1 Introduction

Exoplanets are a new frontier for astronomical observation, providing insight into the structure of solar systems beyond our own. They are studied closely, with many questions in mind such as how did the planet form? Is it similar to our own? Could there be life there? In order to study exoplanets and ask these questions it is necessary to employ a variety of methods to observe exoplanets, with new methods being researched constantly. Some observations of exoplanets, however, come from indirect observations. By observing the star of a planet, it is possible to draw conclusions about the planets in orbit around it. By using spectra to analyze the elements within the host star we can determine the composition of the planet and its interior structure. To demonstrate this, we take a look at the Kepler-105 system, and its orbiting planet Kepler-105c.

Kepler-105 is an F-type star about 459.2 parsecs away and is too faint to be seen with the naked eye. It is orbited by two exoplanets, Kepler-105b and Kepler-105c. Kepler-105b is a neptune-like gas giant planet. Kepler-105c is a planet that is a potentially rocky world larger than that of Earth. From the NASA Exoplanet Archive (NEA), it has a mass of $4.6 \pm 0.9\ M_{\oplus}$ and a radius of $\sim 1.3\ R_{\oplus}$ (Unterborn at al, 2023). Measurements of density for Kepler-105c have varied greatly between papers. On the NEA, two listed values of density are $2.7^{+3.9}_{-1.8}$ g/cm^3 (Hadden & Lithwick 2017) and $11.2^{+3}_{-2.56}$ g/cm^3 (Jontof-Hutter et al. 2016). This is a significant difference, and the true density likely falls between these two values, which is where the density of Earth falls at about 5.5 g/cm^3.

The parameters of Kepler-105c indicate that it is a rocky exoplanet. It has a radius of about 1.3 Earth radii. This places it within the parameters of likely being a super-earth, being in the range of 1-1.75 $R_{\oplus}$ (Fulton et al., 2017). This makes Kepler-105c likely rocky. While the density is not well defined, it is likely between ~11g/cm^3 and ~ 2.7 g/cm^3. This puts the likely true density close to that of Earth, or even more so. This further indicates that Kepler-105c is rocky. It is also very close to its host star, with a semi-major axis of 0.0731 AU and an orbital period of 7.126 days. This makes it unlikely to be gassy as the gas would be blown away by the star. While the parameters of Kepler-105c indicate it is rocky, it can be more thoroughly shown by analyzing its composition through the use of stellar composition. To do this, we will be using a program called Exoplex.

Exoplex is software that can be used to calculate the composition of exoplanets. By inputting the compositions found in the host star and the planet's mass, ExoPlex can calculate the structure and radius of the exoplanet. It will output the core mass fraction, interior mineralogy, interior pressure, adiabatic temperature, and gravity as a function of depth. We will be using this software to find the interior structure and abundances of Kepler-105c using the stellar abundances of its host star Kepler-105.

2 Methods

Kepler-105 c host star's compositions are taken from Unterborn et al. (2023). Next, in order to place Kepler-105 c's size and orbital distance in context, the literature is reviewed. The NASA Exoplanet Archive is used to place the orbital distance, mass, and radius into context. Lastly, the atmospheric radiation information is taken from ExoKyoto, an exoplanet database created by Kyoto University.

Next, using the values obtained in Unterborn et al. (2023), and the mass found in the NEA, ExoPlex was used to compute a possible radius for a planet with these characteristics. To find the uncertainties, the uncertainties found in the NEA are entered into Exoplex to determine the radii found at each end of the mass uncertainty. Once the radius is found, we then compute the volume of a planet with the given radius using $V = \frac{4}{3}\pi R^3$. Using this volume, the density can easily be calculated using $\rho = \frac{M}{V}$, with the mass being the observed value. The error of the density is calculated by propagating fractional uncertainties in quadrature..

Next, in order to find a model with ExoPlex that describes both the observed mass and radius, several inputs are changed until a radius and mass is computed that is similar to the observed values. These inputs include the following mantle abundance ratios: [Fe/O], [Fe/Mg] (thereby altering the core mass fraction), and core composition. Further, we are able to determine a likely range of planet structure that accurately describes the planet's radius and mass. The range of planetary composition values are then compared to that of Earth's.

The values obtained from the NEA for the various characteristics of Kepler-105c are summarized below.

Mass (M⊕)	Radius (R⊕)	Density (Earth Density)	Host [Fe/H]	Host [Mg/H]	Host [Si/H]	Host [Fe/Mg]	Host [Si/Mg]
4.60 +0.92 -0.85	1.31±0.07	2.05 +0.52 -0.49	-0.12±0.04	-0.09±0.07	-0.08±0.05	0.76±0.22	0.98±0.27

Based on the findings in Unterborn et al. (2023), it can be determined that Kepler-105c appears to have the characteristics of a Super-Mercury rich in iron, based on both radius and density values. Further, ExoKyoto provides a star radiation at atmospheric boundary value of $314598.6\ W/m^2$.

3 Results

Imput: Mass (M⊕)

Mass (M⊕)	Radius (R⊕)	Density (kg/m^3)	[Fe/Mg]	Fe in mantle	Si in core	[Si/Mg]	O in core	S core
3.75	1.446	6815.27	.83	0.1	0	.98	0	0
4.6	1.53	7057.34	.83	0.1	0	.98	0	0
4.8	1.547	7124.06	.83	0.1	0	.98	0	0
<4.8	N/A		.83	0.1	0	.98	0	0

Table 1

Input: Core Mass Fraction and Core Composition

Mass ($M_\oplus$)	Radius ($R_\oplus$)	[Fe/Mg]	[Si/Mg]	Fe in Mantle	O in the core	S in the core	Si in the core	Mantle Composition (%)	Core Composition (%)
3.75	1.421	1.1	0.98	0.1	0	0	0.1	FeO=19.88	Fe=90
								SiO2=40.03	Si=10
								MgO=33.09	O=0
								CaO=3.22	S=0
								Al2O3=3.47	
3.75	1.421	1.1	0.98	2	0	0	0.1	FeO=19.88	Fe=90
								SiO2=40.03	Si=10
								MgO=33.09	O=0
								CaO=3.22	S=0
								Al2O3=3.47	
3.75	1.431	1.1	0.98	0.1	.02	.02	0.1	FeO=19.96	Fe=86
								SiO2=39.79	Si=10
								MgO=33.22	O=2
								CaO=3.23	S=2
								Al2O3=3.78	
3.75	1.456	1.1	2.17	0.1	0	0	0.1	FeO=12.52	Fe=90
								SiO2=62.21	Si=10
								MgO=20.85	O=0
								CaO=2.03	S=0
								Al2O3=2.37	

Table 2

Mantle Mineralogy Comparison

	FeO	SiO2	MgO	CaO	Al2O3
Best fitting parameter (105-c)	19.88	40.03	33.09	3.22	3.47
ExoPlex (Earth)	0.49	57.23	34.9	3.39	3.97
Allegre; et al. 1995 (Earth)	7.48	46.11	37.77	3.23	4.09

Table 3

In table 3, the first row is the mantle mineralogy of Kepler-105c with the best fitting parameters that match the observed radius. The best fitting parameters of Kepler-105c are in bold in table 2. Next is the composition of Earth computed by ExoPlex, and finally the actual composition calculated in the paper by Allegre; et al in 1995.

4 Discussion

Through the course of the project, it was found that mass was the most influential factor in determining the radius of a planet. With more mass, the planet has more material which leads to a greater radius. One of the other influential factors was the ratio between Iron and Magnesium. With a higher ratio, the smaller the radius. However, increasing the Fe mantle does not increase the radius of the planet. When adding less dense elements to the planet, the radius increases. Further, seeing as ExoPlex's mantle composition values for the Earth do not agree with the values found in the literature, it can be said that ExoPlex's analysis of Kepler-15c's mantle may not be accurate. Finally, we found that there is a mass cutoff of 4.8 Earth masses.

5 Conclusion

We have used ExoPlex to find the composition of the planet Kepler-105c by using its host star's, Kepler-105, stellar abundances. The mass and ratio of Fe/Mg had the greatest impact on the radius. The best fitting parameters found were generally very close to what ExoPlex believed them to be for Earth as well as Earth's likely true values, except for FeO and SiO2. The best fitting parameter was found to be 19.88 while ExoPlex thought it to be very small at 0.49. For SiO2, the difference is lesser but still significant. ExoPlex does not match well on FeO and SiO2 with the likely true values. This shows that ExoPlex can struggle with accuracy when the values begin to stray outside of its ideal operating range. In its ideal operating range it can be accurate but quickly becomes less reliable. The Kepler-105c abundances compared to the more accurate Earth values are similar as well, with Kepler-105c having similar values of SiO2, MgO, CaO, and Al2O3. However FeO is very abundant on Kepler-105c compared to Earth, with 19.88 on Kepler-105c and 7.48 on Earth. This analysis can be repeated to learn about the compositions of any exoplanet when given its mass and the compositions of the host star. Being able to determine the composition of an exoplanet with very little information on the planet itself is an extremely useful tool in studying exoplanets and learning about their formation.

6 Contribution Statement

Greg and Omar worked together on the code, the methods/results of the paper, and the methods/results slides. Matthew also worked on the code with Omar and Greg, while writing the introduction and conclusion to the paper and creating the introduction and conclusion slides. Dax worked on the discussion of the paper and created the discussion slides.

Sources Cited

Allegre, C. J., Poirier, J.-P., Humler, E., & Hofmann, A. W. (1995). The chemical composition of the Earth. Earth and Planetary Science Letters, [Volume(Issue)], [Page Range]. Retrieved from https://www.geol.umd.edu/~mcdonoug/KITP%20Website%20for%20Bill/papers/Earth_Models/Allegre_etal_(EPSL_95).pdf

Griffith, E., Weinberg, D. H., Johnson, J. A., Beaton, R., García-Hernández, D. A., Hasselquist, S., Holtzman, J., Johnson, J. W., Jonsson, H., Lane, R. R., Nataf, D. M., & Roman-Lopes, A. (2021). The Similarity of Abundance Ratio Trends and Nucleosynthetic Patterns in the Milky Way Disk and Bulge. Retrieved April 16, 2023, from https://arxiv.org/abs/2009.05063

Exoplanet Kyoto Project. (n.d.). Kepler-105 c. Exoplanet Kyoto. Retrieved April 12, 2023, from http://www.exoplanetkyoto.org/exohtml/Kepler-105_c.html

NASA Exoplanet Science Institute. (NASA). Kepler-105 c: Overview. NASA Exoplanet Archive. Retrieved April 12, 2023, from https://exoplanetarchive.ipac.caltech.edu/overview/Kepler-105c

National Space Science Data Center. (NASA). Planetary Fact Sheet - Ratio to Earth Values. NASA Goddard Space Flight Center. Retrieved April 14, 2023, from https://nssdc.gsfc.nasa.gov/planetary/factsheet/planet_table_ratio.html

Unterborn, C. T., Desch, S. J., Haldemann, J., Lorenzo, A., Schulze, J. G., Hinkel, N. R., & Panero, W. R. (2023). The Nominal Ranges of Rocky Planet Masses, Radii, Surface Gravities, and Bulk Densities. The Astrophysical Journal, 944(1), 42. https://doi.org/10.3847/1538-4357/acaa3b