# Chapter 14

# SOLUTIONS OF TRIGONOMETRIC EQUATIONS

## EXERCISE 14.1

**Trigonometric Equations:**

The equations containing at least one trigonometric functions are called trigonometric equations.

e.g., $\sin x = \frac{2}{5}$, $\sec x = \tan x$

**Q.1 Find the solutions of the following equation which lie in $[0, 2\pi]$**

**(i)** $\sin x = \frac{-\sqrt{3}}{2}$

**(ii)** $\operatorname{cosec}\theta = 2$ **(Gujranwala Board 2005, Lahore Board 2006)**

**(iii)** $\sec x = -2$

**(iv)** $\cot\theta = \frac{1}{\sqrt{3}}$ **(Lahore Board 2010)**

**Solution:**

**(i)** $\sin x = \frac{-\sqrt{3}}{2}$

$$\boxed{x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ = \frac{\pi}{3}}$$

Since $\sin x$ is $-ve$ in III & IV Quadrants with the reference angle $\pi/3$ thus we have

**For III-Quadrant**

$x = \pi + \theta$

$x = \pi + \frac{\pi}{3}$ ,

$x = \frac{4\pi}{3}$ ,

**For IV-Quadrant**

$x = 2\pi - \theta$

$x = 2\pi - \frac{\pi}{3}$

$x = \frac{5\pi}{3}$

So thus the required solution is $x = \frac{4\pi}{3}, \frac{5\pi}{3}$

**(ii)** $\operatorname{cosec}\theta = 2$

$$\frac{1}{\sin\theta} = 2$$

$$\Rightarrow \quad \sin\theta = \frac{1}{2} \qquad \boxed{\theta = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ = \frac{\pi}{6}}$$

Since $\sin\theta$ is +ve in I and II Quadrants with reference angle $\frac{\pi}{6}$ Thus

**For I-Quadrant**

$$x = \frac{\pi}{6}$$

**For II-Quadrant**

$$x = \pi - \theta$$

$$x = \pi - \frac{\pi}{6}$$

$$x = \frac{5\pi}{6}$$

So thus the required solution is $x = \frac{\pi}{6}, \frac{5\pi}{6}$.

**(iii)** $\sec x = -2$

$$\frac{1}{\cos x} = -2 \Rightarrow \cos x = -\frac{1}{2} \qquad \boxed{x = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ = \frac{\pi}{3}}$$

Since $\cos x$ is –ve in II & III Quadrants with reference angle $\frac{\pi}{3}$ thus we have

**For II-Quadrant**

$$x = \pi - \theta$$

$$x = \pi - \frac{\pi}{3}$$

$$x = \frac{2\pi}{3}$$

**For III-Quadrant**

$$x = \pi + \theta$$

$$x = \pi + \frac{\pi}{3}$$

$$x = \frac{4\pi}{3}$$

Thus the required solution is $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

**(iv)** $\cot\theta = \frac{1}{\sqrt{3}}$

$$\tan\theta = \sqrt{3} \qquad \boxed{\theta = \tan^{-1}(\sqrt{3}) = 60^\circ = \frac{\pi}{3}}$$

Since $\tan\theta$ is +ve in I & III Quadrants with reference angle $\frac{\pi}{3}$ thus we have.



**For I-Quadrant**

$$x = \frac{\pi}{3}$$

**For III-Quadrant**

$$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

Thus the required solution is $x = \frac{\pi}{3}, \frac{4\pi}{3}$ Ans.

**Q.2 Solve the following trigonometric equations:**

**(i)** $\tan^2\theta = \frac{1}{3}$ **(ii)** $\operatorname{cosec}^2\theta = \frac{4}{3}$

**(iii)** $\sec^2\theta = \frac{4}{3}$ **(iv)** $\cot^2\theta = \frac{1}{3}$ **(Lahore Board 2007)**

**Solution:**

(i) $\tan^2\theta = \frac{1}{3}$

$$\tan\theta = \pm\frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} \quad \text{and} \quad \tan\theta = \frac{-1}{\sqrt{3}} \qquad \boxed{\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ = \frac{\pi}{6}}$$

Since $\tan\theta$ is +ve in I & III Quadrants with reference angle $\frac{\pi}{6}$

Therefore

**For I-Quad**

$$\theta = \frac{\pi}{6}, \qquad \theta = \frac{\pi}{6} + n\pi,$$

**For III-Quad**

$$\theta = \pi + \frac{\pi}{6}, \qquad \theta = \frac{7\pi}{6}, \qquad \theta = \frac{7\pi}{6} + n\pi$$

Since $\tan\theta$ is –ve in II & IV Quadrants with reference angle $\frac{\pi}{6}$

**For II-Quad**

$$\theta = \pi - \frac{\pi}{6}, \qquad \theta = \frac{5\pi}{6}, \qquad \theta = \frac{5\pi}{6} + n\pi,$$

**For IV-Quad**

$$\theta = 2\pi - \frac{\pi}{6}, \qquad \theta = \frac{11\pi}{6}, \qquad \theta = \frac{11\pi}{6} + n\pi \quad \forall n \in z$$

$$\left\{\frac{\pi}{6} + n\pi\right\} \cup \left\{\frac{7\pi}{6} + n\pi\right\} \cup \left\{\frac{5\pi}{6} + n\pi\right\} \cup \left\{\frac{11\pi}{6} + n\pi\right\}, \forall n \in z$$

**(ii)** $\operatorname{cosec}^2\theta = \frac{4}{3}$ $\boxed{\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ = \frac{\pi}{3}}$

$$\operatorname{cosec}\theta = \pm\frac{2}{\sqrt{3}}$$

$$\operatorname{cosec}\theta = \frac{2}{\sqrt{3}}$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

Since $\sin\theta$ is +ve in I & II Quadrants with reference angle $\frac{\pi}{3}$

$$\operatorname{cosec}\theta = \frac{-2}{\sqrt{3}}$$

$$\sin\theta = \frac{-\sqrt{3}}{2}$$

Since $\sin\theta$ is –ve in III & IV Quadrants with reference angle $\frac{\pi}{3}$

| For I-Quad | For II-Quad | For III-Quad | For IV-Quad |
|---|---|---|---|
| $\theta = \frac{\pi}{3}$ , | $\theta = \pi - \frac{\pi}{3}$ | $\theta = \pi + \frac{\pi}{3}$ , | $\theta = 2\pi - \frac{\pi}{3}$ |
| $\theta = \frac{\pi}{3} + 2n\pi$ | $\theta = \frac{2\pi}{3} + 2n\pi$ | $\theta = \frac{4\pi}{3} + 2n\pi$ , | $\theta = \frac{5\pi}{3} + 2n\pi$ |

$$S.S = \left\{\frac{\pi}{3} + 2n\pi\right\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\} \cup \left\{\frac{5\pi}{3} + 2n\pi\right\}, \forall n \in z \quad \text{Ans.}$$

**(iii)** $\sec^2\theta = \frac{4}{3}$  $\boxed{\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ = \frac{\pi}{6}}$

$$\sec\theta = \pm\frac{2}{\sqrt{3}}$$

$$\sec\theta = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$$

Since $\cos\theta$ is +ve in I & IV Quadrants, with reference angle $\frac{\pi}{6}$ therefore we have

| For I-Quad | For IV-Quad |
|---|---|
| $\theta = \frac{\pi}{6}$ , | $\theta = 2\pi - \frac{\pi}{6}$ |
| $\theta = \frac{\pi}{6} + 2n\pi$ , | $\theta = \frac{11\pi}{6} + 2n\pi$ |

$$\sec\theta = \frac{-2}{\sqrt{3}}$$

$$\cos\theta = \frac{-\sqrt{3}}{2}$$

Since $\cos\theta$ is –ve in II & III Quadrants with reference angle $\frac{\pi}{6}$ therefore we have

| For II-Quad | For III-Quad |
|---|---|
| $\theta = \pi - \frac{\pi}{6}$ , | $\theta = \pi + \frac{\pi}{6}$ |
| $\theta = \frac{5\pi}{6} + 2n\pi$ , | $\theta = \frac{7\pi}{6} + 2n\pi$ |

$$S.S = \left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{4\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\} \cup \left\{\frac{7\pi}{6} + 2n\pi\right\}, \forall n \in z \quad \text{Ans.}$$



**(iv)** $\cot^2\theta = \frac{1}{3}$ **(Lahore Board 2007)**

$$\Rightarrow \cot\theta = \pm\frac{1}{\sqrt{3}}$$

$$\tan\theta = \pm\sqrt{3}$$

$\boxed{\theta = \tan^{-1}(\sqrt{3}) = 60^\circ = \frac{\pi}{3}}$

$$\tan\theta = \sqrt{3}$$

Since $\tan\theta$ is +ve in I & III Quadrants with reference angle $\frac{\pi}{3}$ therefore we have

| For I-Quad | For III-Quad |
|---|---|
| $\theta = \frac{\pi}{3}$ , | $\theta = \pi + \frac{\pi}{3}$ |
| $\theta = \frac{\pi}{3} + n\pi$ , | $\theta = \frac{4\pi}{3} + n\pi$ |

$$\tan\theta = -\sqrt{3}$$

Since $\tan\theta$ is –ve in II & IV Quadrants, with reference angle $\frac{\pi}{3}$ therefore we have

| For II-Quad | For IV-Quad |
|---|---|
| $\theta = \pi - \frac{\pi}{3}$ , | $\theta = 2\pi - \frac{\pi}{3}$ |
| $\theta = \frac{2\pi}{3} + n\pi$ , | $\theta = \frac{5\pi}{3} + n\pi$ |

$$S.S = \left\{\frac{\pi}{3} + n\pi\right\} \cup \left\{\frac{4\pi}{3} + n\pi\right\} \cup \left\{\frac{2\pi}{3} + n\pi\right\} \cup \left\{\frac{5\pi}{3} + n\pi\right\}, \forall n \in z \text{ Ans.}$$

**Q.3 Find the values of $\theta$ satisfying the following equations:**

**$3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$ (Gujranwala Board 2006)**

**Solution:**

$$3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$$

$a = 3, \quad b = 2\sqrt{3}, \quad c = 1$ by quadratic formula

$$\tan\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 - 4(3)(1)}}{2(3)}$$

$$= \frac{-2\sqrt{3} \pm \sqrt{12 - 12}}{6} = \frac{-2\sqrt{3}}{6} = \frac{-1}{\sqrt{3}}$$

$\boxed{\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ = \frac{\pi}{6}}$

$$\Rightarrow \tan\theta = -\frac{1}{\sqrt{3}}$$

Since $\tan\theta$ is –ve in II & IV Quadrants, with reference angle $\frac{\pi}{6}$ therefore we have

**For II-Quad** | **For IV-Quad**

$\theta = \pi - \frac{\pi}{6}$ , $\theta = 2\pi - \frac{\pi}{6}$

$\theta = \frac{5\pi}{6} + n\pi$ , $\theta = \frac{11\pi}{6} + n\pi$

$S.S = \left\{\frac{5\pi}{6} + n\pi\right\} \cup \left\{\frac{4\pi}{6} + n\pi\right\}, \forall n \in z$ Ans.

**Q.4** $\tan^2\theta - \sec\theta - 1 = 0$

**Solution:**

$\tan^2\theta - \sec\theta - 1 = 0$

$\sec^2\theta - 1 - \sec\theta - 1 = 0$

$\sec^2\theta - \sec\theta - 2 = 0$

$\sec^2\theta - 2\sec\theta + \sec\theta - 2 = 0$

$\sec\theta(\sec\theta - 2) + 1(\sec\theta - 2) = 0$

$(\sec\theta - 2)(\sec\theta + 1) = 0$

$\sec\theta = 2$

$\cos\theta = \frac{1}{2}$

Since $\cos\theta$ is +ve in I & IV Quadrants

With reference angle $\frac{\pi}{3}$

**For I-Quad.** **For IV-Quad.**

$\theta = \frac{\pi}{3}$ , $\theta = 2\pi - \frac{\pi}{3}$

$\theta = \frac{\pi}{3} + 2n\pi$, $\theta = \frac{5\pi}{3} + 2n\pi$

$\sec\theta = -1$

$\cos\theta = -1$

$\Rightarrow \theta = \cos^{-1}(-1)$

$\theta = \cos^{-1}(-1)$

$\theta = \pi + 2n\pi$

$S.S = \left\{\frac{\pi}{3} + 2n\pi\right\} \cup \left\{\frac{5\pi}{3} + 2n\pi\right\} \cup \{(2n+1)\pi\}, \forall n \in z$ Ans.

**Q.5** $2\sin\theta + \cos^2\theta - 1 = 0$

**Solution:**

$2\sin\theta + 1 - \sin^2\theta - 1 = 0$

$2\sin\theta - \sin^2\theta = 0$

$\sin\theta(2 - \sin\theta) = 0$



$\Rightarrow \sin\theta = 0 , 2 - \sin\theta = 0$

$\sin\theta = 0$

$\Rightarrow \theta = n\pi$

$\sin\theta = 2$

$2 - \sin\theta = 0$

$\sin\theta = 0$

Which is not possible because $-1 \le \sin\theta \le 1$

$S.S = \{n\pi, \forall n \in z\}$

**Q.6** $2\sin^2\theta - \sin\theta = 0$

**Solution:**

$\sin\theta(2\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = 0$ and $2\sin\theta - 1 = 0$

$2\sin\theta = 1$

$\sin\theta = 1/2$

$\sin\theta = 0$

$\Rightarrow \theta = n\pi$

$\sin\theta = 1/2$

Since $\sin\theta$ is positive in I and II quadrants with the reference angle $\pi/6$.

**I quad.**

$q = \pi/6 + 2n\pi, \forall n \in z$

**II quad.**

$\theta = \pi - \pi/6$

$\theta = 5\pi/6$

$\theta = 5\pi/6 + 2n\pi, \forall n \in z$

$S.S = \{n\pi\} \cup \{\pi/6 + 2n\pi\} \cup \{5\pi/6 + 2n\pi\}, \forall n \in z$

**Q.7** $3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$

**Solution:**

$3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - \sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$

$3\cos\theta(\cos\theta - \sqrt{3}\sin\theta) + \sqrt{3}\sin\theta(\cos\theta - \sqrt{3}\sin\theta) = 0$

$(\cos\theta - \sqrt{3}\sin\theta)(3\cos\theta + \sqrt{3}\sin\theta) = 0$

$\cos\theta - \sqrt{3}\sin\theta = 0$

$\cos\theta = \sqrt{3}\sin\theta$

$\tan\theta = \frac{1}{\sqrt{3}}$

Since $\tan\theta$ is positive in I to III quadrants with the reference angle $\pi/6$

**I quad.**

$\theta = \pi/6 + n\pi$

**III quad**

$\theta = \pi + \pi/6$

$\theta = 7\pi/6$

$\theta = 7\pi/6 + n\pi$

$3\cos\theta + \sqrt{3}\sin\theta = 0$

$\sqrt{3}\sin\theta = -3\cos\theta$

$\tan\theta = -\sqrt{3}$

Since $\tan\theta$ is negative in III and IV quadrants with the reference angle $\pi/3$.

**I quad.**

$\theta = \pi - \pi/3$

$\theta = 2\pi/3$

$\theta = 2\pi/3 + n\pi$

**III quad**

$\theta = 2\pi - \pi/3$

$\theta = 5\pi/3$

$\theta = 5\pi/3 + n\pi$

$S.S = \{\pi/6 + n\pi\} \cup \{7\pi/6 + n\pi\} \cup \{2\pi/3 + n\pi\} \cup \{5\pi/3 + n\pi\}, \forall n \in z$

**Q.8 Find the values of $\theta$ $4\sin^2\theta - 8\cos\theta + 1 = 0$**

**Solution:**

$$4\sin^2\theta - 8\cos\theta + 1 = 0$$
$$4(1-\cos^2\theta) - 8\cos\theta + 1 = 0$$
$$4 - 4\cos^2\theta - 8\cos\theta + 1 = 0$$
$$4\cos^2\theta + 8\cos\theta - 5 = 0$$
$$4\cos^2\theta + 10\cos\theta - 2\cos\theta - 5 = 0$$
$$2\cos\theta(2\cos\theta + 5) - 1(2\cos\theta + 5) = 0$$
$$(2\cos\theta + 5)(2\cos\theta - 1) = 0$$

$$2\cos\theta + 5 = 0$$
$$\cos\theta = \frac{-5}{2}$$

i.e. solution is impossible.

$$2\cos\theta - 1 = 0$$
$$\cos\theta = \frac{1}{2} \qquad \boxed{\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ = \frac{\pi}{3}}$$

Since $\cos\theta$ is +ve in I & IV Quadrants with reference angle $\frac{\pi}{3}$

**For I-Quad.** $\theta = \frac{\pi}{3}$ , $\theta = \frac{\pi}{3} + 2n\pi$ ,

**For IV-Quad.** $\theta = 2\pi - \frac{\pi}{3}$ $= \frac{5\pi}{3} + 2n\pi$

$$S.S = \left\{\frac{\pi}{3} + 2n\pi\right\} \cup \left\{\frac{5\pi}{3} + 2n\pi\right\}, \forall\, n \in z \text{ Ans.}$$

**Q.9 Find the solution set of the following equations.**

**$\sqrt{3}\tan x - \sec x - 1 = 0$** **(Gujranwala Board 2004)**

**Solution:**

$$\sqrt{3}\tan x = 1 + \sec x$$
$$(\sqrt{3}\tan x)^2 = (1+\sec x)^2$$
$$3\tan^2 x = 1 + \sec^2 x + 2\sec x$$
$$3(\sec^2 x - 1) = 1 + \sec^2 x + 2\sec x$$
$$3\sec^2 x - 3 - 1 - \sec^2 x - 2\sec x = 0$$
$$2\sec^2 x - 2\sec x - 4 = 0$$
$$\sec^2 x - \sec x - 2 = 0$$



$$\sec^2 x - 2\sec x + \sec x - 2 = 0$$
$$\sec x(\sec x - 2) + 1(\sec x - 2) = 0$$
$$(\sec x - 2)(\sec x + 1) = 0$$

$$\sec x - 2 = 0$$
$$\sec x = 2$$
$$\Rightarrow \quad \cos x = \frac{1}{2}$$

Since $\cos x$ is +ve in I & IV Quadrant with reference angle $\frac{\pi}{3}$

**For I-Quad.** $x = \frac{\pi}{3}$ , $x = \frac{\pi}{3} + 2n\pi$ ,

**For II-Quad.** $x = 2\pi - \frac{\pi}{3}$ , $x = \frac{5\pi}{3} + 2n\pi$

$$\sec x + 1 = 0$$
$$\sec x = -1$$
$$\cos x = -1$$
$$x = \cos^{-1}(-1)$$
$$x = \pi$$
$$x = \pi + 2n\pi, \forall n \in z$$

Solution set is $\left\{\frac{\pi}{3} + 2n\pi\right\} \cup \left\{\frac{5\pi}{3} + 2n\pi\right\} \cup \{\pi + 2n\pi\}$ , $n \in z$

**Q.10 $\cos 2x = \sin 3x$**

**Solution:**

$$\cos 2x = \sin 3x$$
$$\cos^2 x - \sin^2 x = 3\sin x - 4\sin^3 x$$
$$1 - \sin^2 x - \sin^2 x - 3\sin x + 4\sin^3 x = 0$$
$$4\sin^3 x - 2\sin^2 x - 3\sin x + 1 = 0$$

The above equation is satisfied by $\sin x = 1$

By synthetic division, we have

$$\begin{array}{c|cccc} & 4 & -2 & -3 & 1 \\ 1 & & 4 & 2 & -1 \\ \hline & 4 & 2 & -1 & 0 \end{array}$$

$$4\sin^2 x + 2\sin x - 1 = 0$$

Here $a = 4$, $b = 2$, $c = -1$

By quadratic formula

$$\sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sin x = \frac{-2 \pm \sqrt{4 - 4(4)(-1)}}{2(4)} = \frac{-2 \pm \sqrt{4+16}}{8}$$

$$\sin x = \frac{-2 \pm \sqrt{20}}{8}$$

$$x = \sin^{-1}\left(\frac{\sqrt{20}-2}{8}\right)$$

$$x = 18^\circ = \frac{\pi}{10}$$

Since $\sin x$ is +ve in the I and II Quadrants with reference angle $\frac{\pi}{10}$

**For I-Quad.** **For II-Quad.**

$$x = \frac{\pi}{10} \quad , \quad x = \pi - \frac{\pi}{10}$$

$$x = \frac{\pi}{10} + 2n\pi \, , \; x = \frac{9\pi}{10} + 2n\pi$$

$$\sin x = \frac{-\sqrt{20}-2}{8} = -\left(\frac{\sqrt{20}+2}{8}\right)$$

$$x = \sin^{-1}\left(\frac{\sqrt{20}+2}{8}\right)$$

$$x = 54^\circ = \frac{3\pi}{10}$$

Since $\sin x$ is –ve in III & IV Quadrants with reference angle $\frac{3\pi}{10}$

**For III-Quad.** **For IV-Quad.**

$$x = \pi + \frac{3\pi}{10} \quad , \quad x = 2\pi - \frac{3\pi}{10}$$

$$x = \frac{13\pi}{10} + 2n\pi \quad , \quad x = \frac{17\pi}{10} + 2n\pi$$

Also $\sin x = 1$

$$x = \frac{\pi}{2} + 2n\pi \, , \; \forall\, n \in z$$

Hence solution set is

$$\left\{\frac{\pi}{2} + 2n\pi\right\} \cup \left\{\frac{\pi}{10} + 2n\pi\right\} \cup \left\{\frac{9\pi}{10} + 2n\pi\right\} \cup \left\{\frac{13\pi}{10} + 2n\pi\right\} \cup \left\{\frac{17\pi}{10} + 2n\pi\right\} \quad n \in z \quad \text{Ans.}$$

**Q.11** $\sec 3\theta = \sec \theta$

**Solution:**

$$\sec 3\theta = \sec\theta$$

$$\frac{1}{\cos 3\theta} = \frac{1}{\cos\theta}$$

$$\cos 3\theta = \cos\theta$$

$$\cos 3\theta - \cos\theta = 0$$



$$-2\sin\left(\frac{3\theta+\theta}{2}\right)\sin\left(\frac{3\theta-\theta}{2}\right) = 0$$

$$-2\sin 2\theta \sin\theta = 0$$

$$\Rightarrow \quad \sin 2\theta \sin\theta = 0$$

$$\Rightarrow \quad \sin 2\theta = 0 \qquad\qquad \sin\theta = 0$$

$$\Rightarrow \quad 2\theta = n\pi \qquad\qquad \Rightarrow \theta = n\pi$$

$$\Rightarrow \quad \theta = \frac{n\pi}{2}$$

Solution set is $\left\{\frac{n\pi}{2}\right\} \cup \{n\pi\}$, $n \in Z$ Ans.

**Q.12** $\tan 2\theta + \cot\theta = 0$

**Solution:**

$$\tan 2\theta = -\cot\theta$$

$$\frac{\sin 2\theta}{\cos 2\theta} = -\frac{\cos\theta}{\sin\theta}$$

$$\sin 2\theta \sin\theta = -\cos 2\theta \cos\theta$$

$$\sin 2\theta \sin\theta + \cos 2\theta \cos\theta = 0$$

$$\Rightarrow \quad \cos(2\theta - \theta) = 0$$

$$\Rightarrow \quad \cos\theta = 0$$

$$\Rightarrow \quad \theta = (2n+1)\frac{\pi}{2}$$

Solution set is $\left\{(2n+1)\frac{\pi}{2}\right\}$, $n \in z$

**Q.13** $\sin 2x + \sin x = 0$

**Solution:**

$$\sin 2x + \sin x = 0$$

$$2\sin x\cos x + \sin x = 0$$

$$\sin x(2\cos x + 1) = 0$$

$$\sin x = 0$$

$$x = n\pi$$

$$2\cos x + 1 = 0$$

$$\cos x = \frac{-1}{2}$$

As $\cos x$ is +ve in I & IV Quadrants with reference angle $\frac{\pi}{3}$

**For I-Quad.** **For II-Quad.**

$$x = \frac{\pi}{3} \quad , \quad x = 2\pi - \frac{\pi}{3}$$

$$x = \frac{\pi}{3} + 2n\pi \, , \quad x = \frac{5\pi}{3} + 2n\pi$$

Therefore solution set is

$$\{n\pi\}\cup\left\{\frac{\pi}{3}+2n\pi\right\}\cup\left\{\frac{5\pi}{3}+2n\pi\right\},\quad n\in z$$

**Q.14** $\sin 4x-\sin 2x=\cos 3x$

**Solution:**

$$\sin 4x-\sin 2x=\cos 3x$$

$$2\cos\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right)=\cos 3x$$

$$2\cos 3x\sin x-\cos 3x=0$$

$$\cos 3x\,[2\sin x-1]=0$$

$$\cos 3x=0$$

$$\Rightarrow\quad 3x=(2n+1)\frac{\pi}{2}$$

$$\Rightarrow\quad x=(2n+1)\frac{\pi}{6}$$

$$2\sin x-1=0$$

$$\Rightarrow\quad 2\sin x=1$$

$$\Rightarrow\quad \sin x=\frac{1}{2}\qquad \boxed{x=\sin^{-1}\left(\frac{1}{2}\right)=30^\circ=\frac{\pi}{6}}$$

Since $\sin x$ is +ve in I and II Quadrants

with reference angle $\frac{\pi}{6}$ so

**For I-Quad.**

$$x=\frac{\pi}{6}$$

$$x=\frac{\pi}{6}+2n\pi,$$

**For II-Quad.**

$$x=\pi-\frac{\pi}{6}$$

$$x=\frac{5\pi}{6}+2n\pi,\ \forall n\in z$$

Hence solution set is

$$\left\{(2n+1)\frac{\pi}{6}\right\}\cup\left\{\frac{\pi}{6}+2n\pi\right\}\cup\left\{\frac{5\pi}{6}+2n\pi\right\},n\in Z\qquad \text{Ans.}$$

**Q.15** $\sin x+\cos 3x=\cos 5x$

**Solution:**

$$\sin x+\cos 3x=\cos 5x$$

$$\sin x=\cos 5x-\cos 3x$$

$$\sin x=-2\sin\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)$$

$$=-2\sin 4x\sin x$$



$$\sin x+2\sin 4x\sin x=0$$

$$\sin x(1+2\sin 4x)=0$$

$$\sin x=0$$

$$x=n\pi$$

$$\sin 4x=-\frac{1}{2}\ \Rightarrow\ \boxed{4x=\sin^{-1}\left(\frac{1}{2}\right)=30^\circ=\frac{\pi}{6}}$$

since $\sin x$ is –ve in III & IV Quadrants

with reference angle $\frac{\pi}{6}$

**For III-Quad.**

$$4x=\pi+\frac{\pi}{6},$$

$$4x=\frac{7\pi}{6}+2n\pi,$$

$$x=\frac{7\pi}{24}+\frac{n\pi}{2},$$

**For II-Quad.**

$$4x=2\pi-\frac{\pi}{6}$$

$$4x=\frac{11\pi}{6}+2n\pi$$

$$x=\frac{11\pi}{24}+\frac{n\pi}{2}$$

Hence solution set $\{n\pi\}\cup\left\{\frac{7\pi}{24}+\frac{n\pi}{2}\right\}\cup\left\{\frac{11\pi}{24}+\frac{n\pi}{2}\right\},\quad n\in z$ Ans.

**Q.16** $\sin 3x+\sin 2x+\sin x=0$

**Solution:**

$$\sin 3x+\sin 2x+\sin x=0$$

$$\sin 3x+\sin x+\sin 2x=0$$

$$2\cos\left(\frac{3x-x}{2}\right)\sin\left(\frac{3x+x}{2}\right)+\sin 2x=0$$

$$2\sin 2x\cos x+\sin 2x=0$$

$$\sin 2x(2\cos x+1)=0$$

$$\sin 2x=0$$

$$2x=n\pi$$

$$x=\frac{n\pi}{2}$$

$$2\cos x+1=0\qquad \boxed{x=\cos^{-1}\left(\frac{1}{2}\right)=60=\frac{\pi}{3}}$$

$$\cos x=\frac{-1}{2}$$

Since cos is –ve in II & III Quadrants

with reference angle $\frac{\pi}{3}$

**For II-Quad.**

$$x=\pi-\frac{\pi}{3},$$

$$x=\frac{2\pi}{3}+2n\pi,$$

**For III-Quad.**

$$x=\pi+\frac{\pi}{3}$$

$$x=\frac{4\pi}{3}+2n\pi,\ \forall n\in z$$

Solution set is $\left\{\frac{n\pi}{2}\right\}\cup\left\{\frac{2\pi}{3}+2n\pi\right\}\cup\left\{\frac{4\pi}{3}+2n\pi\right\},\quad n\in z$ Ans.

**Q.17 $\sin 7x - \sin x = \sin 3x$**

**Solution:**

$$\sin 7x - \sin x = \sin 3x$$
$$2\cos\left(\frac{7x+x}{2}\right)\sin\left(\frac{7x-x}{2}\right) = \sin 3x$$
$$2\cos 4x \sin 3x = \sin 3x$$
$$2\cos 4x \sin 3x - \sin 3x = 0$$
$$\sin 3x(2\cos 4x - 1) = 0$$

$\Rightarrow$

$$\sin 3x = 0$$
$$3x = n\pi$$
$$x = \frac{n\pi}{3}$$

$$2\cos 4x - 1 = 0$$
$$2\cos 4x = 1$$
$$\cos 4x = \frac{1}{2} \qquad \boxed{4x = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ = \frac{\pi}{3}}$$

Since $\cos x$ is +ve in I & IV Quadrants.

with reference angle $\frac{\pi}{3}$

| For I-Quad. | For IV-Quad. |
|---|---|
| $4x = \frac{\pi}{3}$ , | $4x = 2\pi - \frac{\pi}{3}$ |
| , | $4x = \frac{5\pi}{3}$ |
| $4x = \frac{\pi}{3} + 2n\pi$ , | $4x = \frac{5\pi}{3} + 2n\pi$ |
| $x = \frac{\pi}{12} + \frac{n\pi}{2}$ , | $x = \frac{5\pi}{12} + \frac{n\pi}{2}$ |

Therefore solution set is $\left\{\frac{n\pi}{3}\right\} \cup \left\{\frac{\pi}{12} + \frac{n\pi}{2}\right\} \cup \left\{\frac{5\pi}{12} + \frac{n\pi}{2}\right\}$ $n \in z$ Ans.

**Q.18 $\sin x + \sin 3x + \sin 5x = 0$**

**Solution:**

$$\sin 5x + \sin x + \sin 3x = 0$$
$$2\sin\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right) + \sin 3x = 0$$





$$2\sin 3x\cos 2x + \sin 3x = 0$$
$$\sin 3x(2\cos 2x + 1) = 0$$

$$\sin 3x = 0$$
$$3x = n\pi$$
$$x = \frac{n\pi}{3}$$

$$2\cos 2x + 1 = 0$$
$$\cos 2x = \frac{-1}{2}$$
$$\boxed{2x = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ = \frac{\pi}{3}}$$

Since $\cos x$ is –ve in II & III Quadrants,

with reference angle $\frac{\pi}{3}$ so

| For I-Quad. | For III-Quad. |
|---|---|
| $2x = \pi - \frac{\pi}{3}$ , | $2x = \pi + \frac{\pi}{3}$ |
| $2x = \frac{2\pi}{3}$ , | $2x = \frac{4\pi}{3}$ |
| $2x = \frac{2\pi}{3} + 2n\pi$ , | $2x = \frac{4\pi}{3} + 2n\pi$ |
| $x = \frac{\pi}{3} + \frac{2n\pi}{2}$ , | $x = \frac{2\pi}{3} + \frac{2n\pi}{2}$ |
| $x = \frac{\pi}{3} + n\pi$ , | $x = \frac{2\pi}{3} + n\pi$ |

Hence solution set is

$$\left\{\frac{n\pi}{3}\right\} \cup \left\{\frac{\pi}{3} + \frac{2n\pi}{3}\right\} \cup \left\{\frac{2\pi}{3} + \frac{2n\pi}{3}\right\}, \quad n \in z \text{ Ans.}$$

**Q.19 $\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 0$**

**Solution:**

$$[\sin 7\theta + \sin\theta] + [\sin 5\theta + \sin 3\theta] = 0$$
$$\left[2\sin\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right)\right] + \left[2\sin\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)\right] = 0$$
$$2\sin 4\theta\cos 3\theta + 2\sin 4\theta\cos\theta = 0$$
$$2\sin 4\theta(\cos 3\theta + \cos\theta) = 0$$
$$2\sin 4\theta\left[2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)\right] = 0$$
$$2\times 2\sin 4\theta(\cos 2\theta\cos\theta) = 0$$
$$\sin 4\theta\cos 2\theta\cos\theta = 0$$

$\Rightarrow \sin 4\theta = 0$

$4\theta = n\pi$

$\theta = \frac{n\pi}{4}$

$\cos 2\theta = 0$

$2\theta = (2n+1)\frac{\pi}{2}$

$\theta = (2n+1)\frac{\pi}{4}$

$\cos\theta = 0$

$\theta = (2n+1)\frac{\pi}{2}$

Hence solutions set is

$$\left\{\frac{n\pi}{4}\right\} \cup \left\{(2n+1)\frac{\pi}{4}\right\} \cup \left\{(2n+1)\frac{\pi}{2}\right\}, n \in Z \quad \text{Ans.}$$

OR

$$\left\{\frac{n\pi}{4}\right\} \cup \left\{\frac{\pi}{4} + \frac{n\pi}{2}\right\} \cup \left\{\frac{\pi}{2} + n\pi\right\}, n \in z$$

**Q.20** $\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$

**Solution:**

$$[\cos 7\theta + \cos\theta] + [\cos 5\theta + \cos 3\theta] = 0$$

$$2\cos\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right) + 2\cos\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right) = 0$$

$$2\cos 4\theta\cos 3\theta + 2\cos 4\theta\cos\theta = 0$$

$$2\cos 4\theta(\cos 3\theta + \cos\theta) = 0$$

$$2\cos 4\theta\left[2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)\right] = 0$$

$$2\times 2\cos 4\theta(\cos 2\theta\cos\theta) = 0$$

$\Rightarrow \cos 4\theta = 0$

$4\theta = (2n+1)\frac{\pi}{2}$

$\theta = (2n+1)\frac{\pi}{8}$

$\cos 2\theta = 0$

$2\theta = (2n+1)\frac{\pi}{2}$

$\theta = (2n+1)\frac{\pi}{4}$

$\cos\theta = 0$

$\theta = (2n+1)\frac{\pi}{2}$

Hence solution set is $\left\{(2n+1)\frac{\pi}{8}\right\} \cup \left\{(2n+1)\frac{\pi}{4}\right\} \cup \left\{(2n+1)\frac{\pi}{2}\right\}, n \in z$

OR

$$S.S = \left\{\frac{\pi}{8} + \frac{n\pi}{4}\right\} \cup \left\{\frac{\pi}{4} + \frac{n\pi}{2}\right\} \cup \left\{\frac{\pi}{2} + n\pi\right\}, n \in z$$