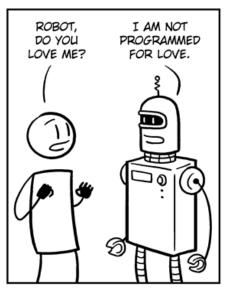
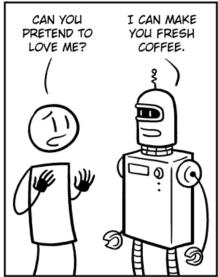
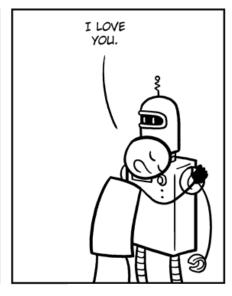
Localization







Where am I?

Where am I going? (Cognition)

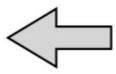
Position, map





Route plan

Where am I? (Perception, localization)



Environment

How do I get there? (Motion control)

Dead Reckoning

- intrinsic sensors

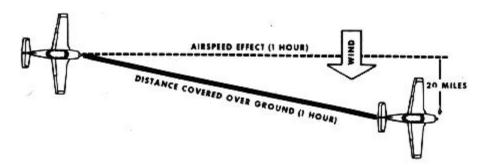
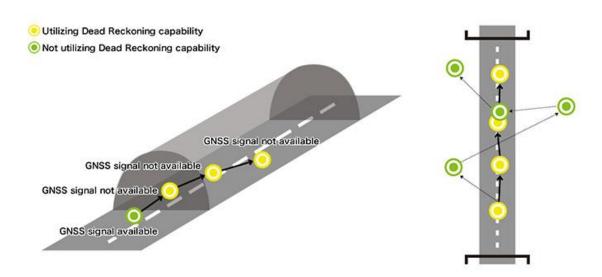


Figure 12-4 Effect of Wind in One Hour



Landmark-based navigation

- Extrinsic sensors





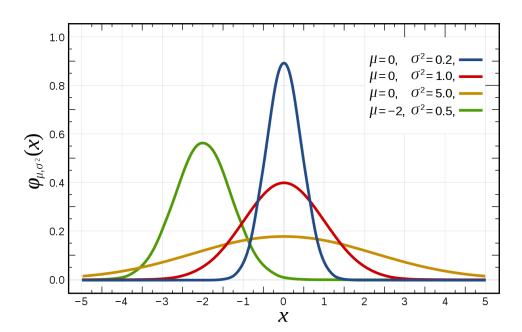
Motion model

$$x(n+1) = x(n) + \delta_x + v_x$$

x(n) - position at time n

 δ_x - "delta x", change in position during time step

 v_{χ} - noise term (e.g. $v_{\chi} \sim N(0, \sigma^2)$)



Motion models

Linear model:

$$x(n+1) = x(n) + \delta_x + v_x$$

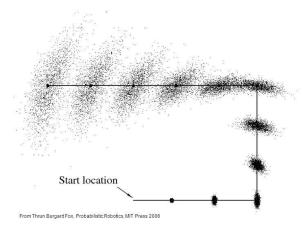
General model:

$$x(n+1) = f(x(n), u(n), v(n))$$

Motion model

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Accumulation of the pose estimation error under the robot motion (only proprioceptive measurements)



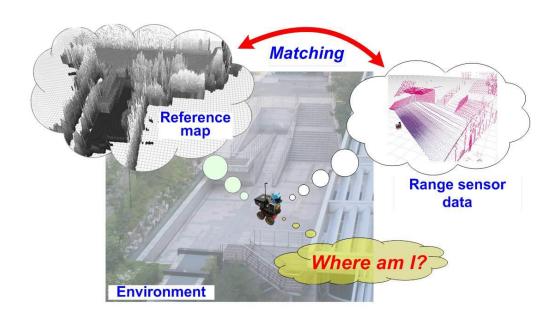
Measurement model and matching/data association

$$z(n) = h(x(n), w(n), map/features)$$

z(n) – measured position

x(n) – true position

w(n) – noise on measurement



1. Simple measurement update

- a) Prediction: Update pose using motion model
- b) Measurement: measure (range, angle)-data to wall with depth sensor
- c) Measurement prediction: find (range, angle)-data from the map and estimated pose
- d) Matching: data association between measurements and measurement predictions
- e) Pose estimation : Use e.g. simple filter to update angle $\hat{\theta}(n+1) = \hat{\theta}(n) + k_1 \left(\theta_{obs} \hat{\theta}(n)\right)$
- $\hat{\theta}(n)$ =current angle estimate, θ_{obs} =observed angle, k_1 = fixed parameter between 0 and 1 (1 means 100% "belief" in θ_{obs})

2. Kalman filter

$$x(n + 1) = Fx(n) + v(n)$$

 $z(n + 1) = Hx(n + 1) + w(n)$

Time Update ("Predict")

(1) Project the state ahead

$$\hat{x}_k = A\hat{x}_{k-1} + Bu_{k-1}$$

(2) Project the error covariance ahead

$$P_k = AP_{k-1}A^T + Q$$



(1) Compute the Kalman gain

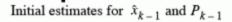
$$K_k = P_k^{\mathsf{T}} H^T (H P_k^{\mathsf{T}} H^T + R)^{-1}$$

(2) Update estimate with measurement z_k

$$\hat{x}_k = \hat{x}_k + K_k(z_k - H\hat{x}_k)$$

(3) Update the error covariance

$$P_k = (I - K_k H) P_k$$



3. Extended Kalman Filter

$$x(n+1) = f(x(n), v(n))$$

$$z(n+1) = h(x(n+1), w(n))$$

Linearization

$$x(n+1) = f(\hat{x}(n)) + F_x(x(n) - \hat{x}(n)) + F_v v(n)$$

$$z(n+1) = h(\hat{x}(n+1)) + H_x(x(n+1) - \hat{x}(n+1)) + H_ww(n)$$

3. Extended Kalman Filter

Prediction step

State prediction and measure estimation

$$egin{aligned} ilde{X}_{i,k} &= f(\hat{X}_{i,k-1}, U_{i,k}, \mathbf{0})igg|_{odom\,etry} \ &Z_{i,k} &= g(\hat{X}_{i-1,k-1}, \mathbf{0})igg|_{laser} \end{aligned}$$

Prediction of error covariance matrix

$$oldsymbol{ ilde{P}}_{i,k} = oldsymbol{J}_{f,X_i} oldsymbol{P}_{i,k-1} oldsymbol{J}_{f,X_i}^T + oldsymbol{J}_{f,W} oldsymbol{\Sigma}_w oldsymbol{J}_{f,W}^T$$

k

Correction step

Kalman gain calculation

$$\begin{split} \boldsymbol{K}_{i,k} &= \\ \boldsymbol{\tilde{P}}_{i,k} \boldsymbol{J}_{g,\boldsymbol{\bar{X}}_i}^T \left(\boldsymbol{J}_{g,\boldsymbol{\bar{X}}_i} \boldsymbol{\tilde{P}}_{i,k} \boldsymbol{J}_{g,\boldsymbol{\bar{X}}_i}^T + \boldsymbol{J}_{g,\boldsymbol{V}} \boldsymbol{\Sigma}_{\boldsymbol{V}} \boldsymbol{J}_{g,\boldsymbol{V}}^T \right)^{-1} \end{split}$$

State correction

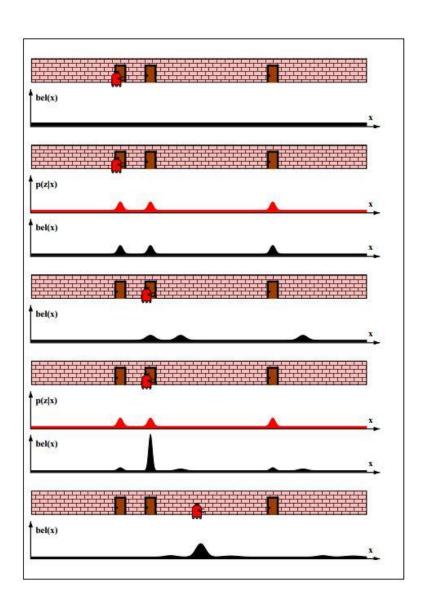
$$\hat{X}_{i,k} = \tilde{X}_{i,k} + \Theta_k K_{i,k} \left[Z_{i,k} - \tilde{X}_{i,k} \right]$$

Covariance matrix update

$$P_{i,k} = \left(I - K_{i,k} J_{g,\overline{X}_i}\right) \tilde{P}_{i,k}$$

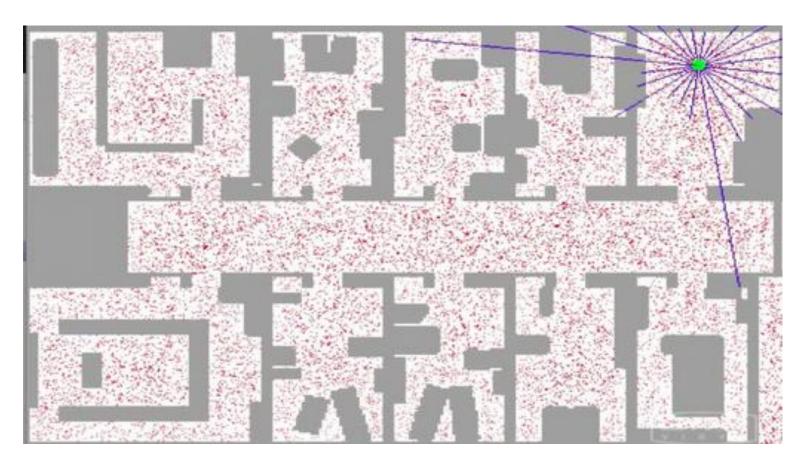
k + 1

Probabilistic Robot Localization



4. Particle Filter (=Sequential Monte Carlo)

P(x(n+1)|x(n),u(n)) P(z(n+1)|x(n+1),map)



4. Particle Filter (Monte carlo simulation)

- a) Initialize: N particles randomly distributed
- b) Motion model for each particle (stochastic)
- c) Step 1b) to 1d) -> how "close" are measurements to map
- d) Update weights for each particle (based on "closeness" from c)
- e) Resampling: Remove/add particles based on weights (stochastic)
- f) Iterate b) to e)

Simultaneous Localisation and Mapping (SLAM)

