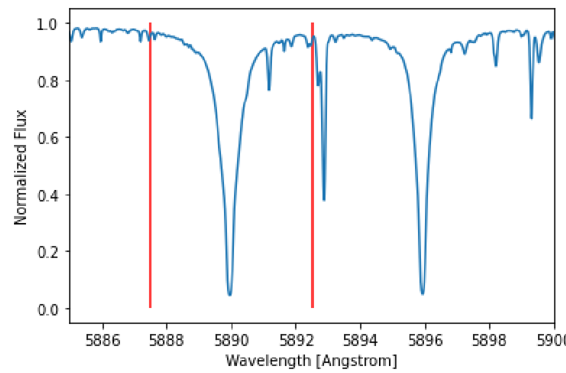
1. Introduction

The composition of planets can be estimated through calculating a star's stellar abundance. This method involves using a spectra of the planet's host star to determine what elements are most likely to be present within a planet. A spectra measures the brightness of light at different wavelengths by showing absorption or emission of certain wavelengths of light. The specific emission or absorption pattern tells us what elements that are located in the object we are measuring and about how much there is based on the intensity. Taking the spectra of a star tells us the stellar abundance, which is what and how much of each element is within the star. The motivation behind this is that this can be used to estimate the elemental structure of exoplanets orbiting it due to a well established relationship between a planet's chemical composition and its host star's composition. To demonstrate performing and analyzing stellar abundance spectra, we will use real data of a solar spectrum to find the abundance of Sodium (Na), compare the different states of Sodium in the abundance, and compare the abundance of Sodium to that of Hydrogen (H).

2. Methods

To analyze the stellar abundance spectrum of Sodium, we first download the solar spectrum as a .txt file from BASS2000. From there, we plot it in Jupyter Notebooks (shown below) between the wavelengths 5885 Å - 5900 Å. Moving forward, we will use various formulae to compute the number density of Sodium atoms in the ground state, neutral Sodium atoms, ionized Sodium atoms, and the total number density of Sodium atoms.

Solar Sodium Spectrum



2.1 Number Density of Sodium in the Ground State

The equivalent width was calculated using the following formula:

$$\log \frac{W}{\lambda} = -3.85 \quad (1)$$

Once the equivalent width is obtained, we calculate the number density of ground state Sodium atoms with

$$log\left(\frac{N^*f^*\lambda}{5000\,\text{Å}}\right) = 14.8. \quad (2)$$

This equation was simply rearranged to solve for N, the number density of Sodium in the ground state. f is the oscillator strength for this transition at 5890 Å, which is given the value 0.65.

2.2 Number Density of Neutral Sodium Atoms

To compute the number density of neutral Sodium atoms, we add the number density of excited Sodium to the number density of ground state Sodium. Since we have already computed the number density of Sodium in the ground state, we use the Boltzmann equation to compute the ratio between the number density of excited Sodium and the number density of ground state Sodium. The equation used is:

$$\frac{N_{excited}}{N_{ground}} = \frac{g_{excited}}{g_{ground}} exp\left(- \frac{E_{excited} - E_{ground}}{kT}\right) \quad (3)$$

The excited subscript refers to the 3p state and the ground subscript refers to the 3s state, g refers to the number of separate and individual states that are degenerate in energy (2 for 3s and 6 for 3p), E refers to the energy, k is the Boltzmann constant, and T is the effective temperature. Note that the g fraction value used is 3. Once the ratio is obtained, we can then solve for the number density in the excited state.

Once this is done, we simply find the number density of neutral Sodium atoms by adding the number densities of excited state Sodium and ground state Sodium using $N_{neutral} = N_{ground} + N_{excited}$. (4)

2.3 Number Density of Ionized Sodium Atoms

To find the number density of ionized Sodium atoms, we must first use the Saha equation to solve for the ratio of the number density of neutral Sodium atoms to that of ionized Sodium atoms. Since we have already computed the number density of neutral Sodium atoms, we can use this ratio to solve for the number density of ionized Sodium atoms. The equation used is

$$\frac{Na_{II}}{Na_I} = \frac{2kT}{P_e} \frac{Z_{II}}{Z_I} \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(-\frac{\chi}{kT}\right) . \quad (5)$$

P_e is the electron pressure, given the value 1 N/m^2. Z is the partition function, with $Z_I = 2.4$ and $Z_{II} = 1$, and χ is the ionization energy, given the value 5.1 ev. Note that N_{aII} is the number density of neutral Sodium atoms, while N_{aI} is the number density of ionized Sodium atoms.

2.4 Total Number Density of Sodium Atoms in All States

To find the total number density of Sodium atoms in all states, we add the number densities of Sodium in the ground state, the excited state, and ionized Sodium using

$$N_{total} = N_{ground} + N_{excited} + N_{ionized} . \quad (6)$$

2.5 Relative Abundance of Sodium to Hydrogen in Various Formats

2.5.1 Physicist's Version

This version of the relative abundance between Sodium and Hydrogen is simply the mole ratio between Sodium and Hydrogen, shown below:

$$N_{Na}/N_H . \quad (7)$$

2.5.2 Galactic Astronomy Version

This version involves taking the log of the mole ratio between Sodium and Hydrogen, with an added offset of 12. The formula for this format is shown below:

$$12 + \log \frac{Na}{H} . \quad (8)$$

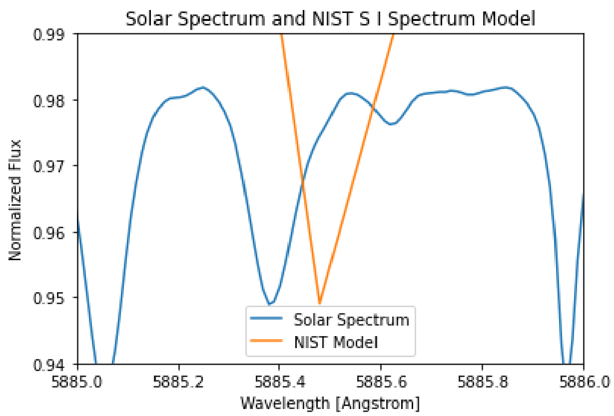
2.5.3 Stellar Astronomy Version

This version involves taking the log of the fraction of the mole ratio of Sodium to Hydrogen, divided by the mole ratio of Sodium to Hydrogen of the Sun. The formula for this format is shown below:

$$\log \frac{\frac{N_{Na}}{N_H}}{(\frac{N_{Na}}{N_H})_{sun}} \quad (9)$$

2.6 Bonus: Sulfur Spectrum

The above steps were repeated using solar Sulfur absorption lines, with the data being retrieved from the NIST database. The solar spectrum plot of Sulfur is shown below.



3. Results

	Sodium (Na)	Sulfur (S)
No. Density of Atoms in Ground State	8.2e14 atoms/cm^2	3.257e14 atoms/cm^2
No. Density of Neutral Atoms	8.6e14 atoms/cm^2	-
No. Density of Ionized Atoms	2.16e18 atoms/cm^2	-
Total No. Density of Atoms in All States	2.16e18 atoms/cm^2	3.07e17 atoms/cm^2
In Physics Terms	1.99e-6	1.47e-5
In Galactic Astronomy Terms	6.5	7.17
In Stellar Astronomy Terms	0	0

	Sodium (Na)
Boltzmann Ratio	**0.0438**
Saha Ratio	**2517.95**

4. Discussion

We found out that we can use the Sodium doublet lines to figure out how much Sodium there is in the Sun. This helps us learn more about what the Sun is made of, which is important to study other distant stars. We can also use this method to study other elements, which we did. We were able to find the abundance of Sodium and Sulfur. This leads us to a better understanding of the Sun's composition.

Table 4. Elemental abundances in the solar photosphere and in CI-chondrites [log N(H) = A(H) = 12]

		Solar Photosphere	σ dex	Orgueil CI-chondrite	σ dex	Sun/Meteorite N_{sun}/N_{met}
1	H	12		8.24	0.04	5710
2	He	10.925	0.02	1.31		4.1E+09
3	Li	1.10	0.10	3.28	0.05	0.01
4	Be	1.38	0.09	1.32	0.03	1.15
5	B	2.70	0.17	2.81	0.04	0.78
6	C	8.39	0.04	7.41	0.04	9.46
7	N	7.86	0.12	6.28	0.06	38.4
8	O	8.73	0.07	8.42	0.04	2.06
9	F	4.56	0.30	4.44	0.06	1.32
10	Ne	8.05	0.10	-1.10		1.4E+09
11	Na	6.30	0.03	6.29	0.02	*1.03*
12	Mg	7.54	0.06	7.55	0.01	*0.98*
13	Al	6.47	0.07	6.45	0.01	*1.05*
14	Si	7.52	0.06	7.53	0.01	*0.97*

Abundance of the Elements in the Solar System; Loders, et al. 2009

By comparing our findings to the table of elemental abundances in the solar photosphere found by Loders, et al, we can see that our results are not too far deviated from the real numbers, assuring that our findings are not wrong.

Looking at the ratio values obtained, we can see that there appears to be about 22.8 ground state Sodium atoms per every excited Sodium atom. Note that this was calculated by taking the reciprocal of the obtained Boltzmann ratio. The Saha ratio tells us that there are about 2517.95 ionized Sodium atoms per every one neutral Sodium atom, which indicates that most of the Sodium found in the Sun is ionized. This is inline with the calculated total number density of Sodium atoms in all states and the number density of ionized Sodium atoms, as the number densities are very close to one another, being almost equal.

5. Conclusion

The growth curve method was instrumental in determining the elemental abundance of elements in the Sun's photosphere. This method allowed us to determine the number of neutral atoms in a lower state based on the equivalent widths of a given line. The Boltzmann equation allowed us to determine the number of neutral atoms in all states and the Saha equation allowed for the number of all atoms of an element to be calculated. The relative abundance for Sodium was found to be 6.5, compared to the actual value of 6.3. Our results match with the determined abundance. Additionally, this process was repeated for Sulfur and the abundance was found to be 7.17, compared to the actual value of 7.17. Overall, this project allowed us to explore atomic physics and to understand how an abundance of an element can be determined from the spectrum of a star.

6. Contribution Statement

The coding for the main part of the project (Sodium abundance) was headed by Omar and Greg and the paper and slides were worked on by everyone. Omar headed the Methods and Discussion sections of both the slides and paper, working closely with Greg who headed the results section of the paper and slides. Dax headed the conclusion section of the paper and slides, while also having done the coding and bonus section for the element of Sulfur. Matthew headed the introduction sections to both the slides and the paper.

References

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