination)
8.	$\neg w \to i \vee m$	7, $x \to y \vdash x \to y \vee z$
9.	$(\neg a \vee \neg w) \to (i \vee m)$	8, $x \to z, y \to z \vdash x \vee y \to z$
10.	$(i \vee m)$	4, 9 Modus Ponens
11.	$e \to \neg(i \vee m)$	P_4 (De Morgan's Law)
12.	$\neg e \vee \neg(i \vee m)$	11, $(A \to B \equiv \neg A \vee B)$
13.	$\neg e$	10, 12 $A \vee B, \neg B \vdash A$

Therefore, the conclusion that Superman does not exist is a valid deduction from the given premises.