Chapter 11: χ^2 Tests

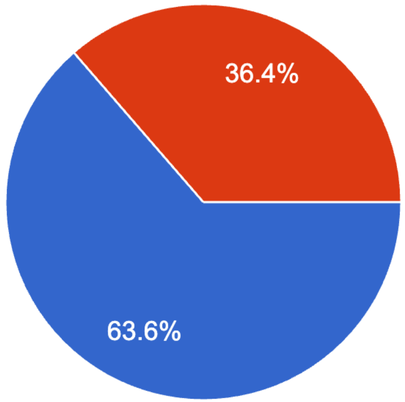
DSCC 462
Computational Introduction to Statistics

Anson Kahng
Fall 2022

Survey Feedback (n = 44 out of 54 so far)

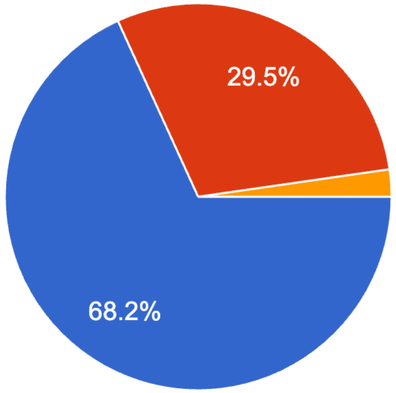
The material is clear

The instructor presents concepts clearly.
44 responses



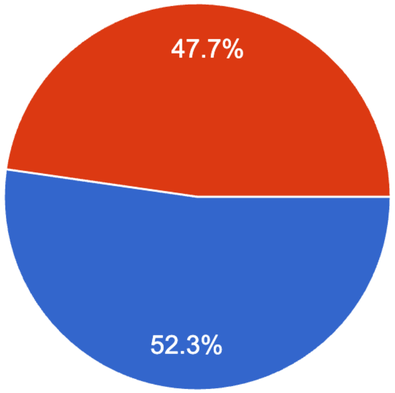
The assignments are useful

The assignments help me apply concepts that we learn
44 responses



You are learning!

I am learning a lot in this class.
44 responses



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What we'll do going forward

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- You like **outlines / summaries** of lecture
- You find **examples** helpful
- You want more **feedback** on homeworks
- You want to see more **R code** on slides

What we'll do going forward

+ instructor slides!

- I will add **summary slides** to lecture
- I will use **examples** to frame concepts
- TAs will give more **detailed feedback** on assignments
- I will add **R code** to slides

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CONTINUE	• Being **responsive** to students' needs • Seeking **feedback** from students • **Answering questions** during class • **Solving examples** in class	• I'll **continue doing these things**!

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- Educational videos:
  - YouTube: zedstatistics, 3blue1brown, jbstatistics

Community Norms

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- Let’s keep side conversations to a minimum

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- Let’s keep side conversations to a minimum
- Let’s stay awake

Lecture Plan for Today

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- Goodness-of-Fit Test

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  - True proportion = expected proportion?
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- Chi-Squared (χ^2) Test of Independence
  - Are variables related or not?

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  - In this setting, a variable could take on one of two values
  - What if the variable had more categories?
- Example: Let's say we are trying to figure out what proportion of people have each season (winter, spring, summer, fall) as their favorite. How can we do inference in this setting?
  - Instead of one p, we have to infer values for p_1, p_2, p_3

Goodness-of-Fit

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- Use the Goodness-of-Fit Test

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  - H_1 : at least one of these equalities does not hold

$$\sum_{i=1}^{2} p_{i_0} = 1$$

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Goodness-of-Fit

$$X_i \sim \text{Binom}(n, p_{i_0}) \approx N\left(np_{i_0}, \sqrt{np_{i_0}(1-p_{i_0})}\right)$$

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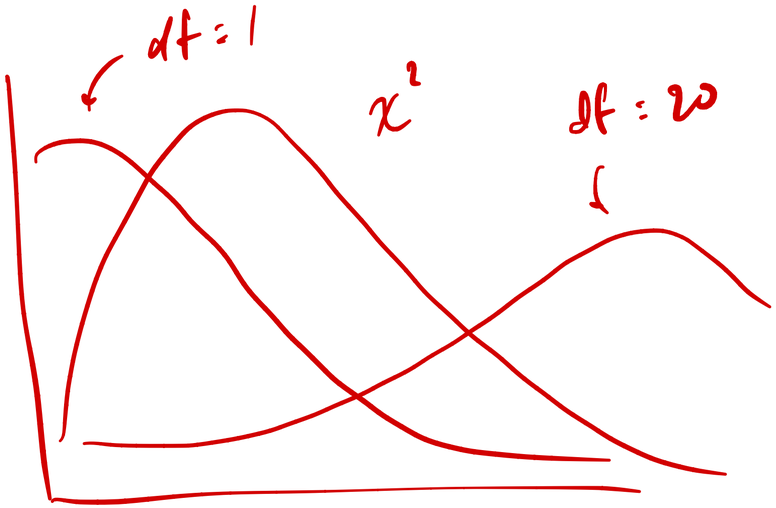
- The test statistic is

$$X^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2_{k-1}$$

(O_i: μ_{X_i}; E_i: np_{i_0})



$$(1-p_{i_0})Z^2 = \left(\frac{X_i - np_{i_0}}{\sqrt{np_{i_0}(1-p_{i_0})}}\right)^2 = \frac{(X_i - E_i)^2}{E_i}$$

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- Here, O_i is the observed number of people who fall in category i
- E_i is the expected number of people who fall into category i under the null hypothesis

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Category	**1**	**2**	**…**	**k**
Observed	O_1	O_2	…	O_k
Expected	$E_1 = n*p_{1_0}$	$E_2 = n*p_{2_0}$	…	$E_k = n*p_{k_0}$

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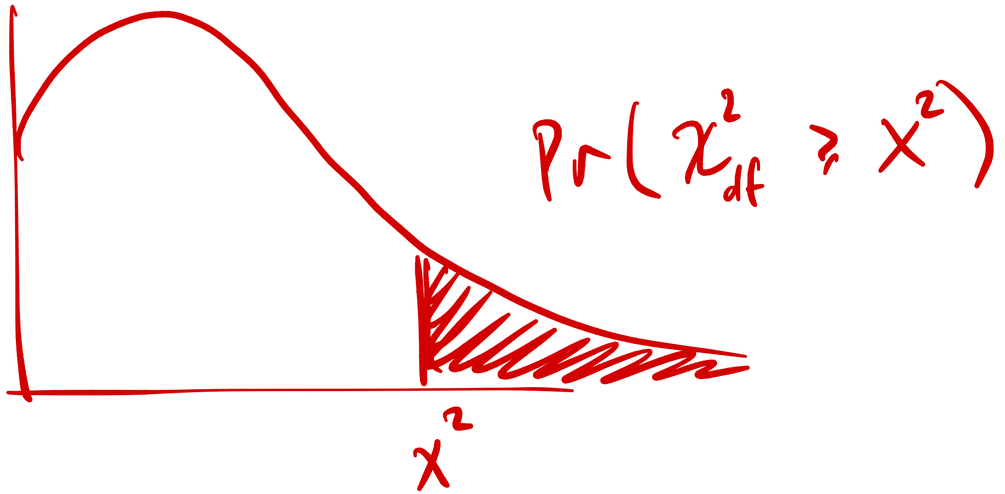
$$X^2 \sim \chi^2_{k-1}$$



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- If $p \leq \alpha$, we reject H_0
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- State your conclusion in the context of the problem
- Note: For this test to be valid, all the expected counts must be at least 5

$np_1, \cdots, np_k \geq 5$

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- To see if this is correct, we go out and take a sample of 200 people and determine their eye color. We get 84 people with brown eyes, 17 people with hazel eyes, 16 people with blue eyes, and 83 people with other color eyes

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- To see if this is correct, we go out and take a sample of 200 people and determine their eye color. We get 84 people with brown eyes, 17 people with hazel eyes, 16 people with blue eyes, and 83 people with other color eyes
- Test at the $\alpha = 0.05$ significance level

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- $H_0 : p_1 = 0.4, p_2 = 0.1, p_3 = 0.05, p_4 = 0.45$ vs. H_1 : at least one of these proportions does not hold

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- Calculate the test statistic:

$n = 200$

Category	Brown	Hazel	Blue	Other
Observed	84	17	16	83
Expected	80	20	10	90

$$X^2 = \frac{(84-80)^2}{80} + \frac{(17-20)^2}{20} + \frac{(16-10)^2}{10} + \frac{(83-90)^2}{90} = 4.79$$

$$p = Pr(\chi^2_3 > 4.79) = 1 - pchisq(4.79, df=3) = 0.188. > \alpha = 0.05$$

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Category	**Brown**	**Hazel**	**Blue**	**Other**
Observed				
Expected				

- $X^2 =$

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- $\Pr(\chi^2 > X^2) =$

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Category	Brown	Hazel	Blue	Other
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- $X^2 =$
- $\Pr(\chi^2 > X^2) =$
- Conclusion:

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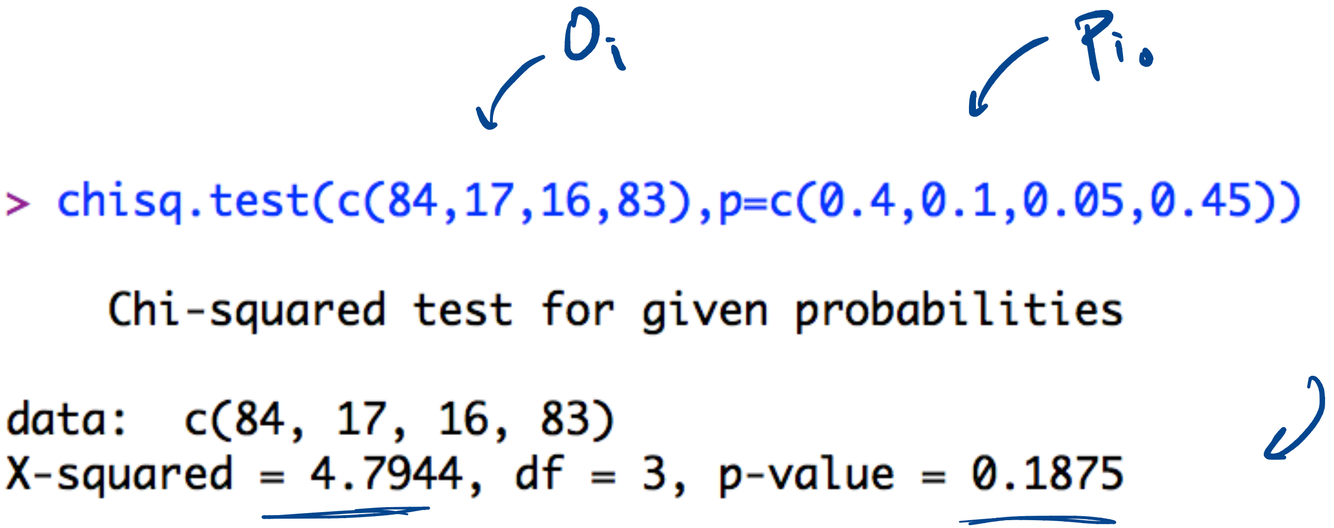

Goodness-of-Fit: Example

- Can do directly in R as well

```
> chisq.test(c(84,17,16,83),p=c(0.4,0.1,0.05,0.45))

    Chi-squared test for given probabilities

data:  c(84, 17, 16, 83)
X-squared = 4.7944, df = 3, p-value = 0.1875
```

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- Now, let's consider the case of two categorical variables
  - We used a normal approximation to the binomial distribution and formed a two-proportion z-test for binary variables
- A generalized technique for testing proportions is through the χ^2 test of independence for contingency tables

Contingency Table

$$O_{11} = \frac{r_1 \times c_1}{n}$$

O_{ij} — i: row, j: col

do you drink? O_{11}

do you smoke?

Variable 1	Variable 2		Total
	Yes	No	
Yes	10 O_{11}	15 O_{12}	25 r_1
No	20 O_{21}	5 O_{22}	25 r_2
Total	30 c_1	20 c_2	n

$\frac{r_1}{n} = Pr(\text{smoke})$

$r_1 = O_{11} + O_{12}$

$\frac{c_1}{n} = Pr(\text{drink})$

$c_1 = O_{11} + O_{21}$

A and B are indep iff

$Pr(A \cap B) = Pr(A) \times Pr(B)$

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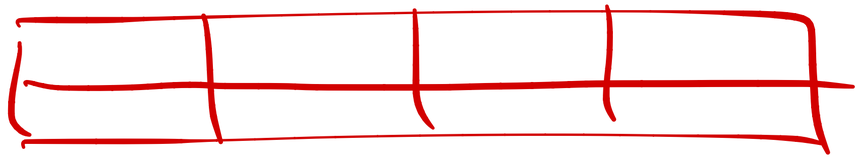

- Setting:
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  - We are interested in whether a population's favorite season is independent of whether or not they are a pet owner
  - How can we test such a hypothesis?
- Idea: Extend the goodness-of-fit test to multiple dimensions

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  - H_1 : the two variables are associated (i.e., not independent)

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  - Let O be the observed frequencies
  - Let E be the expected frequencies under the null hypothesis
- Use the chi-square test to determine whether the deviations between the observed and expected frequencies are too large to be attributed to chance

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$Pr(\text{Yes } V_1)$ $Pr(\text{Yes } V_2)$

$$\left(\frac{r_1}{n}\right)\left(\frac{c_1}{n}\right) \cdot n = \frac{r_1 \times c_1}{n}$$

Variable 1	**Variable 2**		**Total**
	Yes	No	
Yes	O_{11} $E_{11} = \frac{r_1 \times c_1}{n}$	O_{12} $E_{12} = \frac{r_1 \times c_2}{n}$	r_1
No	O_{21} $E_{21} = \frac{r_2 \times c_1}{n}$	O_{22} $E_{22} = \frac{r_2 \times c_2}{n}$	r_2
Total	c_1	c_2	n

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χ^2 Test of Independence

given n

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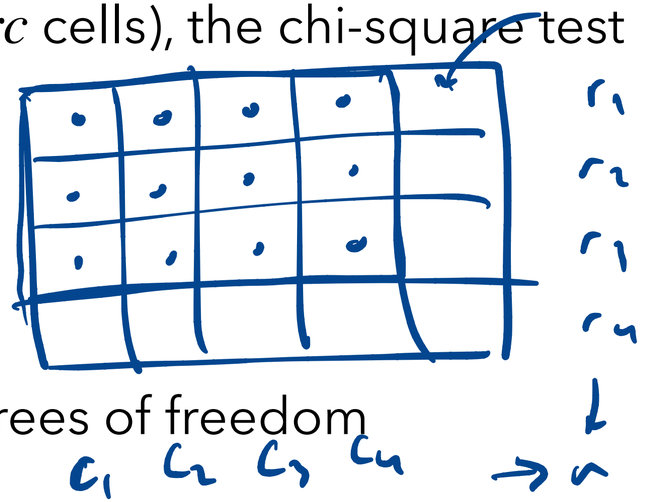

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χ^2 Test of Independence

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- In order for the χ^2 distribution to be appropriate, no cell should have an expected or observed frequency less than 5

χ^2 Test of Independence: Example

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- Test the following hypotheses:
  - H_0 : Flossing and gum disease are independent
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- Perform this test at the $\alpha = 0.05$ significance level

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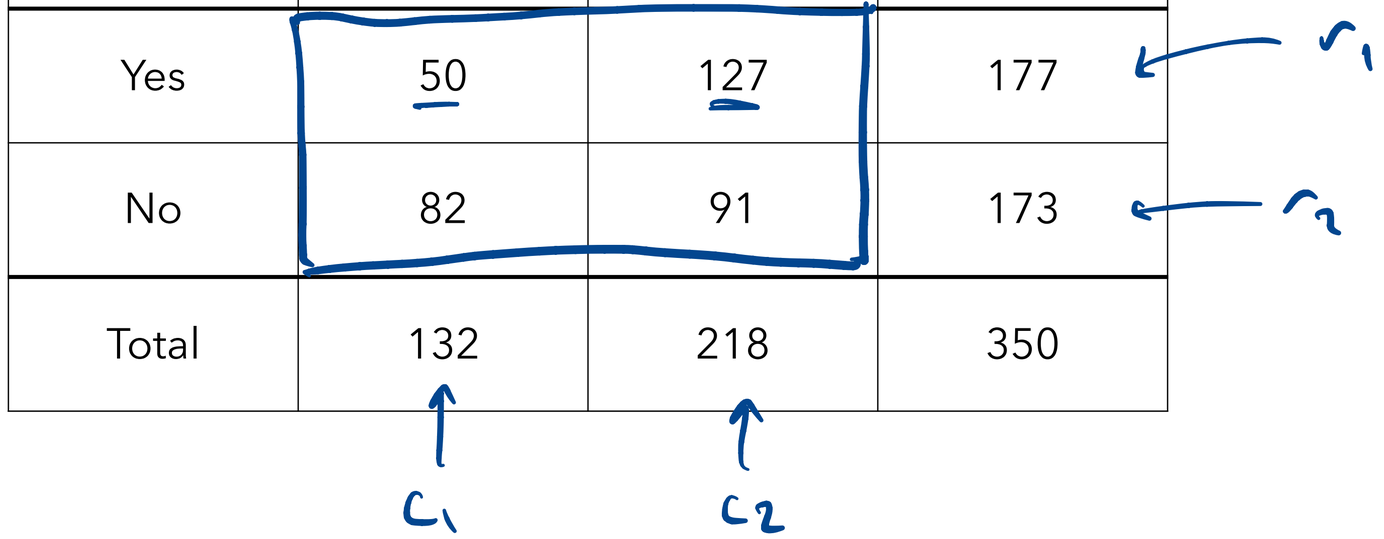
Daily Flossing	**Gum Disease**		**Total**
	Yes	No	
Yes	50	127	177
No	82	91	173
Total	132	218	350



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Yes	50 ↓	127 ↑	177
No	82 ↑	91 ↓	173
Total	132	218	350

Daily Flossing	Gum Disease		Total
	Yes	No	
Yes	66.8	110.2	177
No	65.2	107.8	173
Total	132	218	350

- What is the X^2 statistic?

$$X^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}} = \frac{(50 - 66.8)^2}{66.8} + \cdots + \frac{(91 - 107.8)^2}{107.8} = 13.7.$$

χ^2 Test of Independence: Example

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- How many degrees of freedom do we have? $(2-1)(2-1) = 1$ degree of freedom

χ^2 Test of Independence: Example

$X^2 = 13.7$

- How many degrees of freedom do we have? $(2-1)(2-1) = 1$ degree of freedom
- What is the p value?

$Pr(\chi^2_1 \geq 13.7) = 1 - pchisq(13.7, df=1) = 0.00022$

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- What is the p-value?
  - $P(X^2 \geq 13.659) =$ `1-pchisq(13.659,1)` $= 0.00022$

χ^2 Test of Independence: Example

- How many degrees of freedom do we have?
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- What is the p-value?
  - $P(X^2 \geq 13.659) =$ `1-pchisq(13.659,1)` $= 0.00022$
- Since the p-value is less than $\alpha = 0.05$, we reject the null hypothesis and conclude that there is an association between flossing and gum disease

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  - $P(X^2 \geq 13.659) =$ `1-pchisq(13.659,1)` $= 0.00022$
- Since the p-value is less than $\alpha = 0.05$, we reject the null hypothesis and conclude that there is an association between flossing and gum disease
- Could also use `chisq.test()` in R

χ^2 Test of Independence: R Code

```
> x <- matrix(c(50, 82, 127, 91), nrow=2, ncol=2)
> x
     [,1] [,2]
[1,]   50  127
[2,]   82   91
> chisq.test(x,correct=F)

    Pearson's Chi-squared test

data:  x
X-squared = 13.659, df = 1, p-value = 0.0002192
```

Yates'

"don't do corrections"

$$(O_{ij} - E_{ij} - \tfrac{1}{2})^2$$

Larger Contingency Tables

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  - r categories for the row variable
  - c categories for the column variable
- The inferential procedures are the same for $r \times c$ tables as for 2×2 tables

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- Conduct a study with 260 people who floss

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  - Oral-B, Colgate, and Reach
- Investigate gum disease prevalence for users of each floss type
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  - Record the floss brand they use and whether they have gum disease

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- Test at the $\alpha = 0.05$ significance level

Larger Contingency Tables: Example

Observed

Daily Flossing	**Gum Disease**		**Total**
	Yes	No	
Oral-B	14	70	84
Colgate	25	71	96
Reach	21	59	80
Total	60	200	260

Larger Contingency Tables: Example

Observed

Daily Flossing	Gum Disease		Total
	Yes	No	
Oral-B	14	70	84
Colgate	25	71	96
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Total	60	200	260

Expected

Daily Flossing	Gum Disease		Total
	Yes	No	
Oral-B	19.4	64.6	84
Colgate	22.2	73.8	96
Reach	18.5	61.5	80
Total	60	200	260

r_1 r_2 r_3

c_1 c_2

$$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

Larger Contingency Tables: Example

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- What is the test statistic?

Larger Contingency Tables: Example

- What is the test statistic?

- How many degrees of freedom?

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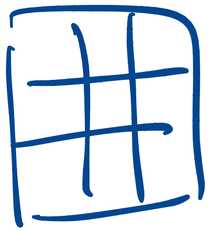
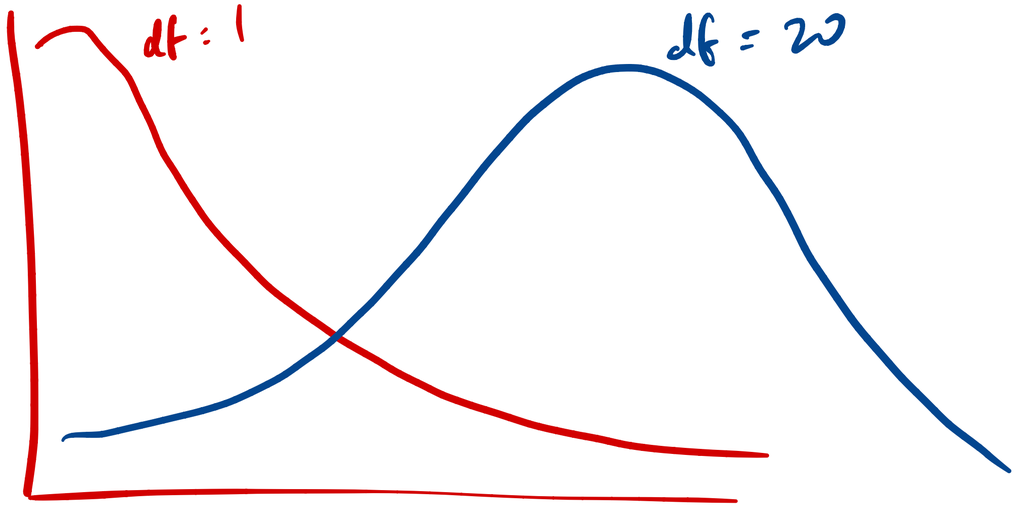
$$X^2 = 2.874$$





- How many degrees of freedom? $(r-1)(c-1) = 2 \cdot 1 = 2$
- What is the p-value?

$$Pr(\chi^2_2 > 2.874) = 0.238 > \alpha = 0.05$$

Fail to reject.

Larger Contingency Tables: Example

- What is the test statistic?

- How many degrees of freedom?
- What is the p-value?

- Conclusion:

Summary

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- Goodness-of-Fit Test

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  - Use this to test if the true proportion = expected proportion

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Summary

- Goodness-of-Fit Test
  - Use this to test if the true proportion = expected proportion
  - Generalization of proportion hypothesis tests
- Chi-Squared (χ^2) Test of Independence
  - Use this to check if variables are independent or not
- Both rely on the χ^2 distribution!

$$X^2 = \sum \frac{(O_i - E_i)^2}{E_i} \sim \chi^2_{df}$$

Practice Midterm:

- NOT exhaustive
- 2x as long as real exam (it will be doable)

Review session next week? ← Thurs.