Measuring Planet Mass, Radius, and Density

Dax Begeny, Omar Kotrach, Matthew Williard and Gregory Costa

1. Introduction

While having previously studied various exoplanet detection methods, and their strengths and weaknesses, we can now comfortably use these methods to understand exoplanets better. We can do this by using various methods that are more suited to calculating a given variable of an exoplanet, such as using the transit method to compute an exoplanet's radius or using the radial velocity method to detect an exoplanet's mass.

In the following paper, we do just that. We compute the mass, radius, and density (and the uncertainties in these values) of the planet HD 209 458 b (also known as Osiris). HD 209 458 b is located roughly 49 parsecs away, orbiting HD 209458 in the Pegasus constellation. This is one of the first planets to have had its atmospheric evaporation observed, as it is a hot-jupiter orbiting very close to its host star.

Our motivation for calculating the aforementioned values is to 1) better understand this exoplanet and its properties, 2) to see where HD 209 458 b sits in comparison to other exoplanets of similar radii and mass, and 3) to see where HD 209 458 b would be in comparison on the mass-radius relation plot found in Chen and Kipping (2016). Some other, smaller motivations include learning to fit data from exoplanet detections and learning to use the NEA.

2. Methods

In order to determine the mass, radius, and density of HD 209 458 b, there are several key attributes that have to be known. In order to know the radius of the planet, the transit depth and the radius of the host star must be known. In order for the mass of the planet to be known, the semi-major axis, mass of the host star, and the amplitude of the radial velocity signal must be known.

2.1 Determining the Radius

Transit detection is utilized to find exoplanets that pass in between its host star and the observer. During this process, the host star's light is dimmed. The transit depth, f is proportional to the square ratio of the host star's radius and the exoplanets radius.

$$f = \frac{R_*^2}{R_p^2} \tag{1}$$

The host star's radius is found first by determining its distance using the host star's absolute and apparent magnitude [1].

$$d = 10^{.2(m-M+5-A_v)} \tag{2}$$

Then, the host star's size can be determined by its angular diameter.

$$\delta = 2arctan(\frac{R_*}{d}) \tag{3}$$

2.2 Determining the Mass

The mass of the exoplanet can be found by using the mass of the host star, the planet's semi-major axis, and its period. Using the relationship between a star's radius and its mass, the semi-major axis can be determined using Kepler's Third Law [1].

$$a = \sqrt[3]{\frac{T^2GM_*}{4\pi^2}} \tag{4}$$

Once the semi-major axis is established, the following relationship between the amplitude of the radial velocity signal, mass of the host star, and the semi-major axis can be used to find the mass of the planet.

$$m_p = KM_* \sqrt{\frac{a}{GM_*}} \qquad (5)$$

2.3 Density

The density of HD 209 458 b can be found by dividing the mass of the planet by the planet's volume, which can be found using the radius.

$$\rho = \frac{3m}{4\pi r^3} \qquad (6)$$

3. Results

Understanding the radial velocity and transit detection methods allow us to accurately determine the radius, mass, and density of any exoplanet. This allows us to represent the population of exoplanets in terms of radius, mass and density.

3.1 The Radius of HD 209 458 b

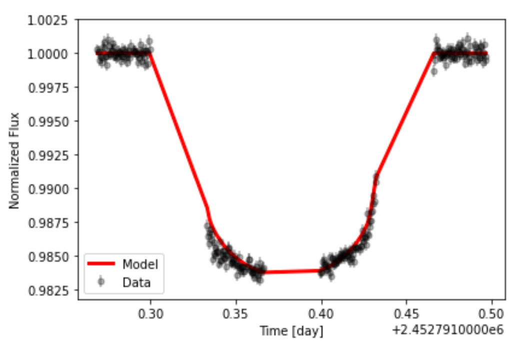

Figure 1.

The plot above shows data from a transit of HD 209 458 b where the time in days is plotted against the normalized flux. From this data, the radius relationship between the host star and the exoplanet can be determined using (1). Using the apparent and absolute magnitudes of HD 209 458, the distance to the star was found to be 159.5 light years. The radius of HD 209 458 is found to be 1.2 ± .06 R_{Sun}. Using the relative flux during the time of the transit, the radius of HD 209 458 b is 1.4 ± .07 $R_{Jupiter}$.

3.2 The Mass of HD 209 458 b

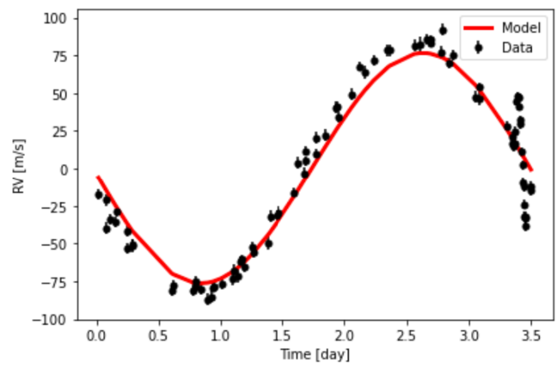

Figure 2.

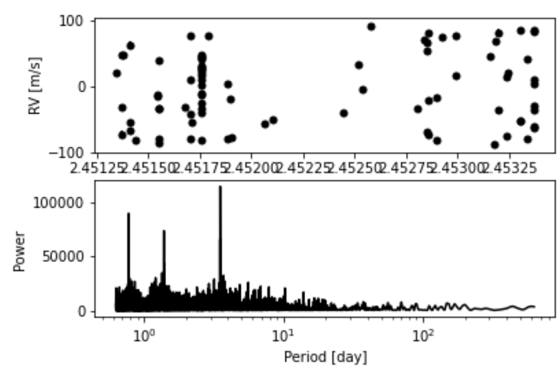

Figure 3.

The period of HD 209 458 b was found by using radial velocity data. Figure 2 shows the period of the planet in days against the detected radial velocity. For finding the period, the Lomb-Scargle periodogram was applied. This tests the significance of weak periodic signals with uneven sampling. The resulting period of HD 209 458 b was found to be 3.525 days, as shown in Figure 3.

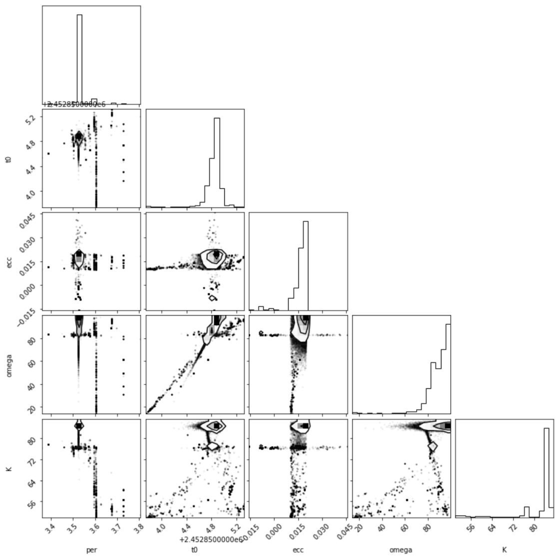

Figure 4.

Using Emcee, which created Figure 4., the radial velocity signal was determined to be around 86 m/s. Emcee is an ensemble sampler that efficiently samples the parameter space With this information, the mass of HD 209 458 b can be determined. Using equation (4), the semi-major axis of HD 209 458 b's orbit can be determined. Finally, using equation (5), the mass of HD 209 458 b was determined to be 0.71 ± .08 $M_{\mathbf{Jupiter}}$.

3.3 The Density of HD 209 458 b

Once the mass and the radius of HD 209 458 b have been determined, the density of the planet is simple to determine. The mass of the HD 209 458 b is divided by the radius of HD 209 458 b. Once this is done, the density of HD 209 458 b is 310.94 ± .04 kg/m^3.

4. Discussion

Having found the mass, radius, and density of exoplanet Osiris and comparing it to the many other exoplanets and Jupiter, we can form an understanding of the planet itself. It is less massive than Jupiter, falling in around 0.71 Jupiter mass, however it is larger than Jupiter itself, with a radius of about 1.4 Jupiter mass. This means that Osiris must be much less dense than Jupiter, and this is shown when plotting density in figure 6. Jupiter has a density of just over 10 grams per cubic centimeter while osiris has about 2 grams per cubic centimeter. This is about 0.2 Jupiter's density. This shows that having a larger mass does not necessarily mean that a planet will have a larger radius. If the density of a planet is lesser, the gas will spread out and have a larger radius. To check if this trend continues beyond Osiris and Jupiter, we check these measurements against other exoplanets

4.1 How do your measurements compare to other exoplanets with similar masses and radii?

We retrieved data from NASA's Exoplanet Archive (NAE) [3] and selected exoplanets with a radius running from 0.01 to 2 $R_{Jupiter}$ and a mass ranging from 0.2 to 1.2 $M_{Jupiter}$. This made a sample of 814 planets.

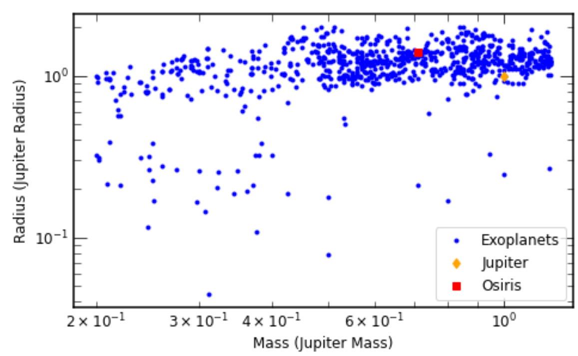

Figure 5.

The graph above plots Mass vs Radius and shows Osiris and Jupiter in comparison to other exoplanets with similar mass and radii

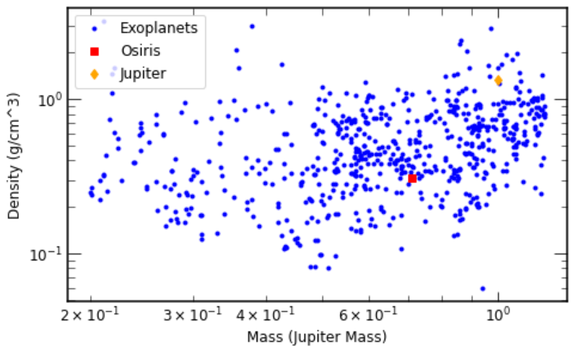

Figure 6.
The graph plots Mass vs Density and shows Osiris and Jupiter in comparison to other exoplanets with similar mass and radii. Osiris's position within the population of exoplanets appears to confirm that our calculations of mass, radius, and density were reasonable and are similar to other exoplanets.

4.1.2 Trends of the Data

The greater trend of the mass and radius relationship appears to be relatively flat. As the mass of exoplanets increases the radius of exoplanets does not seem to increase by much. The trend of mass against density is not as well defined as mass and radius, but focusing on the large cluster on the higher mass end, there appears to be a positive trend. As the mass of exoplanets increases, their density seems to increase. These two trends seem to indicate that more mass added to an exoplanet does not seem to actually increase the radius of the planet, rather the increased gravity from the mass crushes the gas down, increasing the density of the planet. However, it is clear this trend only continues up to a point. Very massive gas giants of several times Jupiter's mass have much larger radii than jupiter. It is likely that this trend only exists over small changes in radii, within one jupiter radius.

4.2 How do measurements compare with the M-R relation from Chen & Kipping (2016)?

In their paper, Chen and Kipping [4] create a model to forecast the mass/radius of an astronomical object based on a measurement of the other. The M-R power law would be described by:

$$R = C\,(M)^{s} \qquad (7)$$

The variables R and M represent the radius and mass, respectively. The parameters C and S are determined by the planet's properties. The values of S (-.044) is given for a jovian planet in the paper, but the value of C is not explicitly stated. Therefore, we used the values of Jupiter to determine the value of C, which was found to be 14.43. Using these values, we calculated the radius of Osiris using the M-R power law, which gave a result of 1.01 $R_{Jupiter}$. This value is 27.85% different from the expected value of 1.4 $R_{Jupiter}$. The paper also includes a graph showing the relationship between mass and radius.

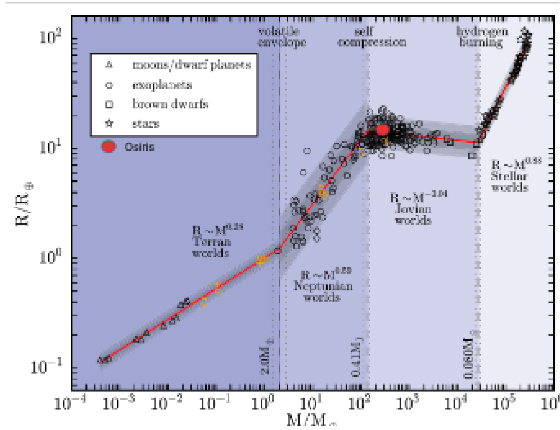

Figure 7
The graph above plots the relationship of Mass-Radius, Osiris is detonated by the red dot, and it shows that our calculations fit the relationship by Chen & Kipping.

5. Conclusion

In conclusion, this project involved analyzing the radial velocity and transit data of an exoplanet to determine its mass, radius and density. The radial velocity was used to measure the planet's mass, while the transit data was used to measure the planet's radius relative to the host star. Then, with the measured radius and mass, we could calculate the planet's density. The analysis revealed that the planet has a radius of 1.4 $R_{Jupiter}$ and mass of .71 $M_{Jupiter}$, with uncertainties of .07 and .08 respectively. The calculated density was found to be 310.94 kg/m^3 with an uncertainty of 0.04. Also, comparing our calculations to other exoplanets with similar masses and radii, it was found that the planet falls within the range of other exoplanets and the values predicted by the mass-radius relation of Chen and Kippin. This suggests that the planet's properties are consistent with current models of exoplanets. Overall, this project provided an opportunity to practice the skills of data analysis and uncertainty propagation in exoplanet observations.

6. Contribution Statement

Dax Begeny led the coding portion of this assignment, while also writing the methods and results and preparing the methods and results section of the presentation. Greg Costa created the plots of the discussion, wrote the conclusion section, prepared the conclusion section of the presentation, and helped to check the code. Matthew Williard aided Dax in the code, wrote the discussion, and prepared the discussion section of the presentation. Omar Kotrach wrote the introduction, prepared the introduction section of the presentation, and helped to check the code.

References

[1] Mazeh, T. (2000). The Spectroscopic Orbit of the Planetary Companion Transiting HD 209458.

[2] Ice plotter. (n.d.). Retrieved February 19, 2023, from https://exoplanetarchive.ipac.caltech.edu/cgi-bin/IcePlotter/nph-icePlotInit?mode=demo&set=confirmed

[3] Exoplanet Archive. (n.d.). Planetary Systems Table. California Institute of Technology. Retrieved February 21, 2023, from https://exoplanetarchive.ipac.caltech.edu/cgi-bin/TblView/nph-tblView?app=ExoTbls&config=PS

[4] Chen, J., & Kipping, D. (2017). Probabilistic forecasting of the masses and radii of other worlds. The Astrophysical Journal, 834(1), 17. https://doi.org/10.3847/1538-4357/834/1/17