Chapter 2: Descriptive Statistics and Displays

DSCC 462

Computational Introduction to Statistics

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Plan for Today

- Visualize datasets using graphs
- Describe important aspects of data using measures of **center** and **spread**
- Transform data to allow for statistical tests

Describing Data

- Once we have data collected, we need to distill it down into summaries that provide essential information
- **Goal**: Describe important attributes of a dataset in a concise manner
- When we summarize, some information is lost, but a great deal of communicative power can be gained
- Often, we want to use both numerical and graphical summaries
- The type of summary statistics you use will depend on the type of data

Describing Data

- Generally, we want to know what the distribution of our variables is
- **Distribution**: the values a variable can take on and how often it takes each value
- Make plots and construct tables to see what the distribution looks like

Describing Categorical Distributions

- **Absolute frequency table**: A table whose values correspond to how often we observe each of the categories of a variable
- Categories and numeric count in each category

Cause of Death	Number of Deaths
Cancer	12
Heart Attack	30
Stroke	10
Car Accident	53
Other	37

Describing Categorical Distributions

- **Relative frequency table**: A table whose values are the percentage of the total number of observations that appear in each category

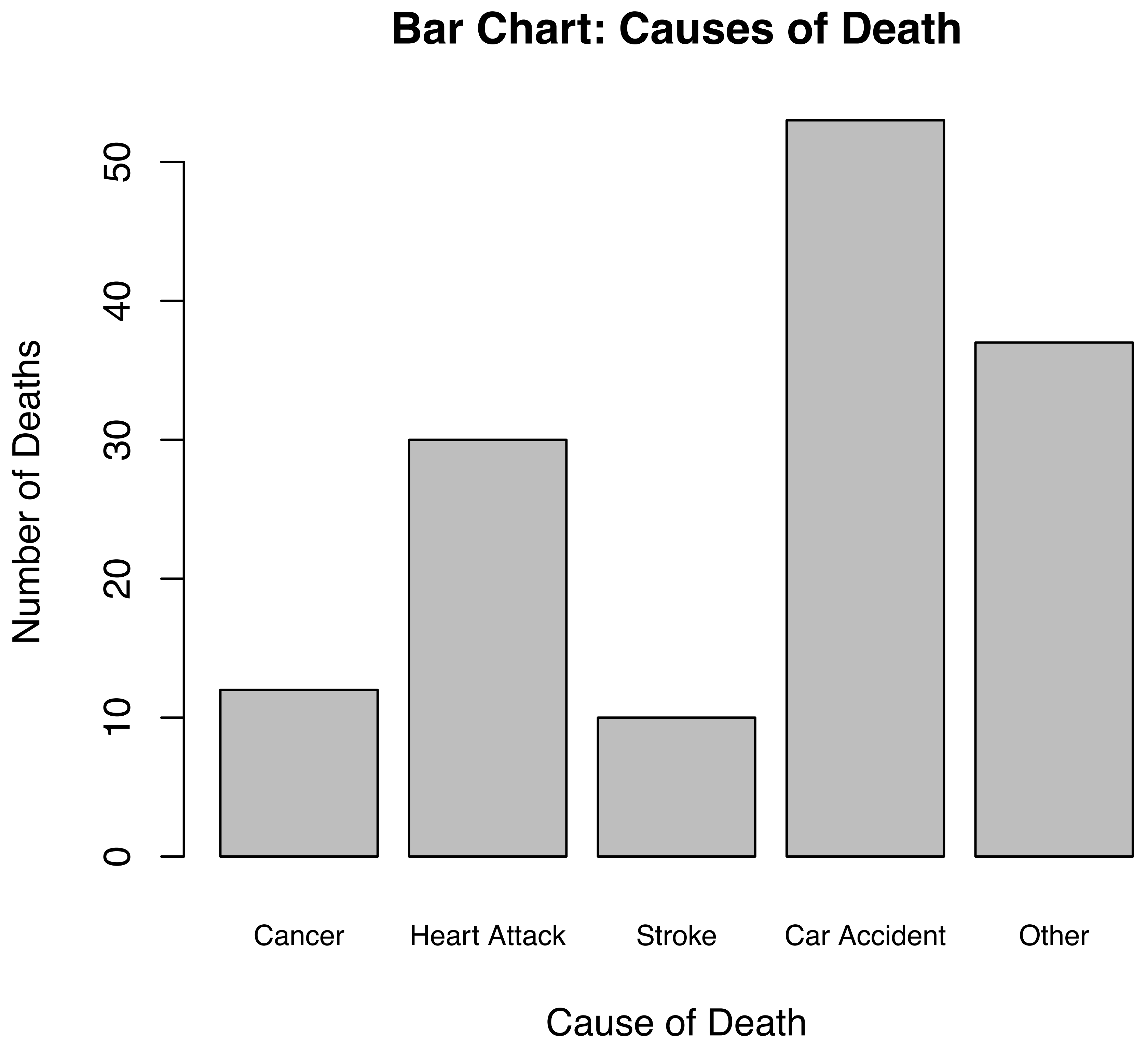
- Categories and proportion (relative frequency) in each category

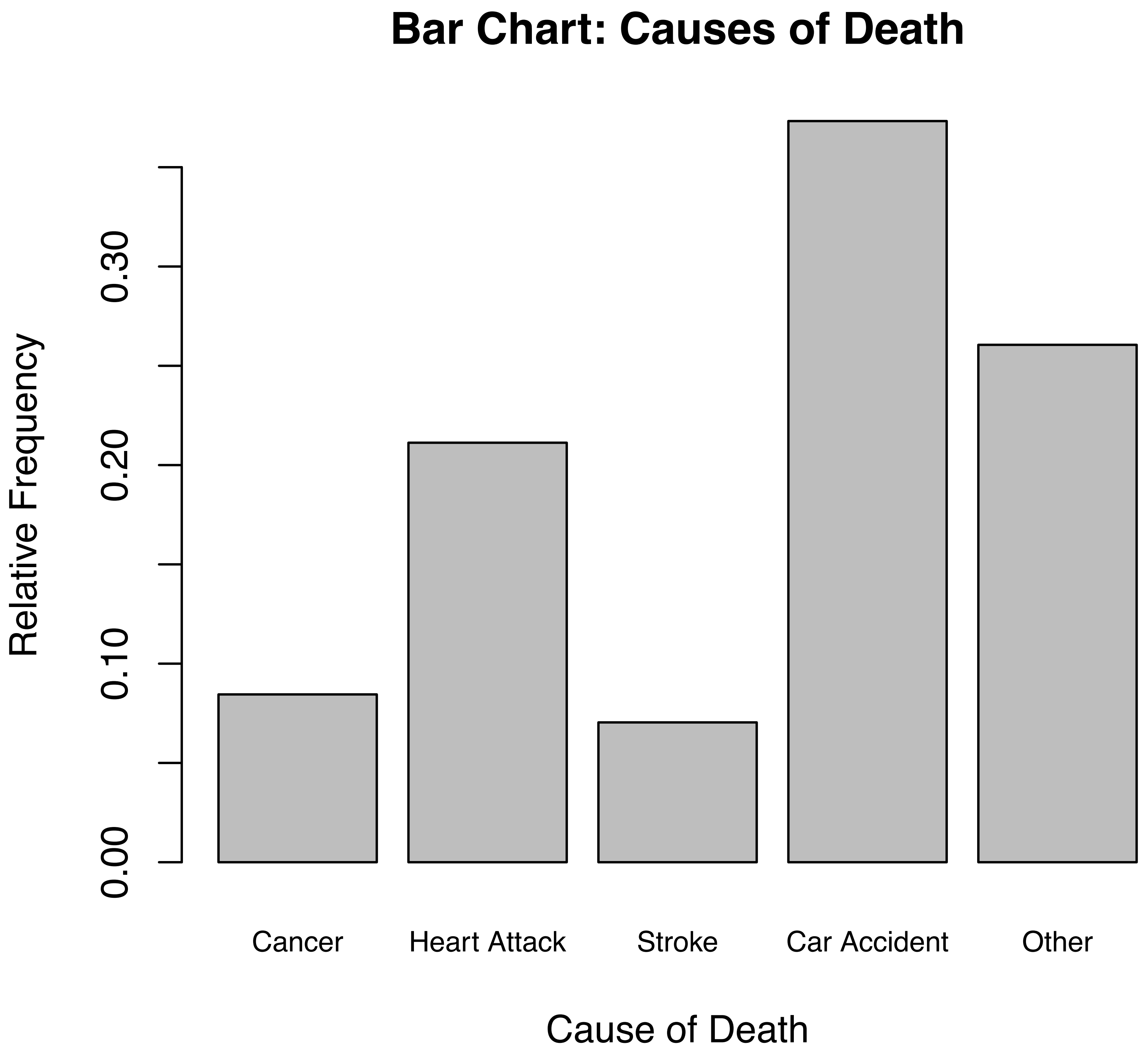
Cause of Death	Number of Deaths	Relative Frequency (%)
Cancer	12	8.45
Heart Attack	30	21.13
Stroke	10	7.04
Car Accident	53	37.32
Other	37	26.06

Describing Categorical Distributions: Graphically

- Graphically: We typically use a bar chart to illustrate the information in a frequency (or relative frequency) table
- **Bar chart**: a graphical display with categories listed on the horizontal axis and vertical bars drawn for each category, where the height of the bar represents the frequency (or relative frequency) of observations within that category
- Bars do not touch and are all of the same width

Bar Charts


Bar Chart: Causes of Death
Number of Deaths
0
10
20
30
40
50
Cancer
Heart Attack
Stroke
Car Accident
Other
Cause of Death


Bar Chart: Causes of Death
Relative Frequency
0.00
0.10
0.20
0.30
Cancer
Heart Attack
Stroke
Car Accident
Other
Cause of Death

Describing Continuous Distributions

- When data are continuous, we will want to describe distributions in a different way
- Typically, we are interested in how much of the data falls within a given range
  - Generally, ranges should be of equal length

Histogram

- **Histogram**: a graphical display of a frequency distribution for discrete or continuous data where the horizontal axis displays intervals for which the data is binned over, and the vertical axis represents the frequency (or relative frequency)

- Must determine *binwidth*, or the length of the intervals used for the horizontal axis

  - No best way for determining binwidth, though many possibilities exist (often, we first determine how many bins we want, and then evenly divide the range into that many bins)

- By default, R uses Sturges' formula: number of bins $k = \lceil \log_2(n) \rceil + 1$

Histogram

- Since the horizontal axis represents a continuous number line, there should *not* be breaks between bars of the histogram
- The only time that there should be space between histogram bars is when 0 entries are observed in a given interval

Histogram Workflow

1. Determine the minimum and maximum of the dataset
2. Calculate binwidth
3. Define a series of class intervals that are equal in size and adjacent to each other
    - The first class should contain the minimum; the last class should contain the maximum
    - If an observation is exactly on the boundary, put it in the lower bin
4. Determine how many observations fall within each bin
5. Indicate position of the class interval on the horizontal axis
6. At each class interval, draw a vertical bar equal in height to the number of observations in that class

Histogram Example

165.5	158.1	154.7	178.9	180.9	194.5	174.1	160.5
202.0	259.8	125.5	160.0	160.9	179.0	195.0	149.3
194.8	148.0	193.9	162.2	159.1	206.3	178.7	164.5
204.4	179.5	126.9	196.6	231.1	202.8	174.0	181.9
186.4	198.0	150.8	172.2	187.1	152.8	169.6	206.6
177.3	148.0	196.7	179.7	180.6	152.6	185.3	158.7
211.3	185.1	177.3	184.6	165.7	170.1	178.4	184.2

Table 1: Weights (lbs) of 56 patients

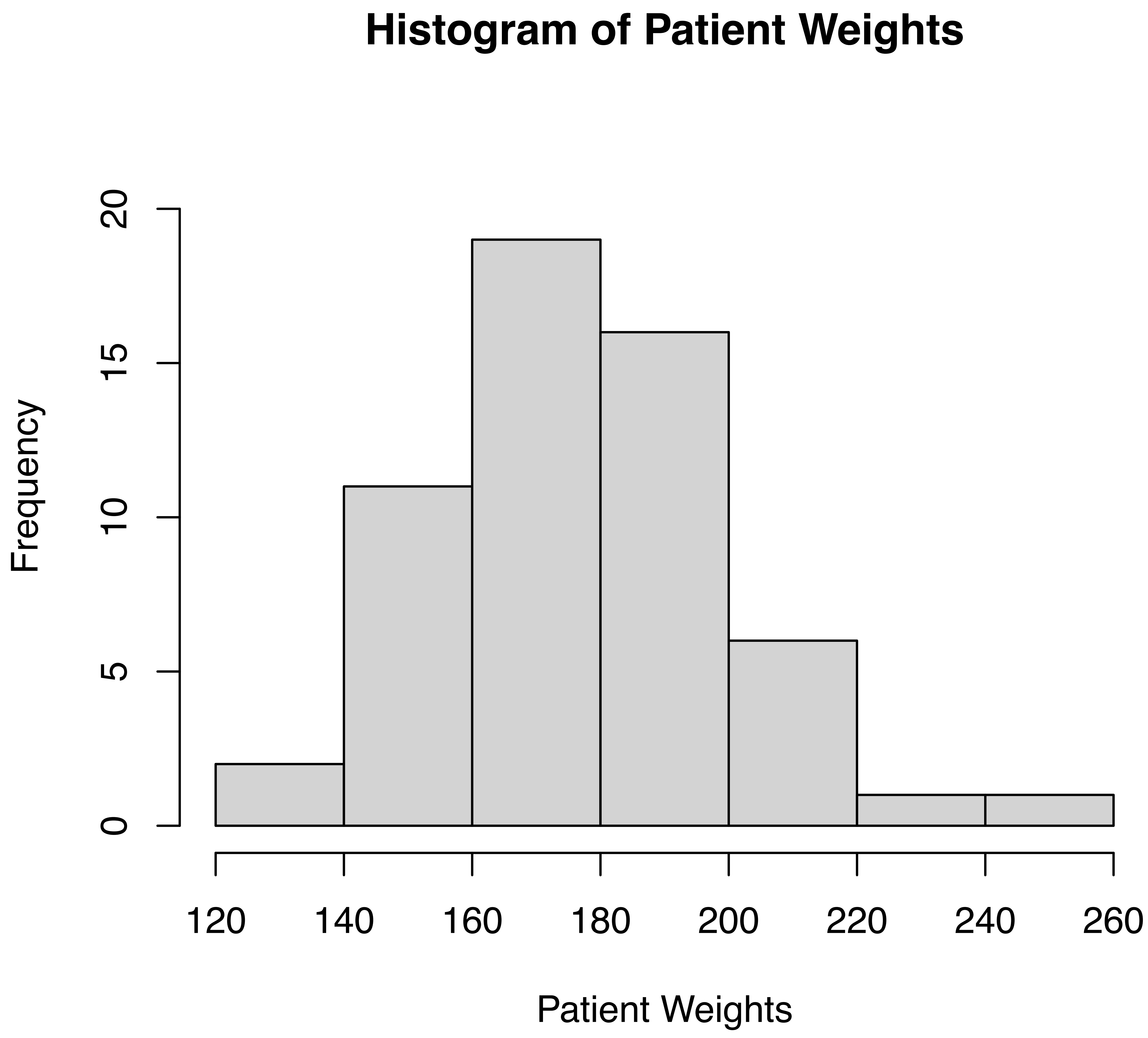
Histogram Example

- Minimum: 125.5
- Maximum: 259.8
- How many bins? $\lceil \log_2 56 \rceil + 1 = 7$ bins
- Range: 120 - 260, so each bin has length 20
- Frequency table

Weight	Frequency
(120, 140]	2
(140, 160]	10
(160, 180]	20
(180, 200]	16
(200, 220]	6
(220, 240]	1
(240, 260]	1

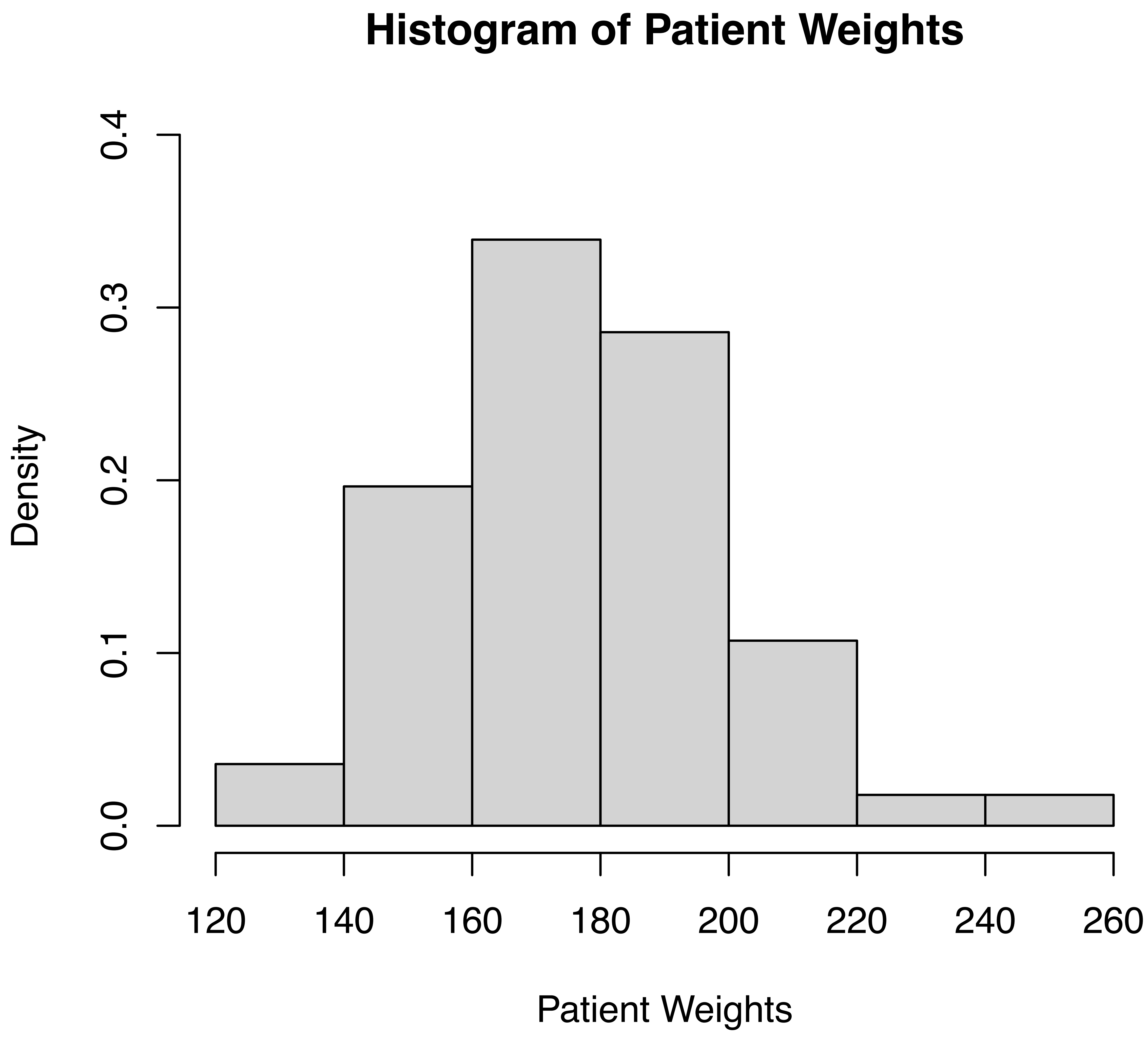
Histogram Example: Count

Weight	Frequency
(120, 140]	2
(140, 160]	10
(160, 180]	20
(180, 200]	16
(200, 220]	6
(220, 240]	1
(240, 260]	1



Histogram Example: Density

Weight	Rel. Frequency
(120, 140]	0.04
(140, 160]	0.20
(160, 180]	0.34
(180, 200]	0.29
(200, 220]	0.11
(220, 240]	0.02
(240, 260]	0.02



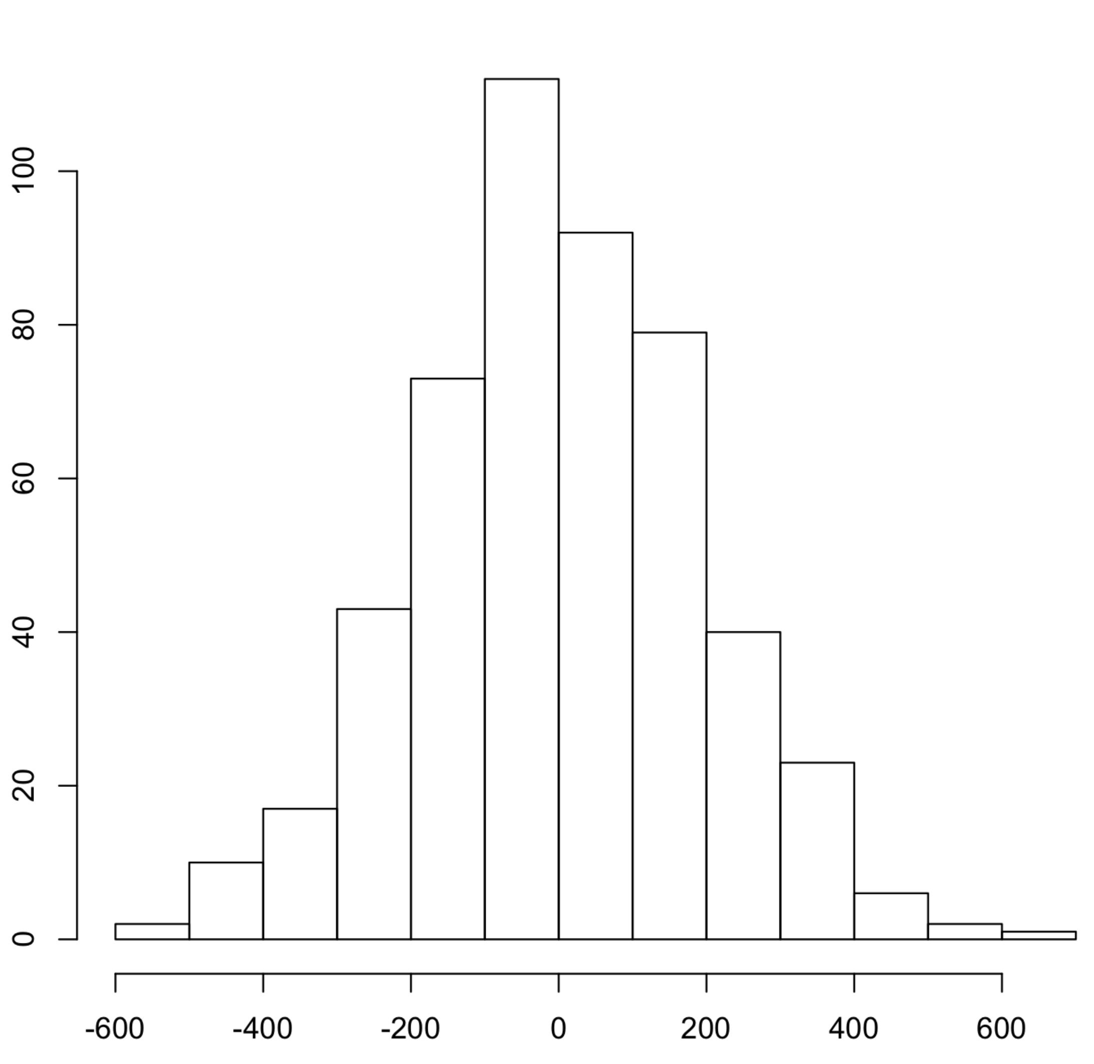
Histogram Properties

- Typically, we want to examine a histogram's:
  - Center
  - Shape
  - Spread
- Be aware of deviations from the typical pattern / extreme points
- **Outliers**: Data that are not typical of the rest of the values in the dataset

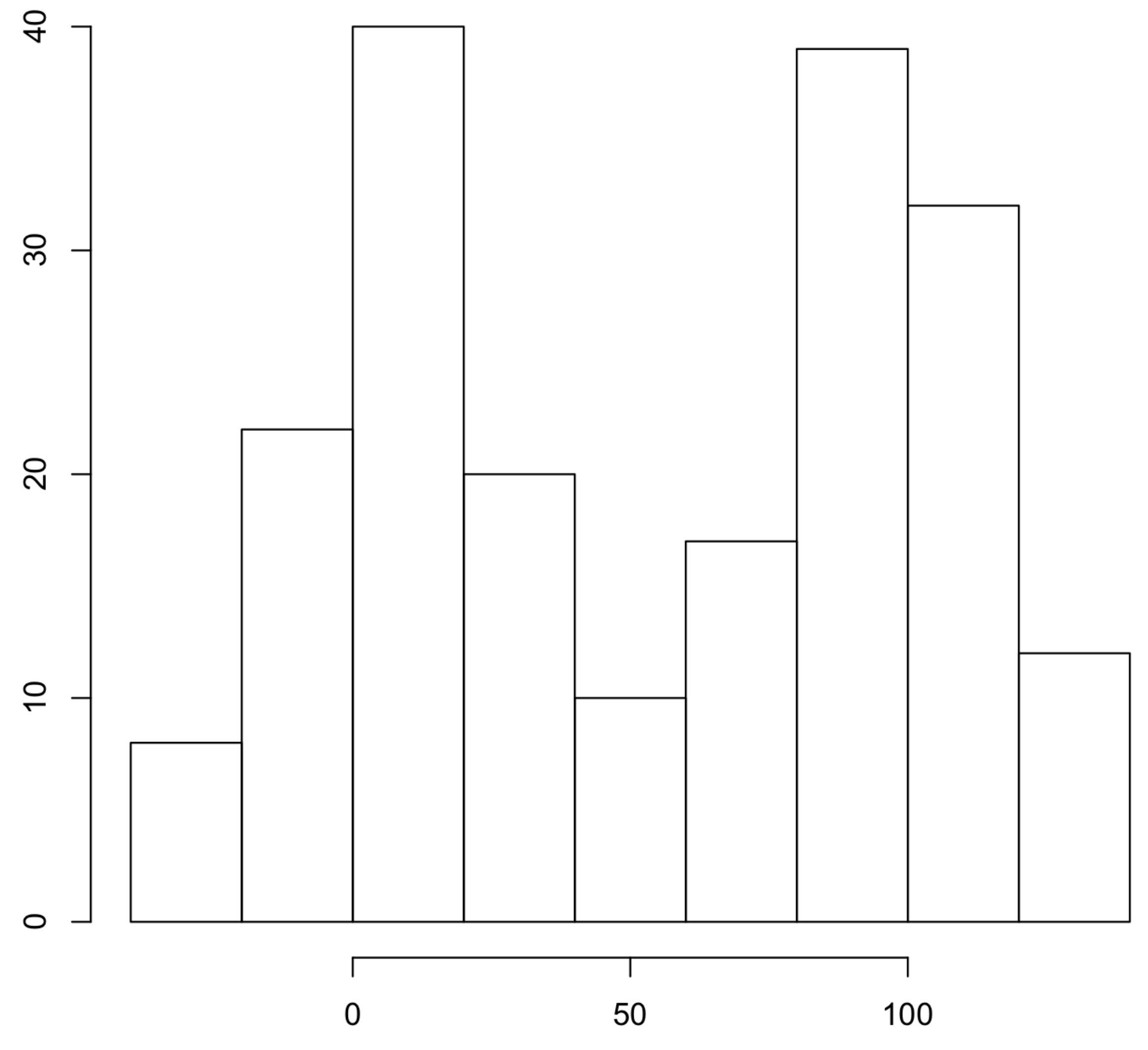
Center, Shape, and Spread

- Does the distribution appear to have one center, or are there multiple peaks?
- Is the distribution symmetric?
- Are the data all close together, or are they largely spread over the domain?
- Do any data points seem to deviate from typical patterns?

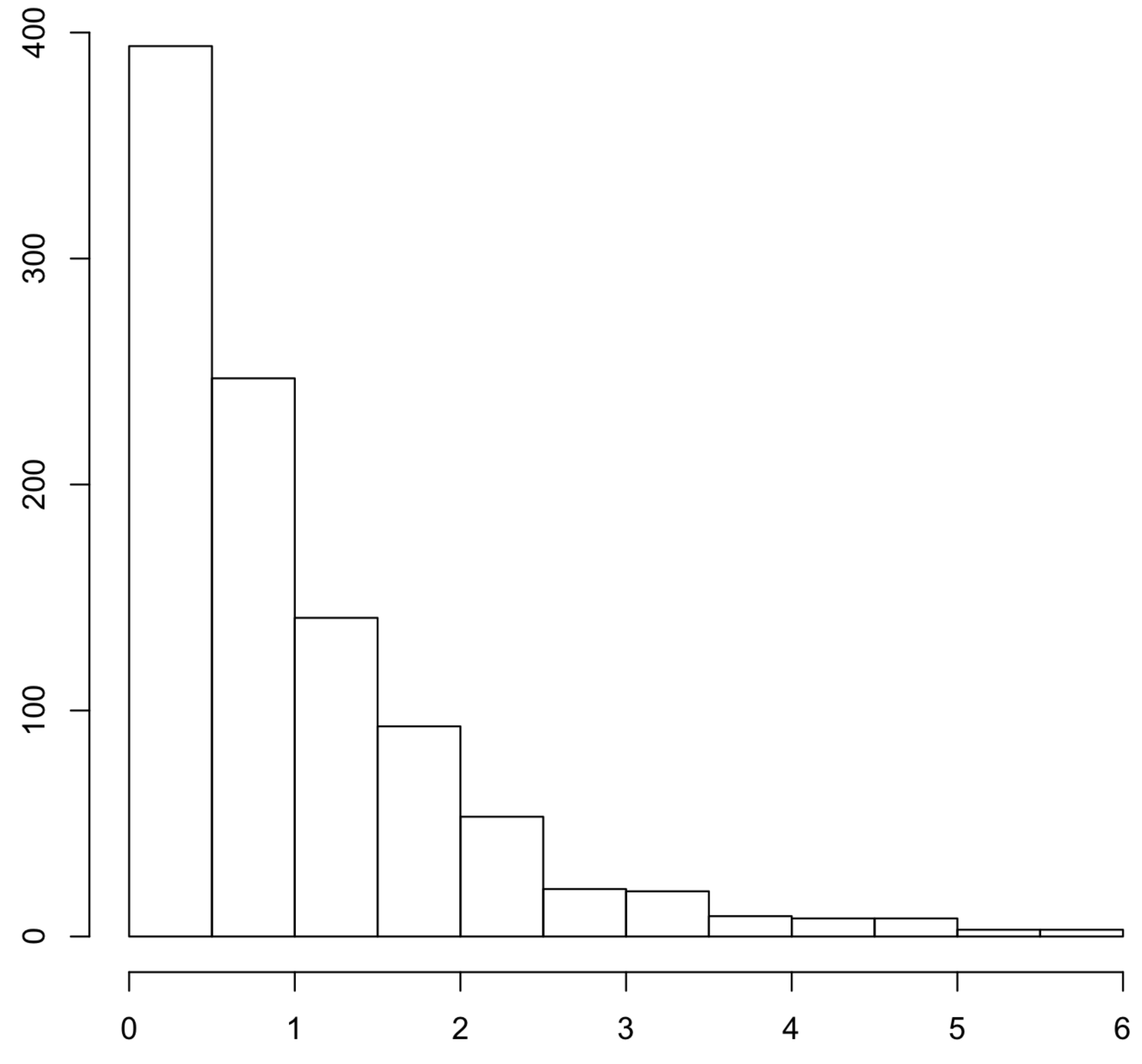
Center, Shape, and Spread: Examples



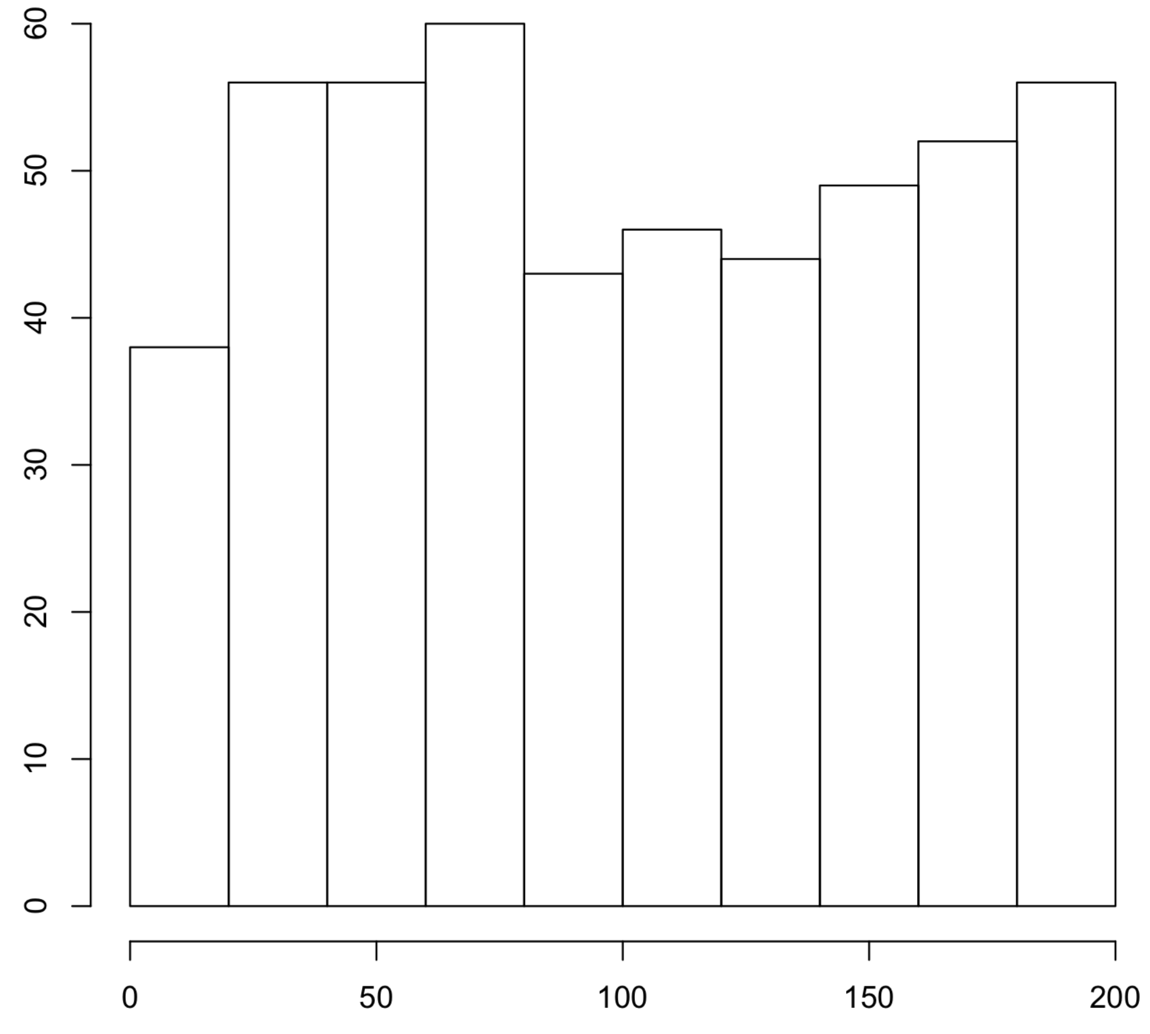
One center
Symmetric



Two centers
Symmetric



One peak
Asymmetric



Large spread
Uniform

Measure of Center: Mean

- Mean ($\bar{x}$): The sum of all observations divided by the total number of observations (n):

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

- The mean is sensitive to outliers
- Example: Find the mean of the following five heights (in): 66, 46, 68, 71, 72
  - R code: `mean(c(66,46,68,71,72))`

Mean for Patient Weights


```
hist(weights, xlab="Patient Weights", main="Histogram of Patient Weights", ylim=c(0,22))
abline(v=mean(weights), col="red")
text(mean(weights)+15,20.5,substitute(paste(bar(x),"=",m),list(m=round(mean(weights),3))),col=2)
```

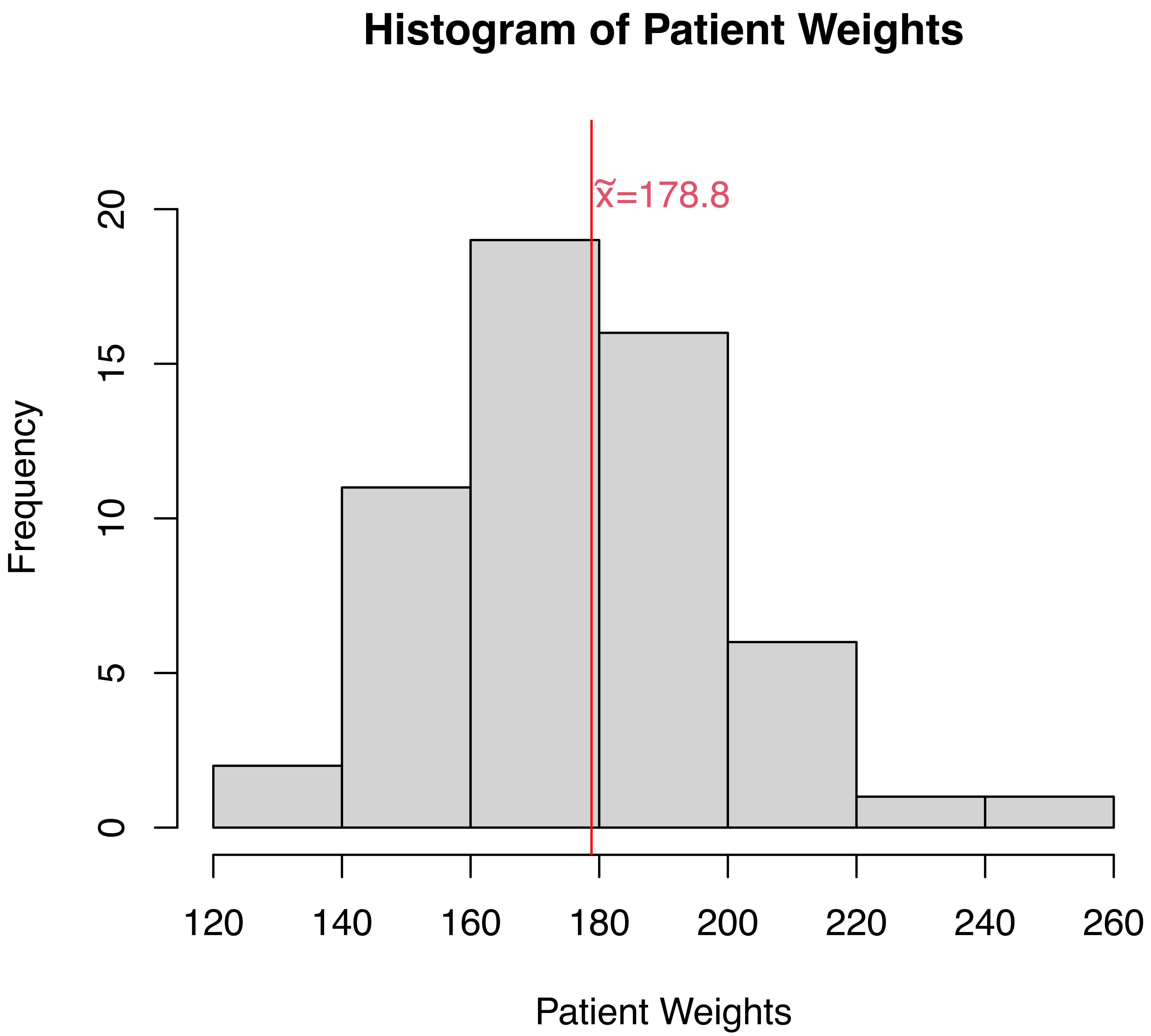
Order Statistics

- Consider data $X_1, X_2, \ldots, X_n$
- We can order these data from smallest to largest
- Let $X_{(k)}$ be the k^{th} largest observation in the dataset
- The order statistics are then $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$

Measure of Center: Median

- **Median**: The middle value of the ordered data, denoted $\tilde{X}$, also called the 50th percentile or Q2 (second quartile)
- Rank the data and find the observation for which half the data are greater than or equal to it and half the data are less than or equal to it
- As an equation: $\tilde{x} = \dfrac{x_{(\lfloor \frac{n+1}{2} \rfloor)} + x_{(\lceil \frac{n+1}{2} \rceil)}}{2}$
- The median is not as sensitive to outliers (it is more robust than the mean)
- Example: Find the median of the following five heights (in): 66, 46, 68, 71, 72
  - R code: `median(c(66,46,68,71,72))`

Median for Patient Weights




```
hist(weights, xlab="Patient Weights", main="Histogram of Patient Weights", ylim=c(0,22))
abline(v=median(weights), col="red")
text(median(weights)+11,20.5,substitute(paste(tilde(x),"=",m),list(m=round(median(weights),3))),col=2)
```

Measure of Center: Trimmed Mean

- ***K*% trimmed mean**: Sample mean of the data remaining after removing the highest *K*% and lowest *K*% of observations

- Example: Find the 10% trimmed mean of the sample 5, 15, 18, 2, 17, 10, 23, 20, 17, 16

  - $\bar{x}_{10\%} =$

- R code:

```
x <- c(5,15,18,2,17,10,23,20,17,16)
mean(x, trim=0.1)
```

Measure of Center: Trimmed Mean

- ***K*% trimmed mean**: Sample mean of the data remaining after removing the highest *K*% and lowest *K*% of observations

- Example: Find the 10% trimmed mean of the sample 5, 15, 18, 2, 17, 10, 23, 20, 17, 16

    - $\bar{x}_{10\%} = \dfrac{5 + 10 + 15 + 16 + 17 + 17 + 18 + 20}{8} = 14.75$

- R code:

```
x <- c(5,15,18,2,17,10,23,20,17,16)
mean(x, trim=0.1)
```

Measure of Center: Trimmed Mean

- K = 0% is the sample mean; K = 50% is the median
- The trimmed mean is a compromise between the mean and median
- Trimmed mean is relatively stable and not too sensitive to outliers

Measure of Center: Mode

- **Mode**: The observation that occurs most frequently
- If all values occur the same number of times, then there is no unique mode
- Example:
    - Find the mode: 3.1, 3.2, 4.5, 5.1, 5.9, 6.0
    - Find the mode: 7.1, 7.8, 7.8, 9.1, 9.3, 9.4, 9.4, 9.4

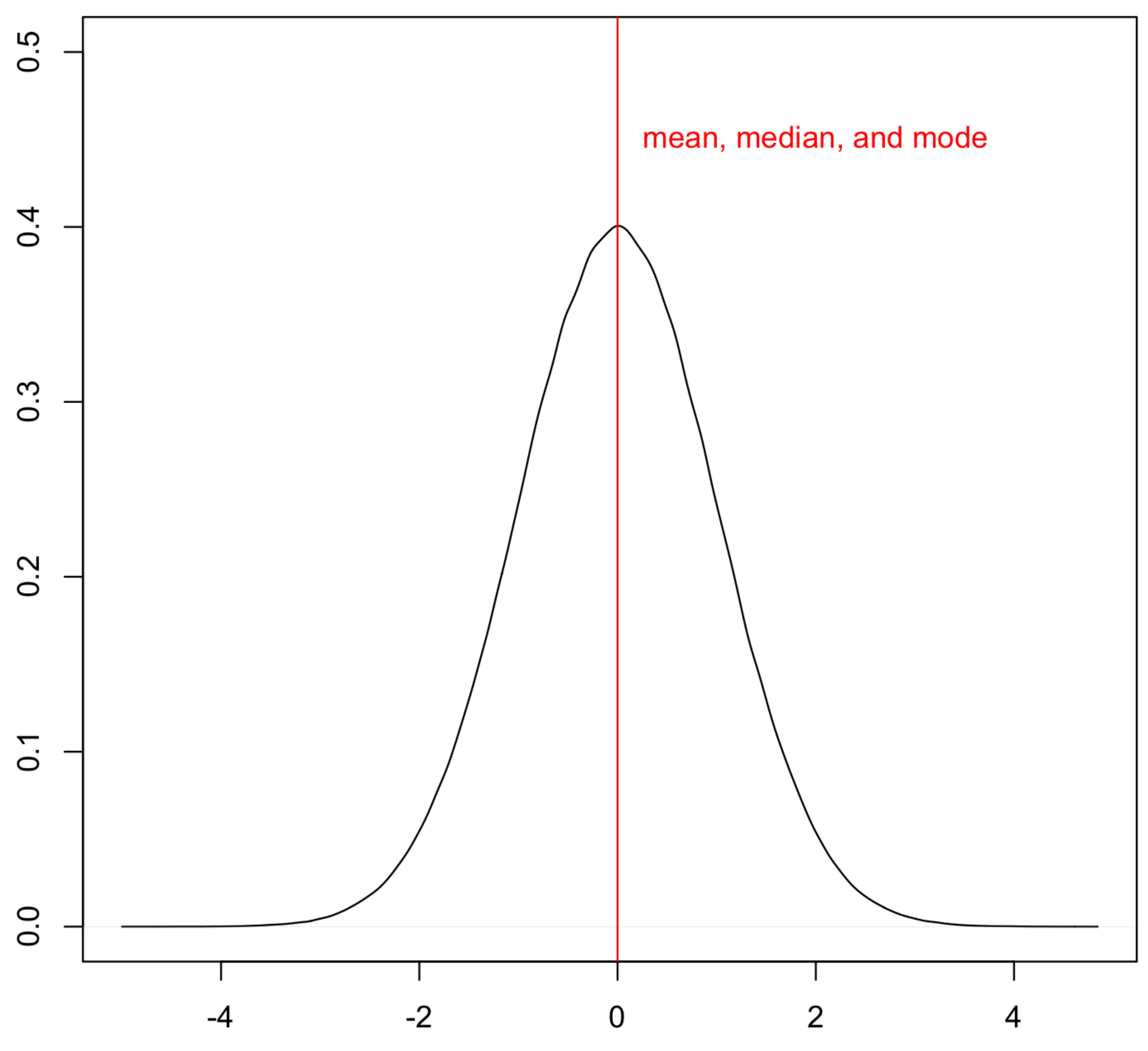
Measure of Center: Mode

- **Mode**: The observation that occurs most frequently
- If all values occur the same number of times, then there is no unique mode
- Example:
  - Find the mode: 3.1, 3.2, 4.5, 5.1, 5.9, 6.0
    - No unique mode
  - Find the mode: 7.1, 7.8, 7.8, 9.1, 9.3, 9.4, 9.4, 9.4
    - Mode = 9.4

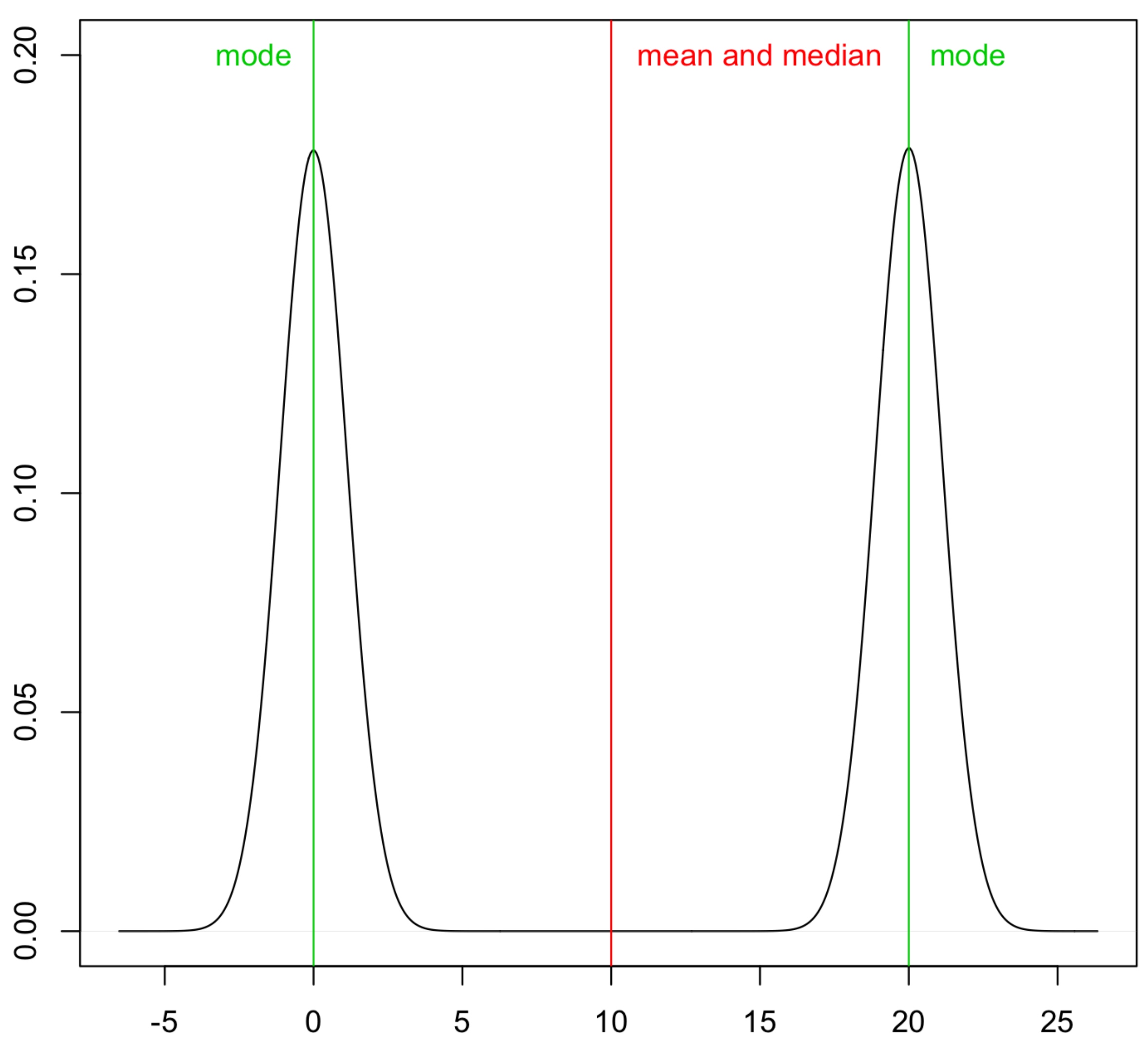
Measures of Center

- The "best" measure of central tendency (mean, trimmed mean, median, mode) generally depends on the distribution of values
- If a distribution is symmetric and unimodal, then the mean, median, and mode should all approximately be the same
- **Unimodal**: A histogram or density plot only has one peak
- **Bimodal**: A histogram or density plot has two peaks
- **Multimodal**: A histogram or density plot has multiple peaks

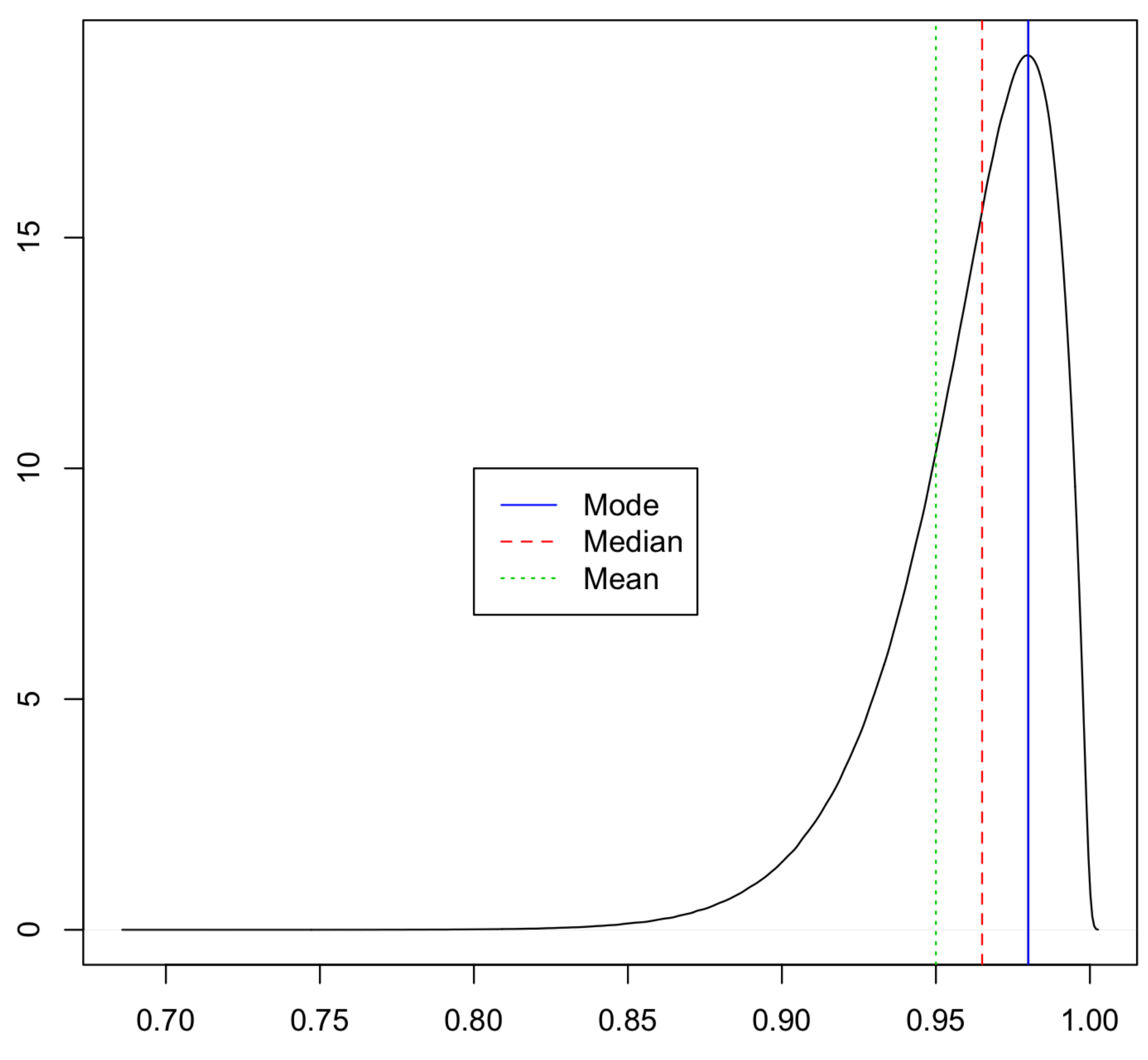
Measures of Center: Comparison



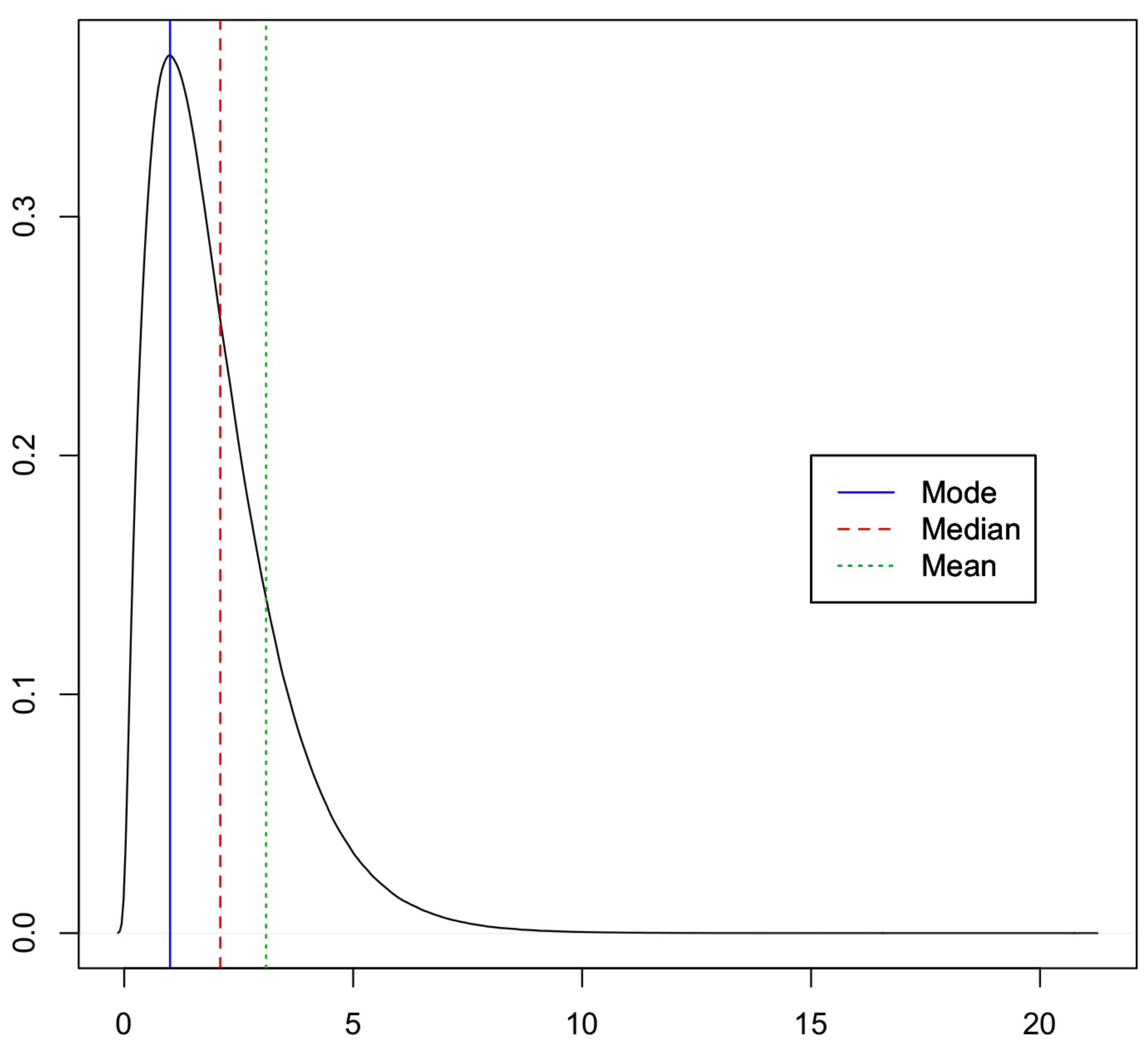
Symmetric
Unimodal



Symmetric
Bimodal

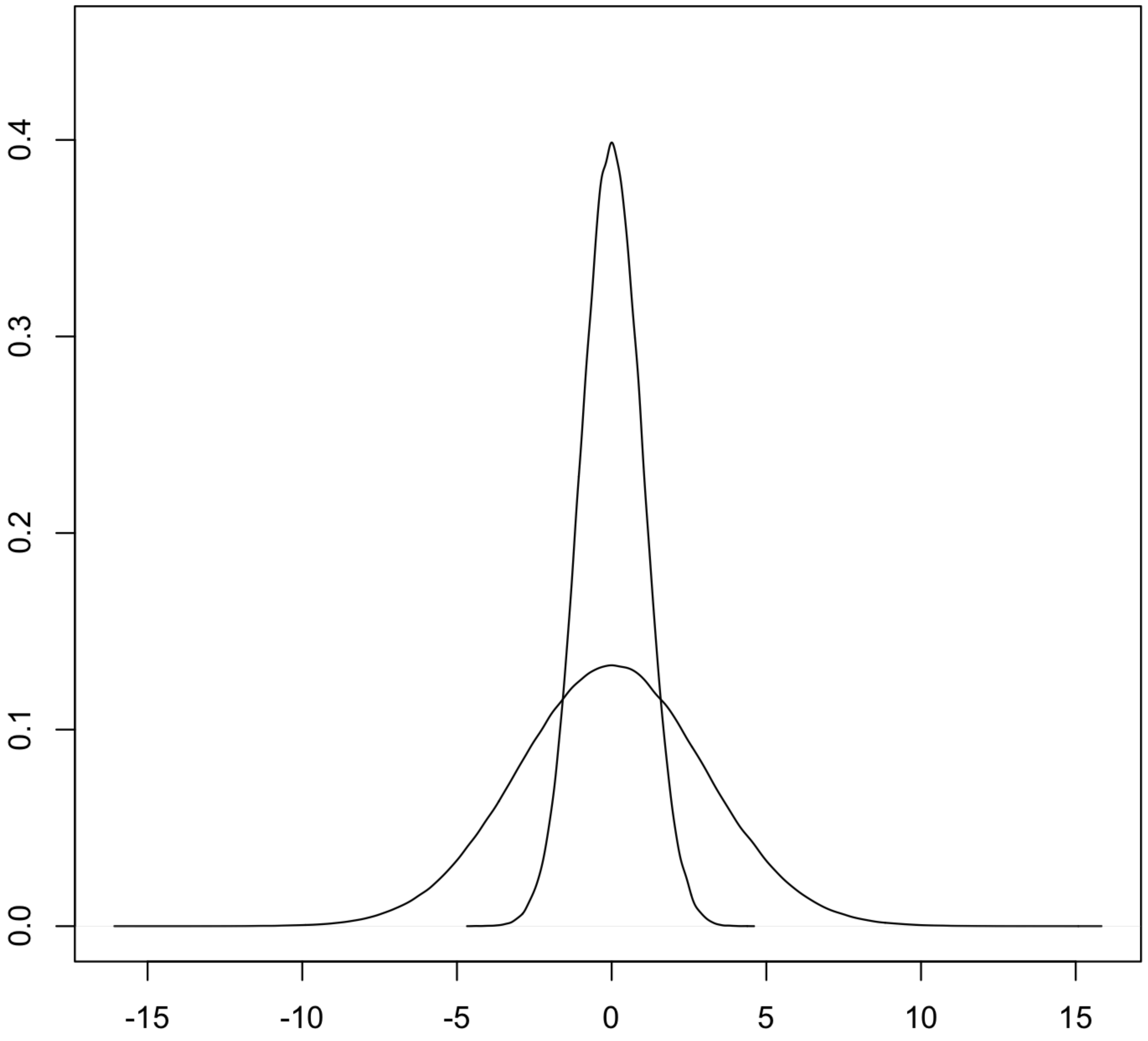


Left-skewed
Unimodal



Right-skewed
Unimodal

Measures of Center: Not the Whole Picture



Measures of Dispersion

- Measures of dispersion give us information regarding the *spread* of the data (i.e., how variable it is)
- While knowing where the center of a distribution is may be important, it is also essential to know how disperse the data is around that center to better understand how the variable acts

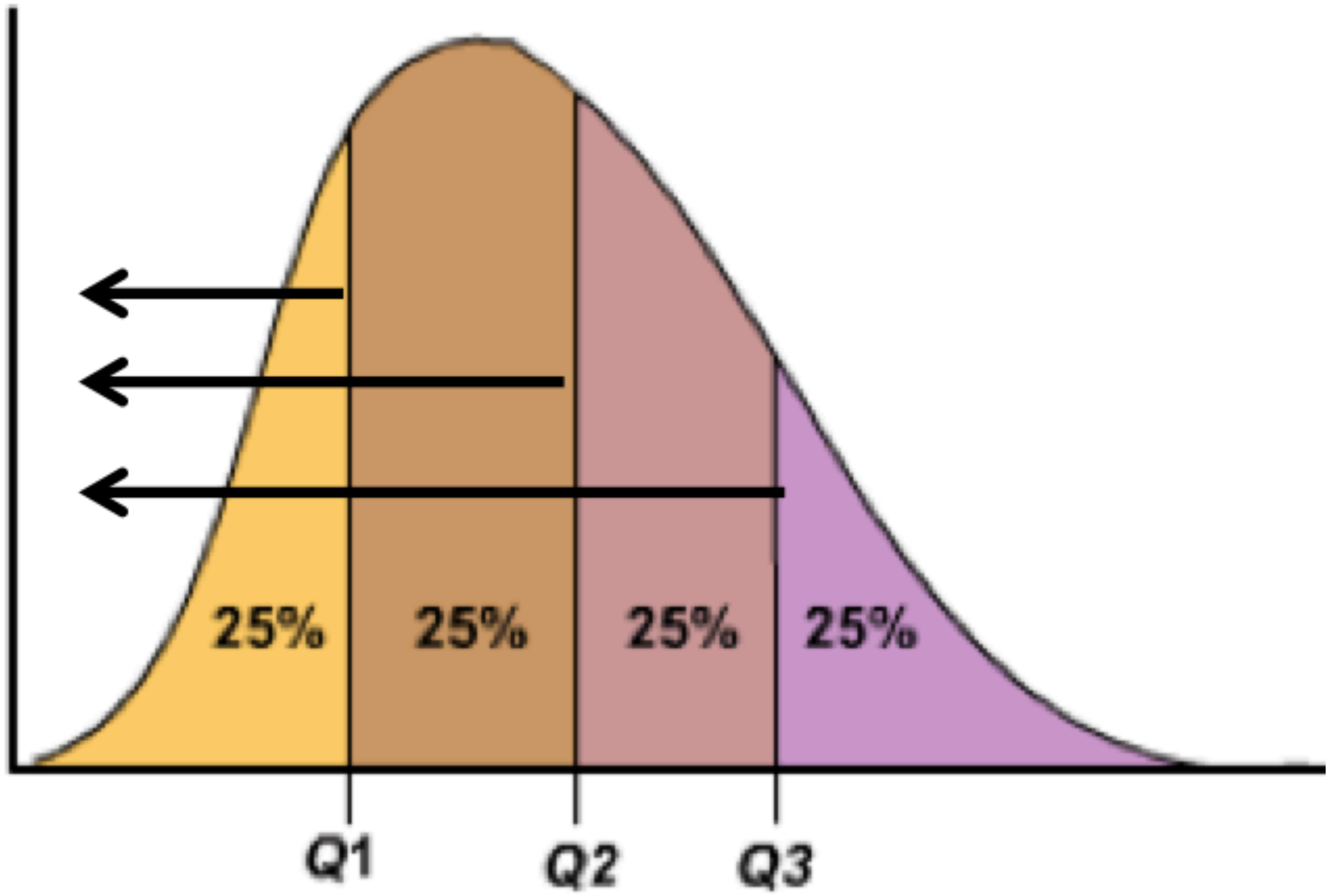
Quantiles (and Percentiles and Quartiles)

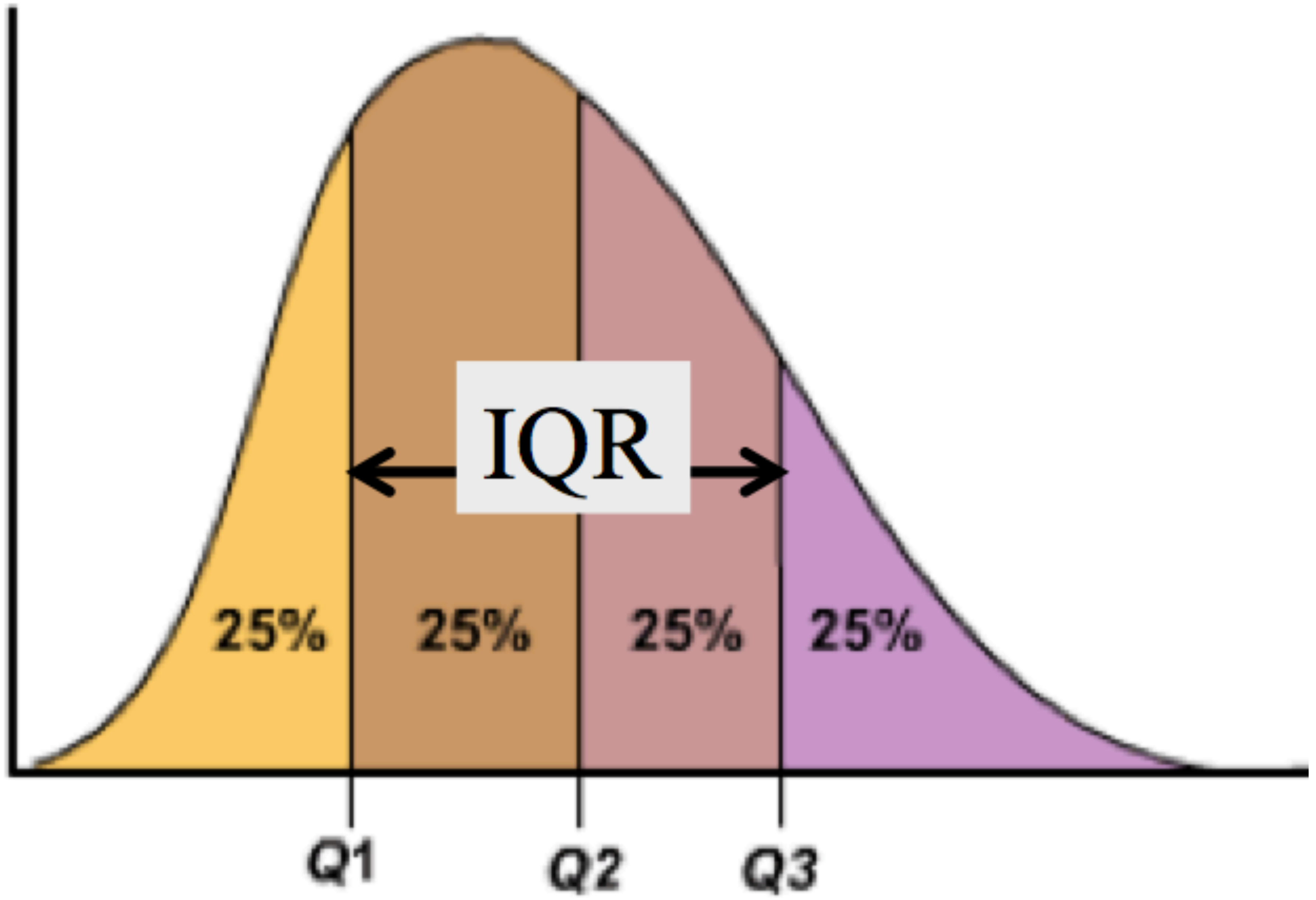
- The *p sample quantile* is the value below which a proportion p of the data are located (and above which a proportion $1 - p$ of the data are located)
- The k^{th} *percentile* is the $(k/100)$ sample quantile
  - E.g., if your birth weight is at the 95th percentile, then you weighed more than 0.95 of all newborn babies
- *Quartiles* correspond to 0.25, 0.50, and 0.75 sample quantiles (split the data into four equal parts)
- Sample quantiles are constructed based on the order statistics
- In R: `quantile()`
  - 9 methods for calculating sample quantiles (types 1-3 are for discrete distributions; types 4-9 are for continuous distributions).
  - Default: `type=7`

Interquartile Range (IQR)

- **Interquartile Range (IQR)**: the difference between the 75^{th} percentile and the 25^{th} percentile
- IQR = Q3 - Q1
- Middle 50% of the observations in a given dataset
- R code: (Option 1) `IQR(data)`
  OR (Option 2) `quantile(data,0.75)-quantile(data,0.25)`

Interquartile Range (IQR)





Interquartile Range (IQR): Step by Step

- Find the median: $\tilde{x} = \dfrac{x_{(\lfloor \frac{n+1}{2} \rfloor)} + x_{(\lceil \frac{n+1}{2} \rceil)}}{2}$
- Lower half of data: Everything less than or equal to median ($\leq \tilde{x}$)
- Upper half of data: Everything greater than or equal to median ($\geq \tilde{x}$)
- Q1 (25^{th} percentile): Median of the lower half of data
- Q3 (75^{th} percentile): Median of the upper half of data
- IQR: Q3 - Q1

Interquartile Range (IQR): Example, $n = 14$

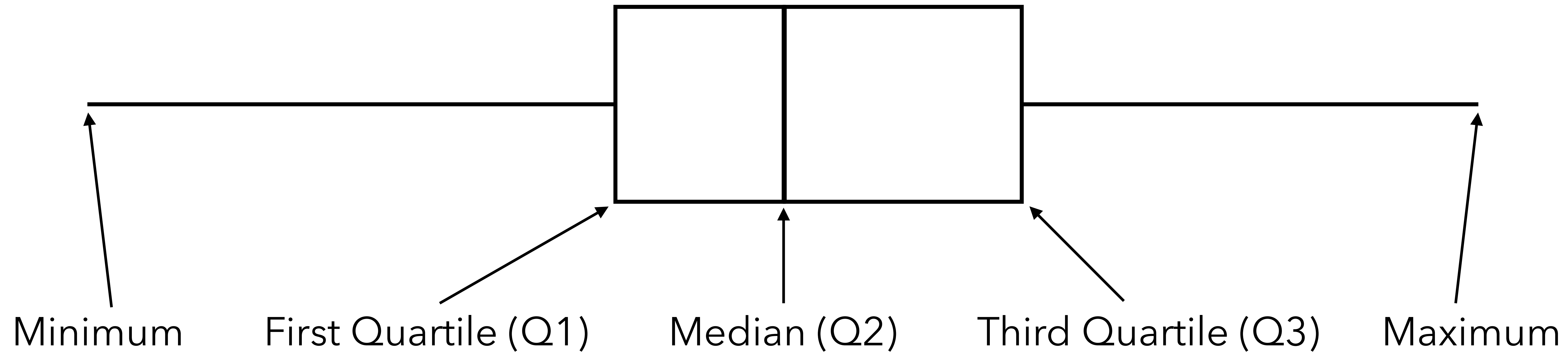
11, 14, 15, 20, 41, 45, 61, 71, 74, 80, 93, 95, 97, 100

Boxplot

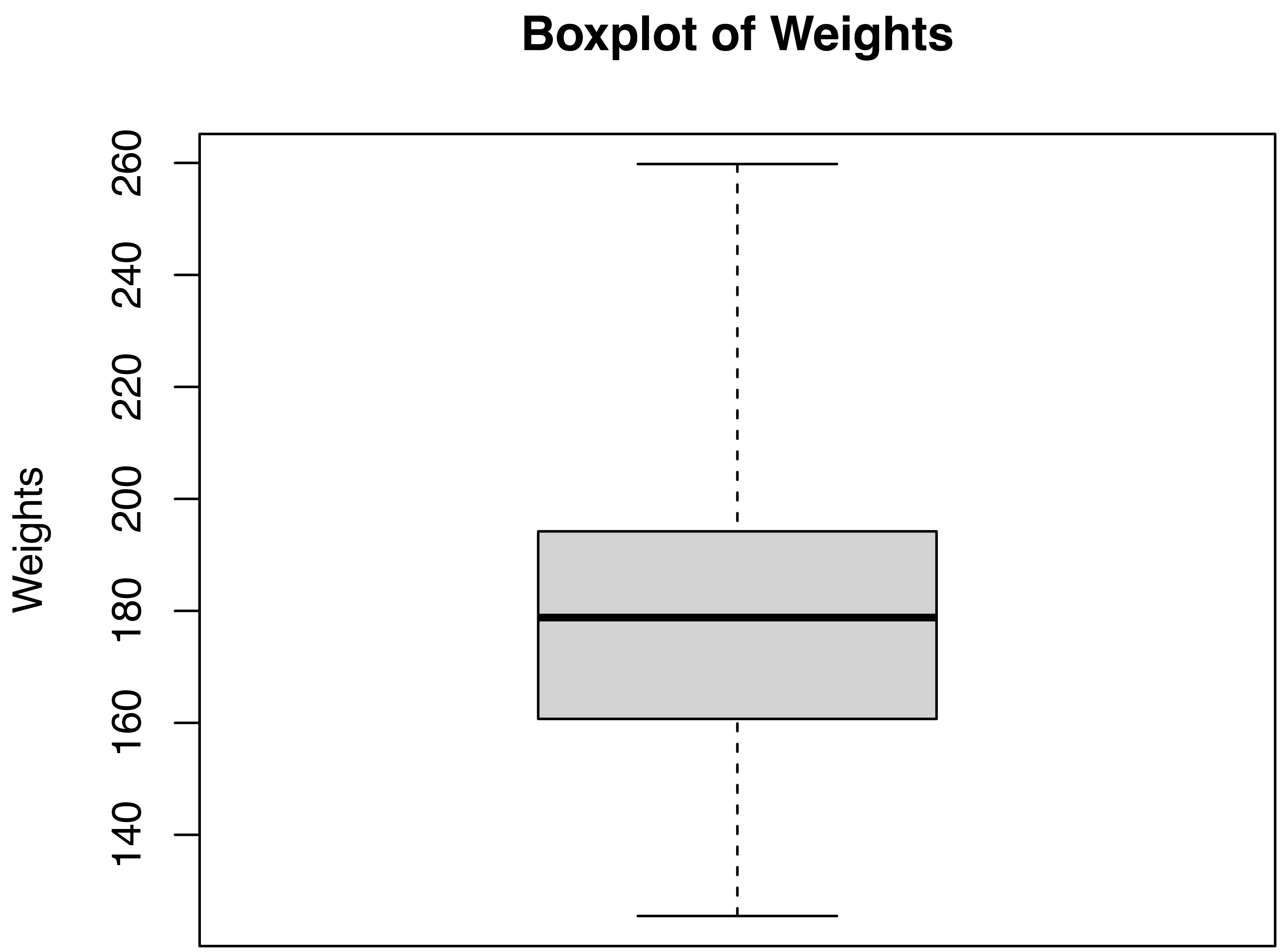
- Boxplots differ from histograms in that they only require one axis and display only summary statistics based on *quartiles*
- **Quartile**: percentiles that break the data into four equal groups
    - 1st quartile (Q1): 25th percentile
    - 2nd quartile (Q2): 50th percentile (median)
    - 3rd quartile (Q3): 75th percentile

Boxplot

- Five number summary: *minimum*, *Q1*, *Q2*, *Q3*, and *maximum*
- A skeletal boxplot uses only the five number summary
- In R: `fivenum(data)`



Boxplot: Example

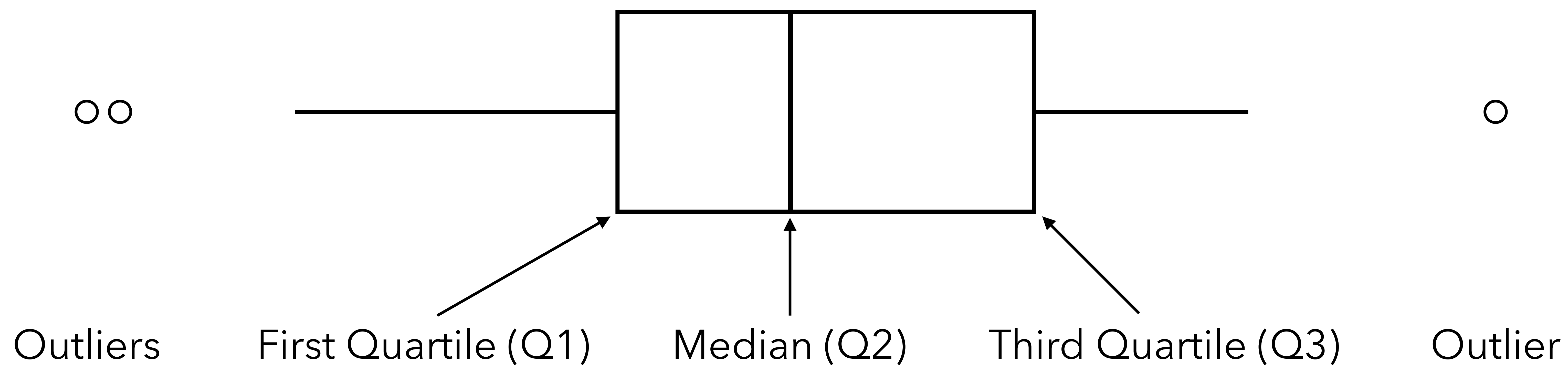



```
boxplot(weights,range=0,main="Boxplot of Weights",ylab="Weights")
```

Modified Boxplot

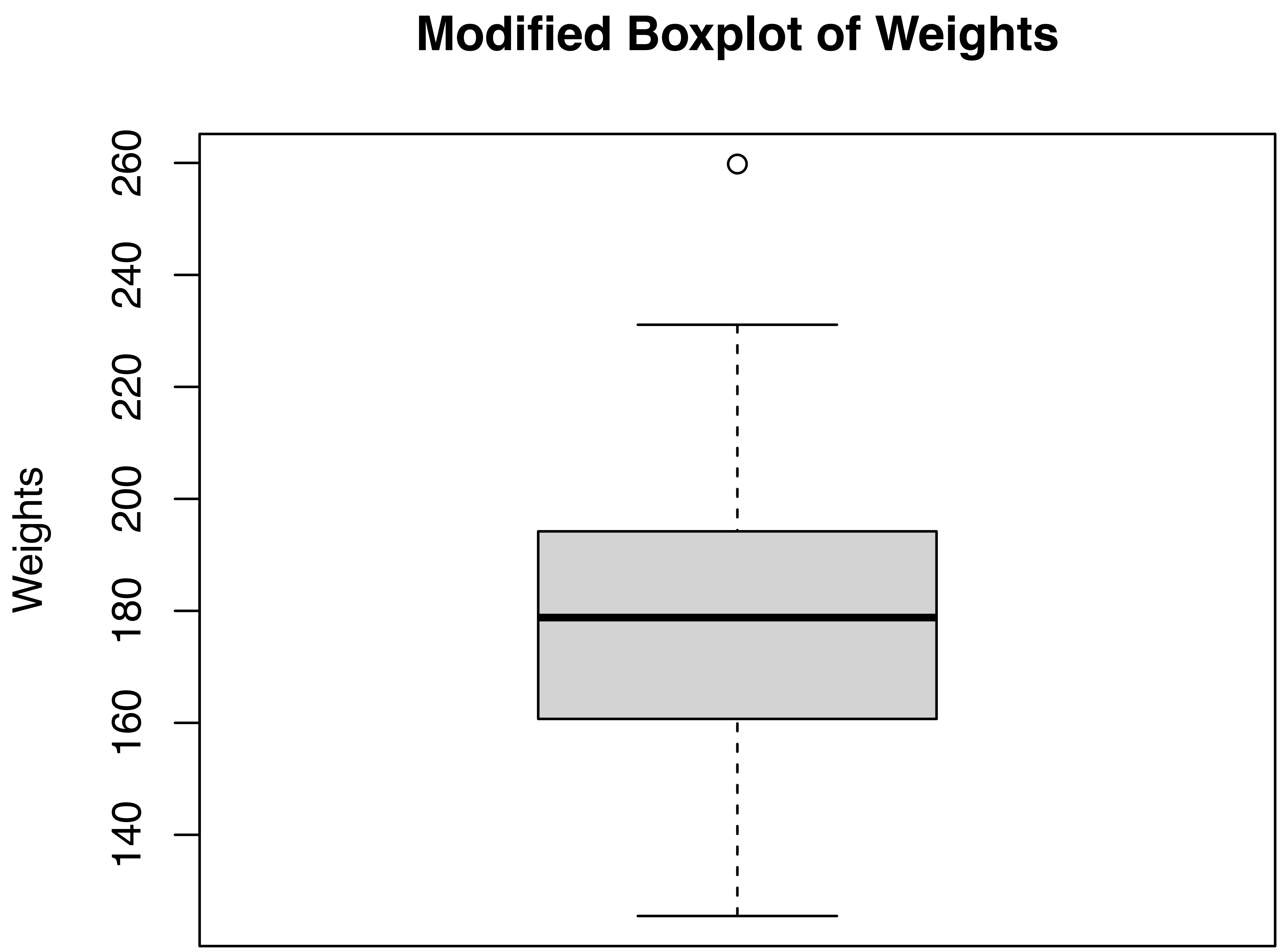
- A **modified boxplot** uses the quartiles, but it also distinguishes outliers from the rest of the data
- Modified boxplots are the standard boxplots that are, generally, the most beneficial to use
- An outlier is a data point that is either:
  - Less than: $Q1 - 1.5 \times (Q3 - Q1)$
  - Greater than: $Q3 + 1.5 \times (Q3 - Q1)$
- Standard span: $1.5 \times (Q3 - Q1) = 1.5 \times IQR$
- Now, whiskers extend only to the highest / lowest non-outlier points, and the outliers are distinguished as separate points

Modified Boxplot



- Note that whiskers extend to the most extreme observation that isn't an outlier, and there can be multiple outliers beyond the whiskers

Modified Boxplot: Example

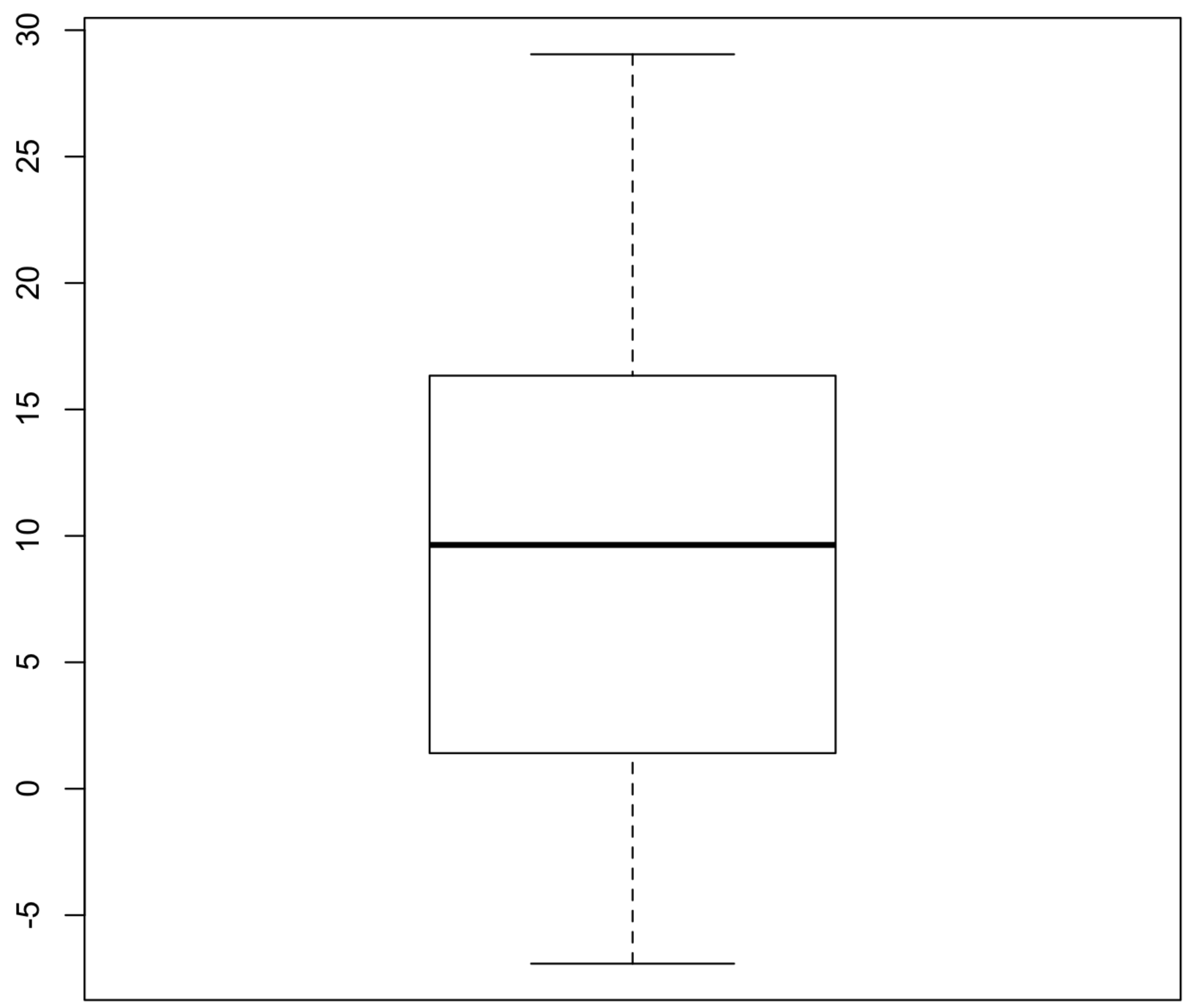



```
boxplot(weights,main="Modified Boxplot of Weights",ylab="Weights")
```

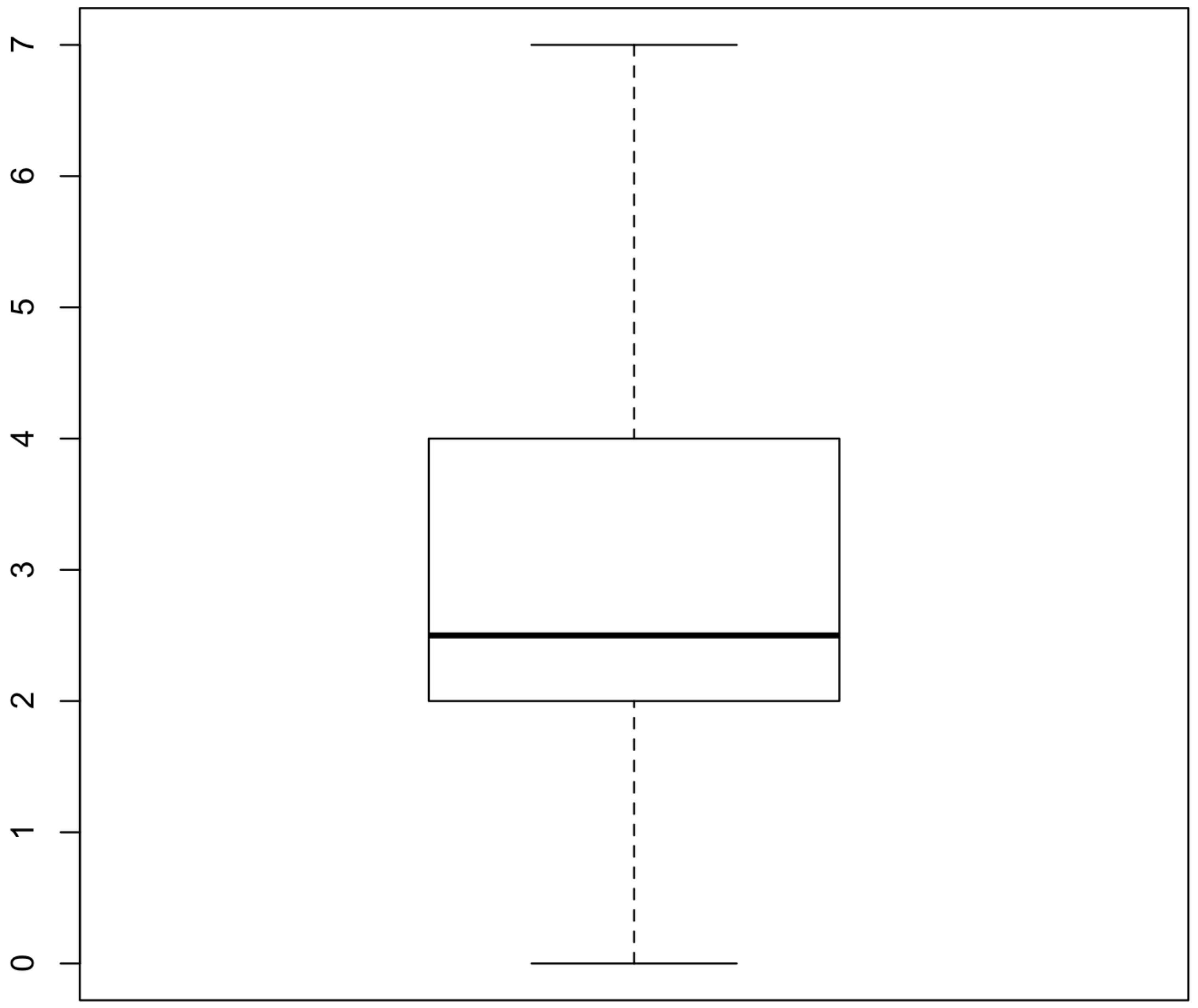
Modified Boxplot: Step by Step

- Construct an axis covering all observations
- Draw a box with lines at Q1, Q2, and Q3
- Locate the *upper fence*: $Q3 + 1.5 \times (Q3 - Q1)$ and *lower fence*: $Q1 - 1.5 \times (Q3 - Q1)$
- Indicate the upper whisker, given by the largest observation that is less than the upper fence
- Indicate the lower whisker, given by the smallest observation that is greater than the lower fence
- Mark outliers as individual points beyond the fences

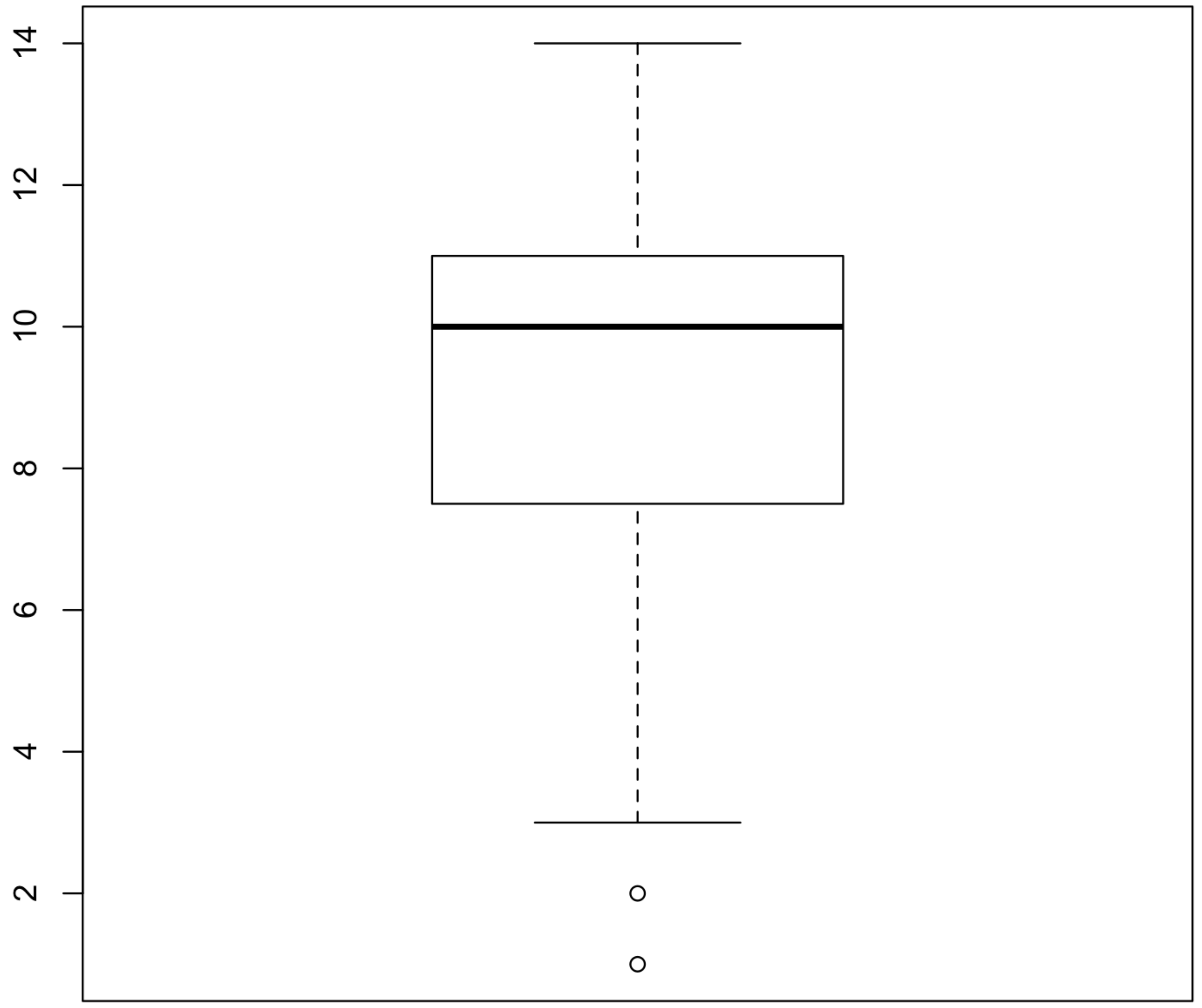
(Modified) Boxplot Examples



Symmetric



Asymmetric
(Right Skew)



Asymmetric
(Left Skew)

Measures of Dispersion: Variance

- **Variance**: The average squared deviation of observations from the mean
- Measured in squared units (e.g., if measuring weights, units are pounds^2)
- We calculate the *sample variance* of our observations as follows:

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

- R code: `var(data)`

Measures of Dispersion: Standard Deviation

- **Standard deviation (SD)**: The positive square root of the variance
- Size of the "typical" deviation from the mean, measured in the same units as observations
- We calculate the *sample standard deviation* of our observations as follows:

$$s = \sqrt{s^2} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

- R code: `sd(data)`

Variance and SD vs. IQR

- In general, variance and standard deviation are more sensitive to outliers than the interquartile range
- Typically, use mean and variance (and standard deviation) when the data is symmetric; when the data is skewed, use median and IQR
- The magnitude of the variance or SD depends on the problem
  - What is large for one group may be small for another
  - Remember that SD is on the same scale as the observations

Updating Sample Mean and Variance

- When a new data point is added to a set of n data points, we don't have to completely recalculate the mean and variance

- Let $\bar{x}_n$ and s_n^2 be the sample mean and variance, respectively, of our n original data points

- Updating for the $(n+1)^{st}$ data point, we get:

$$\bar{x}_{n+1} = \frac{n}{n+1}\bar{x}_n + \frac{1}{n+1}x_{n+1}$$

$$s_{n+1}^2 = \frac{n-1}{n}s_n^2 + \frac{1}{n+1}(x_{n+1} - \bar{x})^2$$

Coefficient of Variation

- The **coefficient of variation** is defined as the ratio of the standard deviation to the mean: $CV = s/\bar{x}$
- Describes dispersion in a way that does not depend on units
- Beneficial for comparing dispersion across different scenarios
- The higher the CV, the higher the dispersion
- Only applicable for data on a ratio scale (meaningful zero)

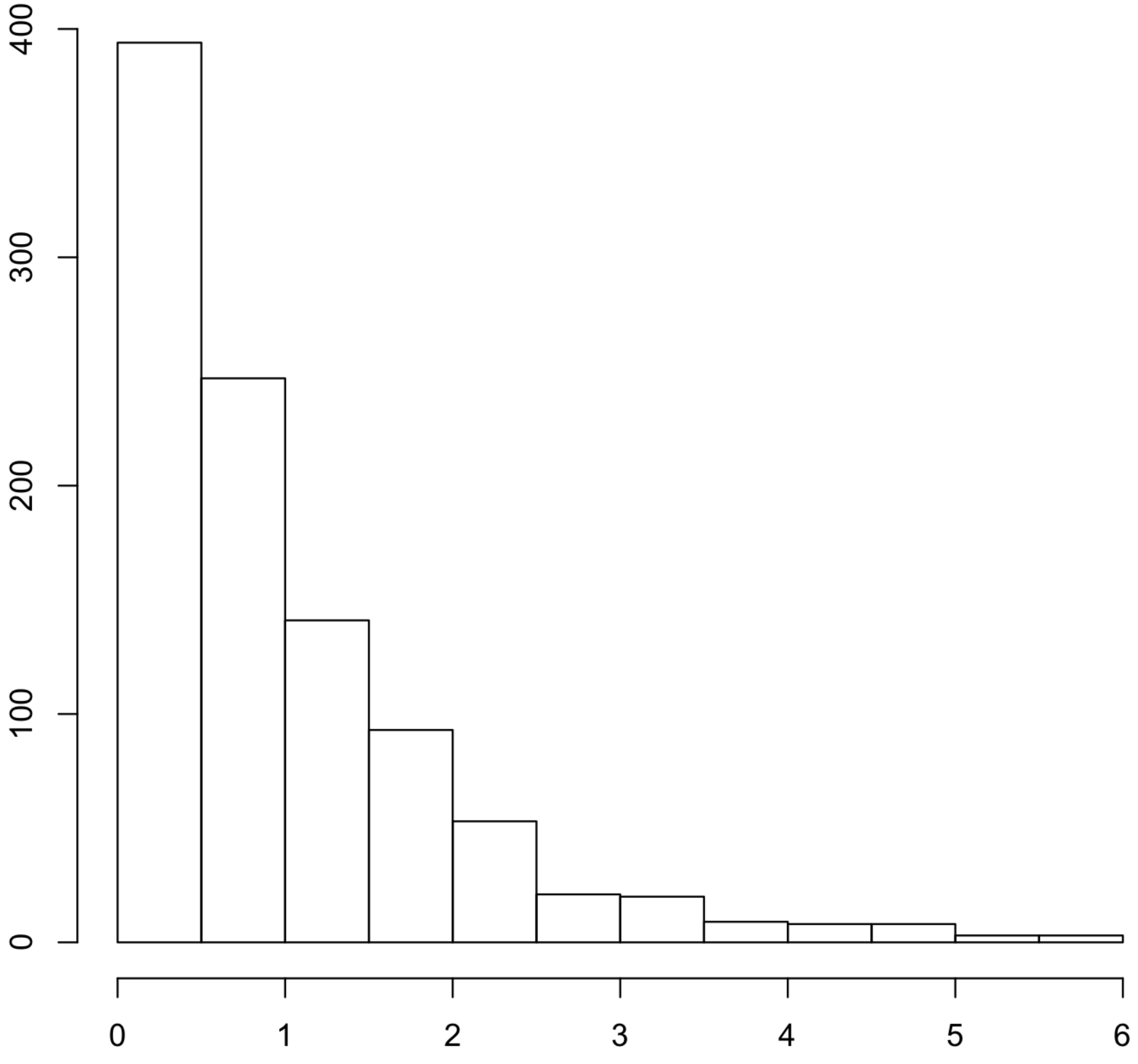
Skewness

- **Skewness**: a measure of asymmetry of a distribution

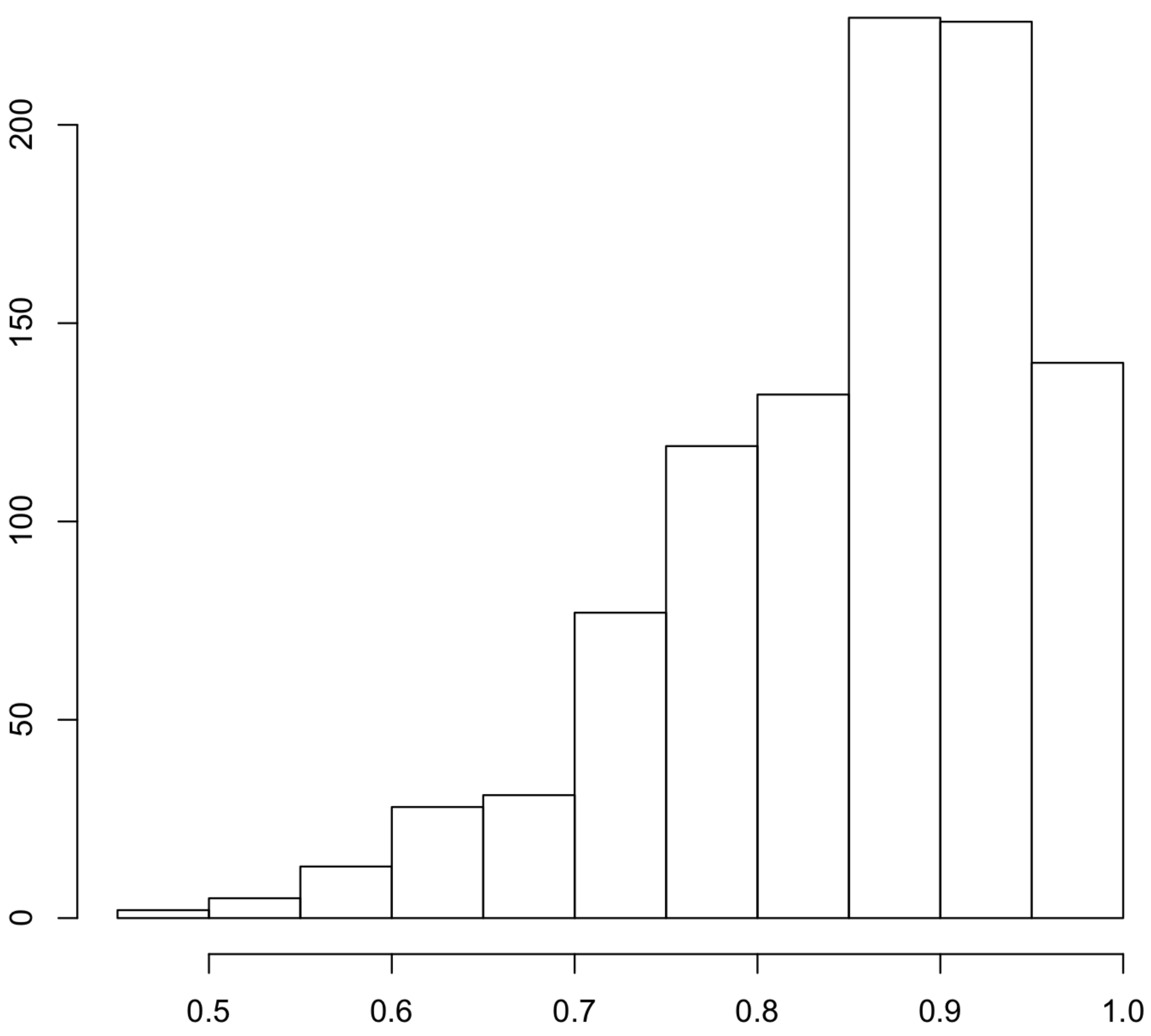
$$\text{skew} = \frac{m_3}{s^3} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^3}{\left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2\right]^{3/2}}$$

- Left (negative) skew: The left tail extends farther out than the right tail
- Right (positive skew): The right tail extends farther out than the left tail
- Symmetric distributions have skew 0
- R code: `library(moments); skewness(data)`

Skewness: Examples



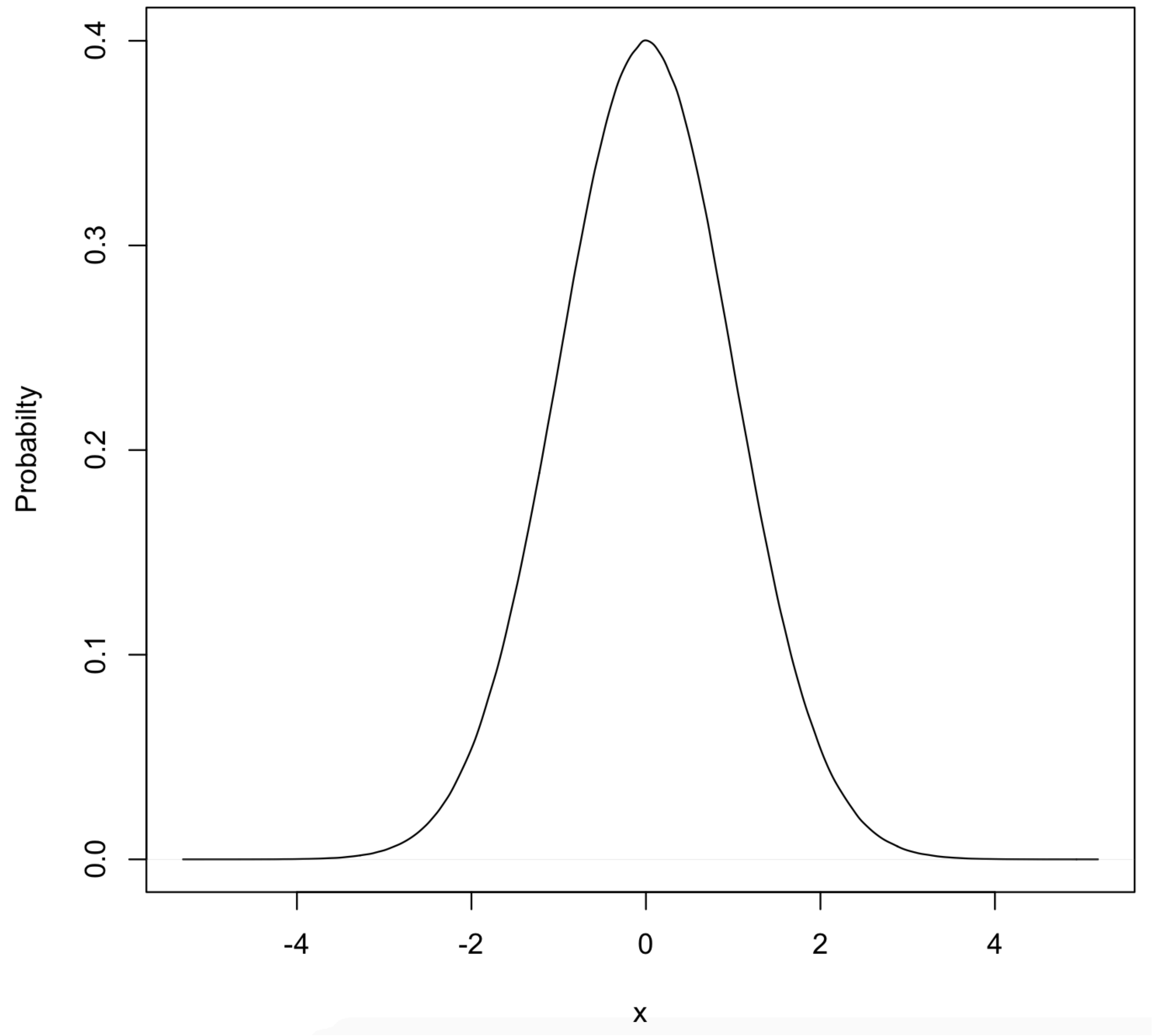
Right Skew (1.890)



Left Skew (-0.963)

Describing Distributions: Normality

- In most statistical applications, *normality* of data is essential
- In particular, in order to apply many common statistical procedures, data should follow a *normal (Gaussian) distribution*
- To see if the data is approximately normal, we can use the *Empirical Rule*



Empirical Rule: z-scores

- To discuss the Empirical Rule, we must first introduce the concept of *z-scores*
- A **z-score** tells us how many standard deviations an observation is from its mean:

$$z = \frac{x - \bar{x}}{s}$$

Example: z-scores

- The mean on Exam 1 is 86, and the standard deviation is 4
- Student A scores 90 on the exam. What is their z-score?
  - $z = \dfrac{90 - 86}{4} = 1$
- Student B scores 78 on the exam. What is their z-score?
  - $z = \dfrac{78 - 86}{2} = -2$

Example: z-scores

- A student takes two tests. The mean on the first test is 86 with a standard deviation of 4. The mean on the second test is 400 with a standard deviation of 15. The student scored 91 on the first test and 425 on the second. Which test did the student score better on relative to the other test takers?

Example: z-scores

- A student takes two tests. The mean on the first test is 86 with a standard deviation of 4. The mean on the second test is 400 with a standard deviation of 15. The student scored 91 on the first test and 425 on the second. Which test did the student score better on relative to the other test takers?

- Step 1: The student's z-score for the first test:

  - $z = \dfrac{91 - 86}{4} = 1.25$

- Step 2: The student's z-score for the second test:

  - $z = \dfrac{425 - 400}{15} = 1.67$

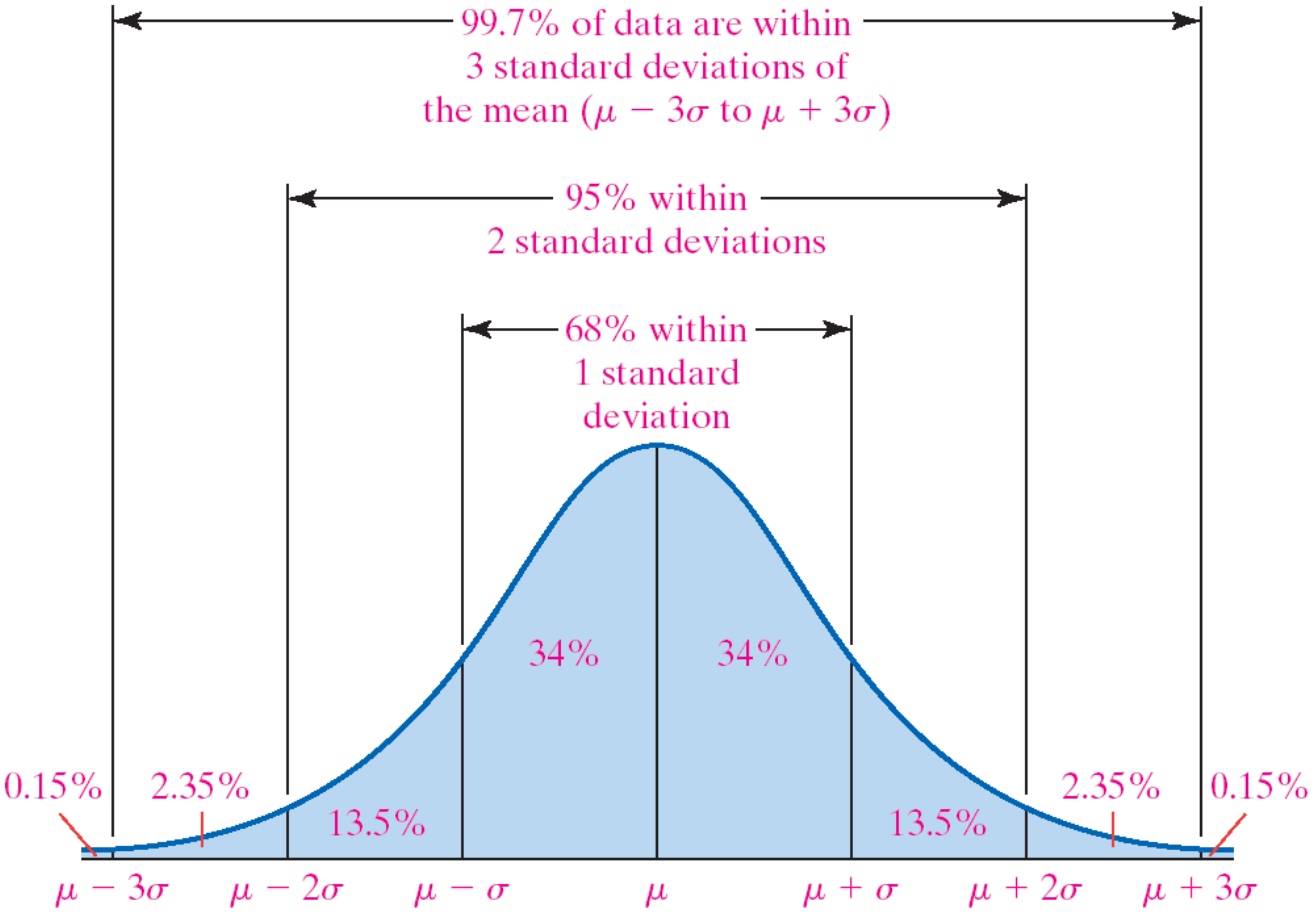
- Step 3: Compare the z-scores:

  - The student did better on the second test relative to their peers since the z-score is higher

Empirical Rule

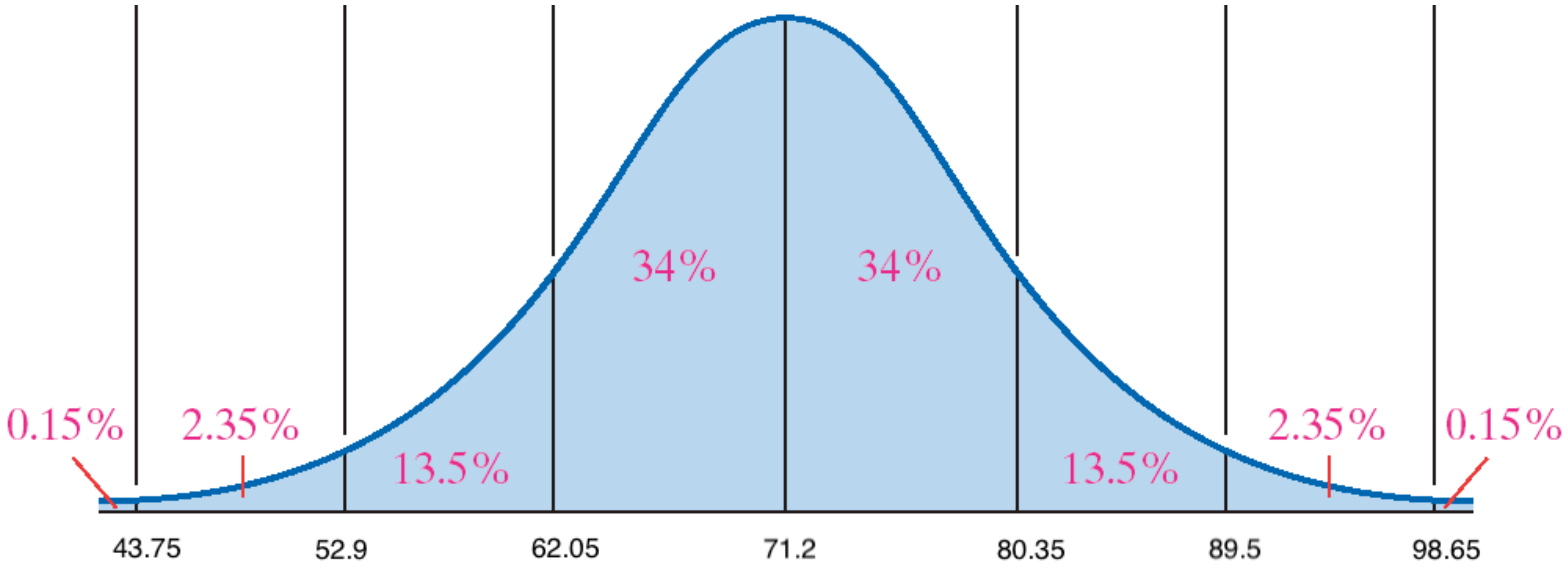
- If a distribution is symmetric, unimodal, and bell-shaped (i.e., normally distributed), then the following hold:
- Approximately 68% of observations fall within one SD of the mean: $\bar{x} \pm s$, or $(\bar{x} - s, \bar{x} + s)$
- Approximately 95% of observations fall within two SDs of the mean: $\bar{x} \pm 2s$, or $(\bar{x} - 2s, \bar{x} + 2s)$
- Approximately 99.7% of observations fall within three SDs of the mean: $\bar{x} \pm 3s$, or $(\bar{x} - 3s, \bar{x} + 3s)$

Empirical Rule



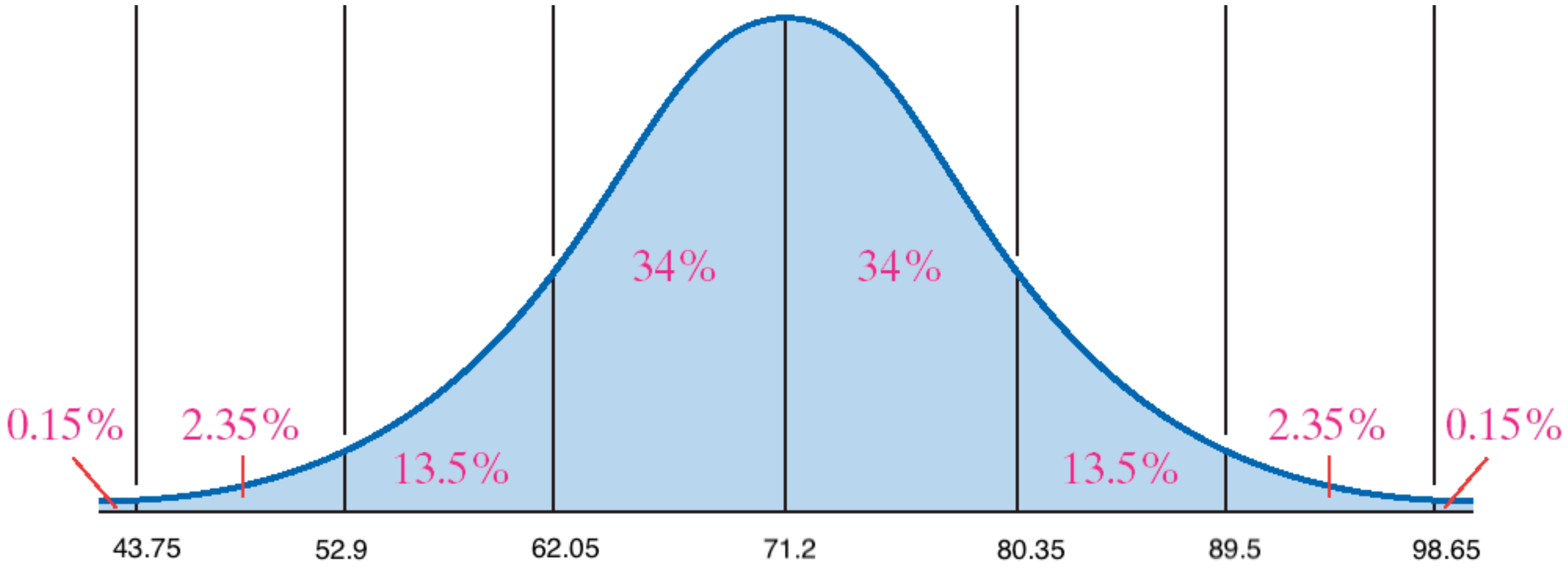
Empirical Rule: Heart Rate Data

- Suppose heart rates have an approximately normal distribution with $\bar{x} = 71.2$ and $s = 9.15$
- Approximately 68% of observations fall in the interval $71.2 \pm 9.15 = (62.05, 80.35)$
- Approximately 95% of observations fall in the interval $71.2 \pm 2 \times 9.15 = (52.9, 89.5)$
- Approximately 99.7% of observations fall in the interval $71.2 \pm 3 \times 9.15 = (43.75, 98.65)$



Empirical Rule: Heart Rate Data

- How fast does a heart rate have to be in order to be at the 97.5th percentile?
- Approximately what percentage of heart rates are greater than 80.35?
- Approximately what percentage of heart rates are between 52.9 and 62.05?



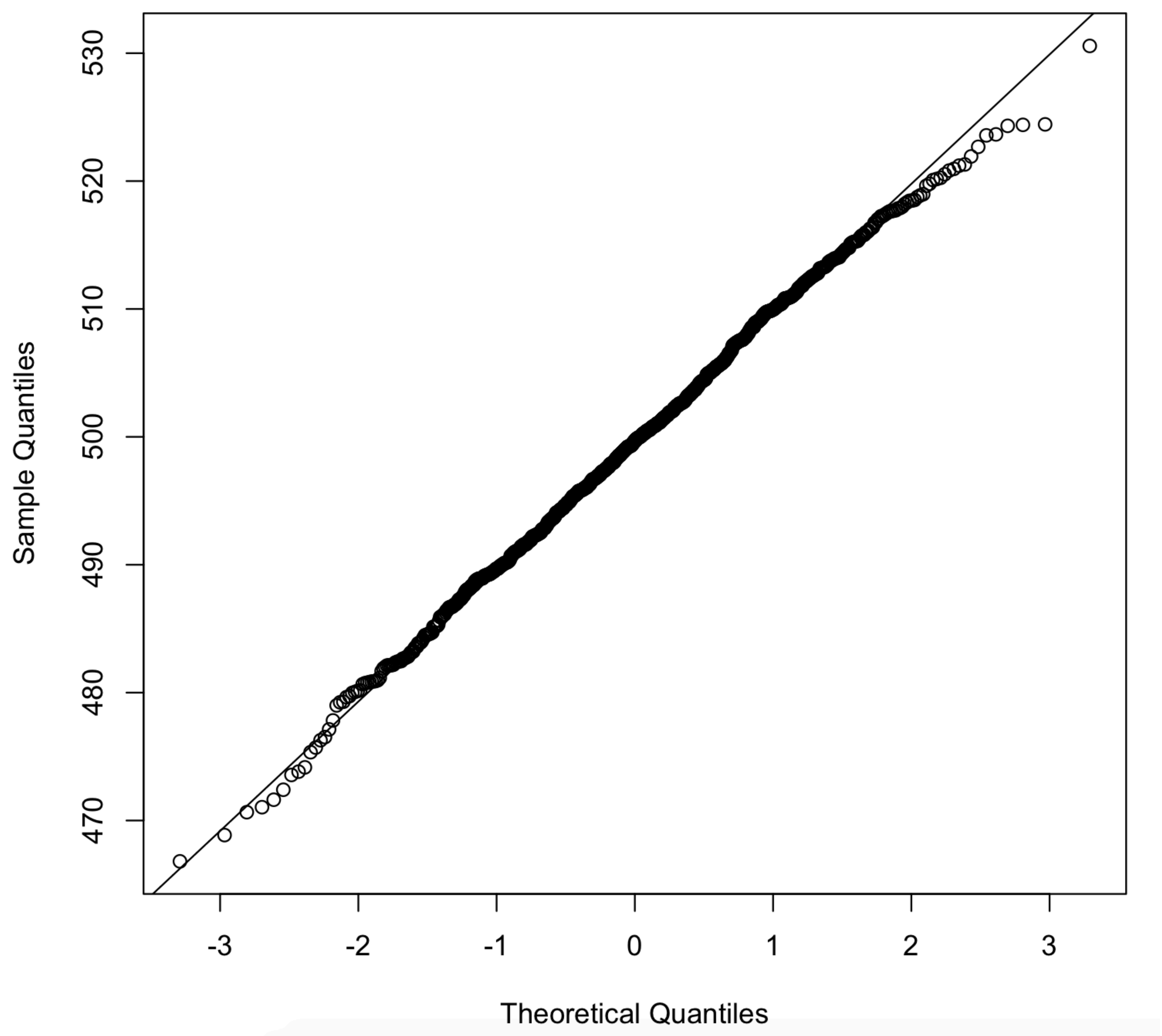
Chebyshev's Inequality

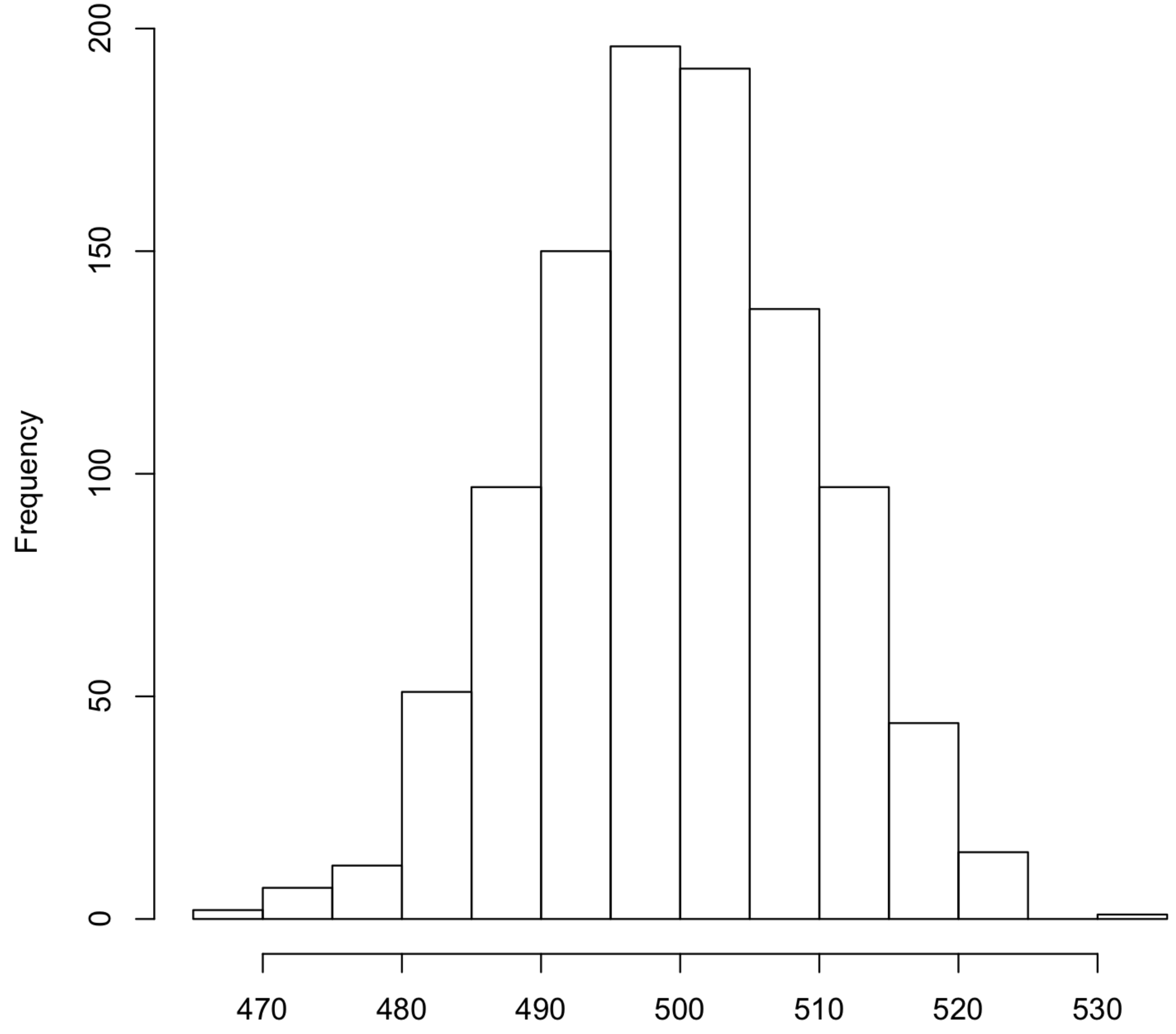
- The empirical rule works for symmetric, unimodal, and bell-shaped data
- If the data is *not* symmetric and unimodal, we may instead use *Chebyshev's Inequality* to summarize the distribution (holds for *any distribution*)
- **Chebyshev's Inequality** (informal): At least $1 - \left(\frac{1}{k}\right)^2$ of observations in a set of data lie within k standard deviations of the mean
- Intuitively: "No more than a certain fraction of values be too far from the mean"
- Weaker than the Empirical Rule because it makes fewer assumptions

Quantile Plots

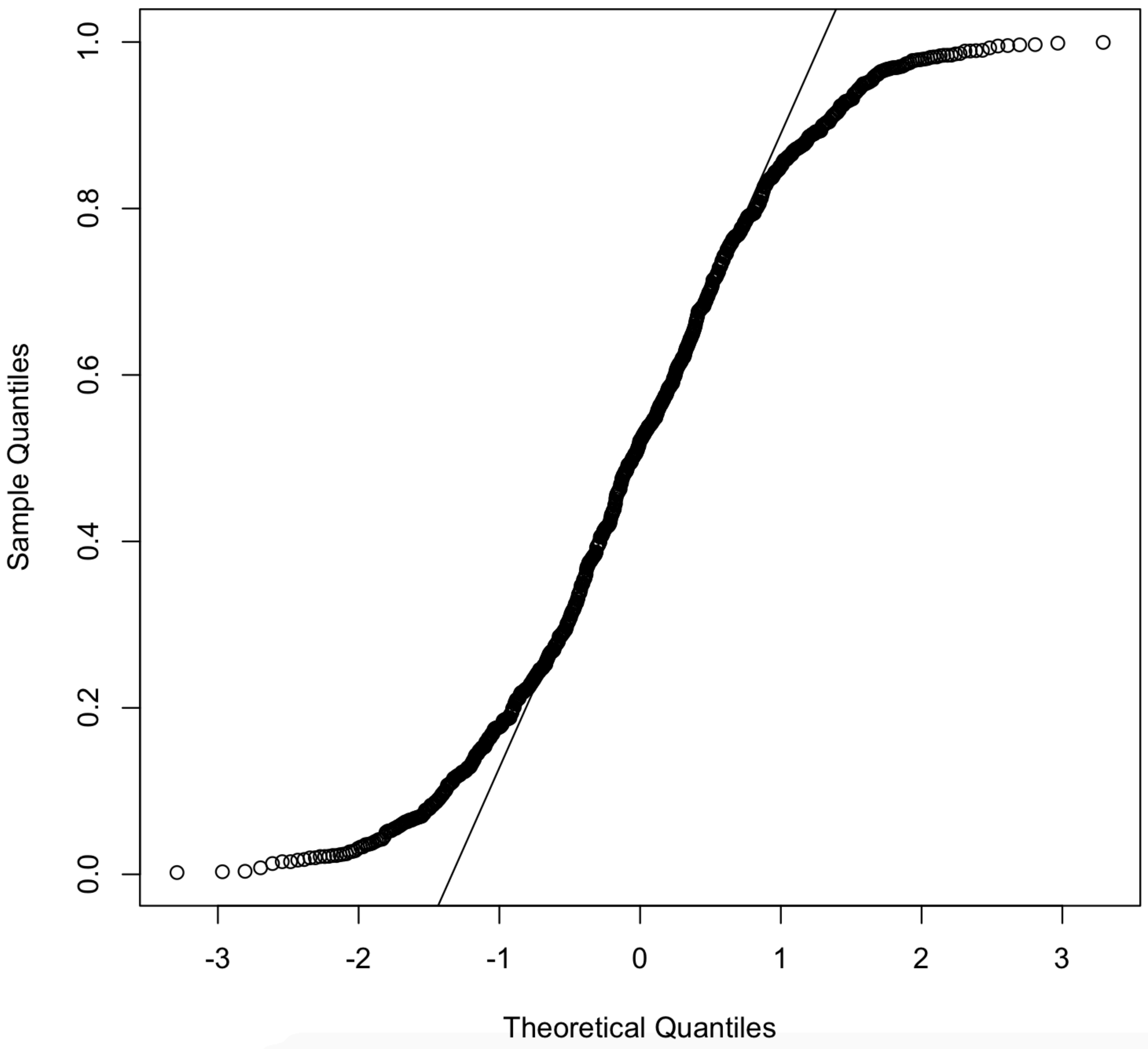
- We may extend the intuition of the empirical rule to a more sophisticated method for examining whether data is in fact normally distributed
- Suppose we have data $x_1, x_2, \ldots, x_n$ and we want to see if they follow a normal distribution
- We can compare the sample quantiles based on the observed data to the theoretical quantiles of the normal distribution
- A *quantile plot* plots the sample quantiles (y-axis) against the theoretical quantiles (x-axis)
- If the points lie approximately on the line $y = x$, then the data is approximately normally distributed
- R code: `qqnorm(data); qqline(data)`

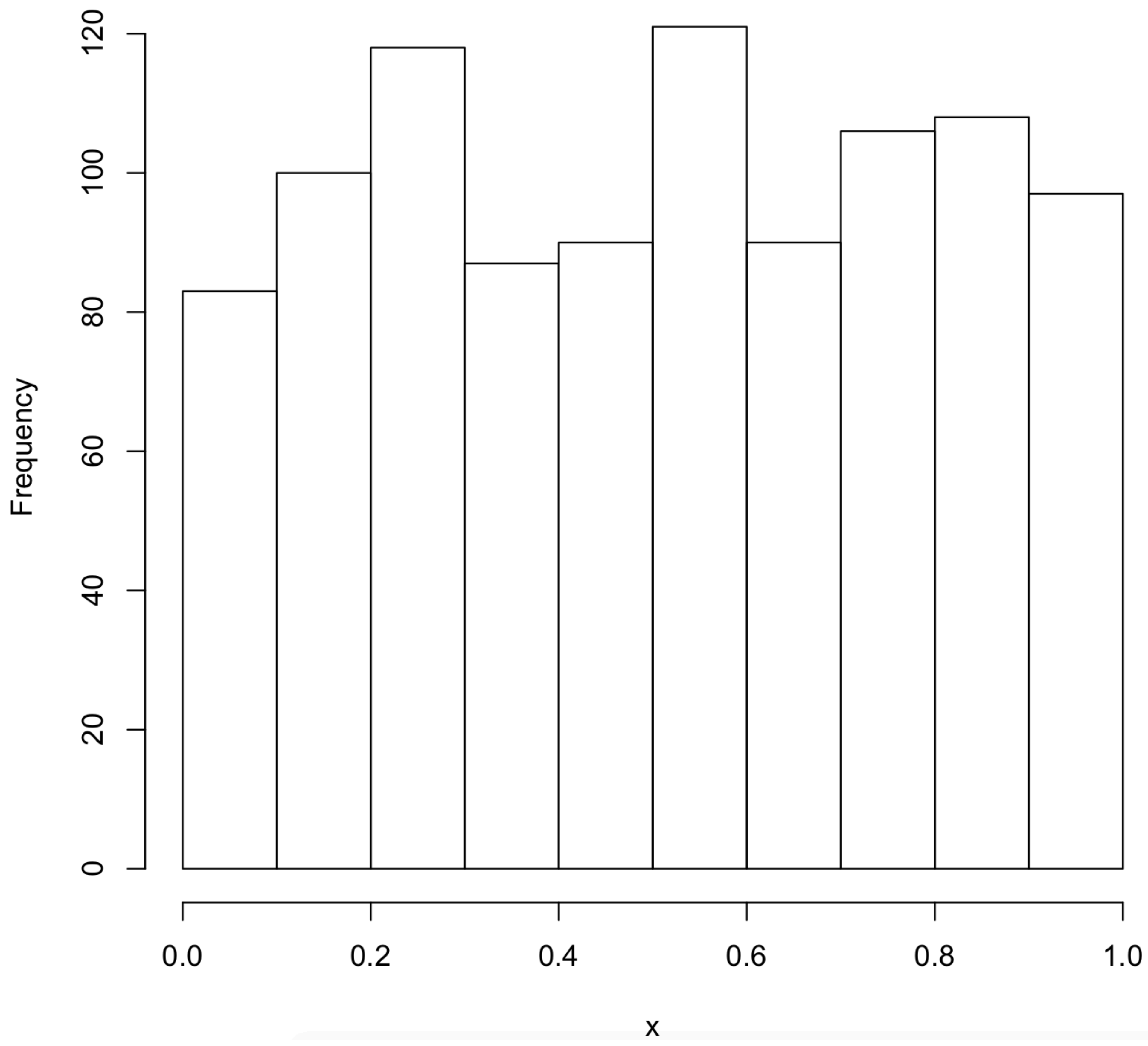
Quantile Plots





Quantile Plots





Transformations

- In some cases, we may need to transform our data to change the measurement units or to make an analytical method simpler, more accurate, or more effective
- Often, when doing a transformation, we attempt to take skewed data and transform it onto a scale in which the data is then approximately normally distributed

Linear Transformations

- *Linear transformations* are typically used to change units
- A linear transformation is of the form $Y = aX + b$
- X is the original data, Y is the transformed data, and a and b are the transforming constants
- Example: When converting Celsius to Fahrenheit, we have $F = 1.8C + 32$ with $a = 1.8$ and $b = 32$

Linear Transformations

- When doing linear transformations, many summary statistics do not need to be explicitly recalculated
- Instead, we can transform each summary statistic to represent the transformed data

Statistic	Original	Transformed
Mean	$\bar{x}$	$a\bar{x} + b$
Median	$\tilde{x}$	$a\tilde{x} + b$
Trimmed Mean	$\bar{x}_{K\%}$	$a\bar{x}_{K\%} + b$
Variance	s^2	$a^2 s^2$
Standard Deviation	s	$\lvert a \rvert s$
Interquartile Range	IQR	$\lvert a \rvert$IQR
Lower Quartile	Q_1	$aQ_1 + b$ if $a > 0$
		$aQ_3 + b$ if $a < 0$
Upper Quartile	Q_2	$aQ_3 + b$ if $a > 0$
		$aQ_1 + b$ if $a < 0$
Quantile	$x_{K\%}$	$ax_{K\%} + b$ if $a > 0$
		$ax_{(100-K)\%} + b$ if $a < 0$

Transformations

- Linear transformations do little to change the overall shape of data
- Other transformations can be used to reduce skewness and thus affect overall shape
- Often, we want to transform data so they are approximately normally distributed in order to apply standard statistical procedures
- Many types of data are not naturally symmetric (e.g., income, age, survival time)
- Transform data to meet assumptions needed for analyses

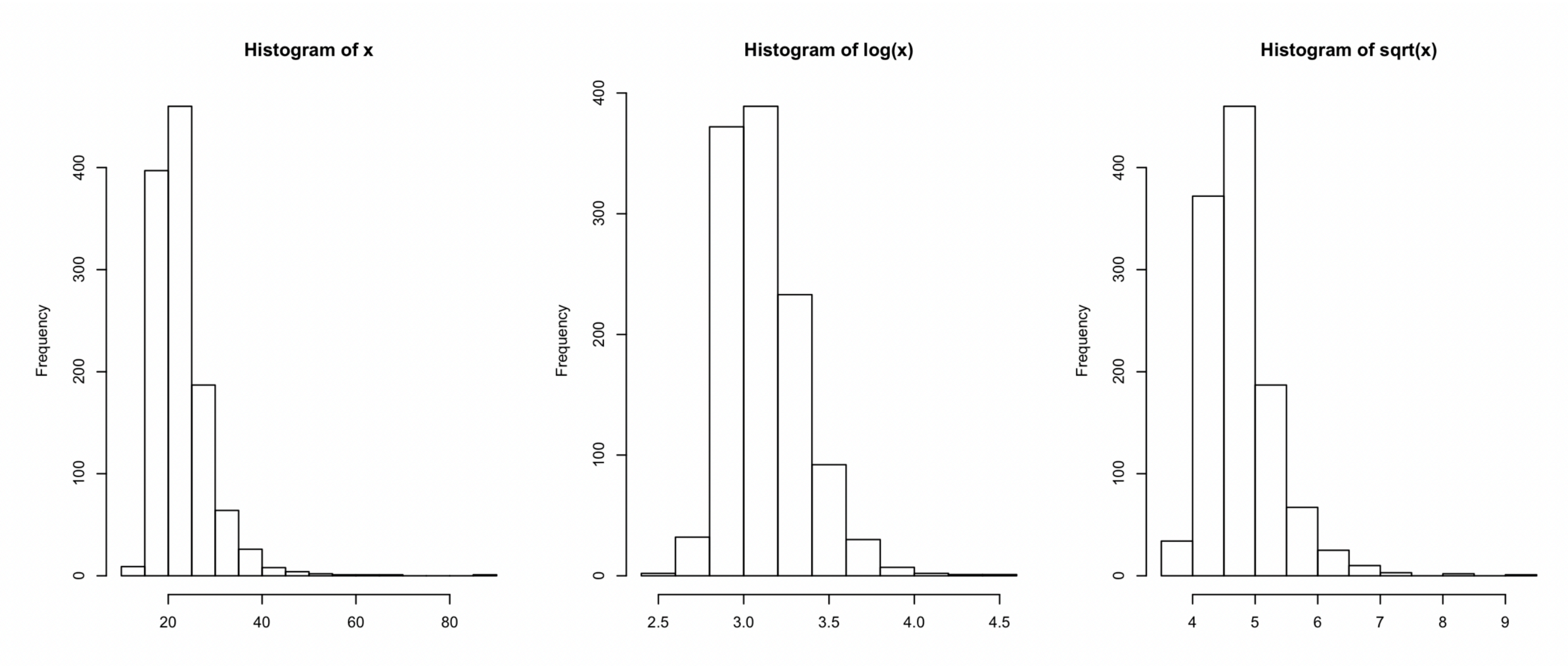
Transformations

- Find an appropriate function $f(x)$ that will transform original data $x_1, x_2, \ldots, x_n$ into $y_1, y_2, \ldots, y_n$ through the transformation $y_i = f(x_i)$
- Often, $f(x)$ must be an increasing function so that if $x_i > x_j$, then $y_i > y_j$
- For right skewed data, use a function that tends to reduce larger values in proportion to smaller ones (i.e., an increasing function whose slope is decreasing)
  - $f(x) = \log(x)$
  - $f(x) = \sqrt{x}$

Transformations

- General Social Survey (GSS) collects data annually on a sample of Americans regarding demographic and lifestyle characteristics
- Consider the 2006 survey, which collected data on the age at which respondents were first married
- Right skew to data
- Apply $f(x) = \log(x)$ and $f(x) = \sqrt{x}$ transformations to see which fits better

Transformations



Transformations

- The $f(x) = \log(x)$ transformation seems to reduce skewness more, although the distribution is not perfectly symmetric

- Check empirical rule:

	k	$\bar{x} - ks$	$\bar{x} + ks$	Theoretical % in Range	Actual % in Range
x	1	17.20	29.45	68	81.6
	2	11.08	35.58	95	96.2
	3	4.95	41.71	99.7	98.4
$\log(x)$	1	2.90	3.34	68	69.6
	2	2.68	3.57	95	95.8
	3	2.46	3.79	99.7	99.1
$\sqrt{x}$	1	4.22	5.37	68	79.7
	2	3.65	5.94	95	96.0
	3	3.08	6.51	99.7	98.6

Box-Cox Power Transformations

- Data is often log-transformed for convenience and interpretability
- A more flexible approach is the *Box-Cox* power transformation, parameterized by a value λ
- For a positive x, the transformed value y_λ is given as follows:

$$y_\lambda = \begin{cases} \frac{x^\lambda - 1}{\lambda}, & \lambda \neq 0 \\ \log(x), & \lambda = 0 \end{cases}$$

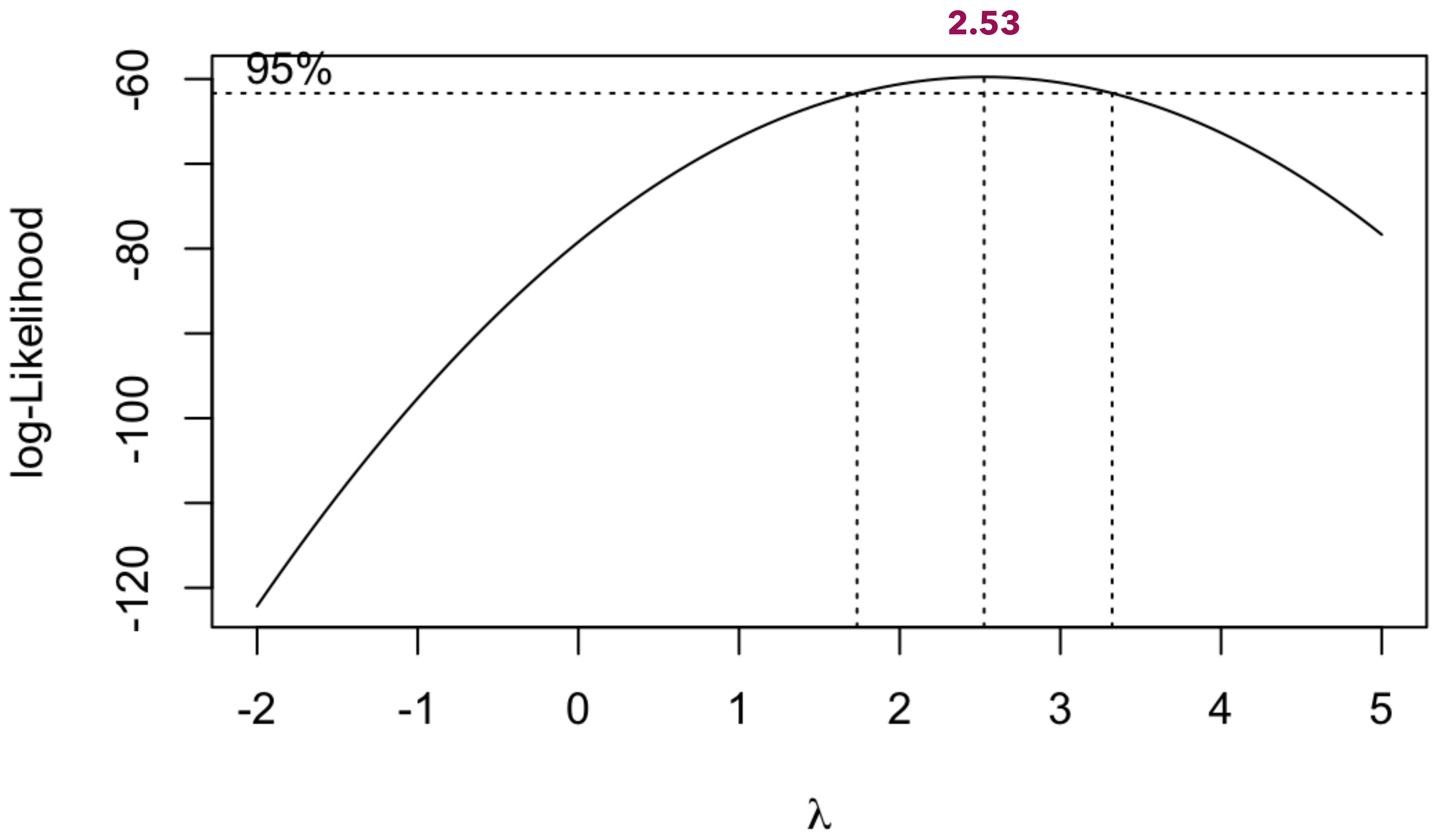
Box-Cox Power Transformations

- R has a built-in function to estimate the optimal parameter $\hat{\lambda}$ given data x
- R code:

```
library(MASS)

bc1 <- boxcox(x~1)

bc1$x[bc1$y==max(bc1$y)]
```



Box-Cox Power Transformations

- Let's examine a histogram and Q-Q plot of the Box-Cox transformed data: