Homework 4

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Problem 1 *Read the first four chapters of Owen (2001).*

Problem 2 *Coding*

Problem 3 *Assume that $(X_i, Y_i), i = 1, \cdots, n$ are i.i.d. samples from population $(\mathbf{X}, \mathbf{Y})$. Parameter of interest is $\tau = \sigma_X^2/\sigma_Y^2$, where σ_X^2 and σ_Y^2 are variance of X and Y respectively. Denote θ by $(\tau, \eta')'$ where η is nuisance parameter. Try to give a set of simultaneous equations to estimate θ and represent the empirical likelihood confidence intercal of τ.*

Solution

To define τ by X and Y, we formulate the following five estimationg equations:

$$\begin{aligned}
0 &= E[X - \mu_x], \\
0 &= E[Y - \mu_y], \\
0 &= E[(X - \mu_x)^2 - \sigma_X^2], \\
0 &= E[(Y - \mu_y)^2 - \sigma_Y^2], \\
0 &= E[\sigma_X^2/\sigma_Y^2 - \tau].
\end{aligned}$$

for the parameter $\theta = (\tau, \mu_x, \mu_y, \sigma_X^2, \sigma_Y^2)'$.

To handle nuisance parameter, write the estimating function as $m(X, Y, \tau, \eta') = 0$, $\tau \in \mathbb{R}, \eta \in \mathbb{R}^4$. The parameters (τ, η') satisfy the equations $E[m(X, Y, \tau, \eta')] = 0$.

Now we define

$$\mathcal{R}(\tau, \eta') = \max \left\{ \prod_{i=1}^{n} n\omega_i | \sum_{i=1}^{n} \omega_i m(X, Y, \tau, \eta') = 0, \omega_i \geq 0, \sum_{i=1}^{n} \omega_i = 1 \right\}$$

and

$$\mathcal{R}(\tau) = \max_{\eta} \mathcal{R}(\tau, \eta').$$

Under mild conditions as described in Chapter 3.10 (Owen, 2001), $-2log\mathcal{R}(\tau) \rightarrow \chi^2_{(1)}$.

Thus, a $100(1-\alpha)\%$ empirical likelihood confidence intercal of τ can be represented as

$$\left\{ \tau | -2log\mathcal{R}(\tau) \leq \chi^{2,1-\alpha}_{(1)} \right\}.$$

■

References

Art B. Owen. *Empirical Likelihood.* 2001.