FIT2014 Assignment 1 Problem 2 – Damien Ambegoda (30594235)

a.

$$F1 = 274$$

$$F2 = 274^2 + 12 = 75088$$

$$F3 = 274^3 + 2 \times 12 \times 274 = 2057400$$

b. The upper bound is:

$$Fn = 274F_{n-1} + 12F_{n-2} \text{ for all n } \geq 3$$

The justification for this is that there are two ways to get a string of n characters. Either add a single character name (274 choices) to every single possible string of size n-1 characters or add a double character (12 choices) name to every single possible string of size n-2 characters. Add them together to get all strings of n characters.

$274\mathrm{F}_{\mathrm{n}-1}$ accounts for adding any of 274 character to all strings of size n-1.

$12\mathrm{F}_{\mathrm{n}-2}$ accounts for adding any of 12 double character names to all strings of size n-2.

The reason why this is an upper bound and not an exact value is because some double character names are 2 single character names in a specific order. For example, 남궁 (Namgung) is the single character name 남 (Nam) plus the single character name 궁 (Gung).[1] This would be counted twice as it would be included as both a double character name as well as two individual single character names. This means that this calculation includes too many strings and is an upper bound and not an exact value.

c. The statement that is trying to be proven is that $Fn \leq 274.044^n$ for all $n \geq 1$. This will be proven by induction. Fn is defined as $274F_{n-1} + 12F_{n-2}$.

<u>Base cases:</u>

$$\mathrm{F}_1 = 274 \leq 274.044^1$$

$$274 \leq 274.044^1$$

$$\text{Therefore, } \mathrm{F}_1 \leq 274.044^1$$

$$\mathrm{F}_2 = 274^2 + 12 = 75088 = 274^2$$

$$274^2 \leq 274.044^2$$

$$\text{Therefore, } \mathrm{F}_2 \leq 274.044^2$$

$$\mathrm{F}_3 = 274F_2 + 12F_1 = 274 \times 75088 + 12 \times 274 = 20577400 = 274.0291939\ldots^3$$

$$274.0291939\ldots^3 \leq 274.044^3$$

$$\text{Therefore, } \mathrm{F}_3 \leq 274.044^3$$

Therefore, the statement is true for n = 1, 2 and 3

[1] Names from: https://en.wikipedia.org/wiki/List_of_Korean_surnames

Inductive hypothesis:

Assume that for some value up until and including n, $F_n \leq 274.044^n$

Inductive Step:

It must be shown that the statement is true for n+1.

$$F_{n+1} = 274F_n + 12F_{n-1}$$

By the hypothesis, F_n must be $\leq 274.044^n$ and F_{n-1} must be $\leq 274.044^{n-1}$.

$$\begin{aligned} 274F_n + 12F_{n-1} &\leq 274(274.044^n) + 12(274.044^{n-1}) \\ &\leq 274.044^n(274 + 12 \times 274.044^{-1}) \\ &\leq 274.044^n(274 + \frac{12}{274.044}) \\ &\leq 274.044^n(274 + 0.0437885\ldots) \\ &\leq 274.044^{n+1} \text{ (as } 274.0437885 \leq 274.044) \end{aligned}$$

Therefore, by the principle of mathematical induction, the statement is true for all $n \geq 1$.