Chapter 12 Non-parametric Test

Daxiang Na

2022-10-25

Contents

1 Questions:

1.1 What is V in the returned result of `wilcox.test`

```
## Wilcoxon signed rank exact test data: data$air and data$sulf.diox
## V = 21, p-value = 0.006653 alternative hypothesis: true location
## shift is not equal to 0
```

1.2 What is the definition of T in `psignrank`?

In the `psignrank` document(https://www.rdocumentation.org/packages/stats/versions/3.6.2/topics/SignRank), the the Wilcoxon signed rank statistic is " the sum of the ranks of the absolute values x[i] for which x[i] is positive". I am wondering if $T = T^+$ instead of $T = min(T^+, T^-)$? This appears to be correct when solving the example problem, where `2*(1 - psignrank(75.5,n=14))` = 0.135 but `2*psignrank(29.5,n=14)` != 0.135

I therefore wonder if $T = T^+$ should be the definition of `psignrank` in R.

Another question regarding this topic: when testing the two-tailed hypothesis, when should we use `2*(1 - psignrank(T,n))` and `2*psignrank(T, n)`? i.e. what is the mean for the wilcoxon signed rank distribution? Is it $n(n+1)/4$ as stated in the document?

This is right.

So when we calculate the p value, we determine whether to use `2*(1 - psignrank(T,n))` or `2*psignrank(T, n)` based on if $T > n(n+1)/4$ or $T < n(n+1)/4$, right?

1.3 How can we decide which probability to calculate in one-sided Wilcoxon Rank Sum test?

In the Wilcoxon Rank Sum test, we always use $W = min(W_1, W_2)$ and $\mu_W = n_{min} * (n_{min} + n_{large} + 1)/2$ no matter if we want to know if $H_1 : \mu_1 - \mu_2 < 0$ or $H_1 : \mu_2 - \mu_1 < 0$, how to we determine if p-value $= 1 - Pr(z < z_W)$ or p-value $= Pr(z < z_W)$?

My understanding: it depends on which group has W_{min}. In the Wilcoxon Rank Sum test we are calculating the probability of getting W_{min} in the distribution of $N(n_{min} * (n_{min} + n_{large} + 1)/2, sigma)$. In the example problem in the slides, H1 is about group 1 has less values, and group 1 turns out to have W_{min}, therefore p-value = Pr(z < 0).

1.4 Question about CLT

For sampling distribution, it can have sampling size n and sampling time m, what determines if it follows CLT? n or m? If we sample for 1 time and 100 times, each time with same size n, does that makes a difference? Another way to ask this question: if we sample for 3 times ($m = 3$), but each time with sample size $n = 1000$, does that follow CLT?

1.5 is it okay to share the Infererence cheat sheet raw file?

Willing to extend it.

2 Wilcoxon Signed Rank Test

- Only for paired sample.
- Evaluate the null hypothesis: $Z_T = (T - \mu_T)/\sigma_T$

- Note:

$$\mu_T = 0$$
$$\sigma_T = \sqrt{\frac{n(n+1)(2n+1)}{6}}$$

- When n is large enough ($n > 12$), we get

$$Z_T \sim N(0,1)$$

- calculate the probability of getting Z_T when $\mu = 0$ is true.
- For two-sided test, follow what we do in the sampling distribution:
  - 2*p when z < 0
  - 2*(1 - p) when z > 0
- if $n > 12$, you can just apply CLT, the R code is: `wilcox.test(before, after, paired = T, exact = F, correct = F)`. `exact =` determines if the statistics follow normal distribution (`exact = F`) or exact distribution (`exact = T`).
- If $n \leq 12$, we cannot use the normal approximation. In that case, we use `psignrank(T,n)` in R to calculate the exact distribution.
  - R requires $T = T^+$ for this to work correctly!

3 Wilcoxon Rank-Sum test (also known as Mann-Whitney U test)

- nonparametric analog to the two-sample t-test
- get W_1 and W_2
- $W = min(W_1, W_2)$
- n_1 = sample size with the smaller sum of ranks.
- n_2 = sample size with the larger sum of ranks.
- $$\mu_W = \frac{n_1(n_1+n_2+1)}{2} \text{ and } \sigma_W = \sqrt{\frac{n_1 n_2 (n_1+n_2+1)}{12}}$$
- $$z_W = \frac{W - \mu_W}{\sigma_W}$$
- $z_W \sim N(0,1)$ when n_1 and n_2 are large enough (n1, n2 > 10).
  - in R: `wilcox.test(..., exact = F, correct = F, paired = F, alt = "")`

- When n_1 and n_2 are very small (i.e. either is less than or equal to 10), we can use the exact distribution to calculate the p-values. In R: `pwilcox(Wobs, n1, n2)`
  - in this case, $W_{obs} = W - n_1(n_1 + 1)/2$
  - `wilcox.test` also works when `exact = T`
- correct: correct the data with continuity correction