Linear Algebra Notes by Dawood Sarfraz

18.06 Linear Algebra
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Text: Introduction to Linear Algebra

http://web.mit.edu/18.06

Lecture # 1

Fundamental problem of LA is solve system of Linear eqs.

CASES

* n linear equations, n unknowns (equal num of eq & unknowns).

(i) Row pic (pic of one eq at a time like 2x2 eq where lines meet).

ii) * Column pic (A column at a time those are rows & columns of a matrix)

iii) Matrix Form

* Two eqs two unknowns.

$$2x - y = 0$$
$$-x + 2y = 3.$$

Matrix: its Rectangular array of numbers or elements arranged in Rows/columns

whats co-efficient matrix??

$$\begin{array}{ccc} A & X & = B \\ \begin{bmatrix} 2 & -1 \\ -1 & +2 \end{bmatrix} & \begin{bmatrix} x \\ y \end{bmatrix} & = \begin{bmatrix} 0 \\ 3 \end{bmatrix} \end{array}$$

Matrix 2x2. co-effient matrix.

unknown vector.

vector.

Pg # 2

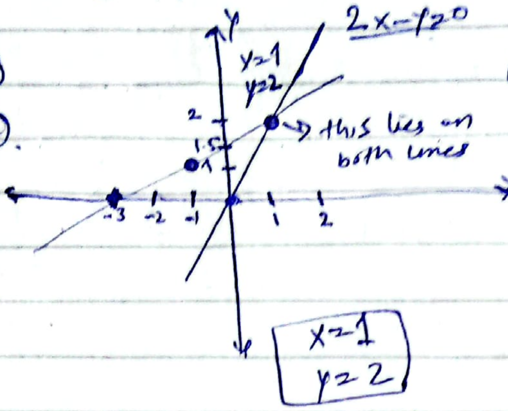
Row picture

$2x - y = 0 \rightarrow (1)$

$-x + 2y = 3 \rightarrow (2)$

$$\begin{bmatrix} 2 & -1 \\ -1 & +2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

A x = B



(1) if x = 1 & y = 2 then will by origin 0. line shows all points that is pass by origin to fulfil $\boxed{2x - y = 0}$

* if $x = \frac{1}{2}$, $y = 1$, also satisfy

If y = 0 we get in $\boxed{-x + 2y = 3}$ (−3) & ~~...~~ solves both eqns

If x = −1 & y should y = 1

$-(-1) + 2(1) = 3$

* There should a point which satisfy equation

* Play with GeoGebra

(2) $-x + 2y = 3$ Not passing through origin.

* Its always important will we pass by origin or NOT

in first $\boxed{2x - y = 0, \text{ Yes}}$

in 2nd $\boxed{-x + 2y = 3, \text{ No}}$

If x & y are zero we don't get three $\boxed{-x + 2y = 3}$

Column Pic

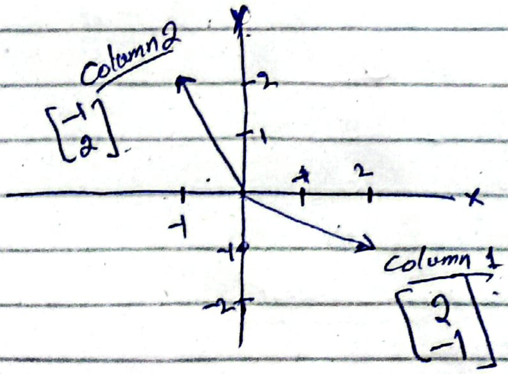
$$x \begin{bmatrix} 2 \\ -1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Colum 1   Columns of equ.   column 2

Need to combine right amount of L.H.S to get R.H.S.

~~Vector~~. Column vector.

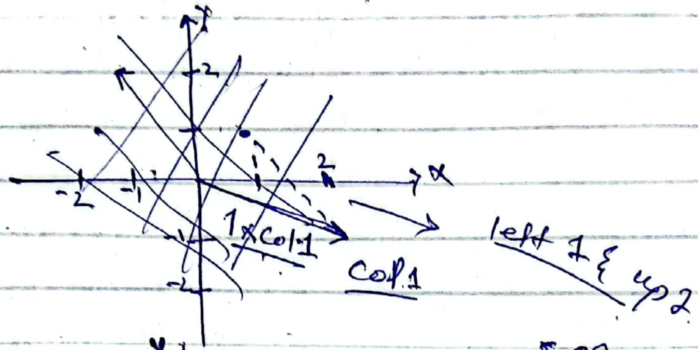
its linear Combinations of columns.



As x & y are +ve so using always +ve sides

Right combination's $x=1$ & $y=2$. bce we already know from Row picture.

$$1\begin{bmatrix}2\\-1\end{bmatrix}+2\begin{bmatrix}-1\\2\end{bmatrix}=\begin{bmatrix}0\\3\end{bmatrix}$$



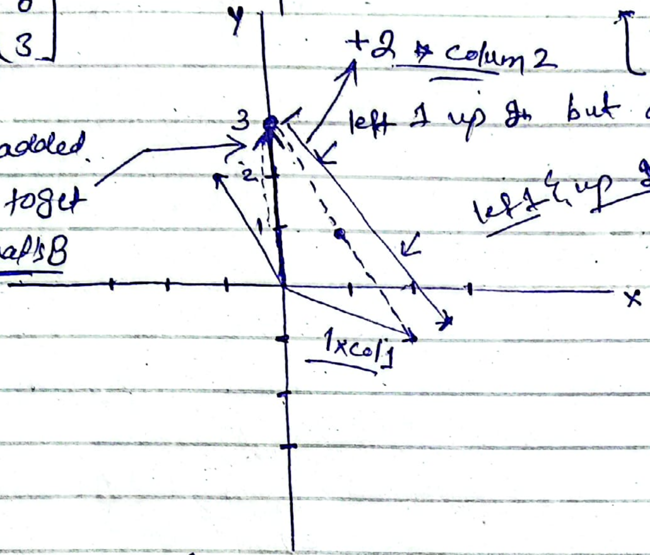
$$\begin{bmatrix}2\\-1\end{bmatrix}+\begin{bmatrix}-2\\4\end{bmatrix}=\begin{bmatrix}0\\3\end{bmatrix}$$

+2 * colum 2 $\begin{bmatrix}0\\3\end{bmatrix}$ $x=0, y=3$

left 1 up 2 but do it from y axis.

where five added --- on y.

but in pic added col 1 & col 2 toget this guy that's B



What are all the combinations

$$x\begin{bmatrix}2\\-1\end{bmatrix}+y\begin{bmatrix}-1\\2\end{bmatrix}=\begin{bmatrix}0\\3\end{bmatrix}$$

If took all x & y combination what'll be results. that could be that i can get any right hand-side at all. It would fill the whole plane.

PG#4

$2x - y = 0 \rightarrow eq1$

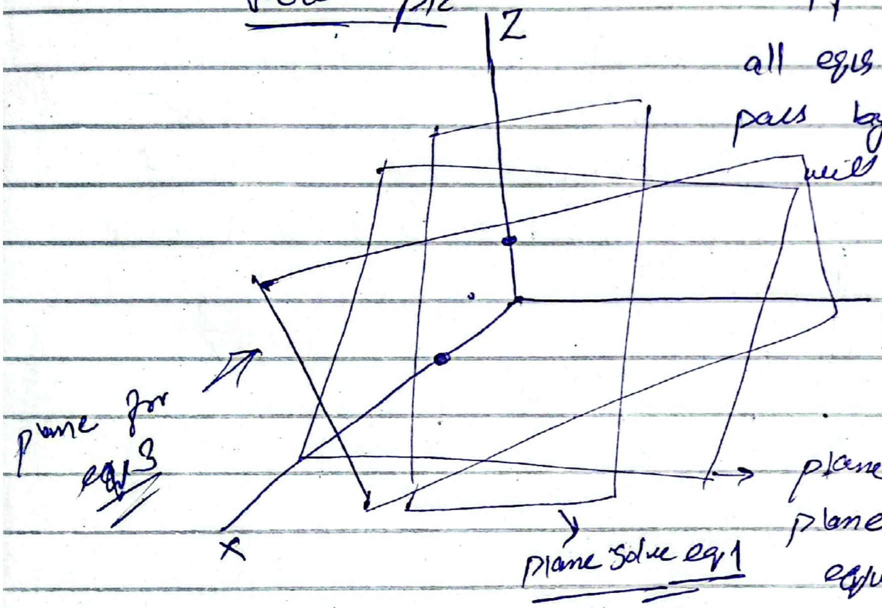
$-x + 2y - z = -1 \rightarrow eq2$

$-3y + 4z = 4 \rightarrow eq3$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

Row pic

If take x,y,z to solve all eqs. then None of them pass by origin, x=0,y=0,z=0 will not solve



eq2 if x=1, y & z =0 could be zero that'd work.

each

eq1 $2x-y=0$ each z can be anything. each row in 3x3 problem gives us a plane in 3 Dimensions.

These two planes meet at a line.

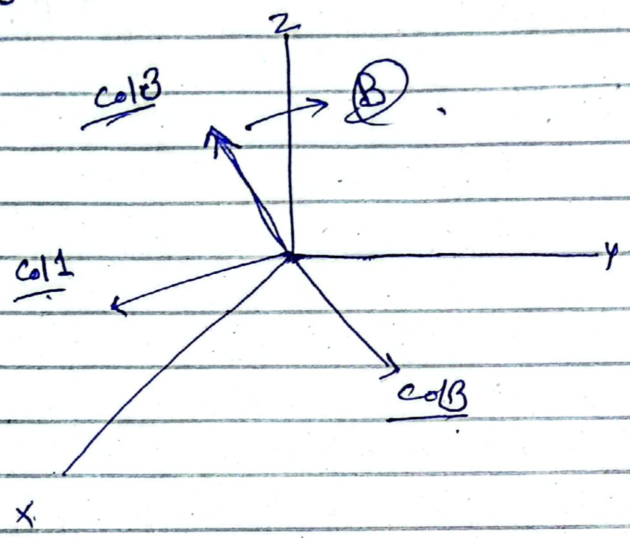
these planes meet at a point but LA will find.

maybe hard to get row pic.

Pg #5

Column pic

$$x\begin{bmatrix}2\\-1\\0\end{bmatrix}+y\begin{bmatrix}-1\\2\\-3\end{bmatrix}+z\begin{bmatrix}0\\-1\\4\end{bmatrix}=\begin{bmatrix}0\\-1\\4\end{bmatrix}$$



None of x, y but z is solution
mean $x=0$, $y=0$, & $z=1$.

so. Column3 is same as B.

$$0\begin{bmatrix}2\\-1\\0\end{bmatrix}+0\begin{bmatrix}-1\\2\\-3\end{bmatrix}+1\begin{bmatrix}0\\-1\\4\end{bmatrix}=\begin{bmatrix}0\\-1\\4\end{bmatrix}.$$

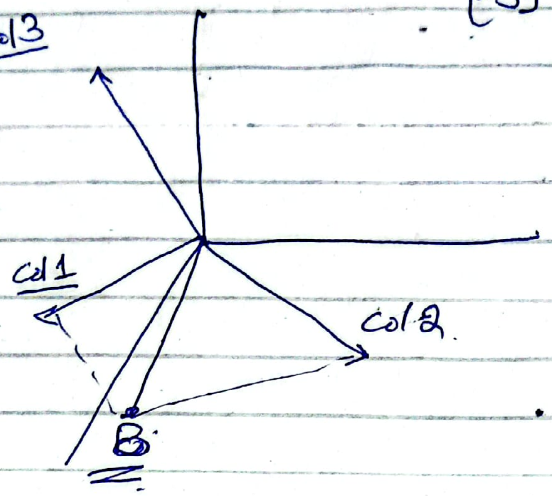
$$0+0+\begin{bmatrix}0\\-1\\4\end{bmatrix}=\begin{bmatrix}0\\-1\\4\end{bmatrix}$$

Now change R.H.S.

$$x\begin{bmatrix}2\\-1\\0\end{bmatrix}+y\begin{bmatrix}-1\\2\\-3\end{bmatrix}+z\begin{bmatrix}0\\-1\\4\end{bmatrix}=\begin{bmatrix}1\\1\\-3\end{bmatrix}$$

Now take $x=1$, $y=1$, & $z=0$
Now in Row pic three new planes.
but in column pic have same three columns.

but combining to produce $\begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}$



- Can I solve $Ax=B$ for every B ??
- Do linear combinations of columns fill 3dimensional space (every B means all B in 3D space).

If multiple matrix by a vector i get combination of columns.

yes. The Ans for This A, is yes

This matrix that i choose for an example is good matrix A Non-Singular matrix, an Investible matrix.
Those will be the matrix that we like best

We will see other matrix where answer become NO - when it could wrong??
If all lies (3 colmns) lie on same plane then their combinations will lie in that same plane.
Then we're in trouble.
If the three columns of my matrix. If those three vectors happens to lie in the same plane

For example, if column three is just sum of column 1 & column 2. It would be in trouble. That be a matrix A where answer would be No

bcz in linear combination, if column three is in the same plane as column One & two. I don't get anything new from that. All combinations are in the ~~same~~ plane and only right hand sides B that I could get would be the Ones in that plane,
Most R.H.S would be out of plane and unreachable. so that would be a singular case. The matrix would be not invertible. There would not be a solution for every B. so Answer would be No

imagine vectors with Nine components. 9D
Nine eqs. 9 unknowns. then we'd have 9-columns. & each One would be a vector in 9-D space. we'd be trying to find combination that hit the correct right hand side B.

And we might also the question can we always do it??
Can we get every ~~hand~~ right hand side B??
Still depends on those Nine columns.

If a picked a random matrix. A would be yes. actually if we MATLAB & just used random command, picked out of ~~Aone~~ 9×9 matrix. It would be Non-singular, & invertible all good.

If choose those columns so that they're Not independent. So that 9th column is same as 8th column.

It contributes nothing new and there so I could not get B. Can you sort of think about Nine vectors in 9-D space and take their combinations??
Those 9 columns & all their combinations may very well fill out the whole 9-D space. But if 9th same as 8th. colums. will gave nothing new, then probably what it would fill out would be a sort of plane & 8th. dimensional plane inside 9th- Dimensional space
And its those 8-D ~~sphere~~ inside 9-D space that we have to work with

Matrix form

$A x = B$
matrix times vector -

$$= \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

1st Method

$$= 1\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 10 \\ 6 \end{bmatrix} = \begin{bmatrix} 2+10 \\ 1+6 \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$

2nd Method

$$\begin{bmatrix} 2(1)+5(2) \\ 1(1)+3(2) \end{bmatrix} = \begin{bmatrix} 2+10 \\ 1+6 \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$