Ph21 Assignment 3
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1 Part I: Coin Tossing

1.1 Uniform prior

We used a set of randomly generated coin tossing results to find the posterior distribution of H (probability of getting heads). The true value of H is used to generate the data set. For each number of trials, the binomial data set is generated randomly for 10 times. b We plotted the posterior results of various number of trials (size of data set) for true values H = 0.5 and H = 0.85, as shown in Figures 1 and 2.

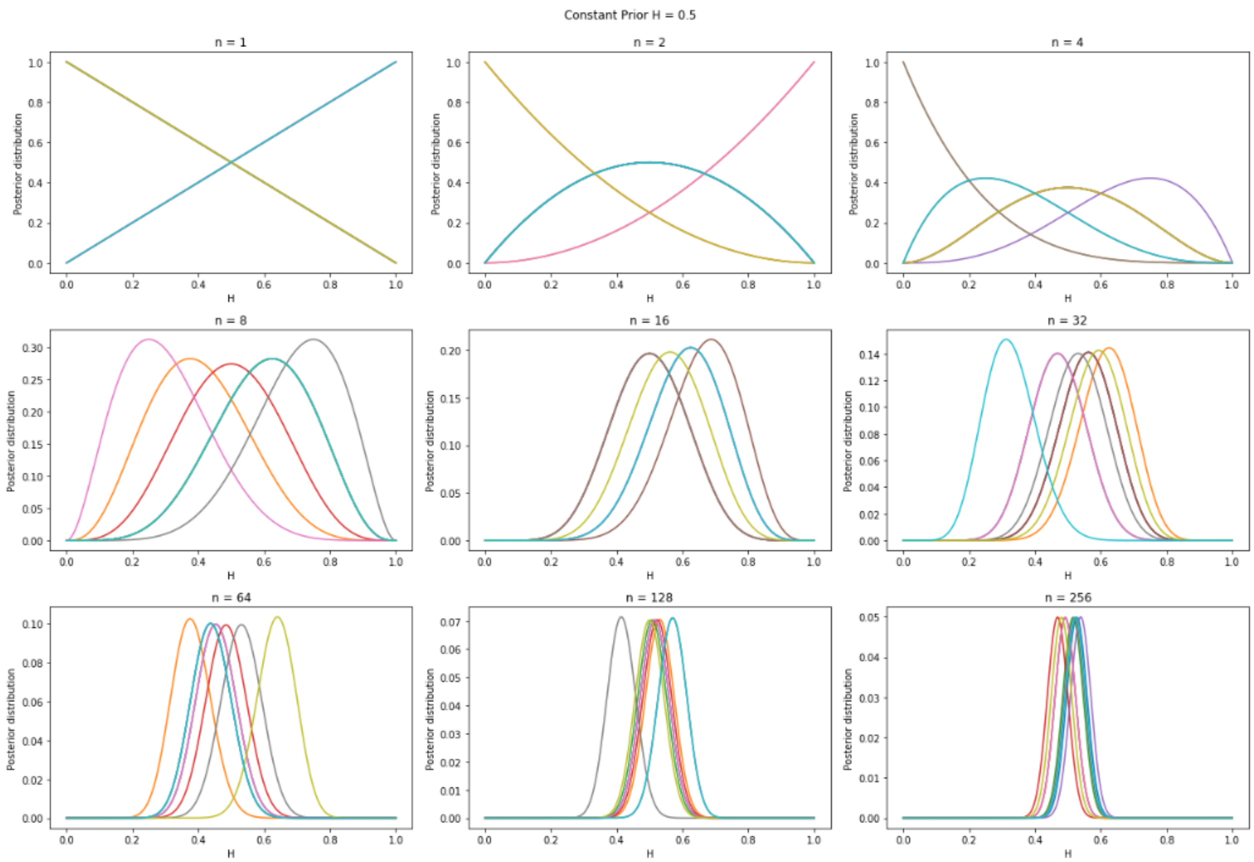

Figure 1: H = 0.5 uniform prior

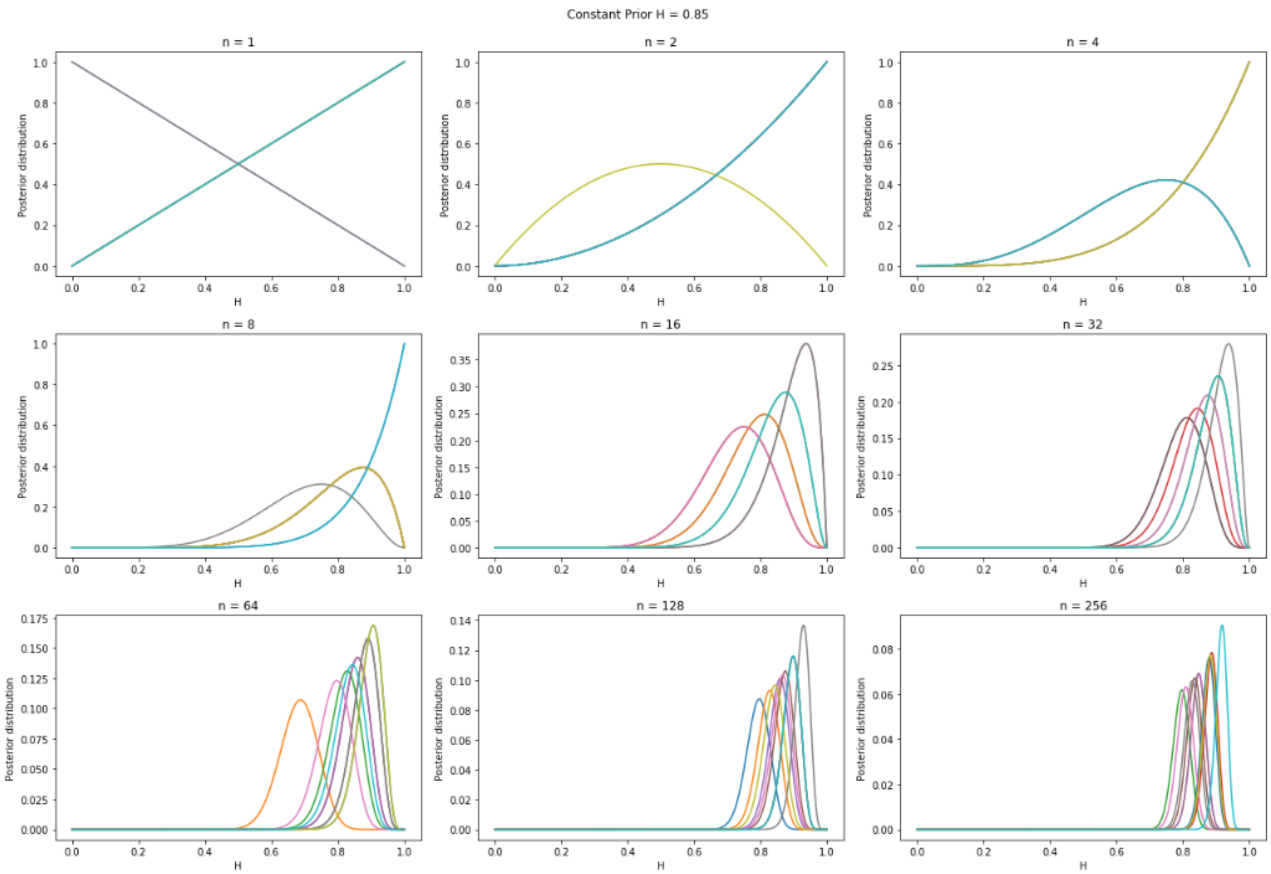

Figure 2: H = 0.85 uniform prior

As the number of trials increase, the H value of the maximum of the posterior distribution approaches the true value.

1.2 Gaussian prior

The H values have a gaussian prior with center at 0.5 and standard deviation 0.15. The true values picked are 0.6 (within 1 standard deviation) and 0.96 (outside of 3 standard deviations). The posterior distribution is plotted for various numbers of trials as in Figures 3 and 4. The location of the maximum value of the posterior distribution approaches the true value of H as the number of trials increases.

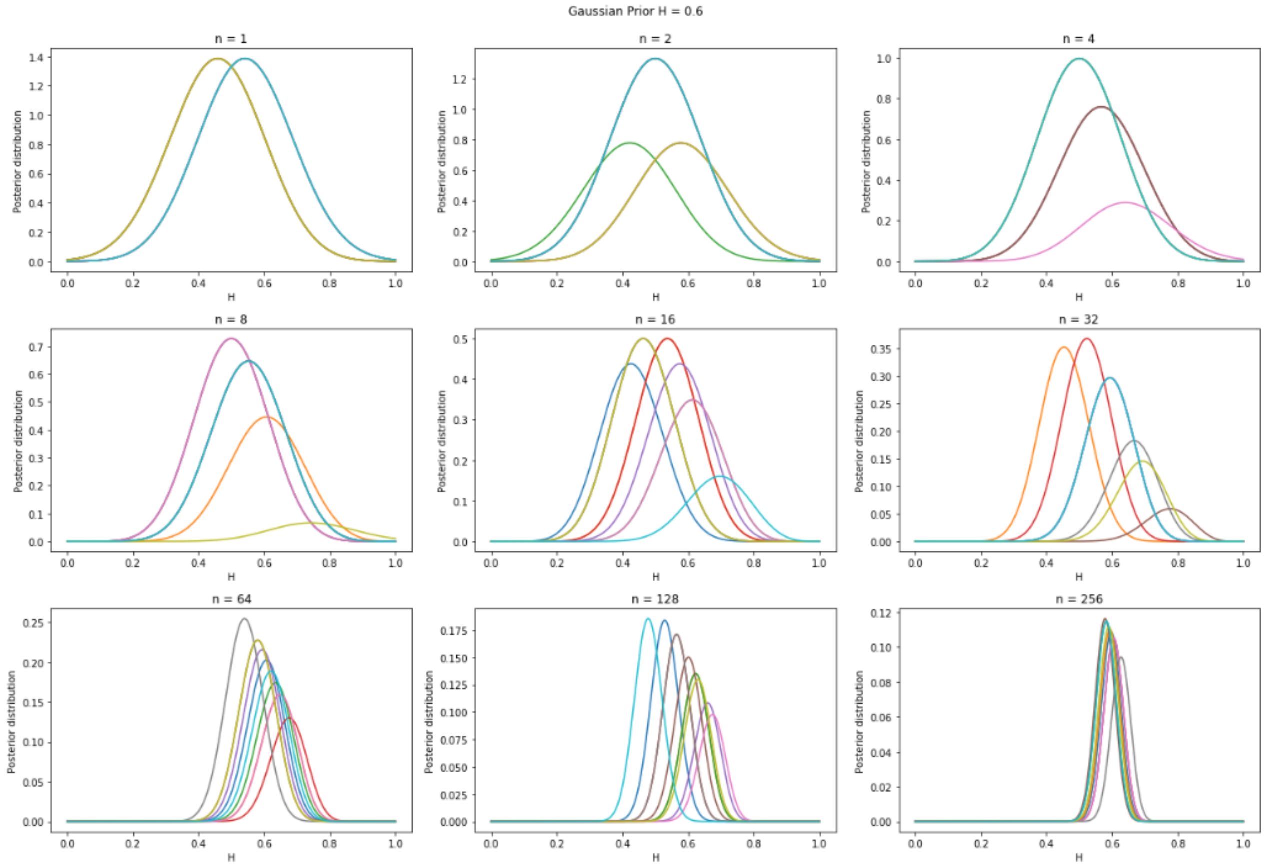

Figure 3: H = 0.6 Gaussian prior

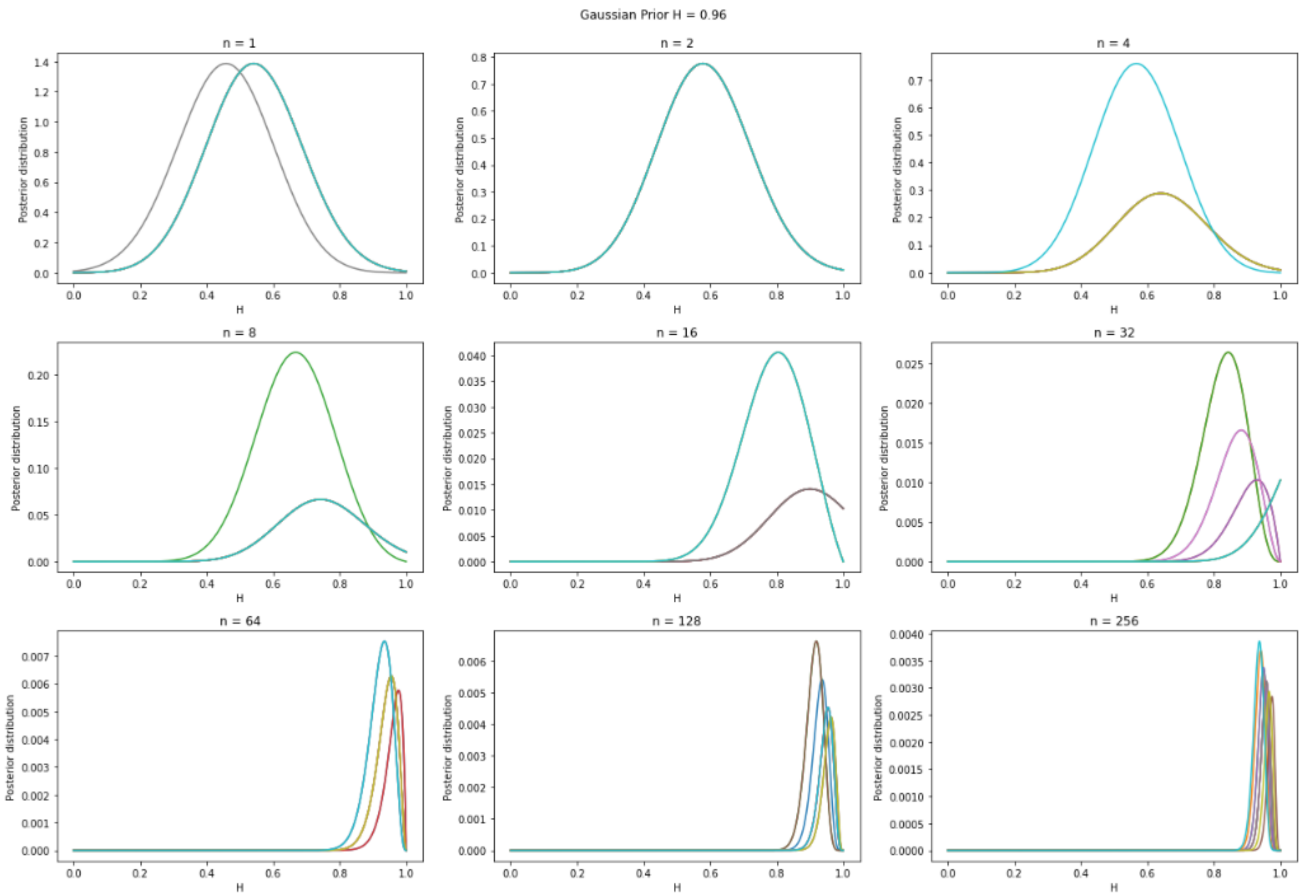

Figure 4: H = 0.96 Gaussian prior

2 Light house problem

2.1 Assumed true values for α and β

The true values are: $\alpha = 1$ km and $\beta =$ 1km. The lorentzian distribution with center at 1 and width equals 1 is used to generate set locations of flashes detected. We assumed that the photodetectors are located at integer values of x. The lorentzian distribution is used because it describes the x-intercept of a ray with a uniformly distributed angle (from Wikipedia). We rounded the data for flashes and counted the number of flashes detected by each detector. The prior distribution of α is uniform. The plots of various numbers of flashes detected are shown in Figure 5. As expected, the location of the maximum value of the posterior distribution slow approaches $\alpha = 1$.

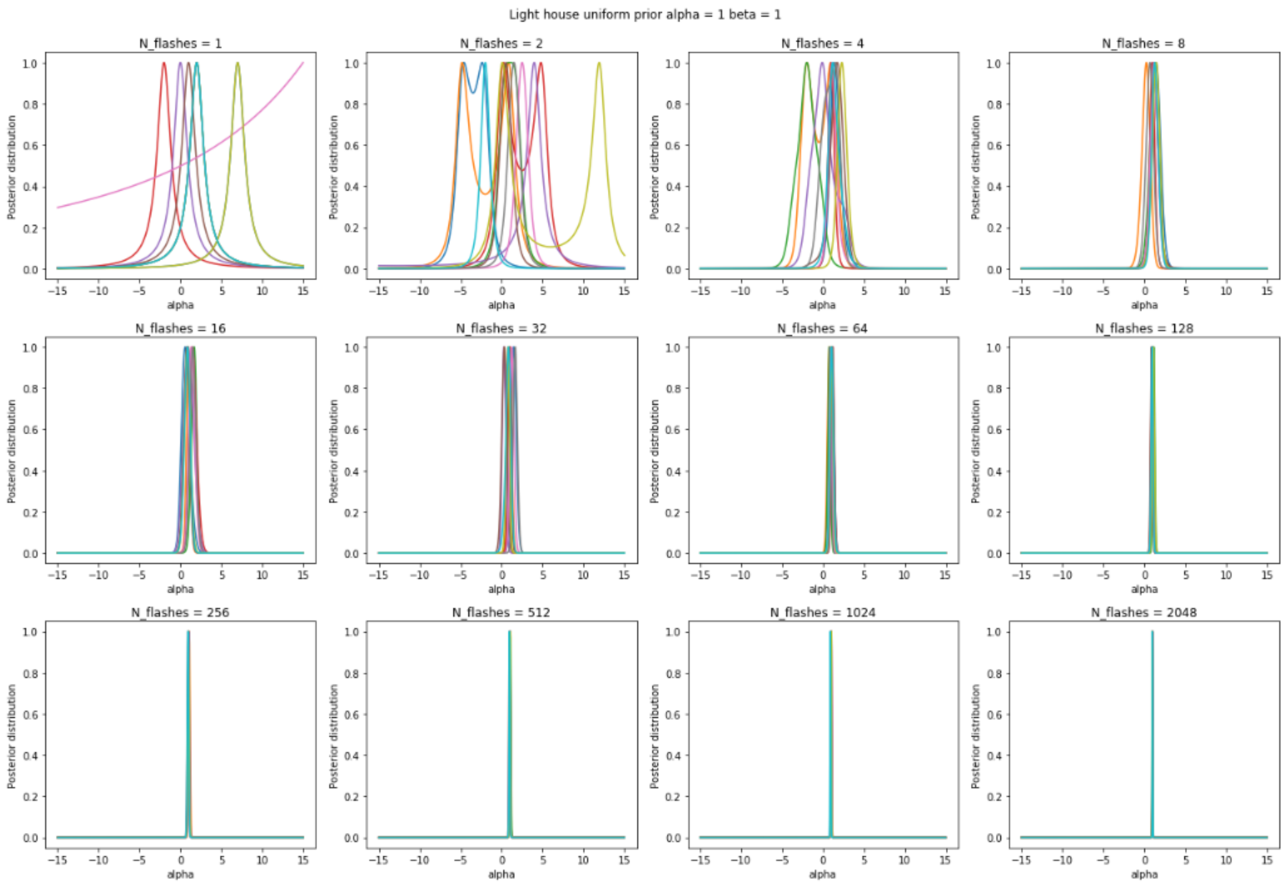

Figure 5: Light house plots with $alpha = 1$ and $beta = 1$.

The mean location and the location of the maximum value of the posterior distribution are shown in Figure 6. We can clearly see that the mean is not the center of the plots, unlike the gaussian distribution.

```
N_flashes = 1
mean = 7.2
most probable alpha = 3.6006006006006017
N_flashes = 2
mean = -0.65
most probable alpha = 1.0780780780780783
N_flashes = 4
mean = 2.55
most probable alpha = 0.6216216216216226
N_flashes = 8
mean = 0.875
most probable alpha = 1.081081081081082
N_flashes = 16
mean = -9.75625
most probable alpha = 1.1081081081081083
N_flashes = 32
mean = 0.15
most probable alpha = 0.9549549549549552
N_flashes = 64
mean = 0.296875
most probable alpha = 1.0000000000000007
N_flashes = 128
mean = 0.5828125
most probable alpha = 0.9849849849849852
N_flashes = 256
mean = 2.884765625
most probable alpha = 0.9849849849849852
N_flashes = 512
mean = -14.1484375
most probable alpha = 1.0090090090090098
N_flashes = 1024
mean = -0.16220703125
most probable alpha = 1.000000000000001
N_flashes = 2048
mean = 1.731201171875
most probable alpha = 1.0000000000000009
```

Figure 6: The mean location and the location of the maximum value of the posterior distribution are shown.

2.2 Unknown α and β

The data generated has $\alpha = 1$ and $\beta = 6$. The number of flashes is 1000. The contour plot is shown in Figure 7. The yellow spot is at $\alpha = 1$ and $\beta = 6$.

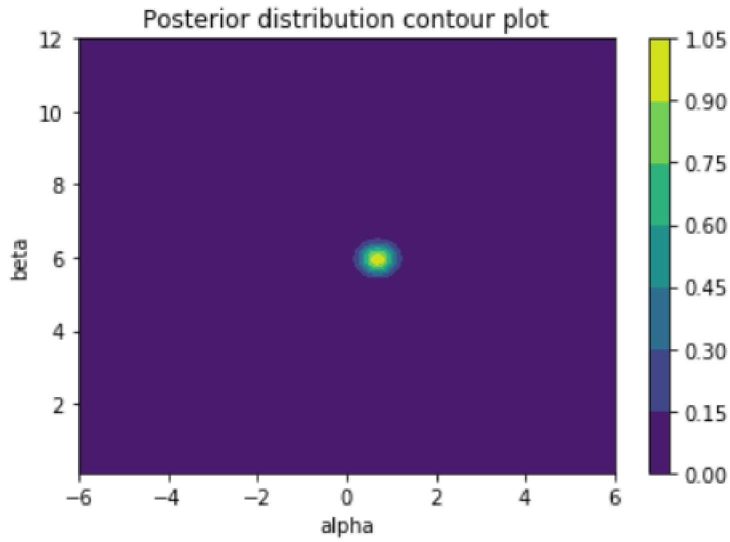

Figure 7: Contour plot of light house