Lab10

AUTHOR
Chenxi Liu 1010615050

PUBLISHED
March 20, 2024

Child mortality in Sri Lanka

In this lab you will be fitting a couple of different models to the data about child mortality in Sri Lanka, which was used in the lecture. Here's the data and the plot from the lecture:

```
library(tidyverse)
```

```
── Attaching core tidyverse packages ──────────────────────── tidyverse 2.0.0 ──
✔ dplyr     1.1.3     ✔ readr     2.1.4
✔ forcats   1.0.0     ✔ stringr   1.5.0
✔ ggplot2   3.4.3     ✔ tibble    3.2.1
✔ lubridate 1.9.3     ✔ tidyr     1.3.0
✔ purrr     1.0.2
── Conflicts ────────────────────────────────────────── tidyverse_conflicts() ──
✖ dplyr::filter() masks stats::filter()
✖ dplyr::lag()    masks stats::lag()
ℹ Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts
to become errors
```

```
library(here)
```

```
here() starts at /Users/dawn/Desktop/uoft/sta2201/HW
```

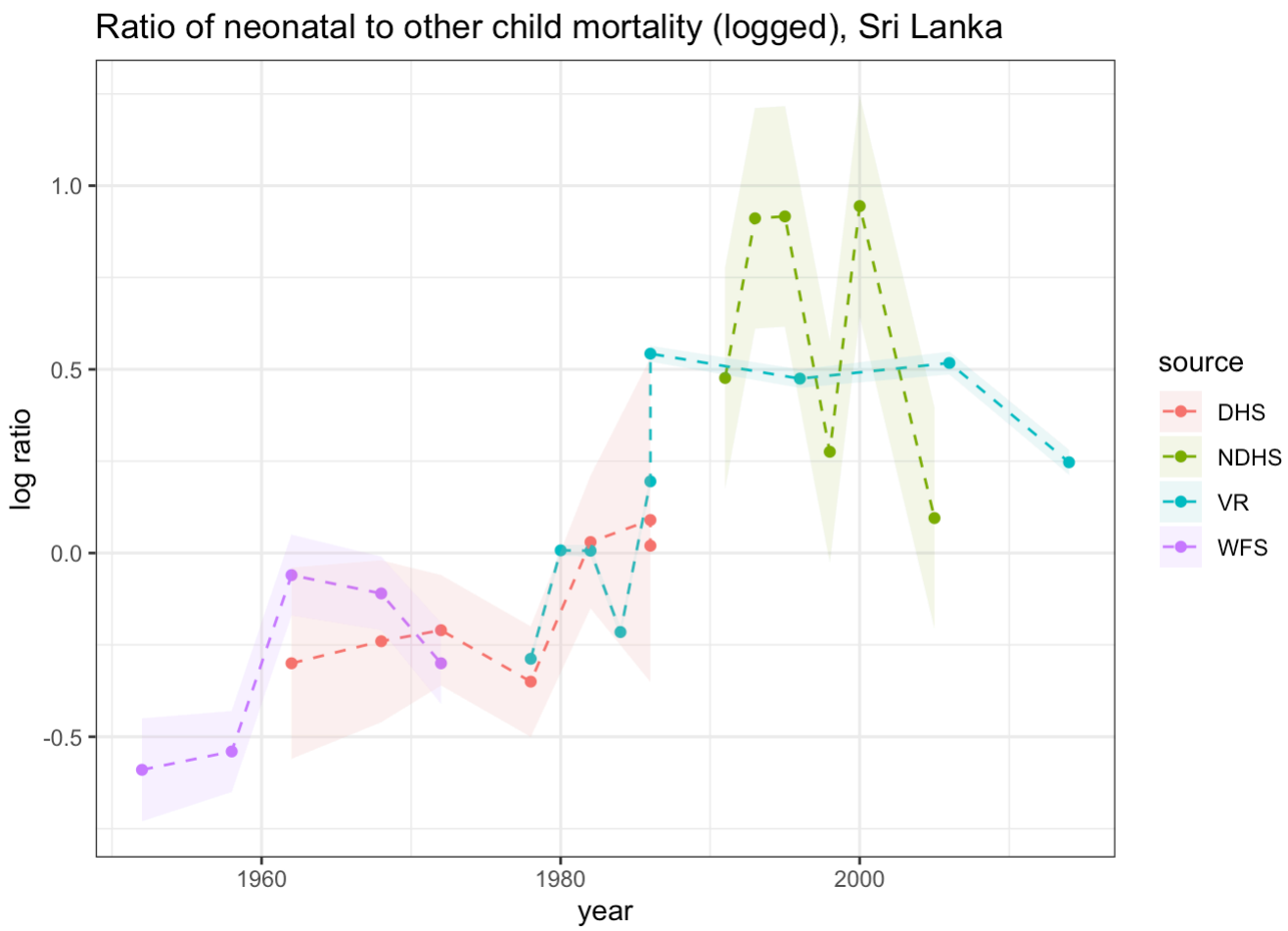
```
library(rstan)
```

```
Loading required package: StanHeaders

rstan version 2.32.5 (Stan version 2.32.2)

For execution on a local, multicore CPU with excess RAM we recommend calling
options(mc.cores = parallel::detectCores()).
To avoid recompilation of unchanged Stan programs, we recommend calling
rstan_options(auto_write = TRUE)
For within-chain threading using `reduce_sum()` or `map_rect()` Stan functions,
change `threads_per_chain` option:
rstan_options(threads_per_chain = 1)


Attaching package: 'rstan'

The following object is masked from 'package:tidyr':

    extract
```

```
library(tidybayes)

library(readr)
lka <- read_csv("/Users/dawn/Desktop/uoft/sta2201/HW/lka.csv")
```

```
Rows: 27 Columns: 7
── Column specification ────────────────────────────────────────────────────────
Delimiter: ","
chr (3): country_name, country_code, source
dbl (4): year, logit_ratio, se, ratio

ℹ Use `spec()` to retrieve the full column specification for this data.
ℹ Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```
ggplot(lka, aes(year, logit_ratio)) +
  geom_point(aes( color = source)) +
  geom_line(aes( color = source), lty = 2) +
  geom_ribbon(aes(ymin = logit_ratio - se,
                  ymax = logit_ratio + se,
                  fill =  source), alpha = 0.1) +
  theme_bw()+
  labs(title = "Ratio of neonatal to other child mortality (logged), Sri Lanka", y = "
```



Fitting a linear model

Let's firstly fit a linear model in time to these data. Here's the code to do this:

```
observed_years <- lka$year
years <- min(observed_years):max(observed_years)
nyears <- length(years)

stan_data <- list(y = lka$logit_ratio, year_i = observed_years - years[1]+1,
                  T = nyears, years = years, N = length(observed_years),
                  mid_year = mean(years), se = lka$se)

mod <- stan(data = stan_data,
            file = "/Users/dawn/Desktop/uoft/sta2201/HW/lka_linear_me.stan")
```

```
Trying to compile a simple C file

Running /Library/Frameworks/R.framework/Resources/bin/R CMD SHLIB foo.c
using C compiler: 'Apple clang version 14.0.3 (clang-1403.0.22.14.1)'
using SDK: 'MacOSX13.3.sdk'
clang -arch arm64 -I"/Library/Frameworks/R.framework/Resources/include" -DNDEBUG   -
I"/Library/Frameworks/R.framework/Versions/4.3-arm64/Resources/library/Rcpp/include/"
-I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/"  -
I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/unsupported"  -
I"/Library/Frameworks/R.framework/Versions/4.3-arm64/Resources/library/BH/include" -
I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/src/"  -
I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/"  -
I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppParallel/include/"  -
I"/Library/Frameworks/R.framework/Versions/4.3-arm64/Resources/library/rstan/include"
-DEIGEN_NO_DEBUG  -DBOOST_DISABLE_ASSERTS  -DBOOST_PENDING_INTEGER_LOG2_HPP  -
DSTAN_THREADS  -DUSE_STANC3 -DSTRICT_R_HEADERS  -DBOOST_PHOENIX_NO_VARIADIC_EXPRESSION
-D_HAS_AUTO_PTR_ETC=0  -include '/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/stan/math/prim/fun/Eigen.hpp'  -
D_REENTRANT -DRCPP_PARALLEL_USE_TBB=1   -I/opt/R/arm64/include    -fPIC  -falign-
functions=64 -Wall -g -O2  -c foo.c -o foo.o
In file included from <built-in>:1:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/stan/math/prim/fun/Eigen.hpp:22:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/Dense:1:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/Core:88:
/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:1: error:
unknown type name 'namespace'
namespace Eigen {
^
/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:16: error:
expected ';' after top level declarator
```

```
namespace Eigen {
              ^
              ;
In file included from <built-in>:1:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/stan/math/prim/fun/Eigen.hpp:22:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/Dense:1:
/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/Core:96:10: fatal error: 'complex'
file not found
#include <complex>
         ^~~~~~~~~
3 errors generated.
make: *** [foo.o] Error 1

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
Chain 1:
Chain 1: Gradient evaluation took 2.1e-05 seconds
Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.21
seconds.
Chain 1: Adjust your expectations accordingly!
Chain 1:
Chain 1:
Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
Chain 1:
Chain 1:  Elapsed Time: 0.019 seconds (Warm-up)
Chain 1:                0.015 seconds (Sampling)
Chain 1:                0.034 seconds (Total)
Chain 1:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
Chain 2:
Chain 2: Gradient evaluation took 3e-06 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.03
seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
```

```
Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
Chain 2:
Chain 2:  Elapsed Time: 0.02 seconds (Warm-up)
Chain 2:                0.017 seconds (Sampling)
Chain 2:                0.037 seconds (Total)
Chain 2:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
Chain 3:
Chain 3: Gradient evaluation took 3e-06 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.03
seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
Chain 3:
Chain 3:  Elapsed Time: 0.02 seconds (Warm-up)
Chain 3:                0.018 seconds (Sampling)
Chain 3:                0.038 seconds (Total)
Chain 3:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
Chain 4:
Chain 4: Gradient evaluation took 2e-06 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.02
seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4:
Chain 4:
Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
```

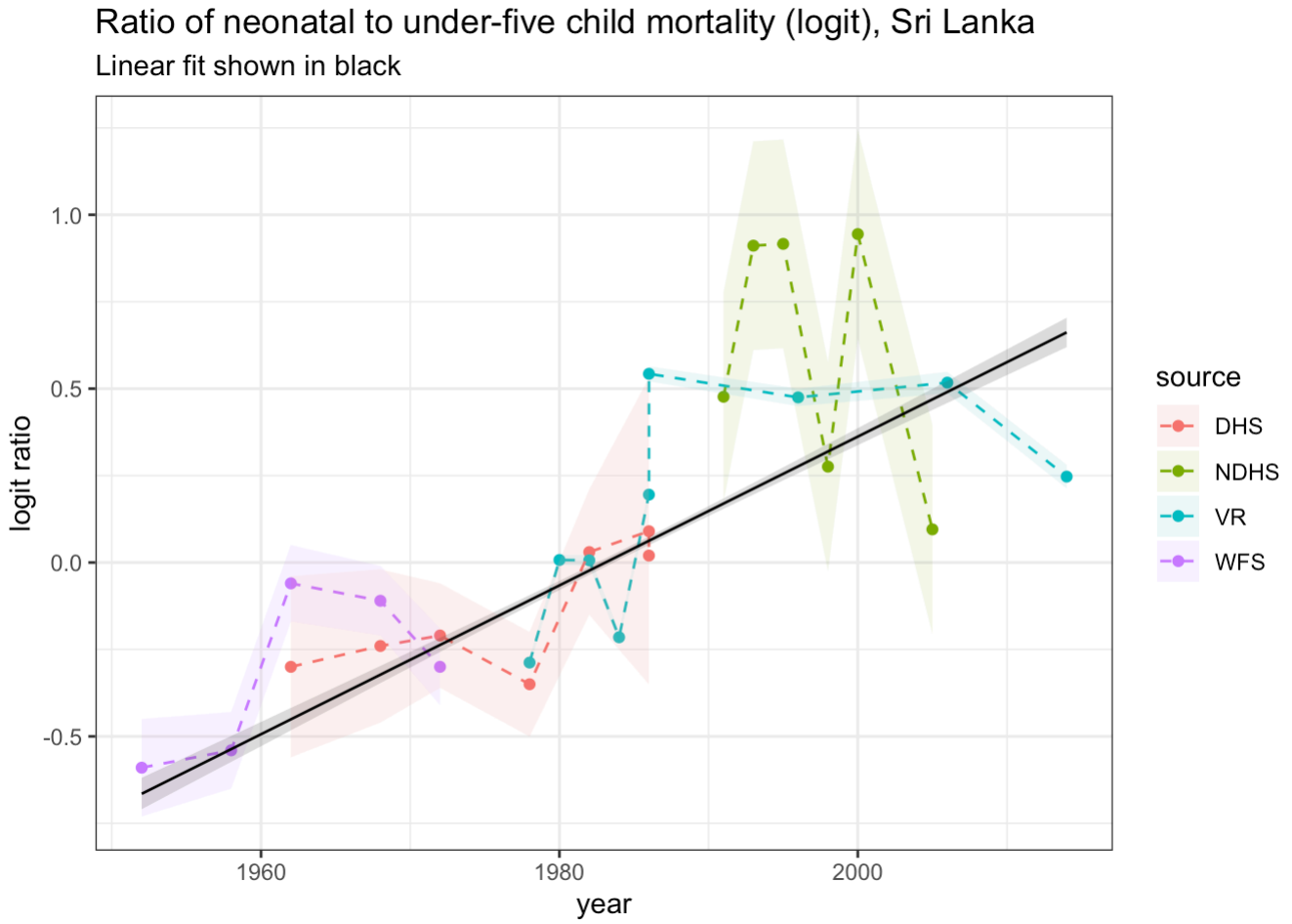
```
Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
Chain 4:
Chain 4:  Elapsed Time: 0.022 seconds (Warm-up)
Chain 4:                0.017 seconds (Sampling)
Chain 4:                0.039 seconds (Total)
Chain 4:
```

Extract the results:

```
res <- mod %>%
  gather_draws(mu[t]) %>%
  median_qi() %>%
  mutate(year = years[t])
```

Plot the results:

```
ggplot(lka, aes(year, logit_ratio)) +
  geom_point(aes( color = source)) +
  geom_line(aes( color = source), lty = 2) +
  geom_ribbon(aes(ymin = logit_ratio - se,
                  ymax = logit_ratio + se,
                  fill =  source), alpha = 0.1) +
  theme_bw()+
  geom_line(data = res, aes(year, .value)) +
  geom_ribbon(data = res, aes(y = .value, ymin = .lower, ymax = .upper), alpha = 0.2)+
  theme_bw()+
  labs(title = "Ratio of neonatal to under-five child mortality (logit), Sri Lanka",
       y = "logit ratio", subtitle = "Linear fit shown in black")
```



Question 1

Project the linear model above out to 2022 by adding a `generated quantities` block in Stan (do the projections based on the expected value μ). Plot the resulting projections on a graph similar to that above.

```
model_code <- "
data {
  int<lower=0> N; // number of observations
  int<lower=0> T; //number of years
  int<lower=0> mid_year; // mid-year of study
  vector[N] y; //log ratio
  vector[N] se; // standard error around observations
  vector[T] years; // unique years of study
  int<lower=0> year_i[N]; // year index of observations

}

parameters {
  real alpha;
  real beta;

}

transformed parameters{
  vector[T] mu;
```

```
  for(t in 1:T){
    mu[t] = alpha + beta*(years[t] - mid_year);
  }
}

model {

  y ~ normal(mu[year_i], se);

  alpha ~ normal(0, 1);
  beta ~ normal(0,1);
}

generated quantities {
  real mu_2022;
  mu_2022 = alpha + beta * (2022 - mid_year);
}
"
```

```
stan_data <- list(y = lka$logit_ratio, year_i = observed_years - years[1]+1,
                  T = nyears, years = years, N = length(observed_years),
                  mid_year = mean(years), se = lka$se)

# Fit the model
fit <- stan(
  model_code  = model_code,  # your Stan model file
  data = stan_data,
  iter = 1000,
  chains = 4
)
```

```
Trying to compile a simple C file

Running /Library/Frameworks/R.framework/Resources/bin/R CMD SHLIB foo.c
using C compiler: ‘Apple clang version 14.0.3 (clang-1403.0.22.14.1)’
using SDK: ‘MacOSX13.3.sdk’
clang -arch arm64 -I"/Library/Frameworks/R.framework/Resources/include" -DNDEBUG   -
I"/Library/Frameworks/R.framework/Versions/4.3-arm64/Resources/library/Rcpp/include/"
-I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/"  -
I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/unsupported"  -
I"/Library/Frameworks/R.framework/Versions/4.3-arm64/Resources/library/BH/include" -
I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/src/"  -
I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/"  -
I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppParallel/include/"  -
I"/Library/Frameworks/R.framework/Versions/4.3-arm64/Resources/library/rstan/include"
-DEIGEN_NO_DEBUG  -DBOOST_DISABLE_ASSERTS  -DBOOST_PENDING_INTEGER_LOG2_HPP  -
DSTAN_THREADS  -DUSE_STANC3 -DSTRICT_R_HEADERS  -DBOOST_PHOENIX_NO_VARIADIC_EXPRESSION
```

```
-D_HAS_AUTO_PTR_ETC=0  -include '/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/stan/math/prim/fun/Eigen.hpp'  -
D_REENTRANT -DRCPP_PARALLEL_USE_TBB=1   -I/opt/R/arm64/include    -fPIC  -falign-
functions=64 -Wall -g -O2  -c foo.c -o foo.o
In file included from <built-in>:1:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/stan/math/prim/fun/Eigen.hpp:22:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/Dense:1:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/Core:88:
/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:1: error:
unknown type name 'namespace'
namespace Eigen {
^
/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:16: error:
expected ';' after top level declarator
namespace Eigen {
               ^
               ;
In file included from <built-in>:1:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/stan/math/prim/fun/Eigen.hpp:22:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/Dense:1:
/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/Core:96:10: fatal error: 'complex'
file not found
#include <complex>
         ^~~~~~~~~
3 errors generated.
make: *** [foo.o] Error 1

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
Chain 1:
Chain 1: Gradient evaluation took 1.7e-05 seconds
Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.17
seconds.
Chain 1: Adjust your expectations accordingly!
Chain 1:
Chain 1:
Chain 1: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 1: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 1: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 1: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 1: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 1: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 1: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 1: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 1: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 1: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 1: Iteration: 900 / 1000 [ 90%]  (Sampling)
```

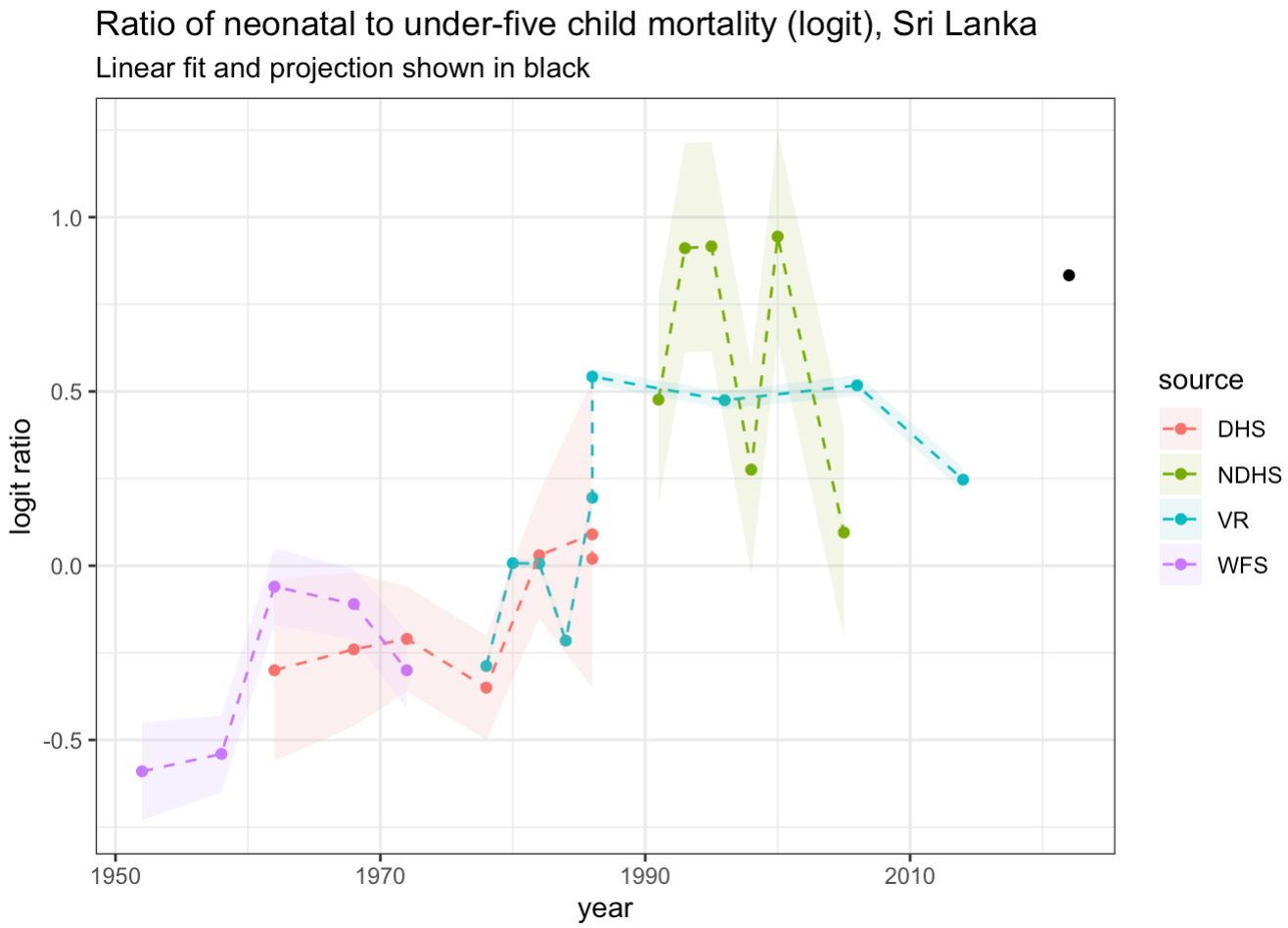
```
Chain 1: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 1:
Chain 1:  Elapsed Time: 0.012 seconds (Warm-up)
Chain 1:                0.01 seconds (Sampling)
Chain 1:                0.022 seconds (Total)
Chain 1:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
Chain 2:
Chain 2: Gradient evaluation took 1e-06 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.01
seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 2: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 2: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 2: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 2: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 2: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 2: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 2: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 2: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 2: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 2: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 2: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 2:
Chain 2:  Elapsed Time: 0.011 seconds (Warm-up)
Chain 2:                0.01 seconds (Sampling)
Chain 2:                0.021 seconds (Total)
Chain 2:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
Chain 3:
Chain 3: Gradient evaluation took 1e-06 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.01
seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 3: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 3: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 3: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 3: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 3: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 3: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 3: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 3: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 3: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 3: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 3: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 3:
```

```
Chain 3:  Elapsed Time: 0.011 seconds (Warm-up)
Chain 3:                0.01 seconds (Sampling)
Chain 3:                0.021 seconds (Total)
Chain 3:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
Chain 4:
Chain 4: Gradient evaluation took 2e-06 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.02
seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4:
Chain 4:
Chain 4: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 4: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 4: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 4: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 4: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 4: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 4: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 4: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 4: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 4: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 4: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 4: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 4:
Chain 4:  Elapsed Time: 0.012 seconds (Warm-up)
Chain 4:                0.009 seconds (Sampling)
Chain 4:                0.021 seconds (Total)
Chain 4:
```

```r
res <- fit %>%
  gather_draws(mu[t]) %>%
  median_qi() %>%
  mutate(year = years[t])

mu_2022 <- extract(fit)$mu_2022
lka_projection <- data.frame(year = 2022, logit_ratio = median(mu_2022))

# Plot the existing data
p <- ggplot(lka, aes(year, logit_ratio)) +
  geom_point(aes(color = source)) +
  geom_line(aes(color = source), lty = 2) +
  geom_ribbon(aes(ymin = logit_ratio - se, ymax = logit_ratio + se, fill = source), al
  theme_bw() +
  labs(title = "Ratio of neonatal to under-five child mortality (logit), Sri Lanka",
       y = "logit ratio", subtitle = "Linear fit and projection shown in black")

# Add the projection for 2022
p <- p + geom_point(data = lka_projection, aes(year, logit_ratio), color = "black") +
  geom_line(data = lka_projection, aes(year, logit_ratio), color = "black", linetype =
```

```
# Print the plot
print(p)
```

```
`geom_line()`: Each group consists of only one observation.
ℹ Do you need to adjust the group aesthetic?
```



Question 2

The projections above are for the logit of the ratio of neonatal to under-five child mortality. You can download estimates of the under-five child mortality from 1951 to 2022 here: https://childmortality.org/all-cause-mortality/data/estimates?refArea=LKA. Use these data to get estimates and projections of neonatal mortality for Sri Lanka, and plot the results.

```
df <- read_csv("/Users/dawn/Desktop/uoft/sta2201/HW/lab10.csv")
```

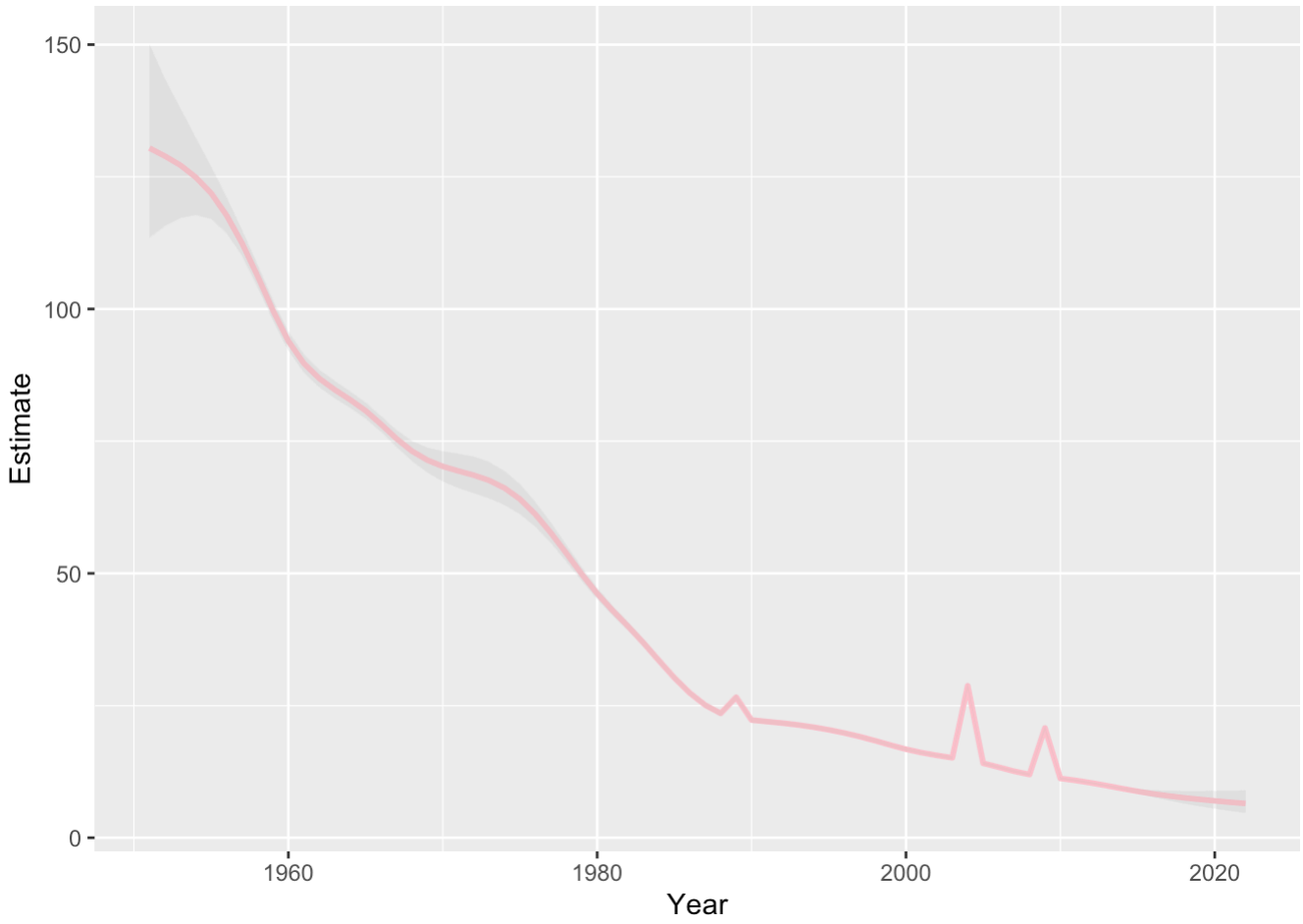
```
Rows: 72 Columns: 4
── Column specification ────────────────────────────────────────────────────────
Delimiter: ","
dbl (4): Year, Estimate, Lower bound, Upper bound

ℹ Use `spec()` to retrieve the full column specification for this data.
ℹ Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```
head(df)
```

```
# A tibble: 6 × 4
   Year Estimate `Lower bound` `Upper bound`
  <dbl>    <dbl>         <dbl>         <dbl>
1  2022     6.49          4.70          8.96
2  2021     6.72          5.08          8.91
3  2020     6.98          5.50          8.87
4  2019     7.27          5.97          8.82
5  2018     7.57          6.49          8.83
6  2017     7.92          7.07          8.89
```

```
df$Year <- as.character(df$Year)
df$Year <- ifelse(grepl("^[0-9]+$", df$Year), as.numeric(df$Year), NA)

# Create the plot
ggplot(data = df, aes(x = Year)) +
  geom_line(aes(y = Estimate), color = "pink", size = 1) +
  geom_ribbon(aes(ymin = `Lower bound`, ymax = `Upper bound`), fill = "gray", alpha =
```

```
Warning: Using `size` aesthetic for lines was deprecated in ggplot2 3.4.0.
ℹ Please use `linewidth` instead.
```




```
  theme_minimal() +
  labs(title = "Neonatal Mortality Estimates and Projections for Sri Lanka",
x = "Year",
       y = "Neonatal Mortality Rate (per 1000 live births)") +
  theme(plot.title = element_text(hjust = 0.5))
```

```
List of 99
 $ line                      :List of 6
  ..$ colour       : chr "black"
  ..$ linewidth    : num 0.5
  ..$ linetype     : num 1
  ..$ lineend      : chr "butt"
  ..$ arrow        : logi FALSE
  ..$ inherit.blank: logi TRUE
  ..- attr(*, "class")= chr [1:2] "element_line" "element"
 $ rect                      :List of 5
  ..$ fill         : chr "white"
  ..$ colour       : chr "black"
  ..$ linewidth    : num 0.5
  ..$ linetype     : num 1
  ..$ inherit.blank: logi TRUE
  ..- attr(*, "class")= chr [1:2] "element_rect" "element"
 $ text                      :List of 11
  ..$ family       : chr ""
  ..$ face         : chr "plain"
  ..$ colour       : chr "black"
  ..$ size         : num 11
  ..$ hjust        : num 0.5
  ..$ vjust        : num 0.5
  ..$ angle        : num 0
  ..$ lineheight   : num 0.9
  ..$ margin       : 'margin' num [1:4] 0points 0points 0points 0points
  .. ..- attr(*, "unit")= int 8
  ..$ debug        : logi FALSE
  ..$ inherit.blank: logi TRUE
  ..- attr(*, "class")= chr [1:2] "element_text" "element"
 $ title                     : chr "Neonatal Mortality Estimates and Projections for
Sri Lanka"
 $ aspect.ratio              : NULL
 $ axis.title                : NULL
 $ axis.title.x              :List of 11
  ..$ family       : NULL
  ..$ face         : NULL
  ..$ colour       : NULL
  ..$ size         : NULL
  ..$ hjust        : NULL
  ..$ vjust        : num 1
  ..$ angle        : NULL
  ..$ lineheight   : NULL
  ..$ margin       : 'margin' num [1:4] 2.75points 0points 0points 0points
  .. ..- attr(*, "unit")= int 8
  ..$ debug        : NULL
  ..$ inherit.blank: logi TRUE
  ..- attr(*, "class")= chr [1:2] "element_text" "element"
 $ axis.title.x.top          :List of 11
  ..$ family       : NULL
  ..$ face         : NULL
  ..$ colour       : NULL
  ..$ size         : NULL
  ..$ hjust        : NULL
```

```
 ..$ vjust        : num 0
 ..$ angle        : NULL
 ..$ lineheight   : NULL
 ..$ margin       : 'margin' num [1:4] 0points 0points 2.75points 0points
 .. ..- attr(*, "unit")= int 8
 ..$ debug        : NULL
 ..$ inherit.blank: logi TRUE
 ..- attr(*, "class")= chr [1:2] "element_text" "element"
$ axis.title.x.bottom       : NULL
$ axis.title.y              :List of 11
 ..$ family       : NULL
 ..$ face         : NULL
 ..$ colour       : NULL
 ..$ size         : NULL
 ..$ hjust        : NULL
 ..$ vjust        : num 1
 ..$ angle        : num 90
 ..$ lineheight   : NULL
 ..$ margin       : 'margin' num [1:4] 0points 2.75points 0points 0points
 .. ..- attr(*, "unit")= int 8
 ..$ debug        : NULL
 ..$ inherit.blank: logi TRUE
 ..- attr(*, "class")= chr [1:2] "element_text" "element"
$ axis.title.y.left         : NULL
$ axis.title.y.right        :List of 11
 ..$ family       : NULL
 ..$ face         : NULL
 ..$ colour       : NULL
 ..$ size         : NULL
 ..$ hjust        : NULL
 ..$ vjust        : num 0
 ..$ angle        : num -90
 ..$ lineheight   : NULL
 ..$ margin       : 'margin' num [1:4] 0points 0points 0points 2.75points
 .. ..- attr(*, "unit")= int 8
 ..$ debug        : NULL
 ..$ inherit.blank: logi TRUE
 ..- attr(*, "class")= chr [1:2] "element_text" "element"
$ axis.text                 :List of 11
 ..$ family       : NULL
 ..$ face         : NULL
 ..$ colour       : chr "grey30"
 ..$ size         : 'rel' num 0.8
 ..$ hjust        : NULL
 ..$ vjust        : NULL
 ..$ angle        : NULL
 ..$ lineheight   : NULL
 ..$ margin       : NULL
 ..$ debug        : NULL
 ..$ inherit.blank: logi TRUE
 ..- attr(*, "class")= chr [1:2] "element_text" "element"
$ axis.text.x               :List of 11
 ..$ family       : NULL
 ..$ face         : NULL
```

```
 ..$ colour       : NULL
 ..$ size         : NULL
 ..$ hjust        : NULL
 ..$ vjust        : num 1
 ..$ angle        : NULL
 ..$ lineheight   : NULL
 ..$ margin       : 'margin' num [1:4] 2.2points 0points 0points 0points
 .. ..- attr(*, "unit")= int 8
 ..$ debug        : NULL
 ..$ inherit.blank: logi TRUE
 ..- attr(*, "class")= chr [1:2] "element_text" "element"
$ axis.text.x.top           :List of 11
 ..$ family       : NULL
 ..$ face         : NULL
 ..$ colour       : NULL
 ..$ size         : NULL
 ..$ hjust        : NULL
 ..$ vjust        : num 0
 ..$ angle        : NULL
 ..$ lineheight   : NULL
 ..$ margin       : 'margin' num [1:4] 0points 0points 2.2points 0points
 .. ..- attr(*, "unit")= int 8
 ..$ debug        : NULL
 ..$ inherit.blank: logi TRUE
 ..- attr(*, "class")= chr [1:2] "element_text" "element"
$ axis.text.x.bottom        : NULL
$ axis.text.y               :List of 11
 ..$ family       : NULL
 ..$ face         : NULL
 ..$ colour       : NULL
 ..$ size         : NULL
 ..$ hjust        : num 1
 ..$ vjust        : NULL
 ..$ angle        : NULL
 ..$ lineheight   : NULL
 ..$ margin       : 'margin' num [1:4] 0points 2.2points 0points 0points
 .. ..- attr(*, "unit")= int 8
 ..$ debug        : NULL
 ..$ inherit.blank: logi TRUE
 ..- attr(*, "class")= chr [1:2] "element_text" "element"
$ axis.text.y.left          : NULL
$ axis.text.y.right         :List of 11
 ..$ family       : NULL
 ..$ face         : NULL
 ..$ colour       : NULL
 ..$ size         : NULL
 ..$ hjust        : num 0
 ..$ vjust        : NULL
 ..$ angle        : NULL
 ..$ lineheight   : NULL
 ..$ margin       : 'margin' num [1:4] 0points 0points 0points 2.2points
 .. ..- attr(*, "unit")= int 8
 ..$ debug        : NULL
 ..$ inherit.blank: logi TRUE
```

```
  ..- attr(*, "class")= chr [1:2] "element_text" "element"
 $ axis.ticks               : list()
  ..- attr(*, "class")= chr [1:2] "element_blank" "element"
 $ axis.ticks.x             : NULL
 $ axis.ticks.x.top         : NULL
 $ axis.ticks.x.bottom      : NULL
 $ axis.ticks.y             : NULL
 $ axis.ticks.y.left        : NULL
 $ axis.ticks.y.right       : NULL
 $ axis.ticks.length        : 'simpleUnit' num 2.75points
  ..- attr(*, "unit")= int 8
 $ axis.ticks.length.x      : NULL
 $ axis.ticks.length.x.top  : NULL
 $ axis.ticks.length.x.bottom: NULL
 $ axis.ticks.length.y      : NULL
 $ axis.ticks.length.y.left  : NULL
 $ axis.ticks.length.y.right : NULL
 $ axis.line                : list()
  ..- attr(*, "class")= chr [1:2] "element_blank" "element"
 $ axis.line.x              : NULL
 $ axis.line.x.top          : NULL
 $ axis.line.x.bottom       : NULL
 $ axis.line.y              : NULL
 $ axis.line.y.left         : NULL
 $ axis.line.y.right        : NULL
 $ legend.background        : list()
  ..- attr(*, "class")= chr [1:2] "element_blank" "element"
 $ legend.margin            : 'margin' num [1:4] 5.5points 5.5points 5.5points
5.5points
  ..- attr(*, "unit")= int 8
 $ legend.spacing           : 'simpleUnit' num 11points
  ..- attr(*, "unit")= int 8
 $ legend.spacing.x         : NULL
 $ legend.spacing.y         : NULL
 $ legend.key               : list()
  ..- attr(*, "class")= chr [1:2] "element_blank" "element"
 $ legend.key.size          : 'simpleUnit' num 1.2lines
  ..- attr(*, "unit")= int 3
 $ legend.key.height        : NULL
 $ legend.key.width         : NULL
 $ legend.text              :List of 11
  ..$ family      : NULL
  ..$ face        : NULL
  ..$ colour      : NULL
  ..$ size        : 'rel' num 0.8
  ..$ hjust       : NULL
  ..$ vjust       : NULL
  ..$ angle       : NULL
  ..$ lineheight  : NULL
  ..$ margin      : NULL
  ..$ debug       : NULL
  ..$ inherit.blank: logi TRUE
  ..- attr(*, "class")= chr [1:2] "element_text" "element"
 $ legend.text.align        : NULL
```

```
 $ legend.title              :List of 11
  ..$ family       : NULL
  ..$ face         : NULL
  ..$ colour       : NULL
  ..$ size         : NULL
  ..$ hjust        : num 0
  ..$ vjust        : NULL
  ..$ angle        : NULL
  ..$ lineheight   : NULL
  ..$ margin       : NULL
  ..$ debug        : NULL
  ..$ inherit.blank: logi TRUE
  ..- attr(*, "class")= chr [1:2] "element_text" "element"
 $ legend.title.align        : NULL
 $ legend.position           : chr "right"
 $ legend.direction          : NULL
 $ legend.justification      : chr "center"
 $ legend.box                : NULL
 $ legend.box.just           : NULL
 $ legend.box.margin         : 'margin' num [1:4] 0cm 0cm 0cm 0cm
  ..- attr(*, "unit")= int 1
 $ legend.box.background     : list()
  ..- attr(*, "class")= chr [1:2] "element_blank" "element"
 $ legend.box.spacing        : 'simpleUnit' num 11points
  ..- attr(*, "unit")= int 8
 $ panel.background          : list()
  ..- attr(*, "class")= chr [1:2] "element_blank" "element"
 $ panel.border              : list()
  ..- attr(*, "class")= chr [1:2] "element_blank" "element"
 $ panel.spacing             : 'simpleUnit' num 5.5points
  ..- attr(*, "unit")= int 8
 $ panel.spacing.x           : NULL
 $ panel.spacing.y           : NULL
 $ panel.grid                :List of 6
  ..$ colour       : chr "grey92"
  ..$ linewidth    : NULL
  ..$ linetype     : NULL
  ..$ lineend      : NULL
  ..$ arrow        : logi FALSE
  ..$ inherit.blank: logi TRUE
  ..- attr(*, "class")= chr [1:2] "element_line" "element"
 $ panel.grid.major          : NULL
 $ panel.grid.minor          :List of 6
  ..$ colour       : NULL
  ..$ linewidth    : 'rel' num 0.5
  ..$ linetype     : NULL
  ..$ lineend      : NULL
  ..$ arrow        : logi FALSE
  ..$ inherit.blank: logi TRUE
  ..- attr(*, "class")= chr [1:2] "element_line" "element"
 $ panel.grid.major.x        : NULL
 $ panel.grid.major.y        : NULL
 $ panel.grid.minor.x        : NULL
 $ panel.grid.minor.y        : NULL
```

```
 $ panel.ontop               : logi FALSE
 $ plot.background           : list()
  ..- attr(*, "class")= chr [1:2] "element_blank" "element"
 $ plot.title                :List of 11
  ..$ family       : NULL
  ..$ face         : NULL
  ..$ colour       : NULL
  ..$ size         : 'rel' num 1.2
  ..$ hjust        : num 0.5
  ..$ vjust        : num 1
  ..$ angle        : NULL
  ..$ lineheight   : NULL
  ..$ margin       : 'margin' num [1:4] 0points 0points 5.5points 0points
  .. ..- attr(*, "unit")= int 8
  ..$ debug        : NULL
  ..$ inherit.blank: logi FALSE
  ..- attr(*, "class")= chr [1:2] "element_text" "element"
 $ plot.title.position       : chr "panel"
 $ plot.subtitle             :List of 11
  ..$ family       : NULL
  ..$ face         : NULL
  ..$ colour       : NULL
  ..$ size         : NULL
  ..$ hjust        : num 0
  ..$ vjust        : num 1
  ..$ angle        : NULL
  ..$ lineheight   : NULL
  ..$ margin       : 'margin' num [1:4] 0points 0points 5.5points 0points
  .. ..- attr(*, "unit")= int 8
  ..$ debug        : NULL
  ..$ inherit.blank: logi TRUE
  ..- attr(*, "class")= chr [1:2] "element_text" "element"
 $ plot.caption              :List of 11
  ..$ family       : NULL
  ..$ face         : NULL
  ..$ colour       : NULL
  ..$ size         : 'rel' num 0.8
  ..$ hjust        : num 1
  ..$ vjust        : num 1
  ..$ angle        : NULL
  ..$ lineheight   : NULL
  ..$ margin       : 'margin' num [1:4] 5.5points 0points 0points 0points
  .. ..- attr(*, "unit")= int 8
  ..$ debug        : NULL
  ..$ inherit.blank: logi TRUE
  ..- attr(*, "class")= chr [1:2] "element_text" "element"
 $ plot.caption.position     : chr "panel"
 $ plot.tag                  :List of 11
  ..$ family       : NULL
  ..$ face         : NULL
  ..$ colour       : NULL
  ..$ size         : 'rel' num 1.2
  ..$ hjust        : num 0.5
  ..$ vjust        : num 0.5
```

```
  ..$ angle        : NULL
  ..$ lineheight   : NULL
  ..$ margin       : NULL
  ..$ debug        : NULL
  ..$ inherit.blank: logi TRUE
  ..- attr(*, "class")= chr [1:2] "element_text" "element"
 $ plot.tag.position         : chr "topleft"
 $ plot.margin               : 'margin' num [1:4] 5.5points 5.5points 5.5points
5.5points
  ..- attr(*, "unit")= int 8
 $ strip.background          : list()
  ..- attr(*, "class")= chr [1:2] "element_blank" "element"
 $ strip.background.x        : NULL
 $ strip.background.y        : NULL
 $ strip.clip                : chr "inherit"
 $ strip.placement           : chr "inside"
 $ strip.text                :List of 11
  ..$ family       : NULL
  ..$ face         : NULL
  ..$ colour       : chr "grey10"
  ..$ size         : 'rel' num 0.8
  ..$ hjust        : NULL
  ..$ vjust        : NULL
  ..$ angle        : NULL
  ..$ lineheight   : NULL
  ..$ margin       : 'margin' num [1:4] 4.4points 4.4points 4.4points 4.4points
  .. ..- attr(*, "unit")= int 8
  ..$ debug        : NULL
  ..$ inherit.blank: logi TRUE
  ..- attr(*, "class")= chr [1:2] "element_text" "element"
 $ strip.text.x              : NULL
 $ strip.text.x.bottom       : NULL
 $ strip.text.x.top          : NULL
 $ strip.text.y              :List of 11
  ..$ family       : NULL
  ..$ face         : NULL
  ..$ colour       : NULL
  ..$ size         : NULL
  ..$ hjust        : NULL
  ..$ vjust        : NULL
  ..$ angle        : num -90
  ..$ lineheight   : NULL
  ..$ margin       : NULL
  ..$ debug        : NULL
  ..$ inherit.blank: logi TRUE
  ..- attr(*, "class")= chr [1:2] "element_text" "element"
 $ strip.text.y.left         :List of 11
  ..$ family       : NULL
  ..$ face         : NULL
  ..$ colour       : NULL
  ..$ size         : NULL
  ..$ hjust        : NULL
  ..$ vjust        : NULL
  ..$ angle        : num 90
```

```
 ..$ lineheight   : NULL
 ..$ margin       : NULL
 ..$ debug        : NULL
 ..$ inherit.blank: logi TRUE
 ..- attr(*, "class")= chr [1:2] "element_text" "element"
$ strip.text.y.right        : NULL
$ strip.switch.pad.grid     : 'simpleUnit' num 2.75points
 ..- attr(*, "unit")= int 8
$ strip.switch.pad.wrap     : 'simpleUnit' num 2.75points
 ..- attr(*, "unit")= int 8
$ x                         : chr "Year"
$ y                         : chr "Neonatal Mortality Rate (per 1000 live births)"
- attr(*, "class")= chr [1:2] "theme" "gg"
- attr(*, "complete")= logi TRUE
- attr(*, "validate")= logi TRUE
```

Random walks

Question 3

Code up and estimate a first order random walk model to fit to the Sri Lankan data, taking into account measurement error, and project out to 2022.

```
model3 <- "
  data {
  int<lower=0> N;          // Number of observations
  vector[N] logit_ratio;  // Observed data (logit ratios)
  vector<lower=0>[N] se;  // Standard errors of observations
  int<lower=0> T;          // Number of years to project beyond the last observed year
  int<lower=0> last_year; // The last observed year
}

parameters {
  vector[N] true_logit;  // True underlying logit ratios
  real<lower=0> sigma;   // Standard deviation of the random walk
}

model {
  true_logit[1] ~ normal(0, 10); // Weakly informative prior for the first logit ratio

  for (n in 2:N) {
    true_logit[n] ~ normal(true_logit[n-1], sigma); // Random walk
  }

  logit_ratio ~ normal(true_logit, se); // Likelihood with measurement error
}

generated quantities {
  vector[N+T] projected_logit;
  projected_logit[1:N] = true_logit;
```

```
  for (t in 1:T) {
    projected_logit[N+t] = normal_rng(projected_logit[N+t-1], sigma);
  }
}

"
```

```
# Prepare data for Stan
stan_data <- list(
  N = nrow(lka),
  logit_ratio = lka$logit_ratio,
  se = lka$se,
  T = 2022 - max(lka$year),
  last_year = max(lka$year)
)

# Fit the model
fit <- stan(model_code = model3, data = stan_data, iter = 1000, chains = 4)
```

```
Trying to compile a simple C file

Running /Library/Frameworks/R.framework/Resources/bin/R CMD SHLIB foo.c
using C compiler: ‘Apple clang version 14.0.3 (clang-1403.0.22.14.1)’
using SDK: ‘MacOSX13.3.sdk’
clang -arch arm64 -I"/Library/Frameworks/R.framework/Resources/include" -DNDEBUG   -
I"/Library/Frameworks/R.framework/Versions/4.3-arm64/Resources/library/Rcpp/include/"
-I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/"  -
I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/unsupported"  -
I"/Library/Frameworks/R.framework/Versions/4.3-arm64/Resources/library/BH/include" -
I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/src/"  -
I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/"  -
I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppParallel/include/"  -
I"/Library/Frameworks/R.framework/Versions/4.3-arm64/Resources/library/rstan/include"
-DEIGEN_NO_DEBUG  -DBOOST_DISABLE_ASSERTS  -DBOOST_PENDING_INTEGER_LOG2_HPP  -
DSTAN_THREADS  -DUSE_STANC3 -DSTRICT_R_HEADERS  -DBOOST_PHOENIX_NO_VARIADIC_EXPRESSION
-D_HAS_AUTO_PTR_ETC=0  -include '/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/stan/math/prim/fun/Eigen.hpp'  -
D_REENTRANT -DRCPP_PARALLEL_USE_TBB=1   -I/opt/R/arm64/include    -fPIC  -falign-
functions=64 -Wall -g -O2  -c foo.c -o foo.o
In file included from <built-in>:1:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/stan/math/prim/fun/Eigen.hpp:22:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/Dense:1:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/Core:88:
/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:1: error:
```

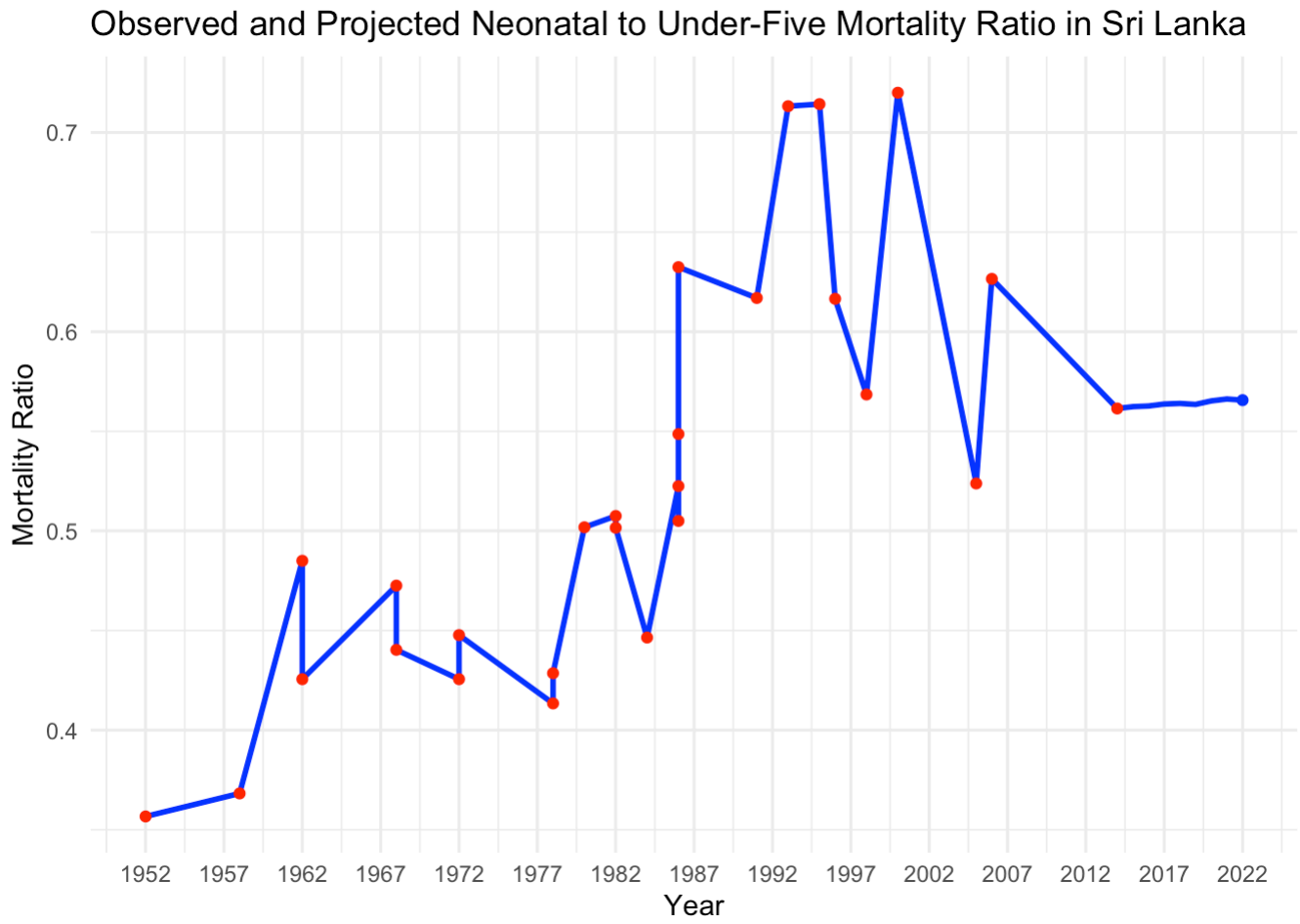
```
unknown type name 'namespace'
namespace Eigen {
^
/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:16: error:
expected ';' after top level declarator
namespace Eigen {
               ^
               ;
In file included from <built-in>:1:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/stan/math/prim/fun/Eigen.hpp:22:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/Dense:1:
/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/Core:96:10: fatal error: 'complex'
file not found
#include <complex>
         ^~~~~~~~~
3 errors generated.
make: *** [foo.o] Error 1

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
Chain 1:
Chain 1: Gradient evaluation took 2.9e-05 seconds
Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.29
seconds.
Chain 1: Adjust your expectations accordingly!
Chain 1:
Chain 1:
Chain 1: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 1: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 1: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 1: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 1: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 1: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 1: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 1: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 1: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 1: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 1: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 1: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 1:
Chain 1:  Elapsed Time: 0.023 seconds (Warm-up)
Chain 1:                0.011 seconds (Sampling)
Chain 1:                0.034 seconds (Total)
Chain 1:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
Chain 2:
Chain 2: Gradient evaluation took 2e-06 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.02
seconds.
Chain 2: Adjust your expectations accordingly!
```

```
Chain 2:
Chain 2:
Chain 2: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 2: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 2: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 2: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 2: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 2: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 2: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 2: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 2: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 2: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 2: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 2: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 2:
Chain 2:  Elapsed Time: 0.025 seconds (Warm-up)
Chain 2:                0.015 seconds (Sampling)
Chain 2:                0.04 seconds (Total)
Chain 2:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
Chain 3:
Chain 3: Gradient evaluation took 2e-06 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.02
seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 3: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 3: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 3: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 3: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 3: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 3: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 3: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 3: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 3: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 3: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 3: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 3:
Chain 3:  Elapsed Time: 0.028 seconds (Warm-up)
Chain 3:                0.011 seconds (Sampling)
Chain 3:                0.039 seconds (Total)
Chain 3:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
Chain 4:
Chain 4: Gradient evaluation took 2e-06 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.02
seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4:
Chain 4:
```

```
Chain 4: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 4: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 4: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 4: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 4: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 4: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 4: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 4: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 4: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 4: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 4: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 4: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 4:
Chain 4:  Elapsed Time: 0.025 seconds (Warm-up)
Chain 4:                0.013 seconds (Sampling)
Chain 4:                0.038 seconds (Total)
Chain 4:
```

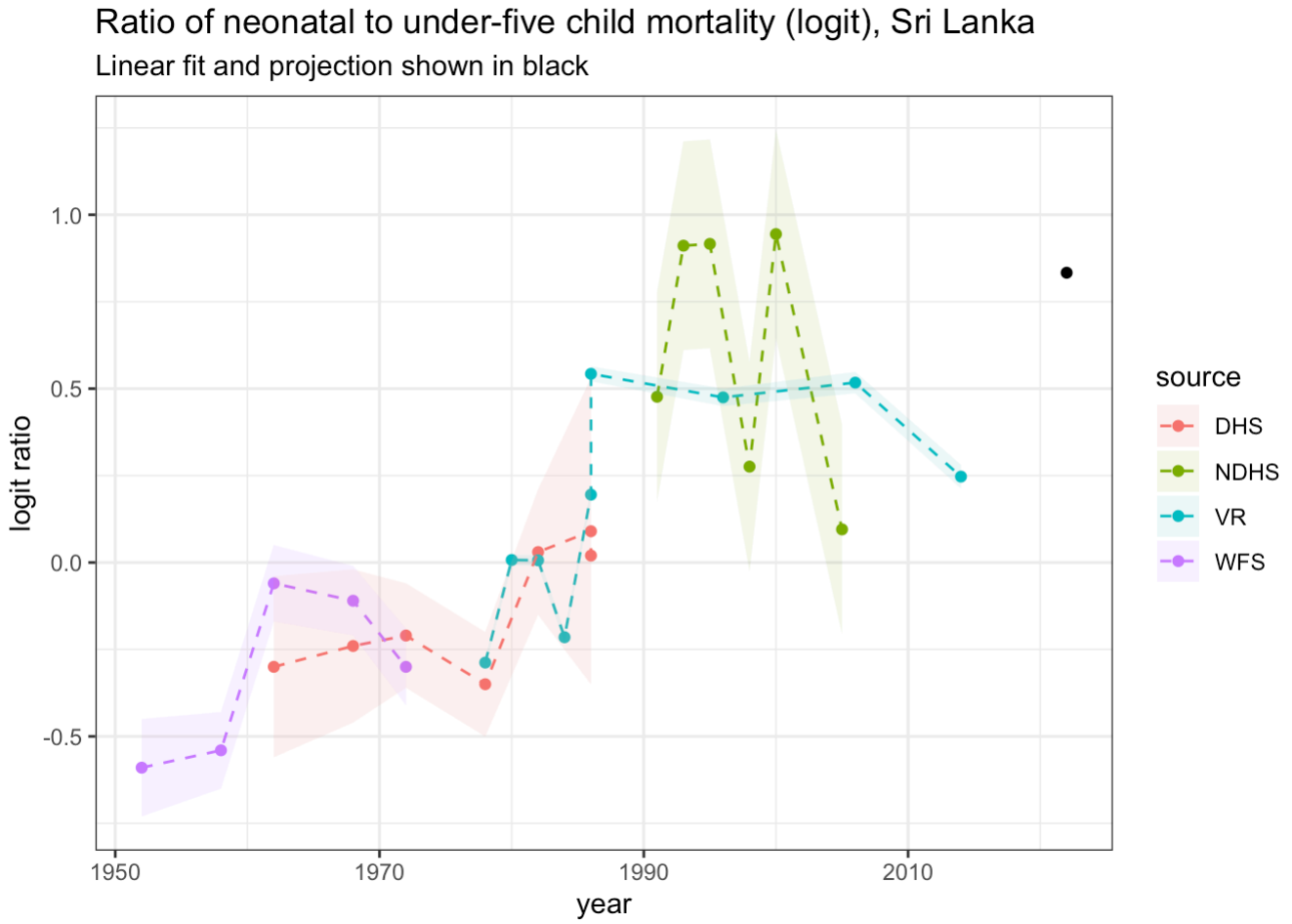
```r
# Extract the projected logits
posterior <- rstan::extract(fit)
projected_logit <- apply(posterior$projected_logit, 2, mean)

# Calculate projected ratios using the logistic function
projected_ratio1<- exp(projected_logit) / (1 + exp(projected_logit))

# Combine the observed and projected data
years <- c(lka$year, (max(lka$year) + 1):2022)
ratios1 <- c(lka$ratio, projected_ratio1[(nrow(lka) + 1):length(projected_ratio1)])
projection_data <- data.frame(year = years, ratio = ratios1)

# Plot the results
ggplot(projection_data, aes(x = year, y = ratio)) +
  geom_line(color = "blue", size = 1) + # Line for both observed and projected
  geom_point(data = lka, aes(x = year, y = ratio), color = 'red') + # Points for obser
  geom_point(data = projection_data[(max(lka$year) + 1):nrow(projection_data), ],
             aes(x = year, y = ratio), color = 'blue') + # Points for projected data
  theme_minimal() +
  labs(title = 'Observed and Projected Neonatal to Under-Five Mortality Ratio in Sri L
       x = 'Year', y = 'Mortality Ratio') +
  scale_x_continuous(breaks = seq(min(projection_data$year), max(projection_data$year)
```

```
Warning: Removed 1980 rows containing missing values (`geom_point()`).
```




```
print(p)
```

```
`geom_line()`: Each group consists of only one observation.
ℹ Do you need to adjust the group aesthetic?
```



Question 4

Now alter your model above to estimate and project a second-order random walk model (RW2).

```
model4 <- "
data {
  int<lower=1> N;           // Number of observations
  vector[N] logit_ratio;    // Observed data (logit ratios)
  vector<lower=0>[N] se;    // Standard errors of observations
  int<lower=0> T;           // Number of years to project beyond the last observed yea
  int<lower=0> last_year;   // The last observed year
}

parameters {
  vector[N] true_logit;       // True underlying logit ratios
  real<lower=0> sigma_walk;   // Standard deviation of the second-order walk
  real<lower=0> sigma_obs;    // Standard deviation of observations (measurement error
}

model {
  // Priors
  true_logit[1] ~ normal(0, 10);  // Weakly informative prior for the first logit rati
  true_logit[2] ~ normal(true_logit[1], sigma_walk); // Prior for the second value

  // Second-order random walk
  for (n in 3:N) {
```

```
    true_logit[n] ~ normal(2 * true_logit[n-1] - true_logit[n-2], sigma_walk);
  }

  // Likelihood
  logit_ratio ~ normal(true_logit, sigma_obs);
}

generated quantities {
  vector[N+T] projected_logit;
  projected_logit[1:N] = true_logit;

  // Projecting out to T years beyond the last observed year
  for (t in 1:T) {
    projected_logit[N+t] = normal_rng(2 * projected_logit[N+t-1] - projected_logit[N+t
  }
}

"
```

```
fit_rw2 <- stan(
  model_code = model4,
  data = stan_data,
  iter = 1000,
  chains = 4
)
```

```
Trying to compile a simple C file

Running /Library/Frameworks/R.framework/Resources/bin/R CMD SHLIB foo.c
using C compiler: ‘Apple clang version 14.0.3 (clang-1403.0.22.14.1)’
using SDK: ‘MacOSX13.3.sdk’
clang -arch arm64 -I"/Library/Frameworks/R.framework/Resources/include" -DNDEBUG   -
I"/Library/Frameworks/R.framework/Versions/4.3-arm64/Resources/library/Rcpp/include/"
-I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/"  -
I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/unsupported"  -
I"/Library/Frameworks/R.framework/Versions/4.3-arm64/Resources/library/BH/include" -
I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/src/"  -
I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/"  -
I"/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppParallel/include/"  -
I"/Library/Frameworks/R.framework/Versions/4.3-arm64/Resources/library/rstan/include"
-DEIGEN_NO_DEBUG  -DBOOST_DISABLE_ASSERTS  -DBOOST_PENDING_INTEGER_LOG2_HPP  -
DSTAN_THREADS  -DUSE_STANC3 -DSTRICT_R_HEADERS  -DBOOST_PHOENIX_NO_VARIADIC_EXPRESSION
-D_HAS_AUTO_PTR_ETC=0  -include '/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/stan/math/prim/fun/Eigen.hpp'  -
D_REENTRANT -DRCPP_PARALLEL_USE_TBB=1   -I/opt/R/arm64/include    -fPIC  -falign-
functions=64 -Wall -g -O2  -c foo.c -o foo.o
In file included from <built-in>:1:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
```

```
arm64/Resources/library/StanHeaders/include/stan/math/prim/fun/Eigen.hpp:22:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/Dense:1:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/Core:88:
/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:1: error:
unknown type name 'namespace'
namespace Eigen {
^
/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:16: error:
expected ';' after top level declarator
namespace Eigen {
               ^
               ;
In file included from <built-in>:1:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/StanHeaders/include/stan/math/prim/fun/Eigen.hpp:22:
In file included from /Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/Dense:1:
/Library/Frameworks/R.framework/Versions/4.3-
arm64/Resources/library/RcppEigen/include/Eigen/Core:96:10: fatal error: 'complex'
file not found
#include <complex>
         ^~~~~~~~~
3 errors generated.
make: *** [foo.o] Error 1

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
Chain 1:
Chain 1: Gradient evaluation took 3e-05 seconds
Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.3
seconds.
Chain 1: Adjust your expectations accordingly!
Chain 1:
Chain 1:
Chain 1: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 1: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 1: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 1: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 1: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 1: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 1: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 1: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 1: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 1: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 1: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 1: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 1:
Chain 1:  Elapsed Time: 0.171 seconds (Warm-up)
Chain 1:                0.265 seconds (Sampling)
Chain 1:                0.436 seconds (Total)
Chain 1:
```

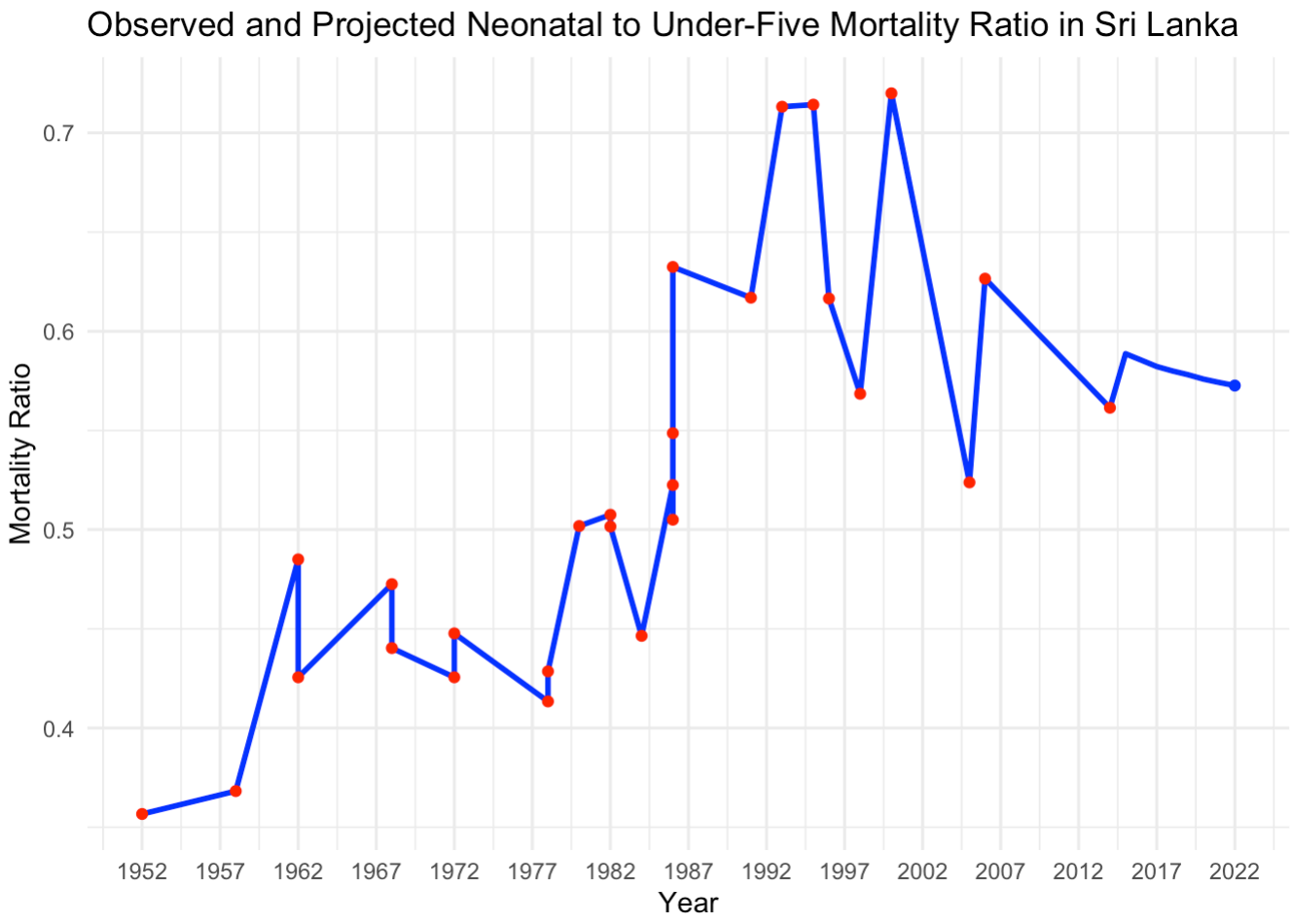
```
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
Chain 2:
Chain 2: Gradient evaluation took 5e-06 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.05
seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 2: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 2: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 2: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 2: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 2: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 2: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 2: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 2: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 2: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 2: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 2: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 2:
Chain 2:  Elapsed Time: 0.178 seconds (Warm-up)
Chain 2:                0.063 seconds (Sampling)
Chain 2:                0.241 seconds (Total)
Chain 2:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
Chain 3:
Chain 3: Gradient evaluation took 4e-06 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.04
seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 3: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 3: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 3: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 3: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 3: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 3: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 3: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 3: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 3: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 3: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 3: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 3:
Chain 3:  Elapsed Time: 0.341 seconds (Warm-up)
Chain 3:                0.172 seconds (Sampling)
Chain 3:                0.513 seconds (Total)
Chain 3:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
```

```
Chain 4:
Chain 4: Gradient evaluation took 3e-06 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.03
seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4:
Chain 4:
Chain 4: Iteration:   1 / 1000 [  0%]  (Warmup)
Chain 4: Iteration: 100 / 1000 [ 10%]  (Warmup)
Chain 4: Iteration: 200 / 1000 [ 20%]  (Warmup)
Chain 4: Iteration: 300 / 1000 [ 30%]  (Warmup)
Chain 4: Iteration: 400 / 1000 [ 40%]  (Warmup)
Chain 4: Iteration: 500 / 1000 [ 50%]  (Warmup)
Chain 4: Iteration: 501 / 1000 [ 50%]  (Sampling)
Chain 4: Iteration: 600 / 1000 [ 60%]  (Sampling)
Chain 4: Iteration: 700 / 1000 [ 70%]  (Sampling)
Chain 4: Iteration: 800 / 1000 [ 80%]  (Sampling)
Chain 4: Iteration: 900 / 1000 [ 90%]  (Sampling)
Chain 4: Iteration: 1000 / 1000 [100%]  (Sampling)
Chain 4:
Chain 4:  Elapsed Time: 0.096 seconds (Warm-up)
Chain 4:                0.061 seconds (Sampling)
Chain 4:                0.157 seconds (Total)
Chain 4:
Warning: There were 98 divergent transitions after warmup. See
https://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup
to find out why this is a problem and how to eliminate them.

Warning: There were 3 chains where the estimated Bayesian Fraction of Missing
Information was low. See
https://mc-stan.org/misc/warnings.html#bfmi-low

Warning: Examine the pairs() plot to diagnose sampling problems

Warning: The largest R-hat is 1.59, indicating chains have not mixed.
Running the chains for more iterations may help. See
https://mc-stan.org/misc/warnings.html#r-hat

Warning: Bulk Effective Samples Size (ESS) is too low, indicating posterior means and
medians may be unreliable.
Running the chains for more iterations may help. See
https://mc-stan.org/misc/warnings.html#bulk-ess

Warning: Tail Effective Samples Size (ESS) is too low, indicating posterior variances
and tail quantiles may be unreliable.
Running the chains for more iterations may help. See
https://mc-stan.org/misc/warnings.html#tail-ess
```

```r
# Extract the projected logits from the fit
posterior_rw2 <- rstan::extract(fit_rw2)
projected_logit_rw2 <- apply(posterior_rw2$projected_logit, 2, mean)

# Calculate projected ratios using the logistic function
projected_ratio2 <- exp(projected_logit_rw2) / (1 + exp(projected_logit_rw2))
```

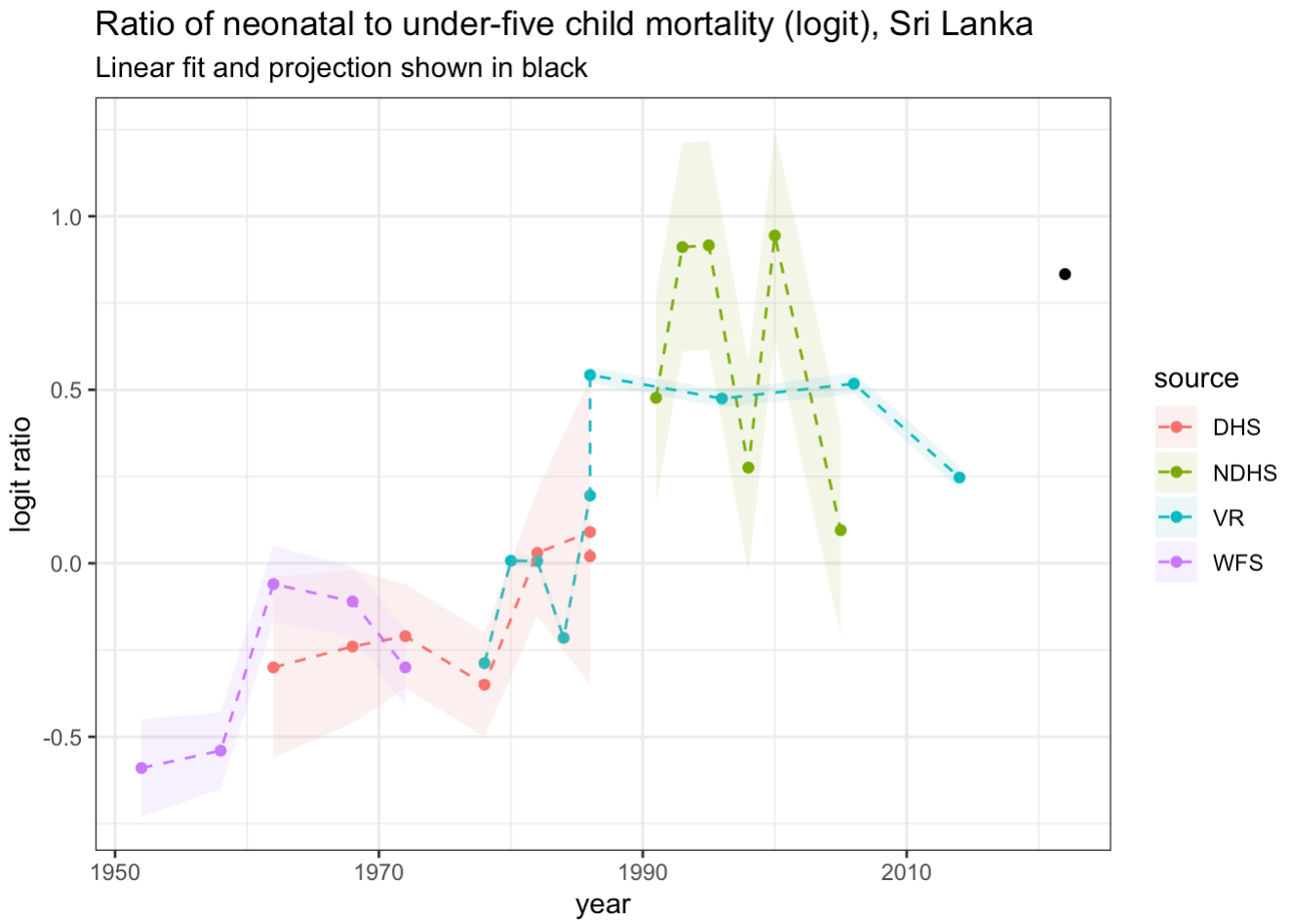
```
# Combine the observed and projected data
years <- c(lka$year, (max(lka$year) + 1):2022)
ratios2 <- c(lka$ratio, projected_ratio2[(nrow(lka) + 1):length(projected_ratio2)])
projection_data2 <- data.frame(year = years, ratio = ratios2)

# Plot the results
ggplot(projection_data2, aes(x = year, y = ratio)) +
  geom_line(color = "blue", size = 1) + # Line for both observed and projected
  geom_point(data = lka, aes(x = year, y = ratio), color = 'red') + # Points for obser
  geom_point(data = projection_data2[(max(lka$year) + 1):nrow(projection_data2), ],
             aes(x = year, y = ratio), color = 'blue') + # Points for projected data
  theme_minimal() +
  labs(title = 'Observed and Projected Neonatal to Under-Five Mortality Ratio in Sri L
       x = 'Year', y = 'Mortality Ratio') +
  scale_x_continuous(breaks = seq(min(projection_data$year), max(projection_data$year)
```

```
Warning: Removed 1980 rows containing missing values (`geom_point()`).
```




```
print(p)
```

```
`geom_line()`: Each group consists of only one observation.
ℹ Do you need to adjust the group aesthetic?
```

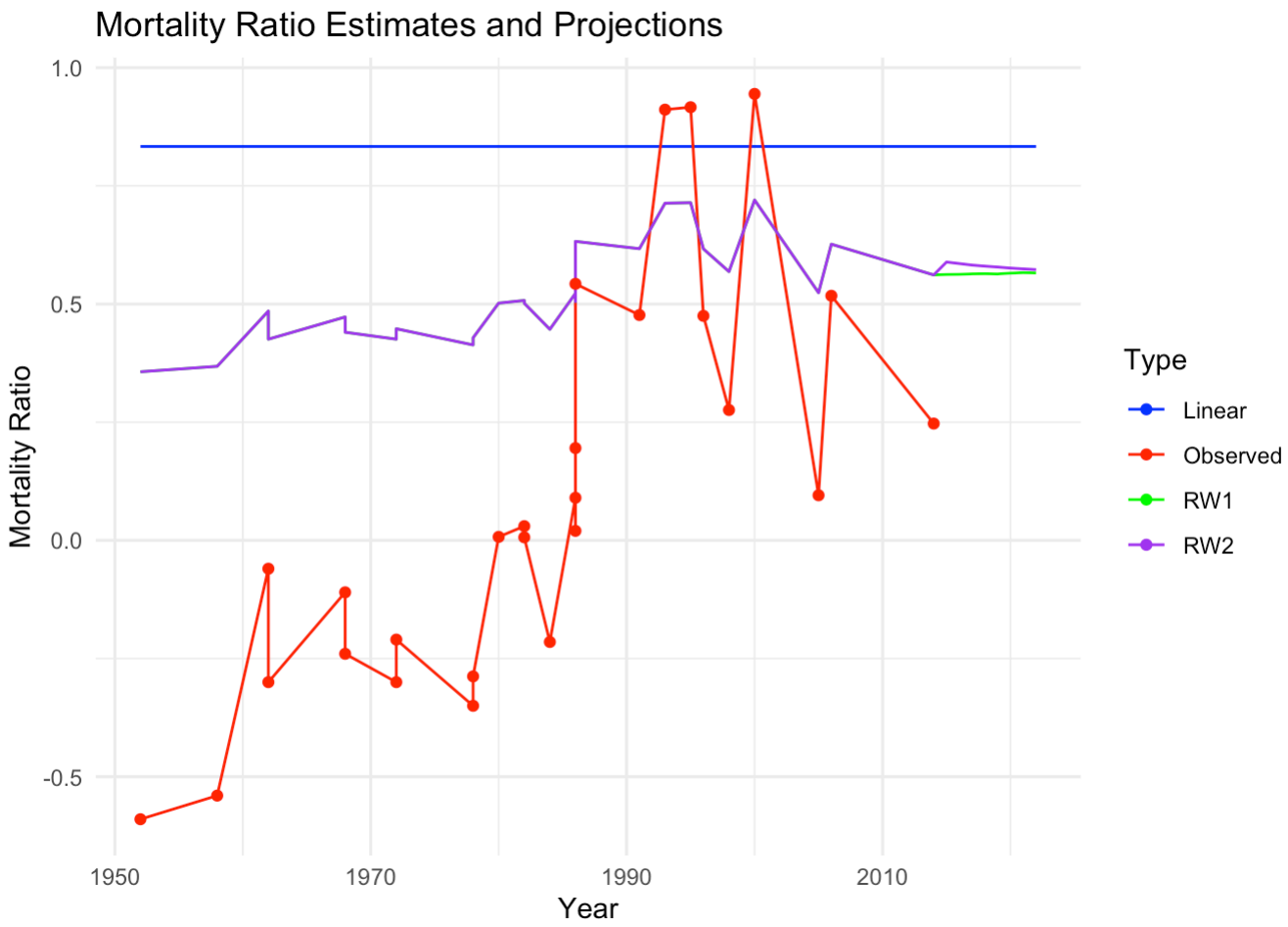


Question 5

Run the first order and second order random walk models, including projections out to 2022. Compare these estimates with the linear fit by plotting everything on the same graph.

```
observed_ratios <- lka$logit_ratio
lka_projection <- lka_projection$logit_ratio
rw1_projections <- projection_data$ratio
rw2_projections <- projection_data2$ratio

# Observed data
observed_data <- data.frame(year = lka$year,
                            ratio = lka$logit_ratio,
                            type = "Observed")

# Projection data
projection_years <- (max(lka$year) + 1):2022
projection_length <- length(projection_years)

# Assuming lka_projection, rw1_projections, rw2_projections are correctly sized for th
linear_data <- data.frame(year = years, ratio = lka_projection, type = "Linear")
rw1_data <- data.frame(year = years, ratio = rw1_projections, type = "RW1")
rw2_data <- data.frame(year = years, ratio = rw2_projections, type = "RW2")

# Combine all data
combined_data <- rbind(observed_data, linear_data, rw1_data, rw2_data)
```

```
# Plot
ggplot(combined_data, aes(x = year, y = ratio, color = type)) +
  geom_line() +
  geom_point(data = subset(combined_data, type == "Observed")) +
  scale_color_manual(values = c("Observed" = "red", "Linear" = "blue", "RW1" = "green"
  theme_minimal() +
  labs(title = 'Mortality Ratio Estimates and Projections',
       x = 'Year',
       y = 'Mortality Ratio',
       color = "Projection Type") +
  guides(color = guide_legend(title = "Type"))
```



Question 6

Briefly comment on which model you think is most appropriate, or an alternative model that would be more appropriate in this context.

Based on the plot alone, if the underlying process is expected to evolve smoothly without abrupt changes, a second-order random walk might strike a good balance between adapting to new data and providing stable projections.The RW2 model adds a smoothing effect compared to RW1, as it assumes the changes in the ratio from year to year follow a random walk themselves. The RW2 model's plot line on the graph looks relatively flat, which may suggest a more stable system.