Chapter 7 Hypothesis Testing

Daxiang Na (那达翔)

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Contents

1 Hypothesis Testing 假设检验

1.1 Definition and Concepts 定义与概念

1. null hypothesis, H_0: "no change"
2. We believe the null hypothesis to be true unless overwhelming evidence exists to the contrary ("innocent until proven guilty")
3. The alternative hypothesis, H_1, or H_A (in this class, we all use H_1), is a second statement that contradicts H_0.
4. Either H_0 or H_1 must be true (mutually exclusive, exhaustive).
5. We need overwhelming evidence to conclude that H_1 is true. - That is why the alpha value, or the "threshold", should be very low, so the chance that H_0 is true is very low.

1.2 Calculation 计算

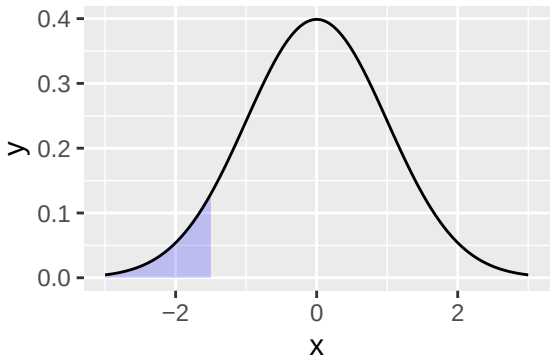
1. We calculate the probability of H_0 is true, which is the probability that you get a mean value from samples that is as extreme or more extreme than $\bar{X}$ if you assume that H_0 is true.
2. For now, we assume the population show normal distribution.
3. z-test:
$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$
4. calculating the p-values for z-tests:

p-values for z-tests

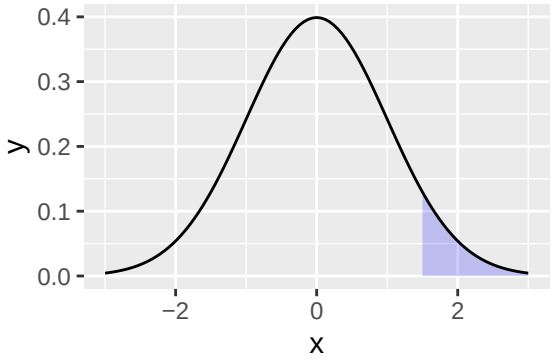
- We calculate our p-value as follows, for each of the three types of tests (*z-tests*):
- One-sided, lower-tailed hypothesis ($H_1 : \mu < \mu_0$):
  - `pnorm(z)`
- One-sided, upper-tailed hypothesis ($H_1 : \mu > \mu_0$):
  - `1-pnorm(z)`
- Two-sided hypothesis ($H_1 : \mu \neq \mu_0$):
  - If $z \leq 0$: `2*pnorm(z)`
  - If $z > 0$: `2*(1-pnorm(z))`

Notes for two sided hypothesis:

when z $<$ 0, you get probability (pnorm(z)) like this:



when z $>$ 0, you get probability (1 - pnorm(z))like this:



2 Hypothesis Testing and Confidence Interval 假设检验与置信区间

2.1 Mathematically equivalent.

3 Type I and Type II errors 一类错误与二类错误

3.1 Definition

	$\mu = \mu_0$	$\mu \neq \mu_0$
Fail to reject	Correct	Incorrect(Type II)
Reject	Incorrect(Type I)	Correct

3.1.1 Type I error 一类错误

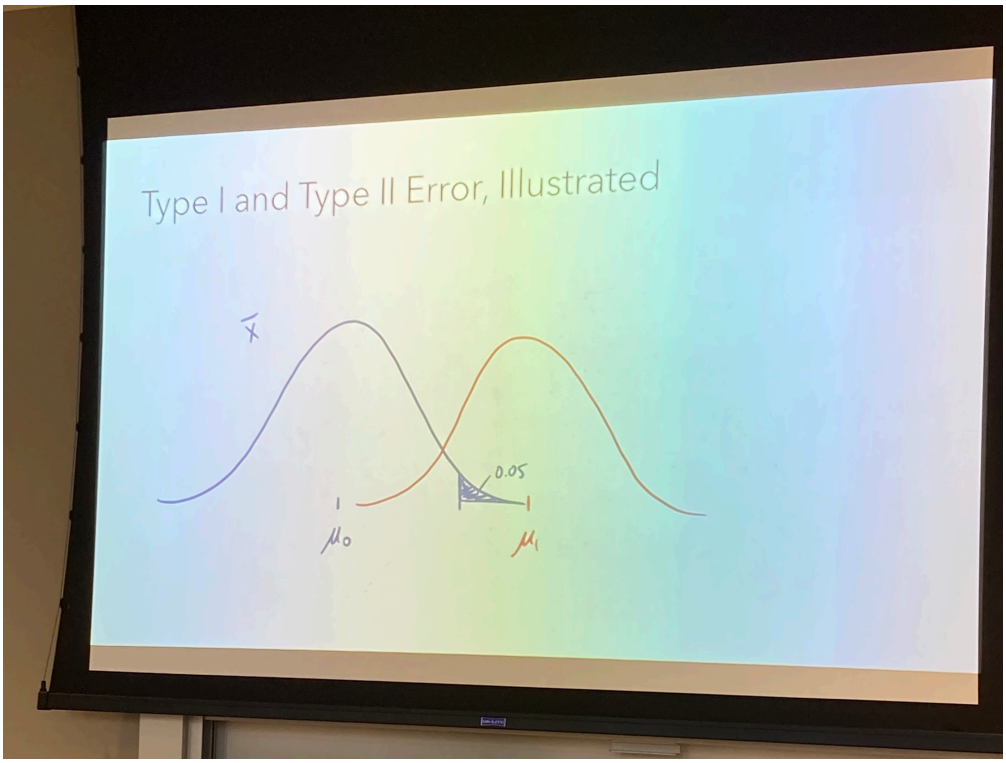
- **Type I error occurs** if we reject a true null hypothesis ("false positive")
  - $H_0 : \mu = \mu_0$ is true, but we reject it.
- The chance of Type I error is Pr(reject $H_0 | H_0$ is true)
- The significance level α is the probability of making a type I error. Thus we decide what α is for our best

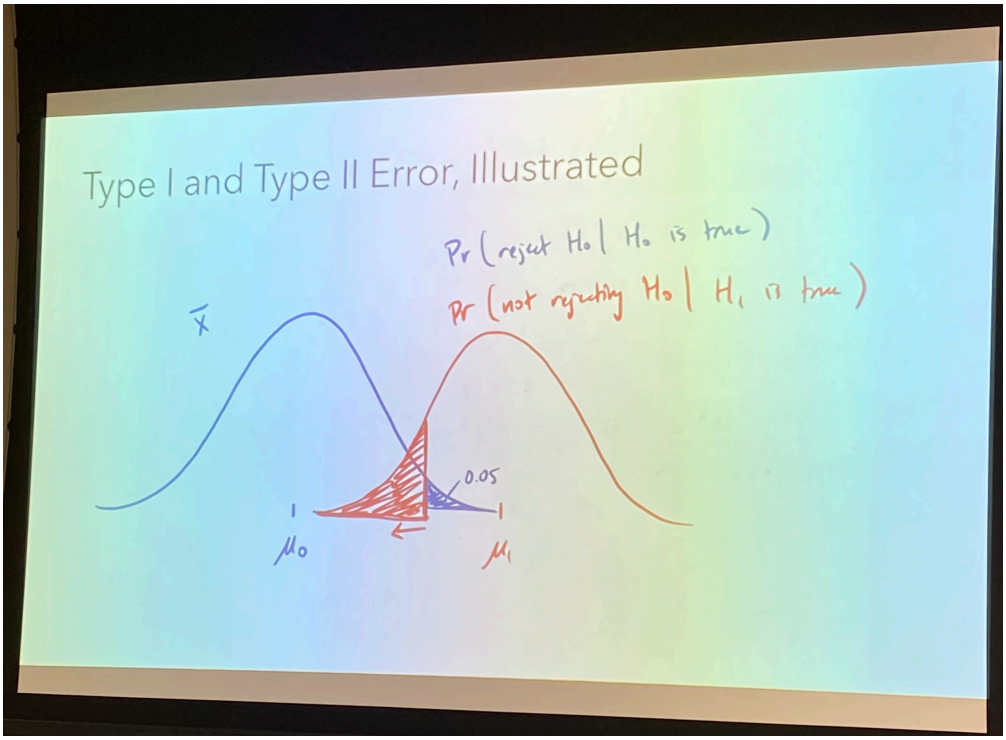
3.1.2 Type II error 二类错误

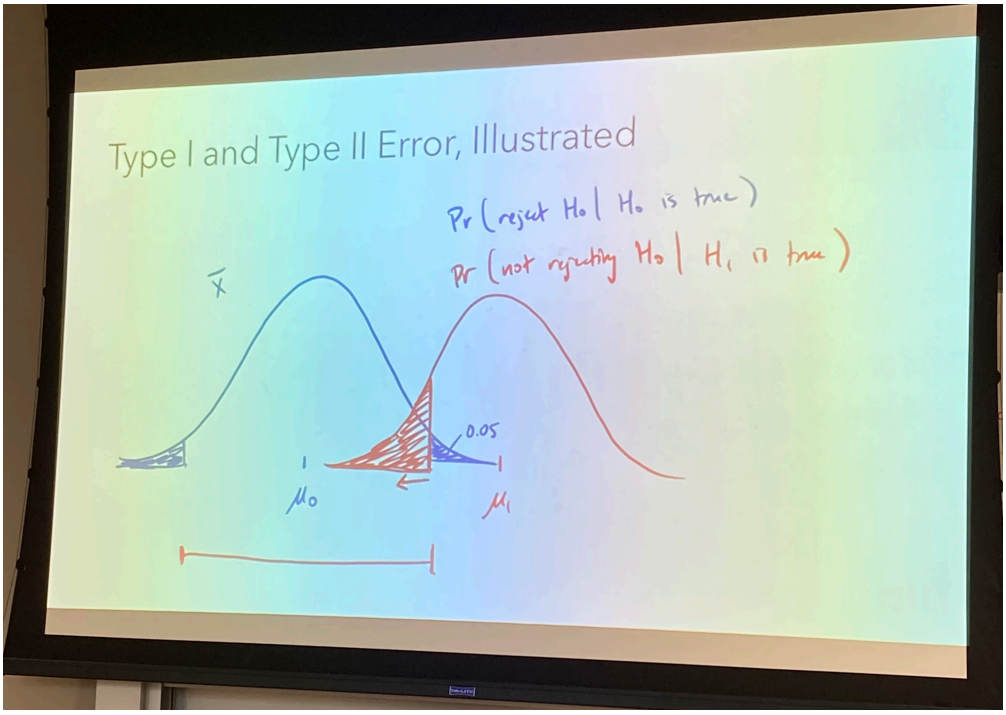
- **Type II error occurs** if we fail to reject a false null hypothesis ("false negative")
  - $H_0 : \mu = \mu_0$ is false, but we fail to reject it.
- The probability of making a type II error is denoted β
- The chance of Type II error is Pr(do not reject $H_0 | H_0$ is false)

3.2 Illustrated

Dr.Kahng's illustrations shown as below:







4 Power

4.1 Definition

- The power of a test is equal to $1 - \beta$

4.2 Calculation