

Homework 6

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Problem 1 Let X_1, \dots, X_m i.i.d $\sim F$, Y_1, \dots, Y_n i.i.d $\sim G$ and $\{X_i\}$ is independent with $\{Y_i\}$, then

- (1) get the U -statistic U_n with kernel $h(x_1, x_2, x_3) = I(x_1 < y_1, x_2 < y_2)$,
- (2) get the limit distribution of U -statistic U_n with $m + n \rightarrow \infty, \frac{m}{n+m} \rightarrow p \in (0, 1)$,
- (3) get the limit distribution of U -statistic U_n under null hypothesis $H_0 : F = G$.

Solution

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Problem 2 Suppose the distribution of X is symmetric about the origin with variance $\sigma^2 > 0$ and $EX^4 < \infty$, consider kernel $h(x, y) = xy + (x^2 - \sigma^2)(y^2 - \sigma^2)$, then

- (1) prove that the U -statistic U_n with kernel h is degenerated of order 1,
- (2) get λ_1, λ_2 and orthogonal functions $\Phi_1(x), \Phi_2(x)$, such that $h(x, y) = \lambda_1 \varphi_1(x) \varphi_1(y) + \lambda_2 \varphi_2(x) \varphi_2(y)$,
- (3) get the limit distribution of nU_n .

Solution

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Problem 3 Prove the Hoeffding decomposition in page 13.

Solution

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Problem 4 Prove the T decomposition in page 12.

Solution

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