Chapter 15: Linear Regression I

DSCC 462
Computational Introduction to Statistics

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Fall 2022

Announcements

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- Midterm grades are out

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- Pickup tomorrow from 3-4 pm in my office (Wegmans 2401) or at office hours next Tuesday

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- Project groups are due tomorrow! Datasets will be released tomorrow (no project proposal necessary)

Plan For Today

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- Basics of regression
- Inference for parameters
- Confidence intervals for true values

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X_i Y_i

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  - Example: If a child is 9 years old, how tall do we expect them to be?

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    - β_0 is the y-intercept and β_1 is the slope for the population

$y = \overset{\beta_1}{m}x + \overset{\beta_0}{b}$

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- **Goal**: Estimate β_0 and β_1 based on a sample in order to model the relationship between y and x

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- When the regression assumptions are met, the use of linear regression is appropriate for describing the relationship between y and x

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predicted value of y_i

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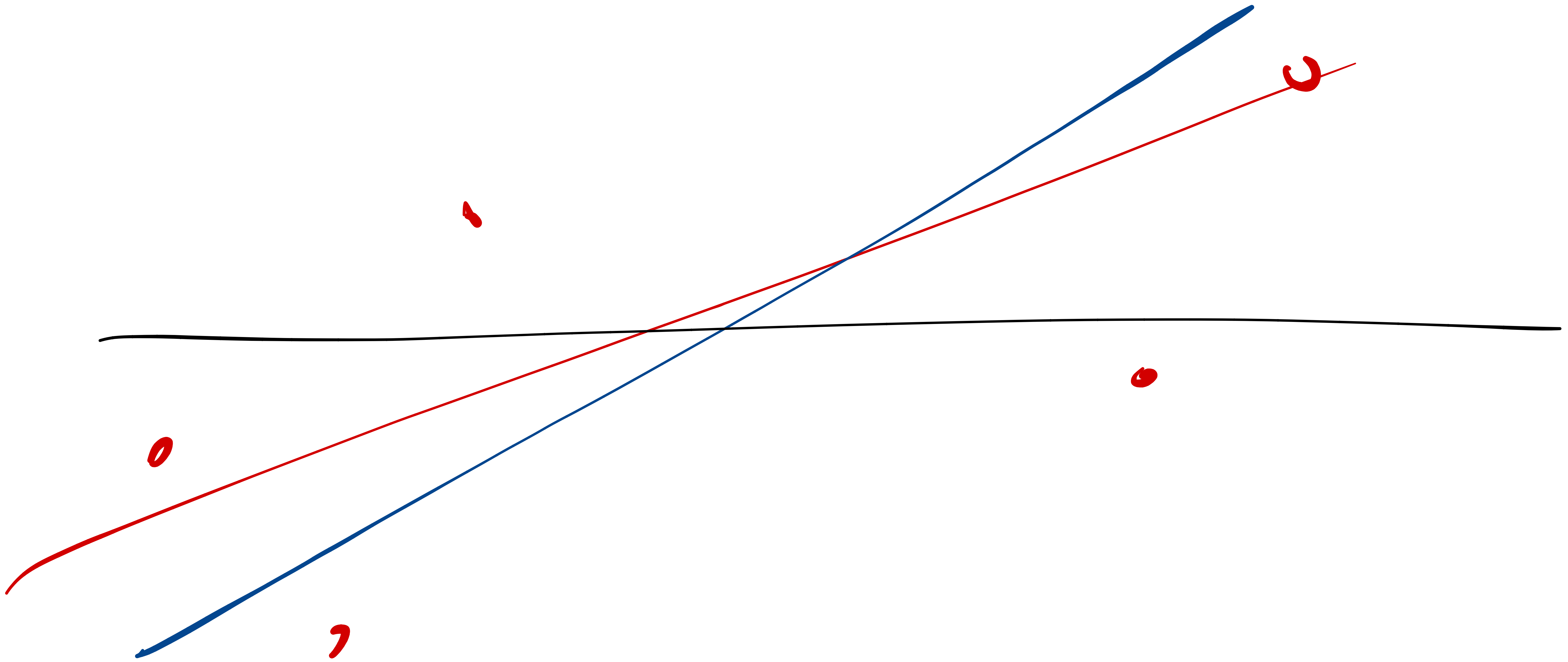
- Once we have estimates $\boxed{\hat{\beta}_0}$ and $\boxed{\hat{\beta}_1}$, we can estimate what y_i would be for a given x_i, under the model
- $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$
- But how do we fit a linear regression model?

Simple Linear Regression

Simple Linear Regression

- Use the *method of least squares* to fit a straight line to a set of points (x_i, y_i)



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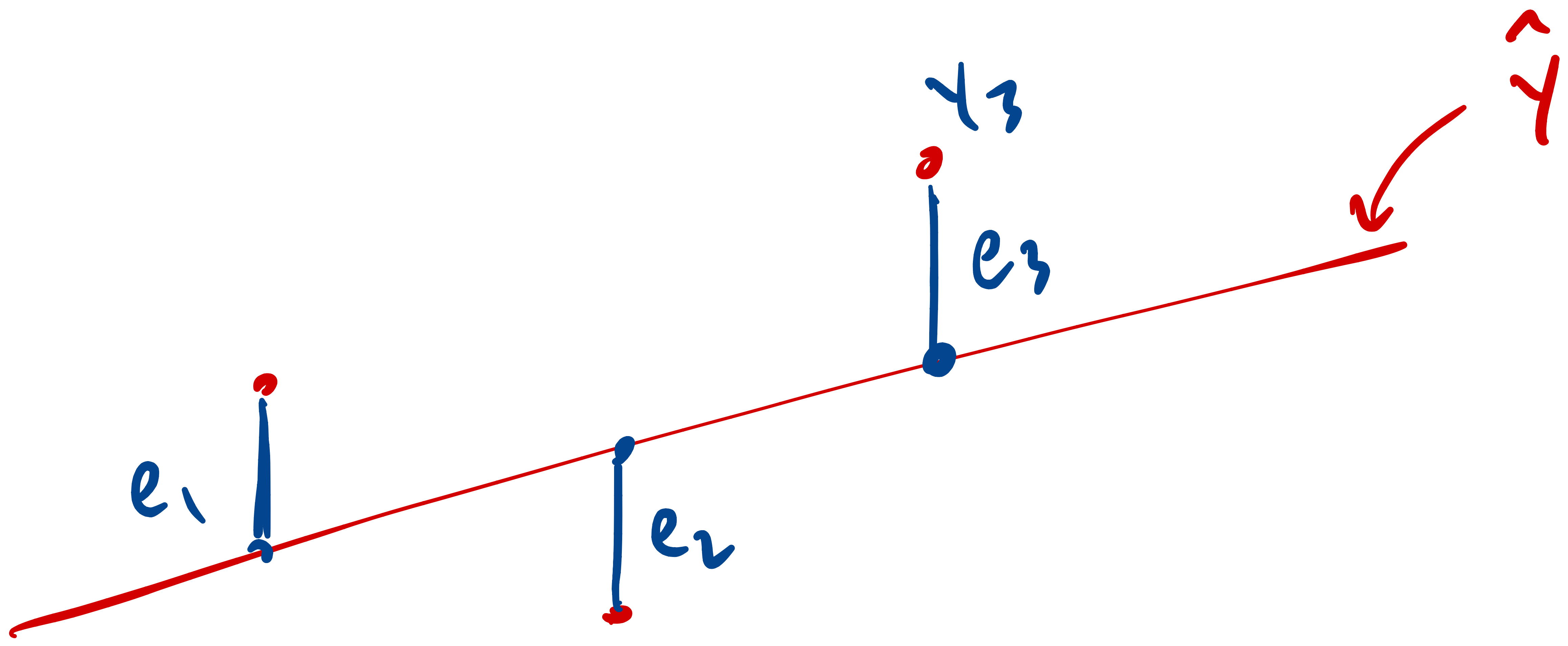

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  - R = A – P (residual = actual – predicted)
- Residuals of $e_i = 0$ indicate that the observed point lies directly on the regression line
- Ideally, we would want every point to lie directly on the line

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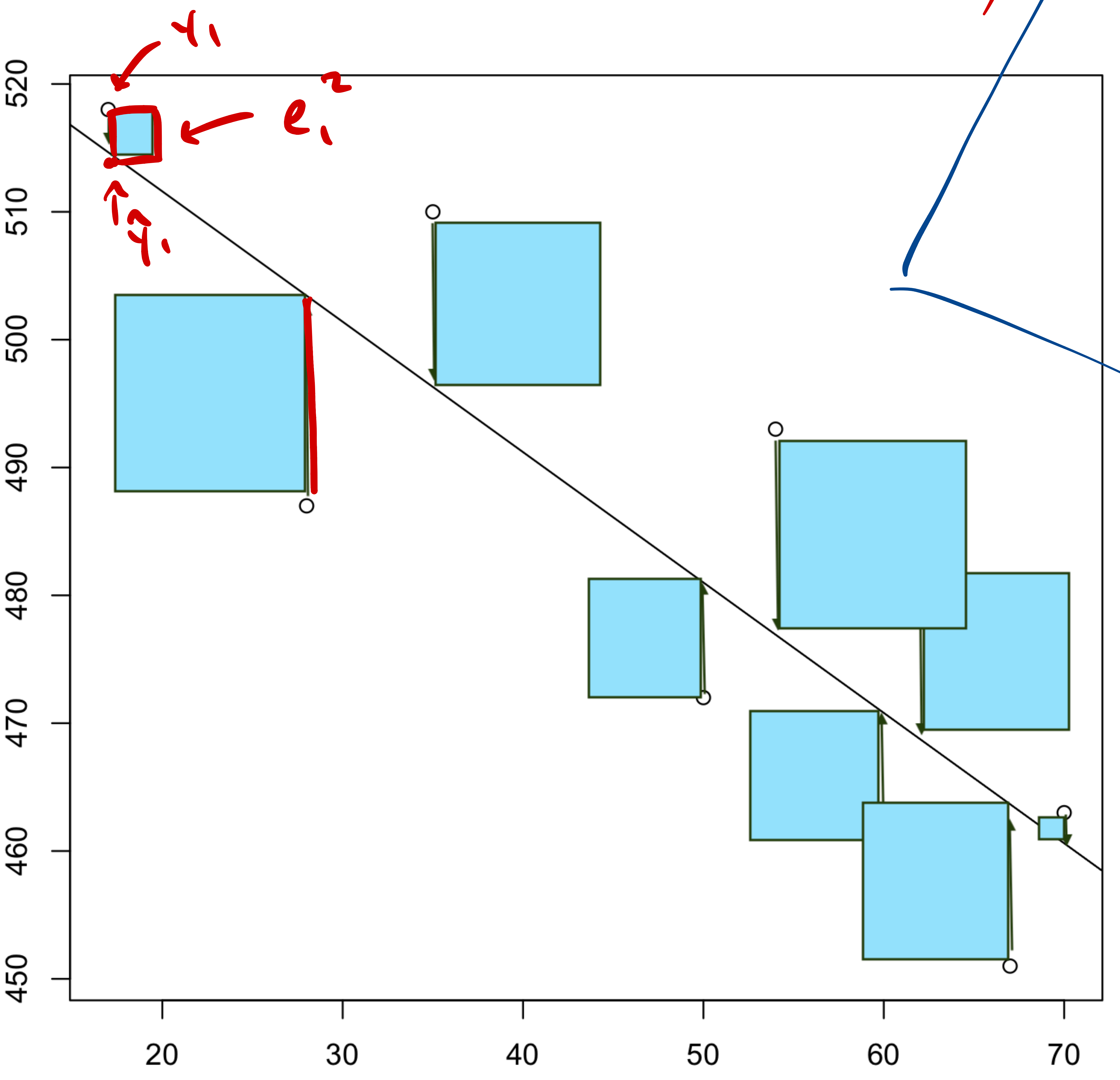
- *Residual sum of squares*:

$$\boxed{\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2}$$

Simple Linear Regression



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$$\sum e_i^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \underline{\sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2}$$

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$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n} (x_i - \overline{x})^2} = \boxed{r\frac{s_y}{s_x}}$$

$$\hat{\beta}_0 = \overline{y} - \hat{\beta}_1 \overline{x}$$

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- Plugging in $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$, we get:

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- Note, r is Pearson's correlation coefficient, s_y is the standard deviation of y, and s_x is the standard deviation of x

Simple Linear Regression

"Run LR on y given x"

```
> set.seed(223542)
> dat1 <- rmvnorm(10,c(11,10), sigma=matrix(c(1,.5, .5, 1),2,2))
> colnames(dat1) <- c("X","Y")
> x <- dat1[,1]
> y <- dat1[,2]
> model1 <- lm(y~x)
> plot(x,y, xlab="X", ylab="Y")
> abline(model1)
> model1

Call:
lm(formula = y ~ x)

Coefficients:
(Intercept)            x
     0.3236       0.8578
```

$\hat{\beta}_0$ $\hat{\beta}_1$

```
> summary(model1)

Call:
lm(formula = y ~ x)

Residuals:
     Min       1Q   Median       3Q      Max
-1.27555 -0.34855 -0.09534  0.52797  1.28676

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.3236     2.3820   0.136  0.89530
x             0.8578     0.2209   3.884  0.00465 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8009 on 8 degrees of freedom
Multiple R-squared:  0.6534,Adjusted R-squared:  0.6101
F-statistic: 15.08 on 1 and 8 DF,  p-value: 0.004651
```

SE($\hat{\beta}_0$)

SE($\hat{\beta}_1$)

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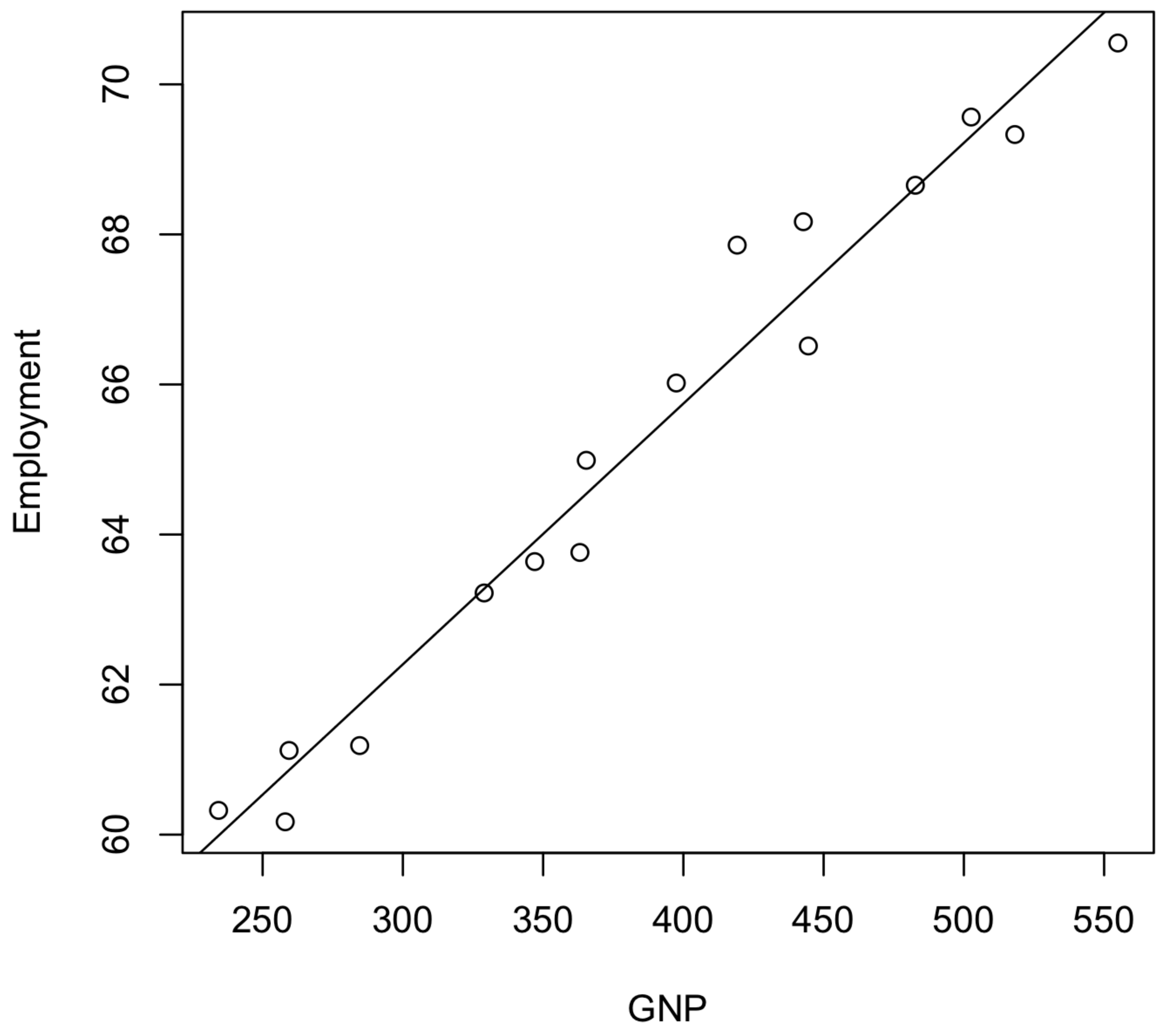
- Include economic variables from mid-1900s America
- GNP: in billions of US dollars
- Employment: in millions of people

$\hat{y}$ vs. y

$n = 16$

$\hat{y} = 51.844 + 0.03475x$



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$r \cdot \frac{s_y}{s_x}$

- $\hat{\beta}_1 = 0.98355\left(\frac{3.512}{99.395}\right) = 0.03475$
- $\hat{\beta}_0 = 65.317 - 0.03475(387.699) = 51.844$

$\bar{y} - \hat{\beta}_1 \bar{x}$

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- $\hat{\beta}_1$ is the slope
  - For each 1 unit increase in x, we expect y to increase by $\hat{\beta}_1$ according to the model
  - E.g., for each $1 billion increase in GNP, we expect the number of people employed to increase by 0.03475 million

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$$\hat{Y} = 51.844 + 0.03475 \underset{\uparrow}{x} \quad 350$$

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Prediction

- We can use our regression line to make predictions
- Suppose we want to predict employment numbers when GNP is \$350 billion
- $\hat{y} = 51.844 + 0.03475x = 51.844 + 0.03475 \cdot 350 = 64.0065$ ~~billion USD~~

mill people

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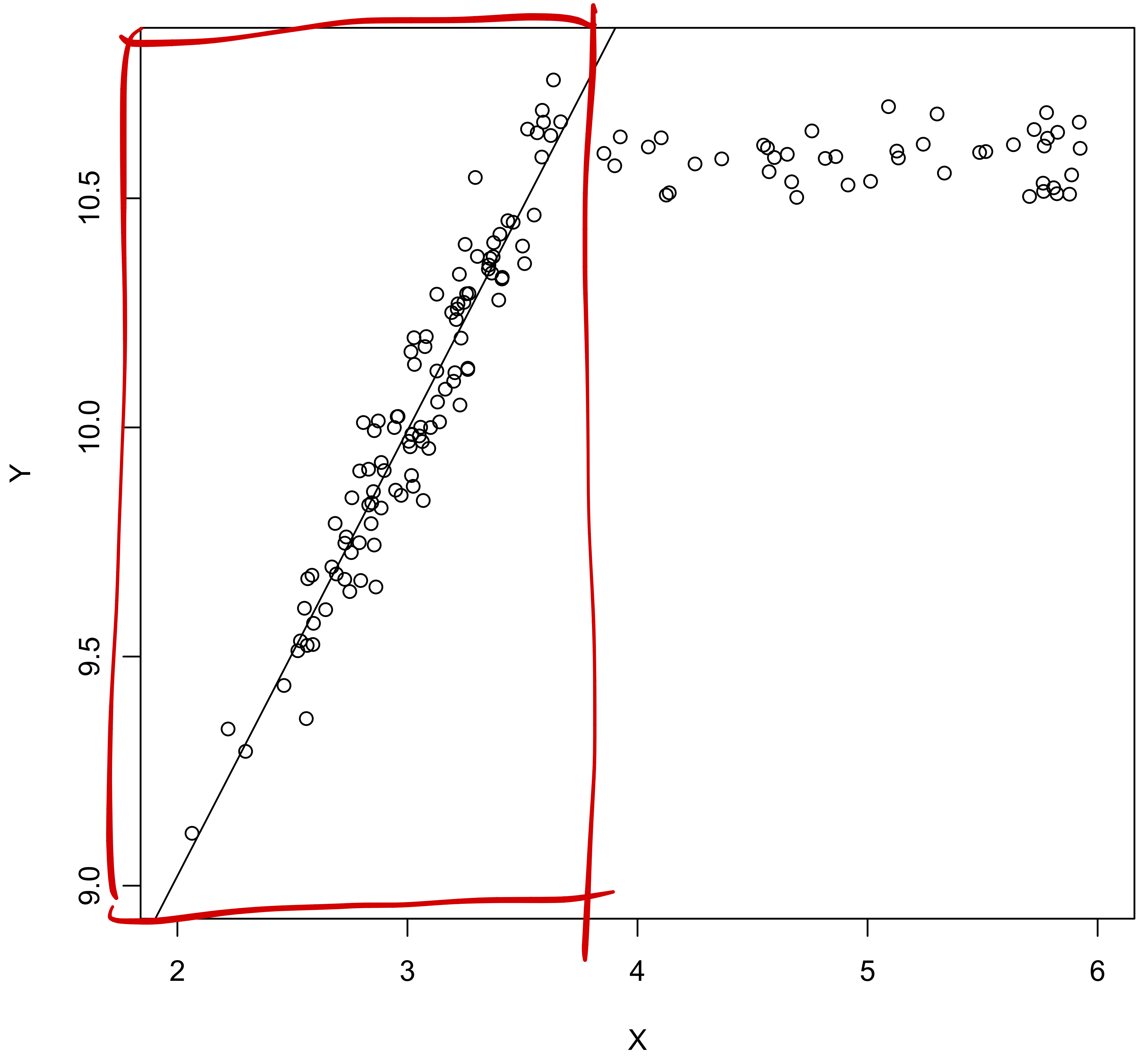
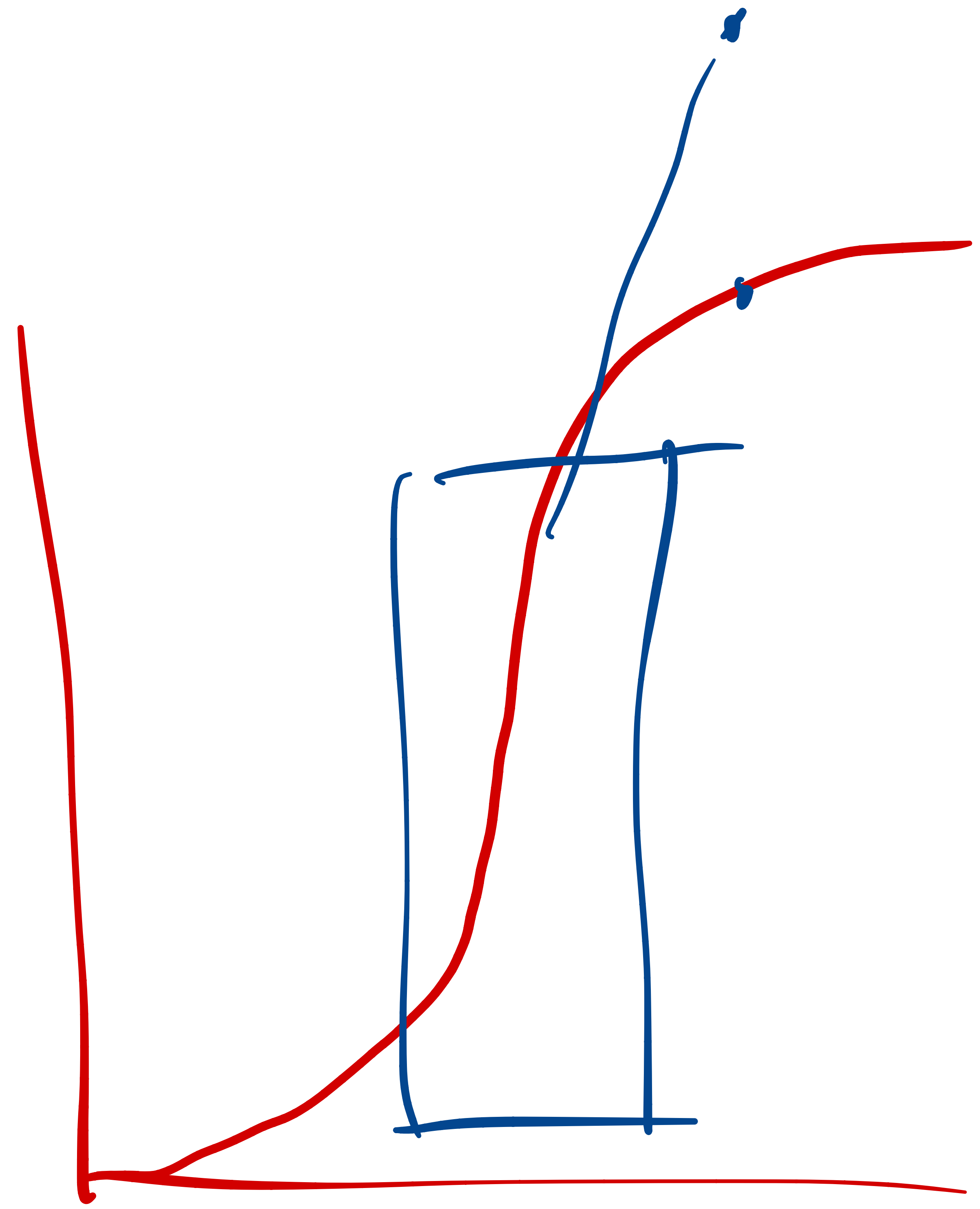
- Intuition: Our model was created only for our range of data, and we do not know what happens outside of this range

Extrapolation





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β_0 β_1

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  - Hypothesis tests for β_0 are often unimportant and have little meaning, so we will focus only on β_1

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- $Var(\hat{\beta}_1) = \dfrac{\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$ $\leftarrow$ $\varepsilon \sim N(0, \sigma^2)$

- $Var(\hat{\beta}_0) = \sigma^2 \left(\dfrac{1}{n} + \dfrac{\bar{x}^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \right)$ $\leftarrow$ x

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$e_i = (y_i - \hat{y}_i)$

- $Var(\hat{\beta}_0) = \sigma^2\left(\dfrac{1}{n} + \dfrac{\bar{x}^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right)$

$df = n-2$

- Note that σ^2 is the variance of the residuals around the predicted regression line

- Estimate σ^2 with sample $s^2 = \underline{Var(e_i)} = \dfrac{1}{n-2}\displaystyle\sum_{i=1}^{n} e_i^2 = \dfrac{1}{n-2}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$

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σ^2

- $$Var(\hat{\beta}_1) = \frac{s^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$SE(\hat{\beta}_1) = \sqrt{Var(\hat{\beta}_1)}$

- $$Var(\hat{\beta}_0) = s^2\left(\frac{1}{n} + \frac{\bar{x}^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right)$$

$SE(\hat{\beta}_0)$

σ^2

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- Test the null hypothesis $H_0 : \beta_1 = \beta_1^*$ vs. $H_1 : \beta_1 \neq \beta_1^*$, where β_1^* is some population slope value, at the α significance level
    - Generally interested in the case of $\beta_1^* = 0$

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- We typically are interested only in hypothesis tests regarding $\hat{\beta}_1$
- The slope tells us of the relationship between x and y
- Test the null hypothesis $H_0 : \beta_1 = \beta_1^*$ vs. $H_1 : \beta_1 \neq \beta_1^*$, where β_1^* is some population slope value, at the α significance level
  - Generally interested in the case of $\beta_1^* = 0$
  - No relationship vs. some relationship

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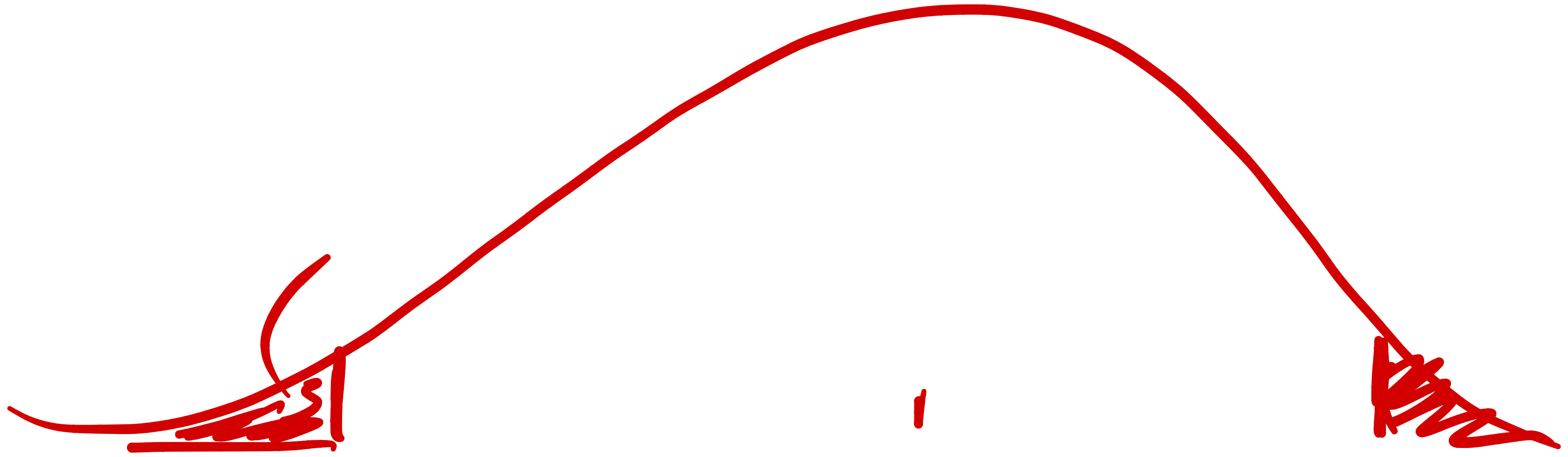

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2x (1− pt(abs(t), df = n-2))

''

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  - Both hypotheses claim that y does not change as x increases

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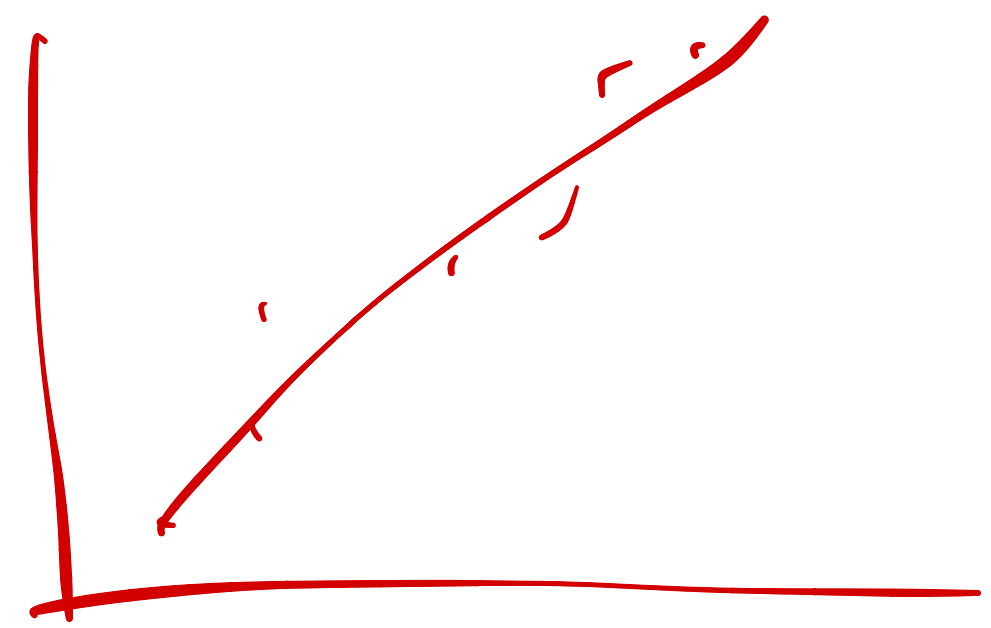

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$n = 16$

$df = n - 2 = 14$

$$t = \frac{\hat{\beta}_1 - \beta_1}{SE(\hat{\beta}_1)} = \frac{0.03475}{0.001706} = 20.$$

$2 \times pt(-20, df = 14) = 0.$

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- Since the p-value is less than 0.05, we reject the null hypothesis and conclude that there is a significant linear relationship between GNP and employment

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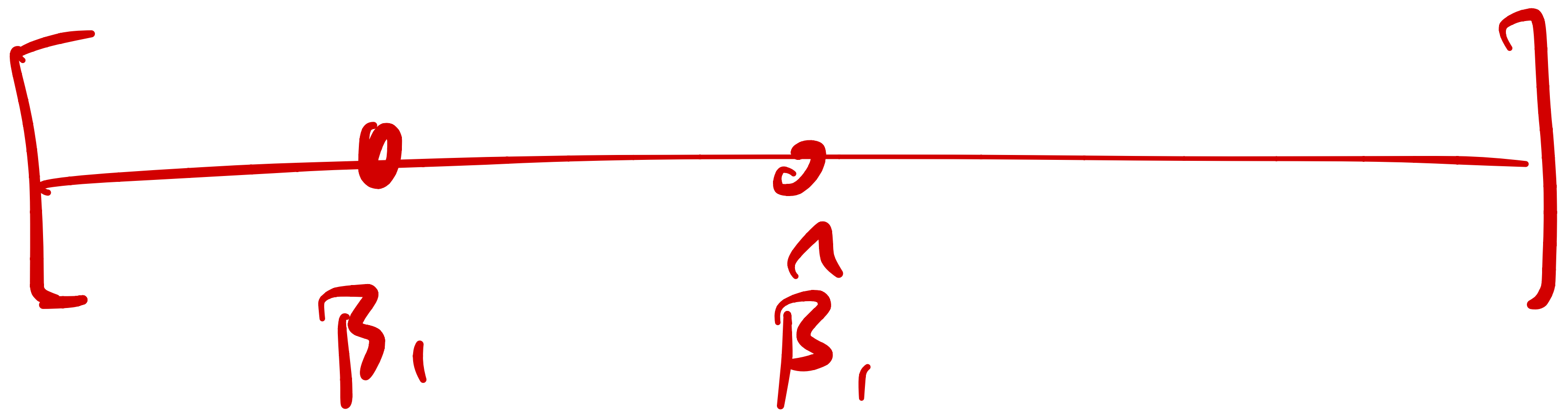

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&= 0.03475 \pm 2.145(0.001706) \\
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- We are 95% confident that the interval $(0.03109,\ 0.03841)$ contains the true population slope

Confidence Intervals for Regression Coefficients

lm(y ~ x)

```
> confint(lm1)
                  2.5 %   97.5 %
(Intercept) -5.1694143 5.816569
dat1[, 1]    0.3484624 1.367178
```

β_0

β_1

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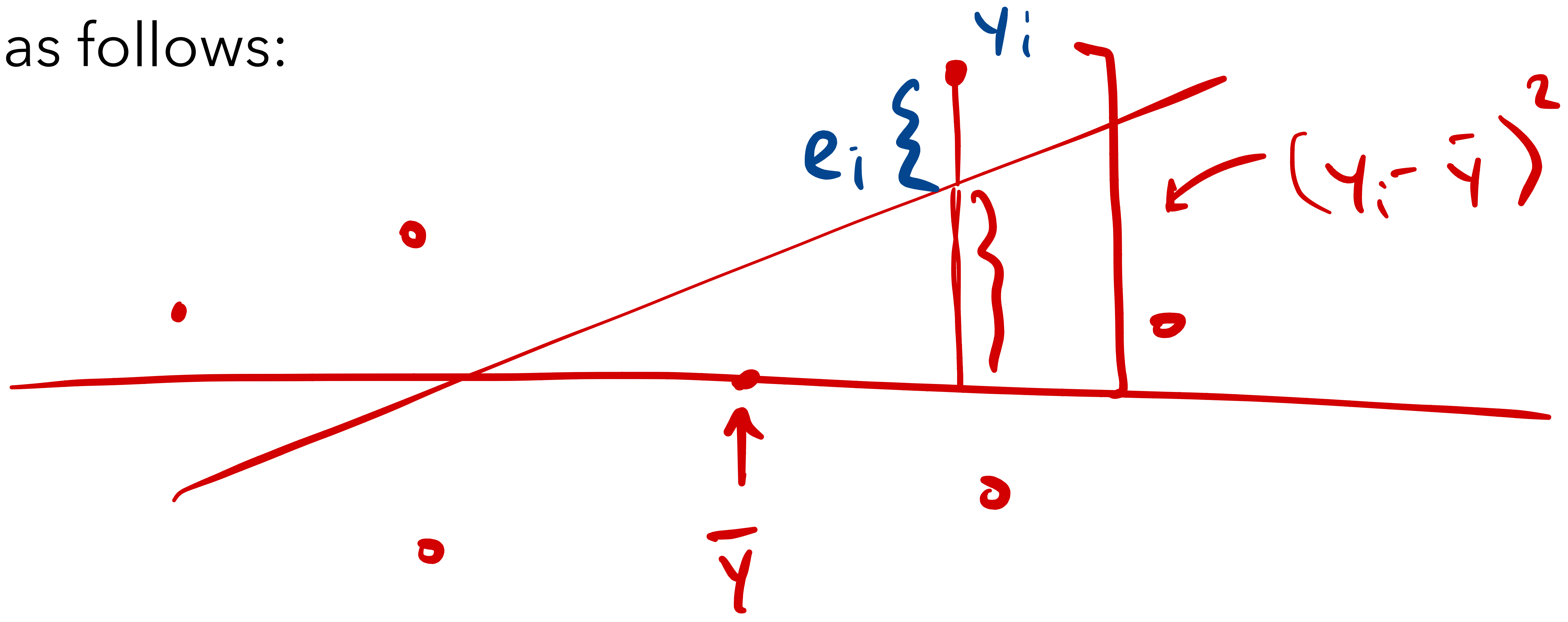

$(y_i - \hat{y})$

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- Since we have two unknown parameters (β_0 and β_1), this is analogous to the $n - k$ degrees of freedom with the MSE in ANOVA

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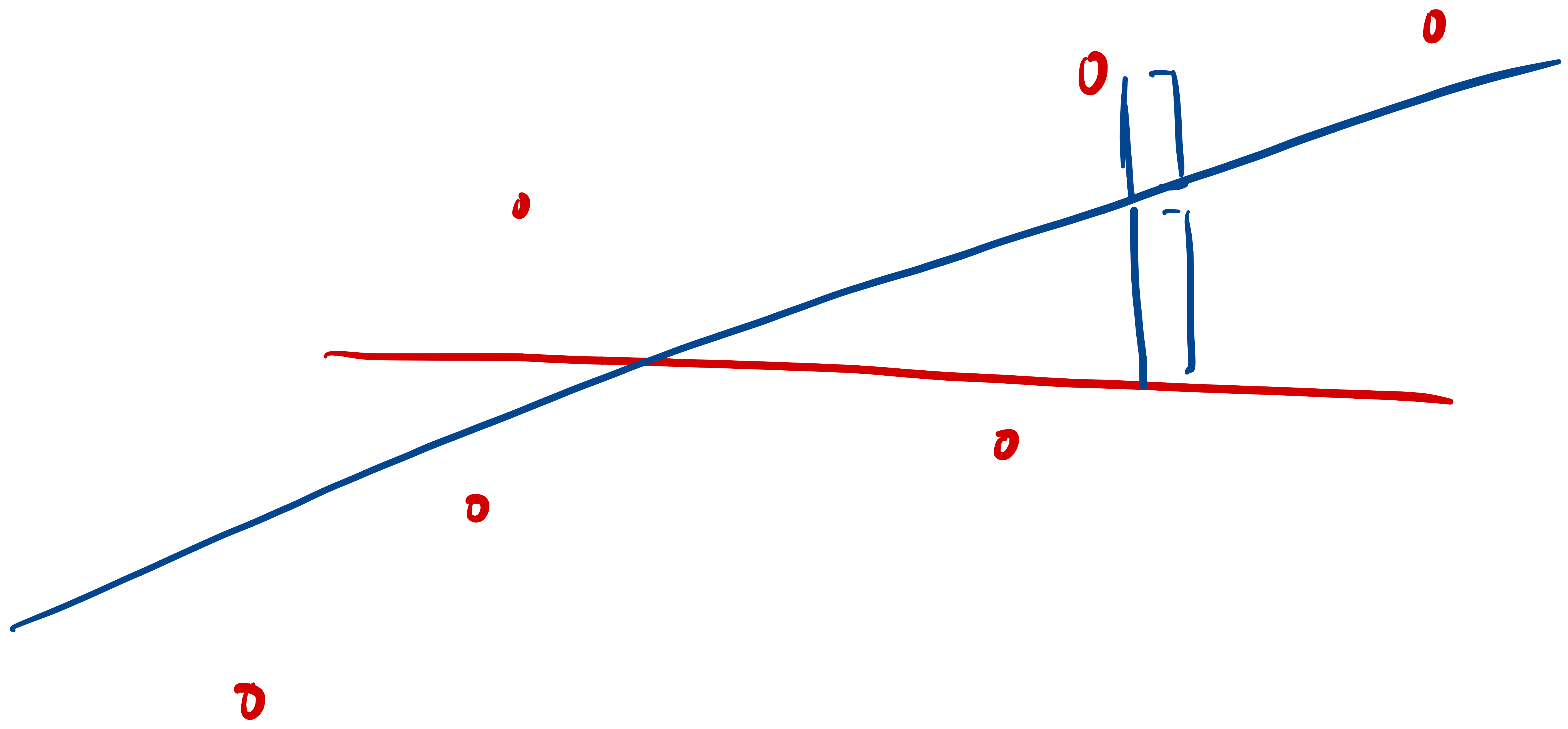

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- The total sum of squares is then $SSTo = SSR + SSE$

$$\sum (y_i - \bar{y})^2 = \sum (\hat{y}_i - \bar{y})^2 + \sum (y_i - \hat{y}_i)^2$$

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- We can use the regression line to estimate the mean value of y corresponding to a particular value of $x = x^*$
  - If we took many samples for a particular x^* and found their average response y, it would be equal to the estimated response from our regression

$x^* \rightarrow \hat{y}$

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- Recall that we predicted, on average, employment to be 64.0065 million given GNP was 350 billion USD
- In actuality, when the GNP is 350 billion USD, the employment will not necessarily equal exactly 64.0065 million, but instead that will be the average response

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- `qt(0.975, df=14)` = 2.145
- CI: $64.0065 \pm 2.145 \cdot 0.17629$
- Conclusion: I am 95% confident that the interval (63.628, 64.385) million contains the true mean employment number when the GNP is 350 billion USD

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- $\hat{y} = 64.0065$
- $SE(y^*) = s\sqrt{\left[1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right]} = 0.6566\sqrt{\left[1 + \frac{1}{16} + \frac{(350 - 387.698)^2}{15 * 99.395^2}\right]} = 0.6799$

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- CI: $64.0065 \pm 2.145 \cdot 0.6799$
- Conclusion: I am 95% confident that the interval (62.548, 65.465) million contains the true employment number when the GNP is 350 billion USD

Inference for Mean and Predicted Response

x^* $1-\alpha$ mean

```
> set.seed(223542)
> x <- rnorm(10, 5, 2)
> y <- rnorm(10, 13, 3)
> predict(lm(y~x), data.frame(x=6), conf.level=0.95, interval="confidence")
       fit      lwr      upr
1 10.41062 7.617309 13.20392
> predict(lm(y~x), data.frame(x=6), conf.level=0.95, interval="prediction")
       fit     lwr      upr
1 10.41062 2.83408 17.98715
```

predicted