Chapter 3: Relationships Between Variables

DSCC 462

Computational Introduction to Statistics

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Fall 2022

Plan for Today

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- Visualize relationships between variables

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- Visualize relationships between variables
- Determine whether variables are correlated

Summaries for Two Variables

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- Recall that we have discussed summaries of center and spread for one variable

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- Much of what we did for one variable can be extended to two variables

Case CQ: Categorical and Quantitative

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- Heights in Group B (in): 61, 62, 64, 67, 69

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- Heights in Group B (in): 61, 62, 64, 67, 69
- Mean for Group A: $\bar{x}_A = 67.7$

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- Heights in Group B (in): 61, 62, 64, 67, 69
- Mean for Group A: $\bar{x}_A = 67.7$
- Mean for Group B: $\bar{x}_B = 64.6$

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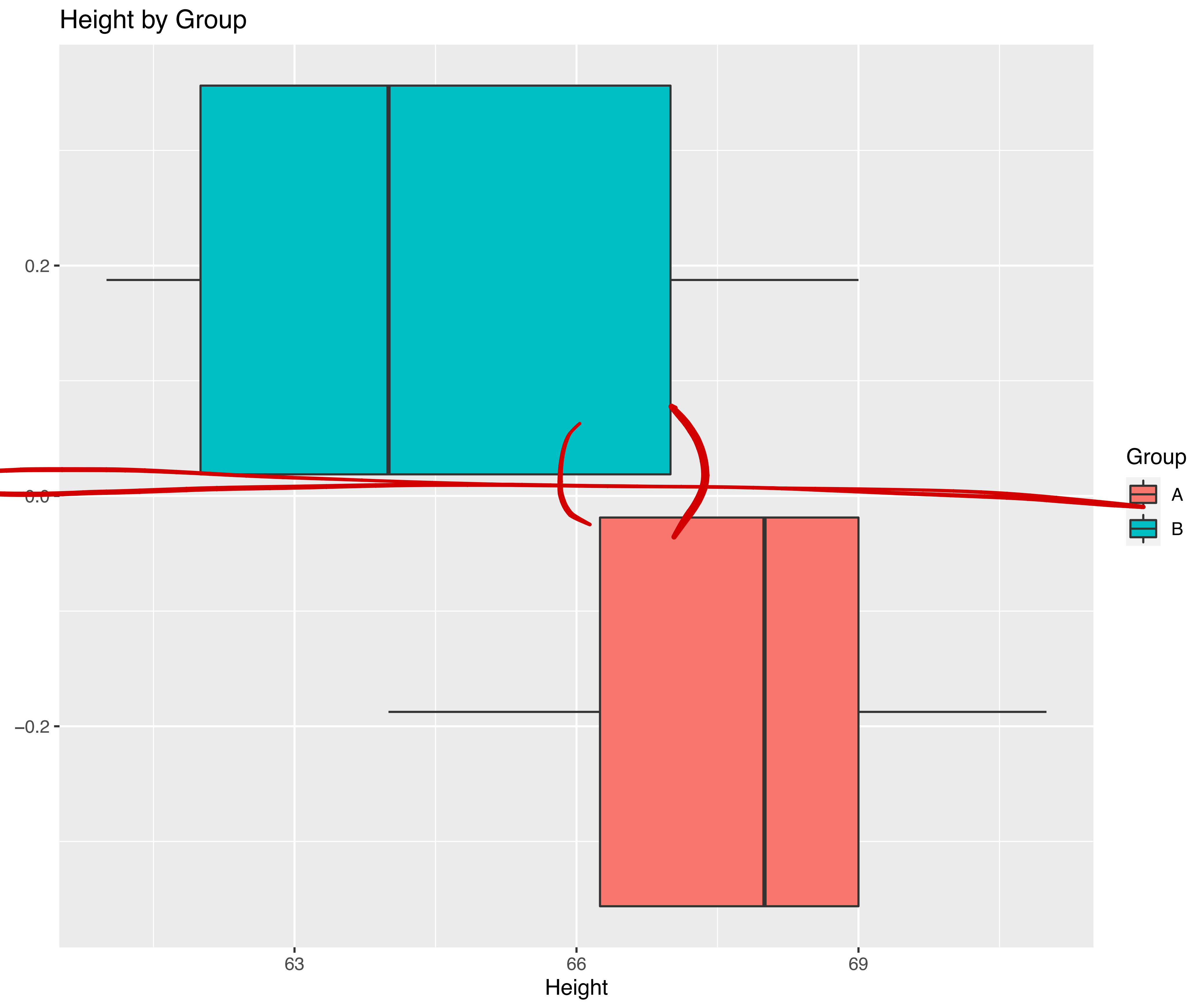

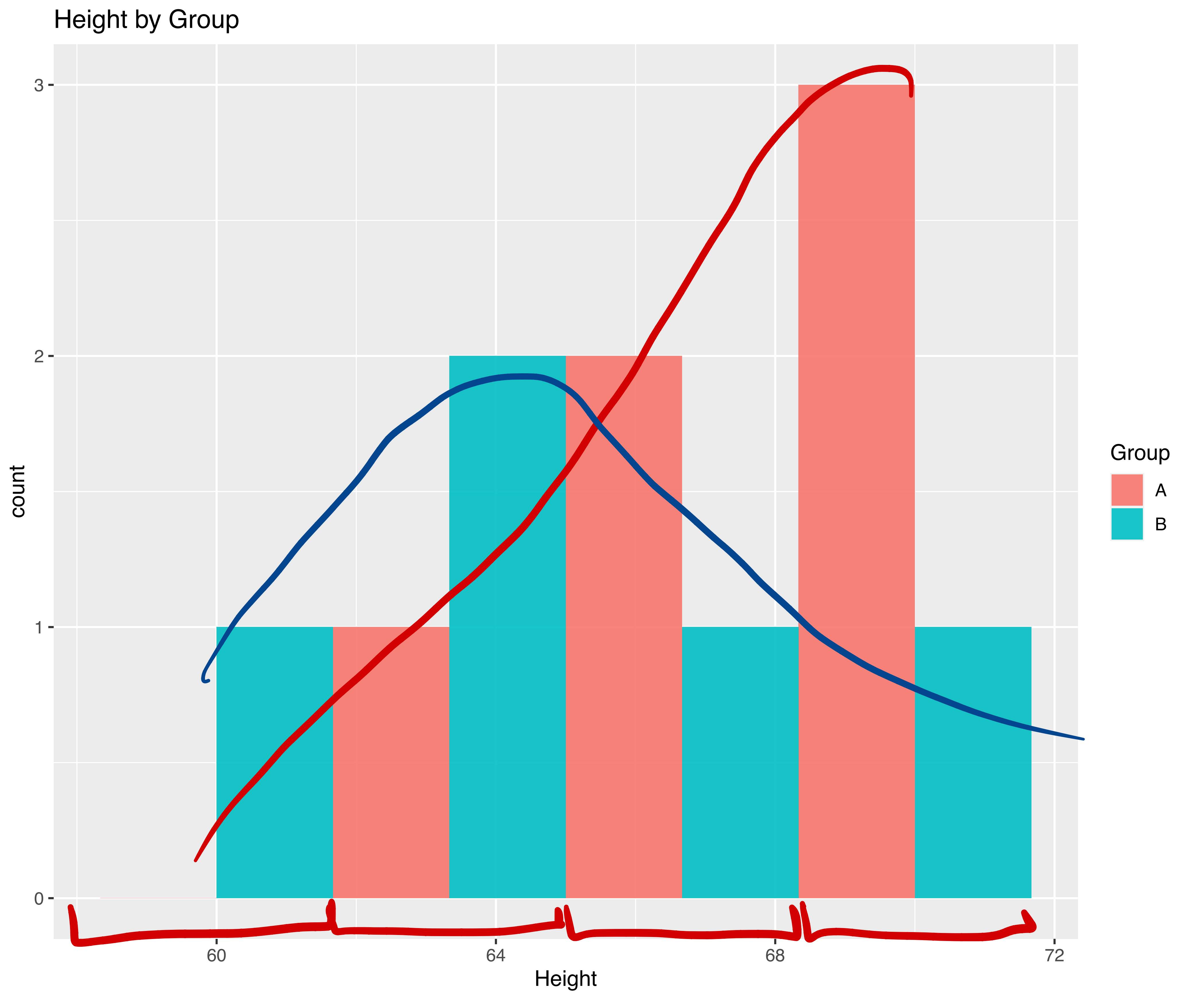

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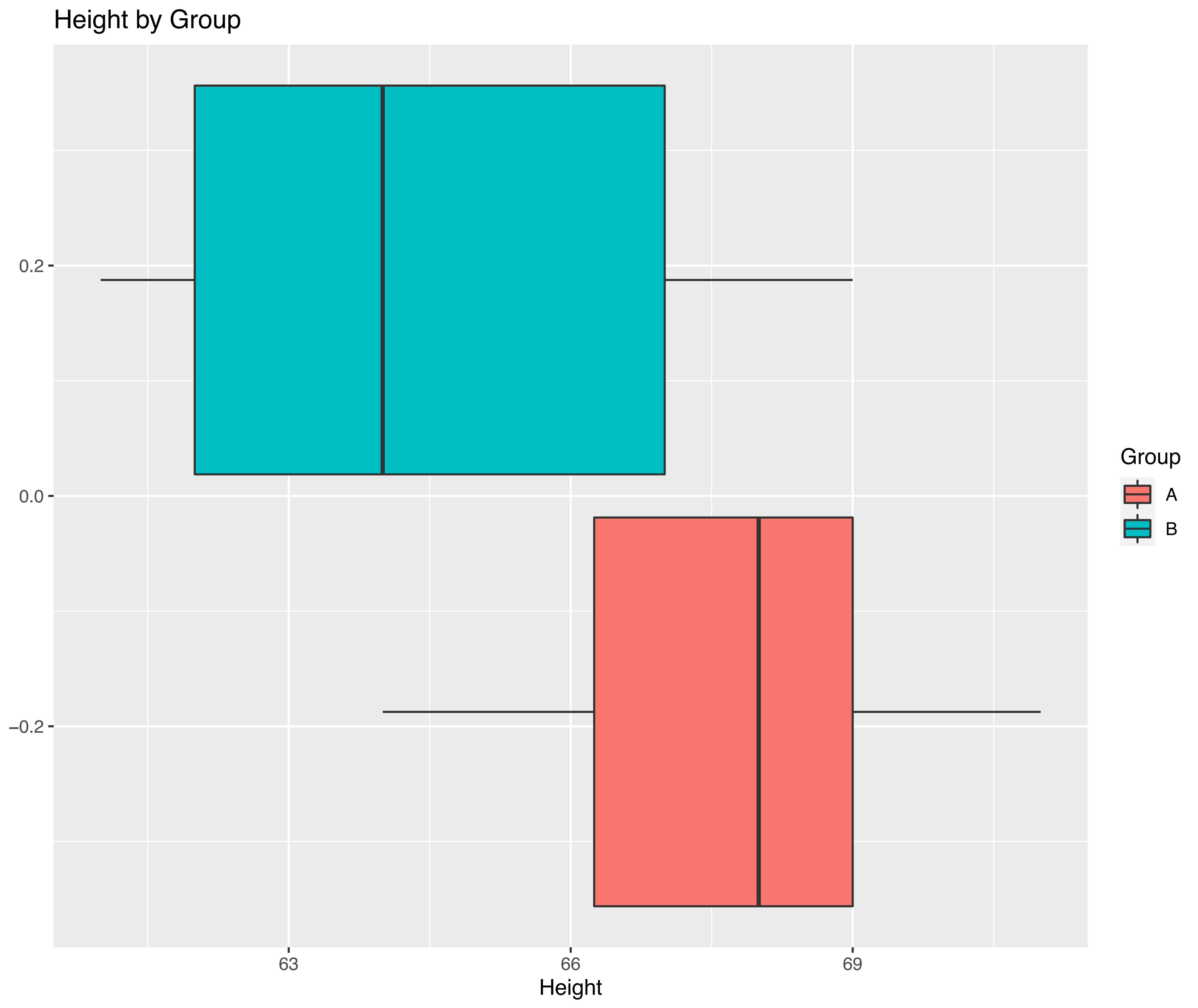

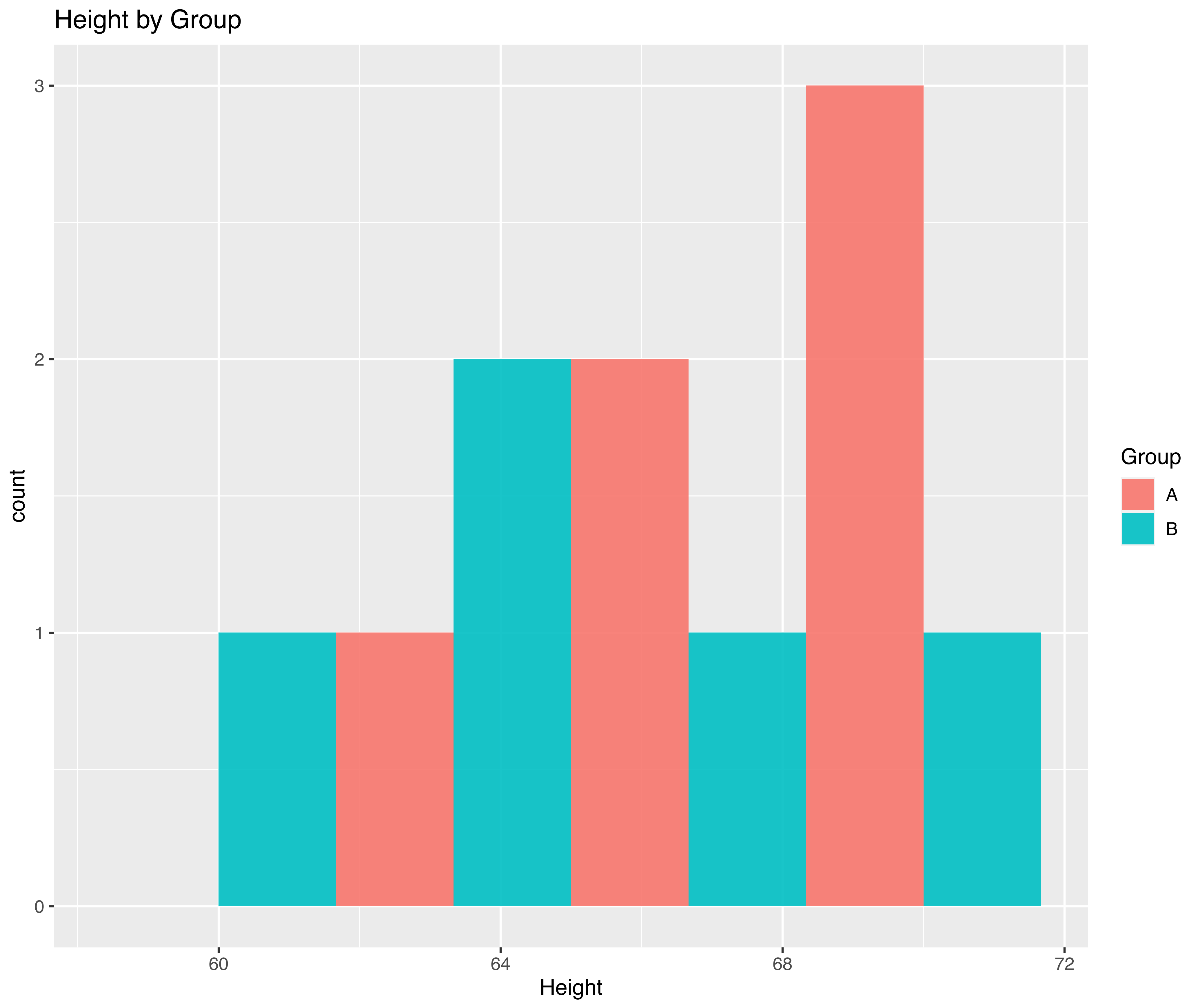


```
dat1<-data.frame(Height=c(64,66,67,69,69,71,61,62,64,67,69), Group=c(rep("A",6),rep("B",5)))
ggplot(dat1, aes(x=Height,fill=Group)) + geom_boxplot() + labs(title="Height by Group")
ggplot(dat1, aes(x=Height,fill=Group)) + geom_histogram(bins=4,alpha=0.9,position='dodge') + labs(title="Height by Group")
```

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	Smoker	Non-Smoker	**Total**
Group A	15	22	**37**
Group B	26	18	**44**
Total	**41**	**40**	**81**

#A

#B

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- What is the probability of being in Group A given that you smoke?

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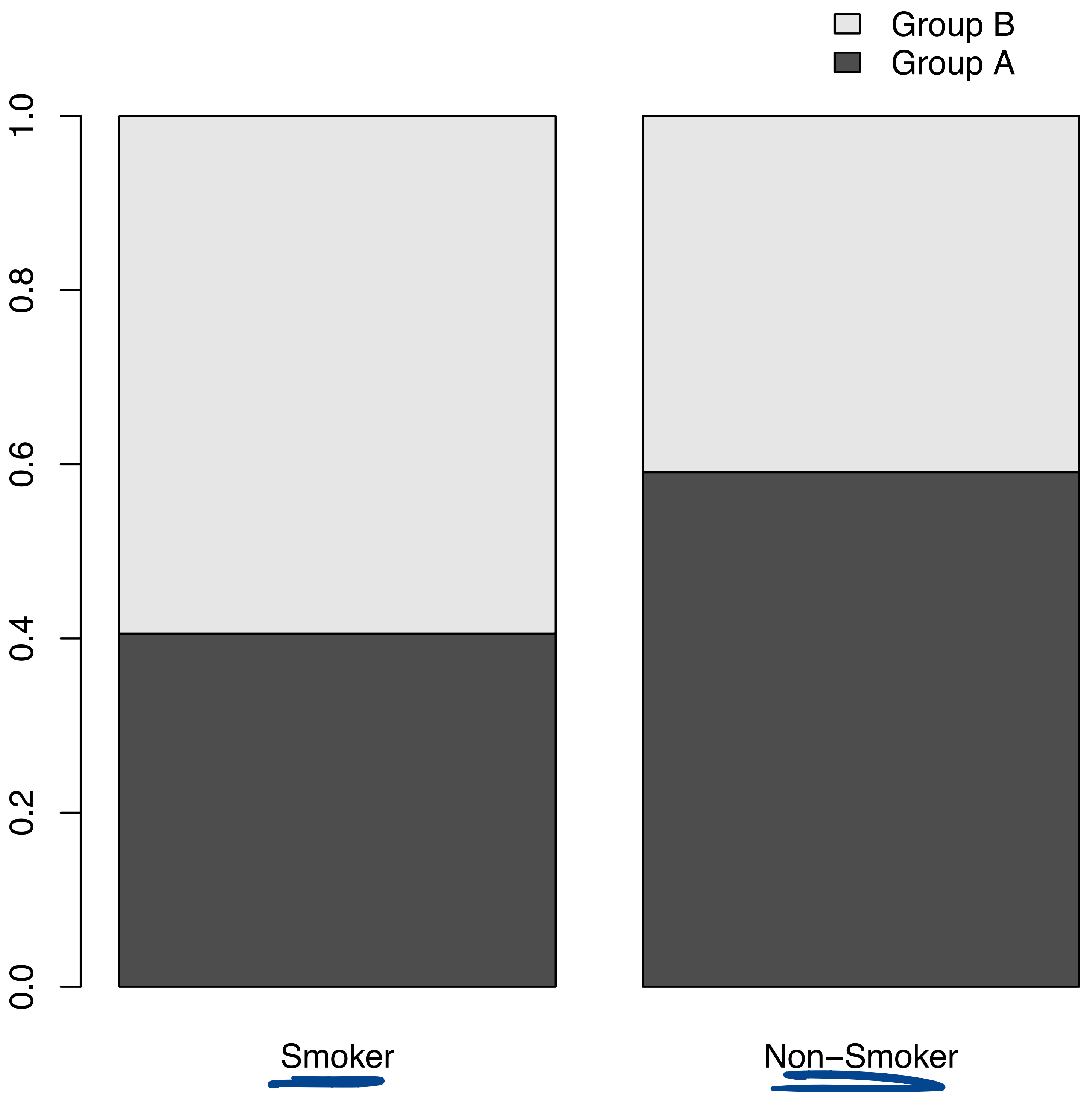

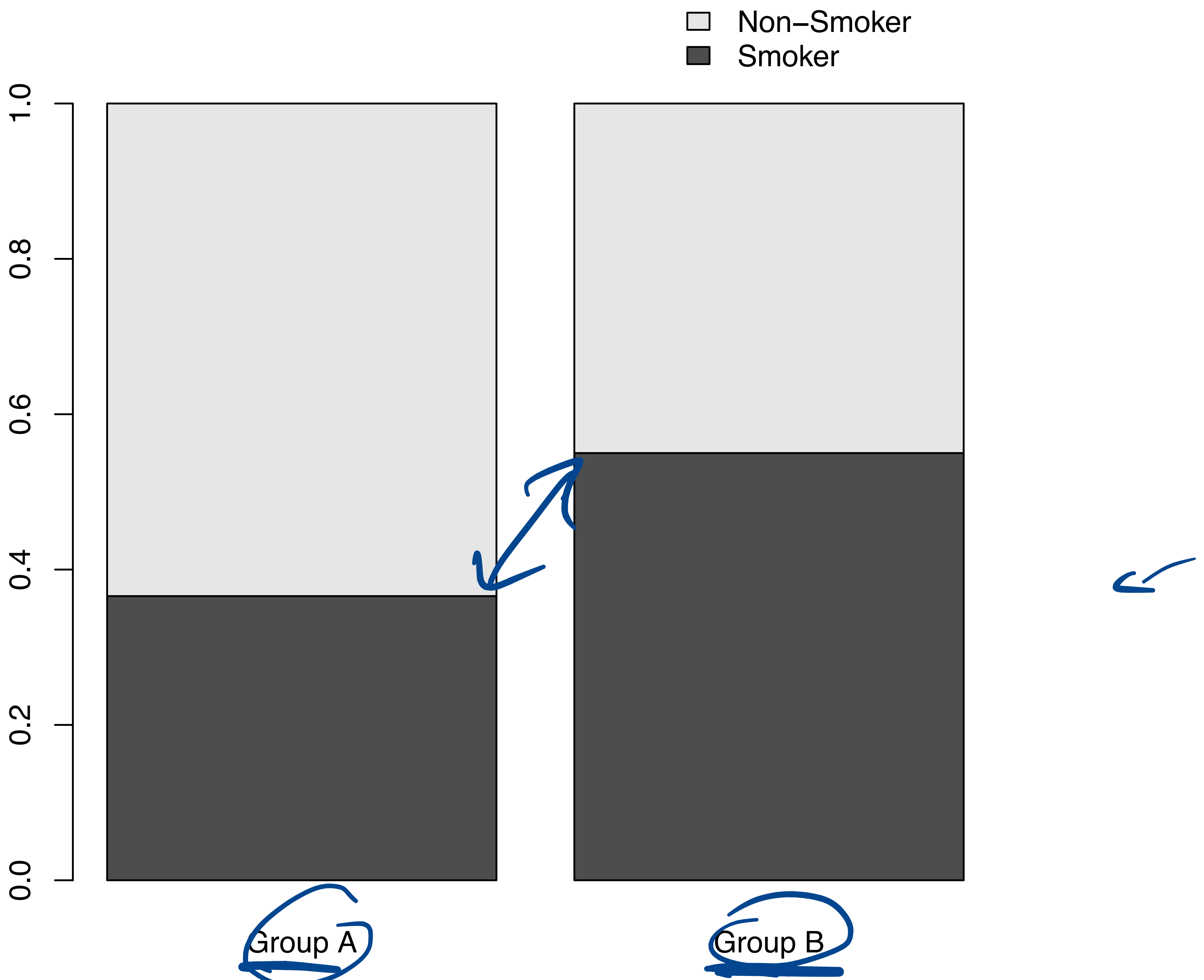

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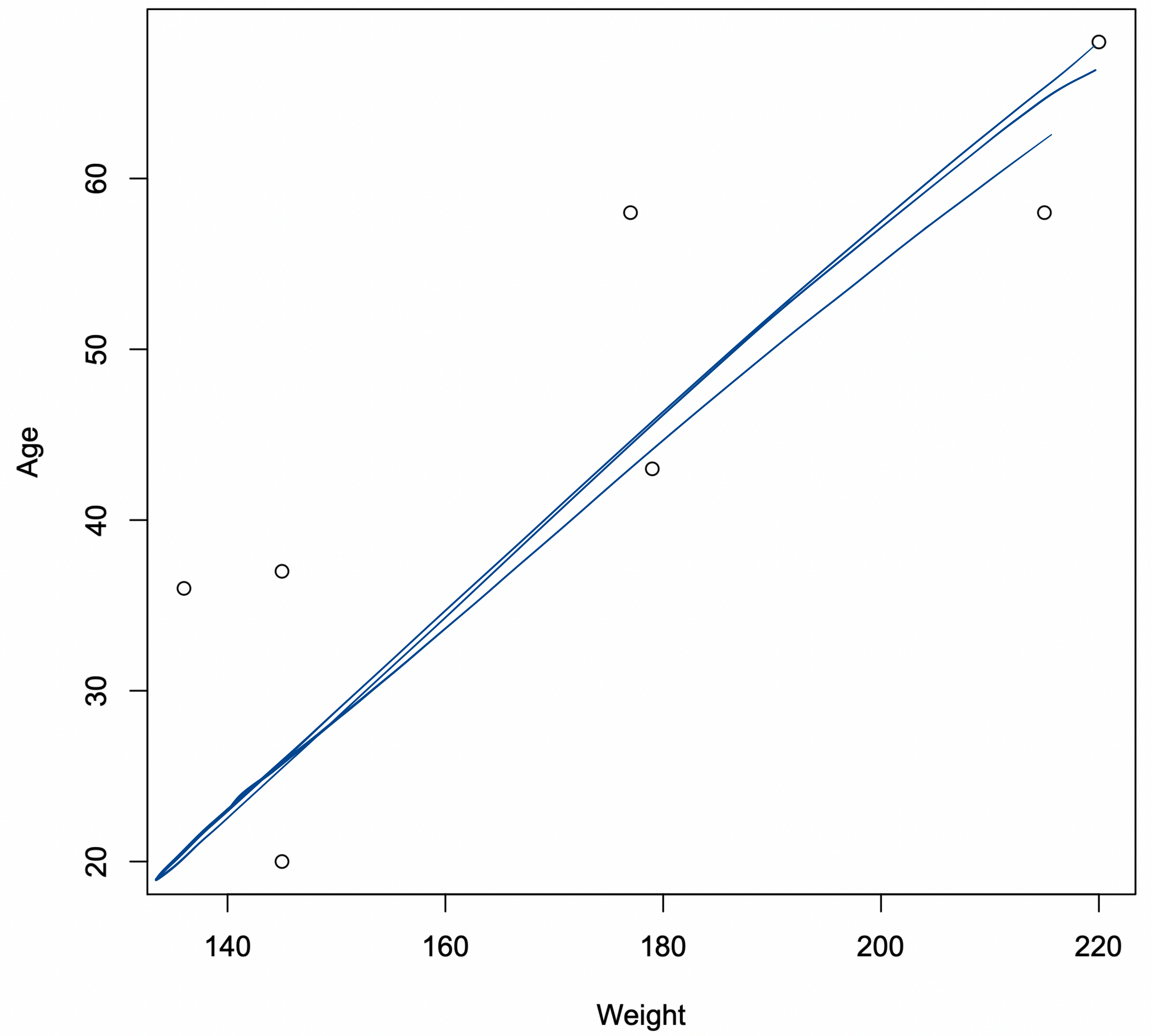
- We can graphically display this relationship with a *two-way scatterplot*
- When we make a scatterplot, we have our two variables as our two axes, and points are plotted based on their corresponding values for each variable
- R code: `plot(x=weight,y=age,xlab="Weight",ylab="Age")`

Scatterplot



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  - ***Direction***: positive, negative, or neither
  - ***Form***: linear, non-linear, or no relationship
  - ***Strength***: strong, weak, or none

Examples: Strength, Direction, and Form

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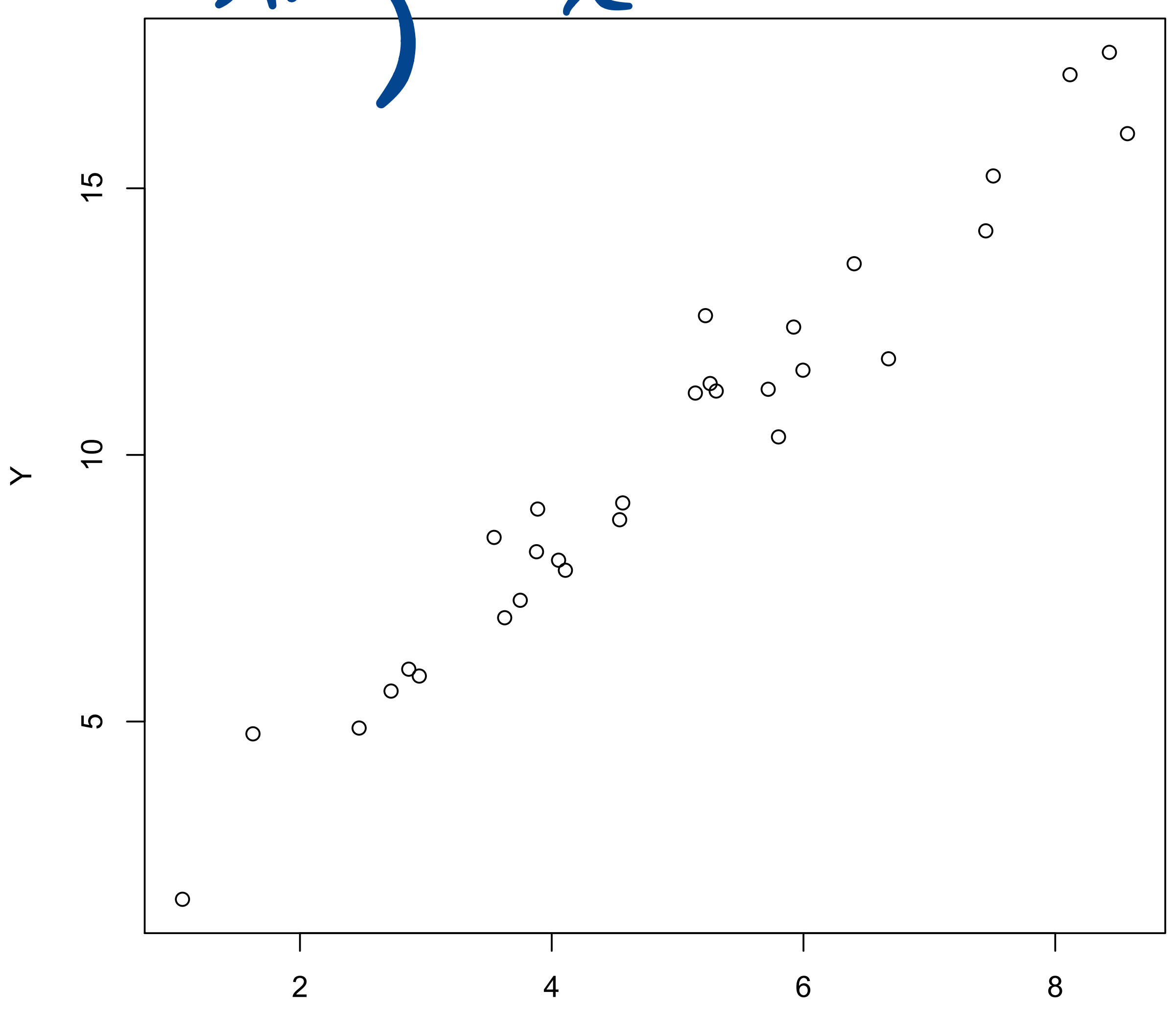

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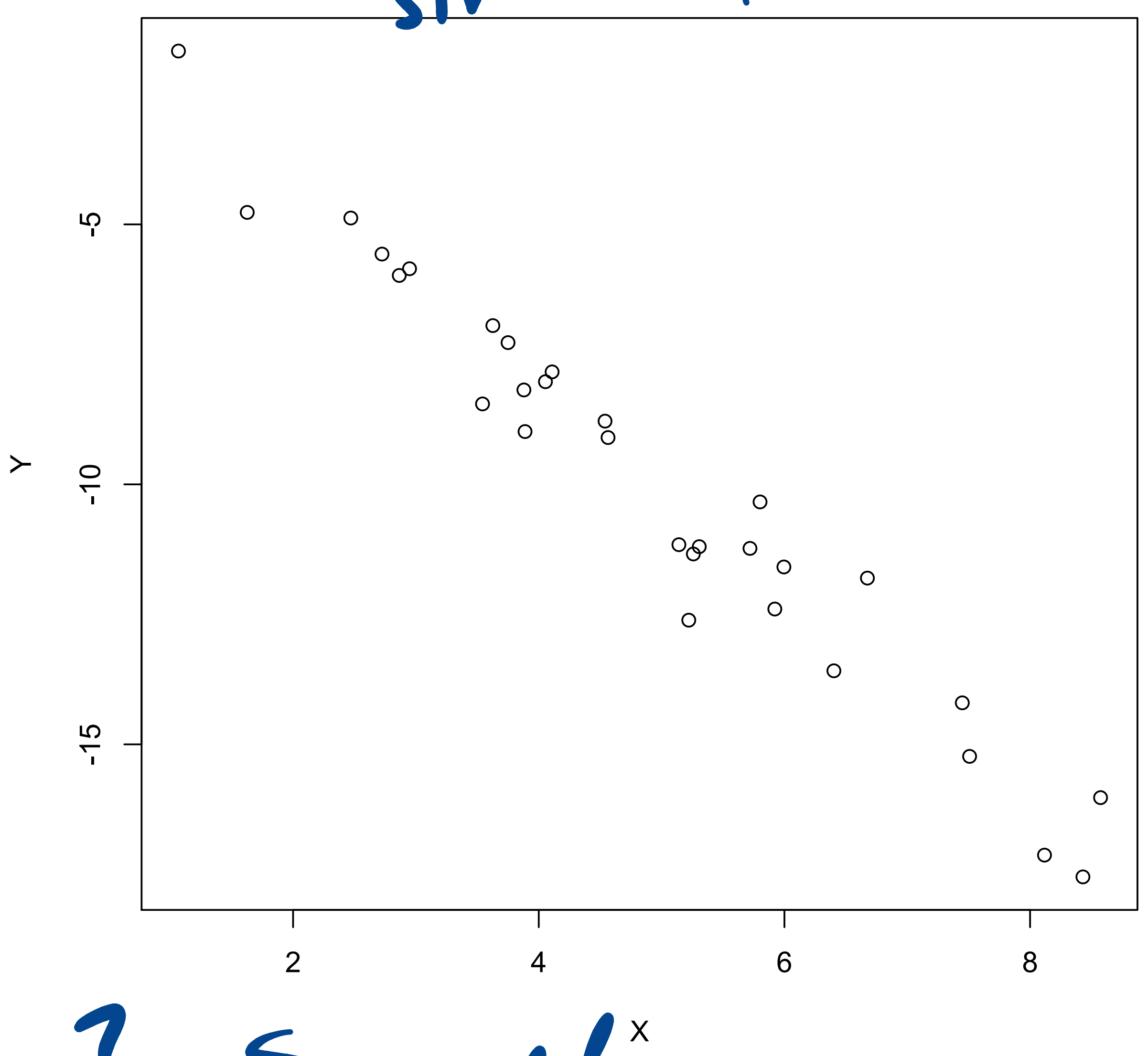

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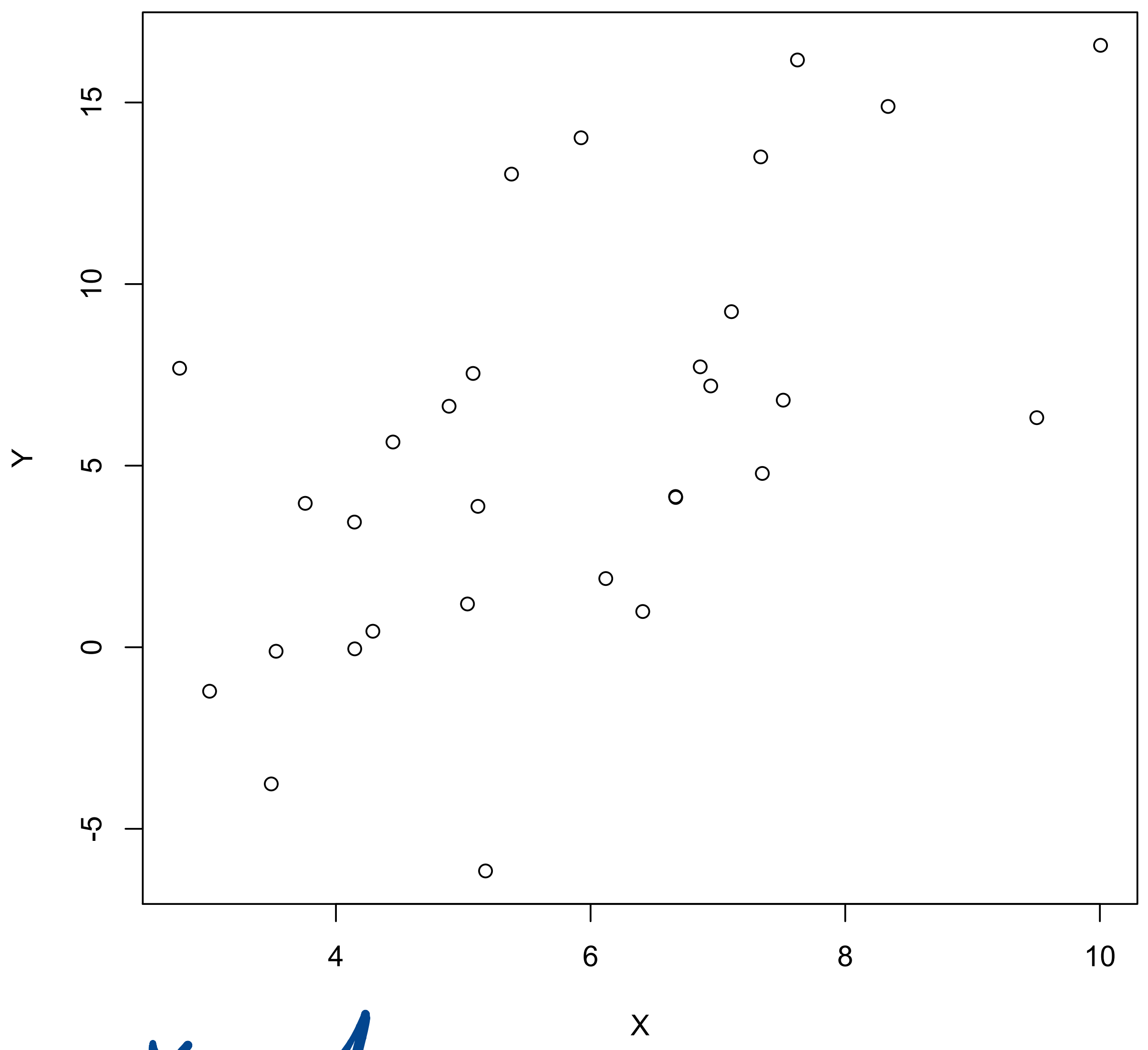

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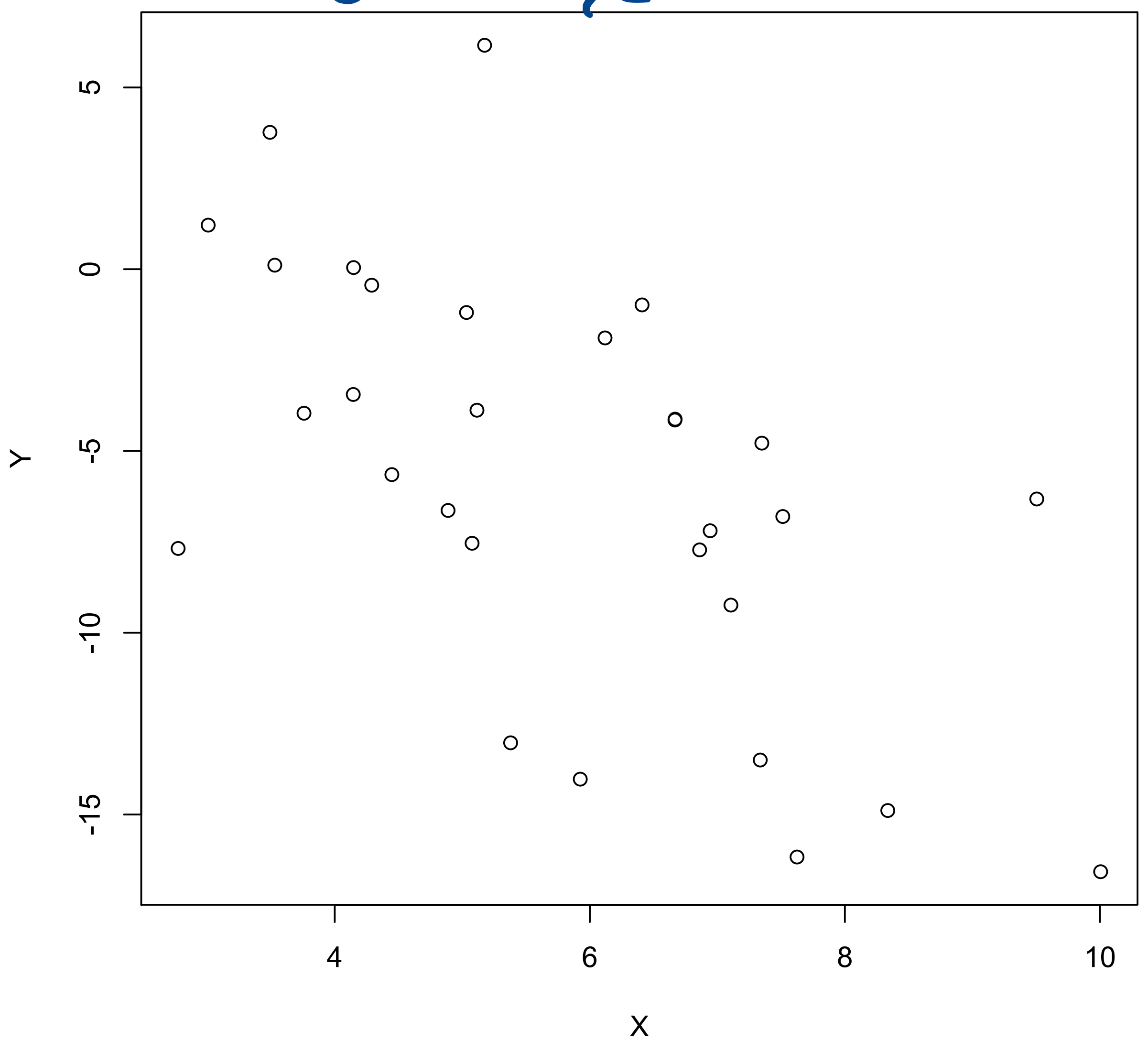

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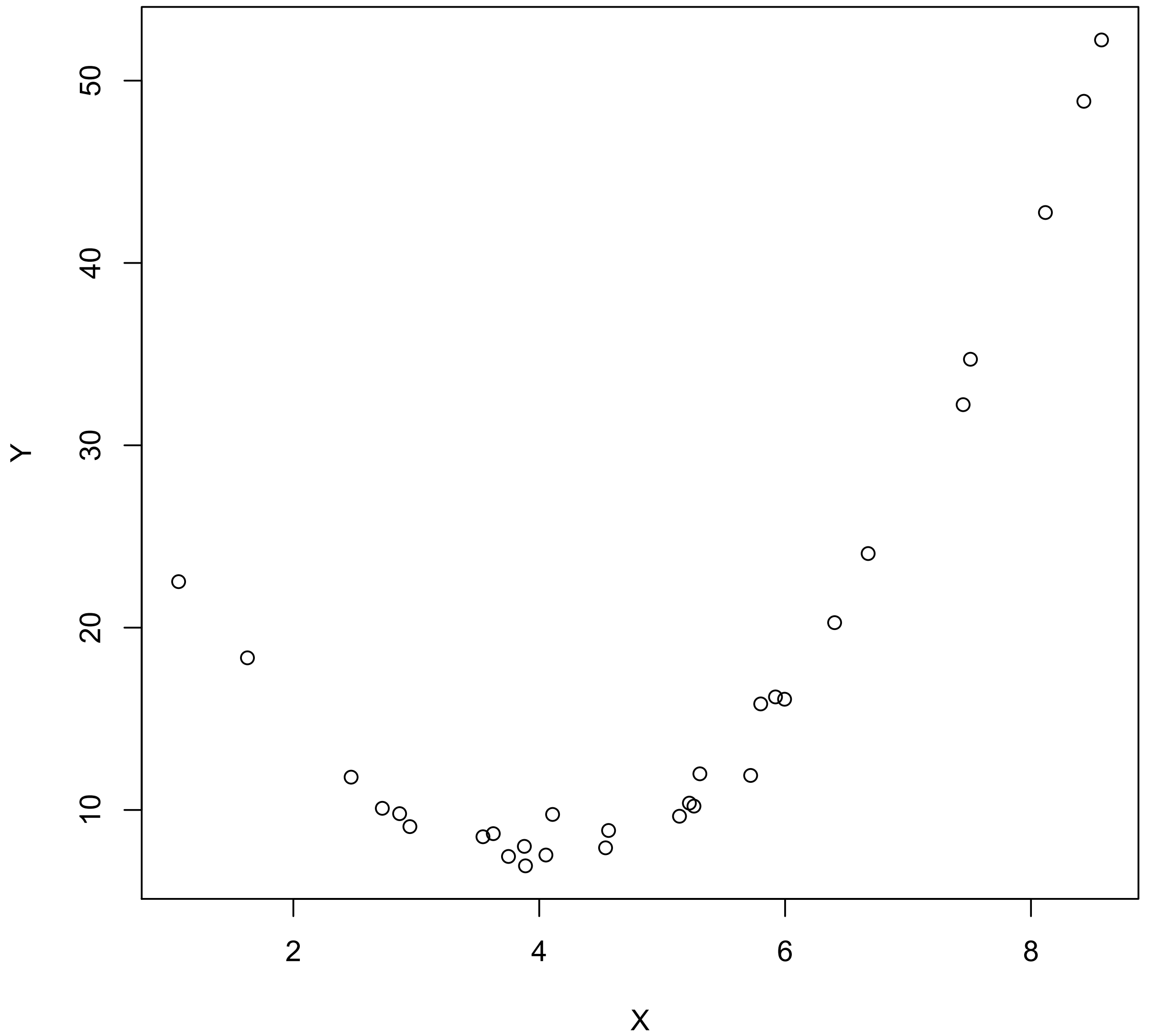

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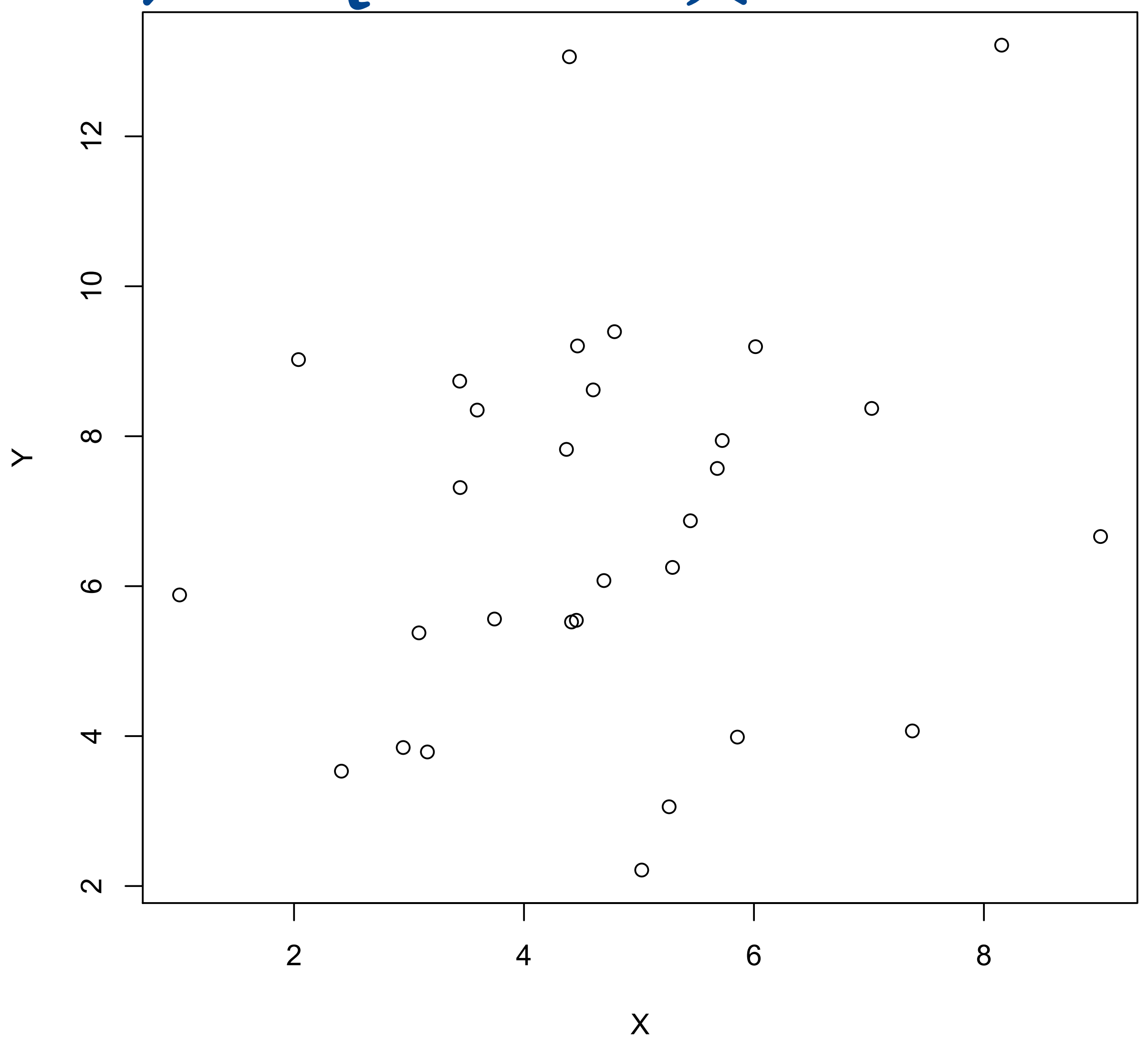
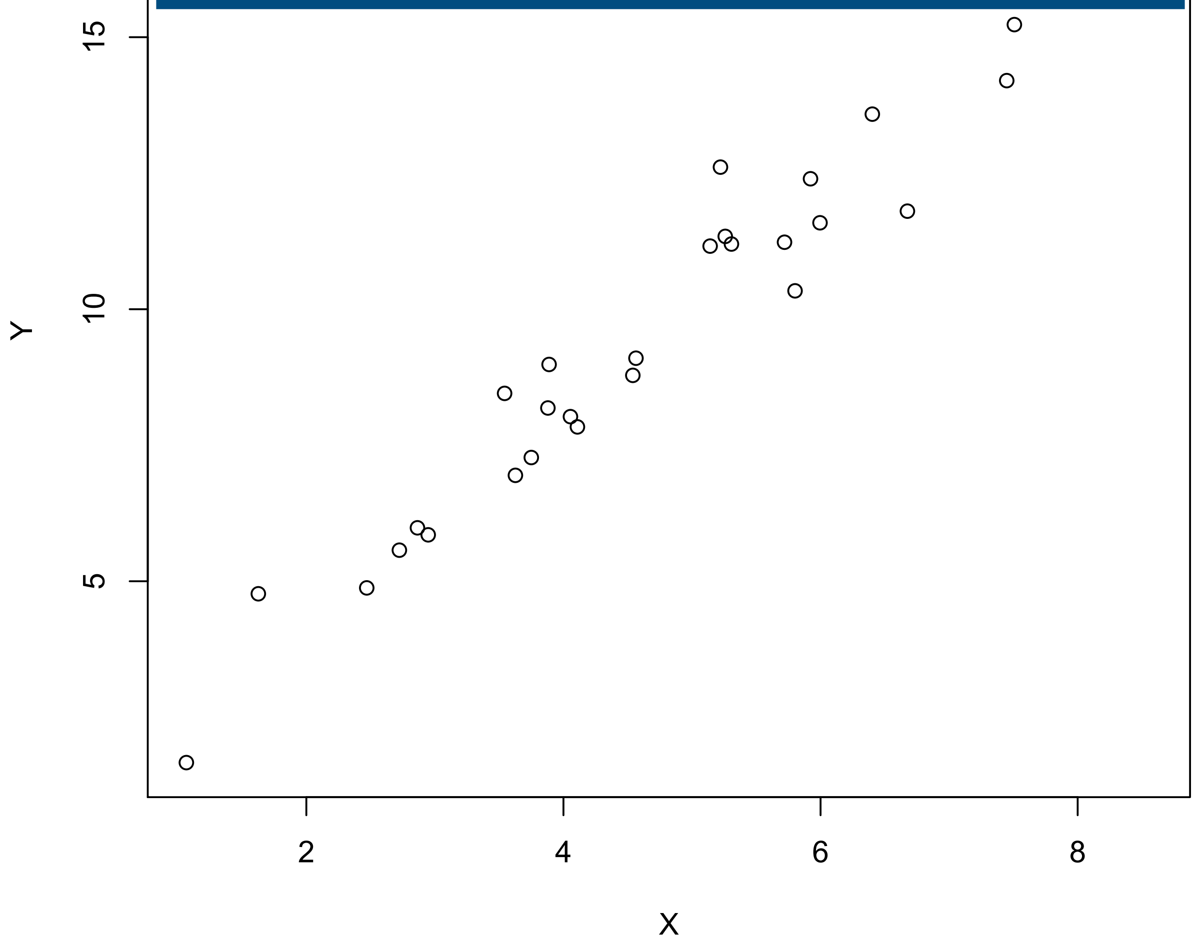
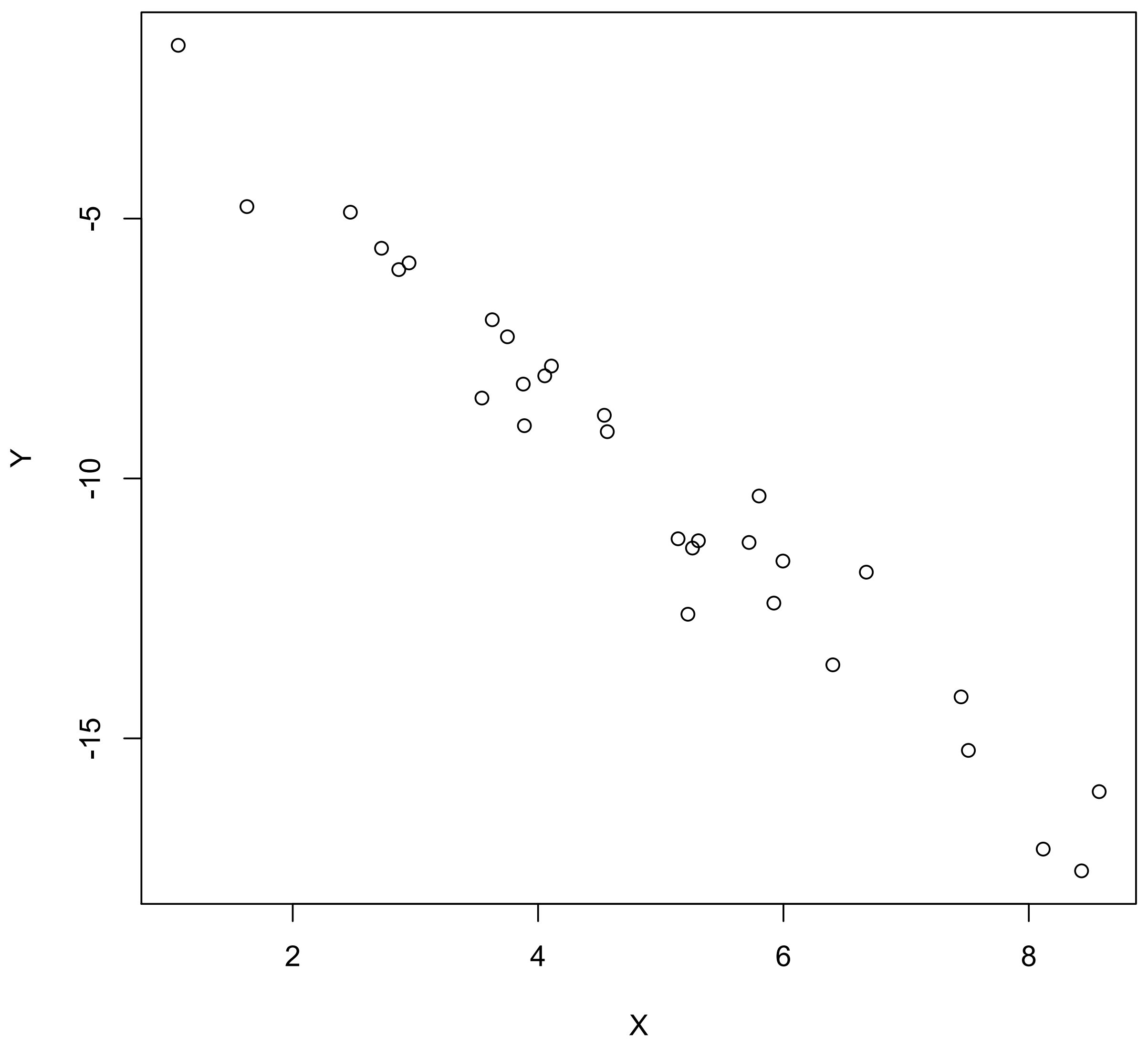
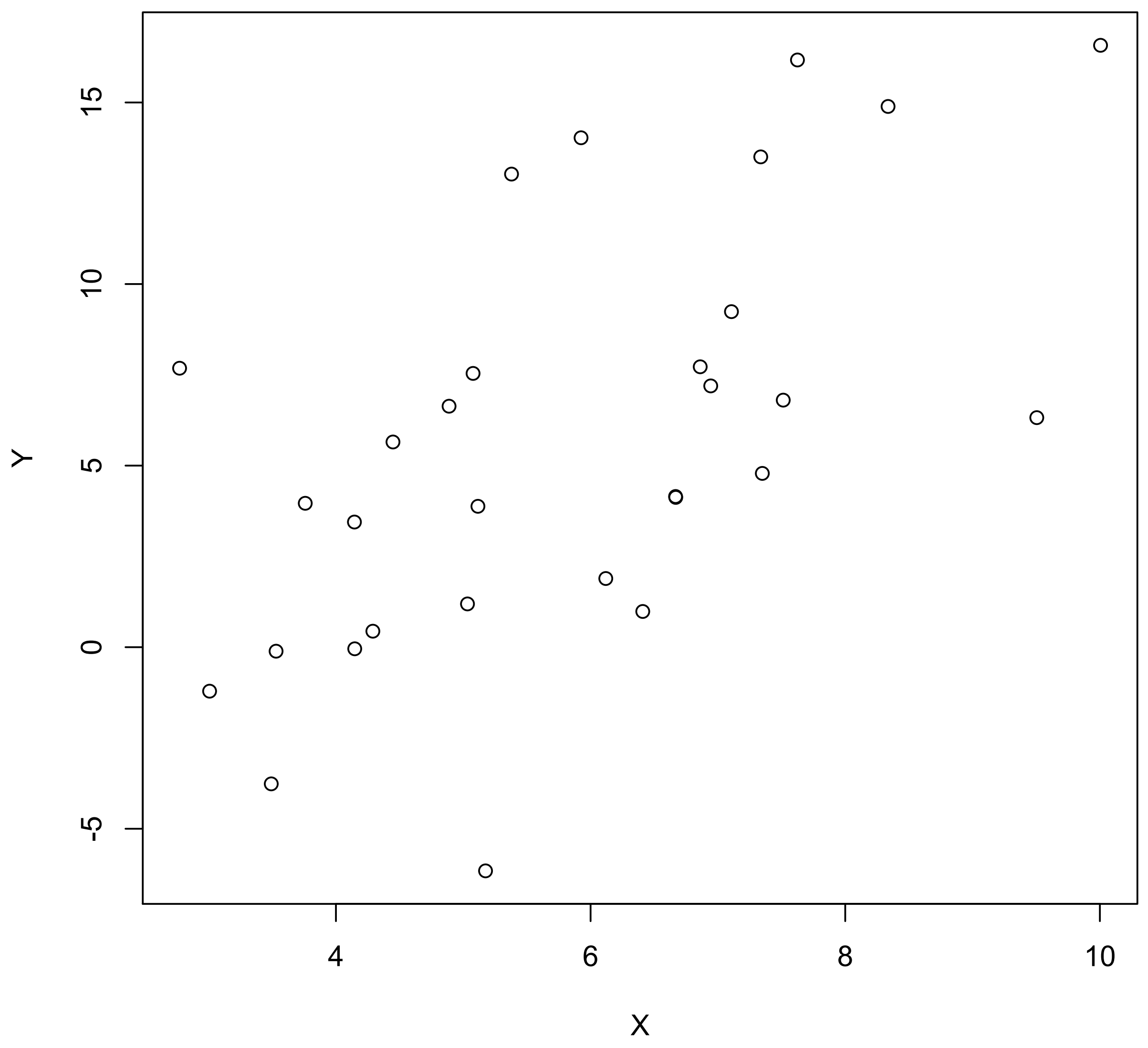


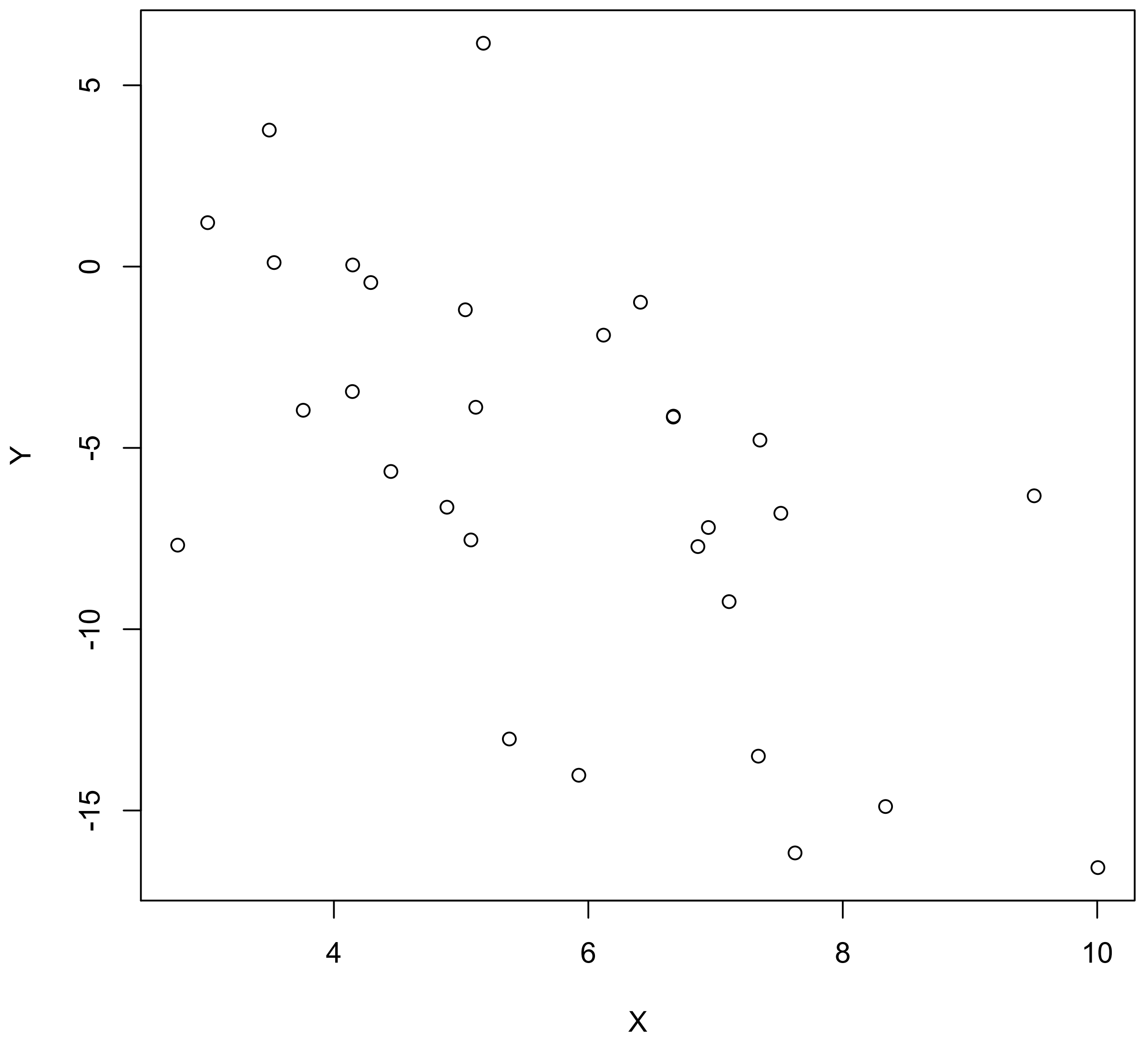
Examples: Strength, Direction, and Form

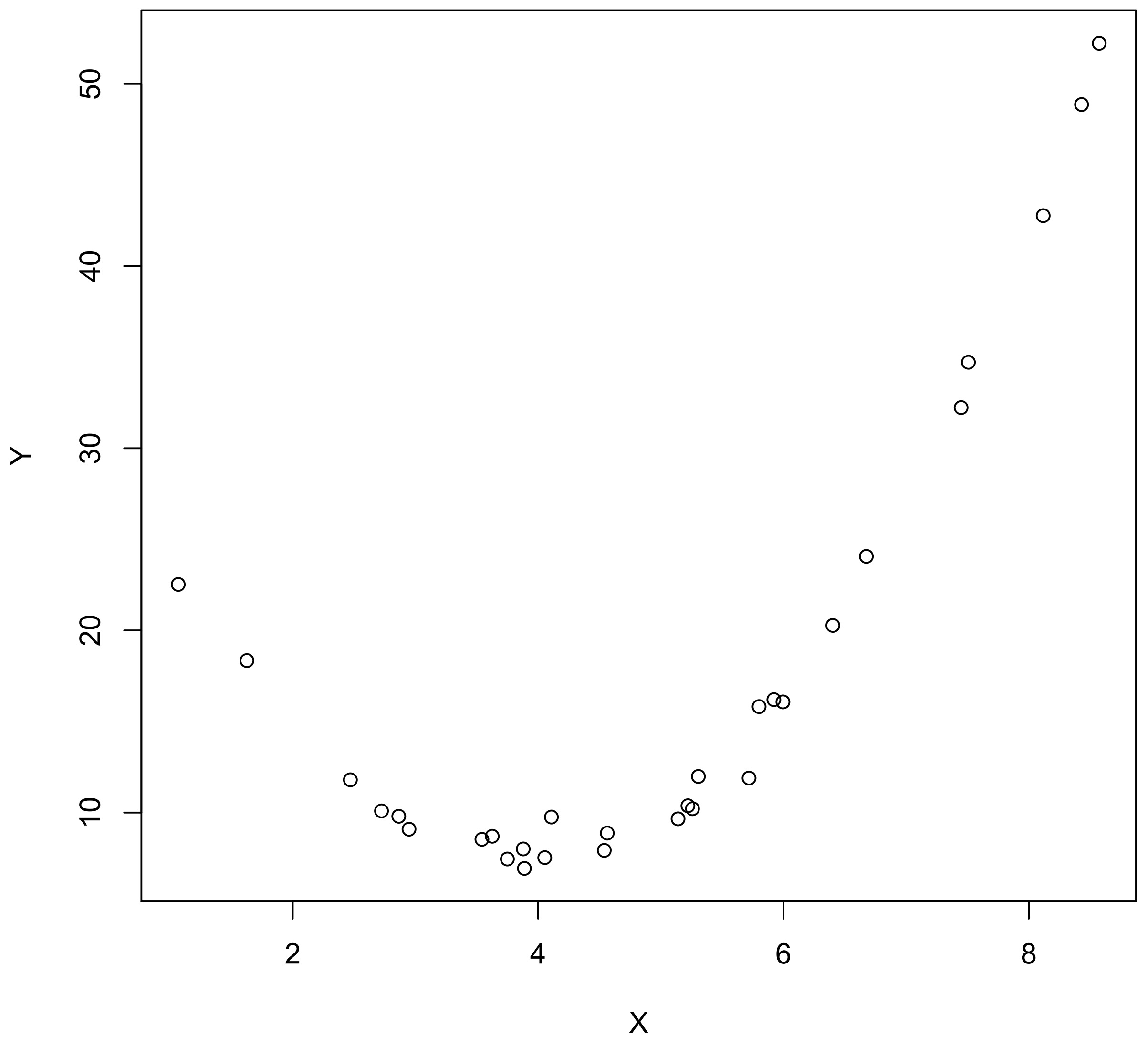
Strong, Positive, Linear

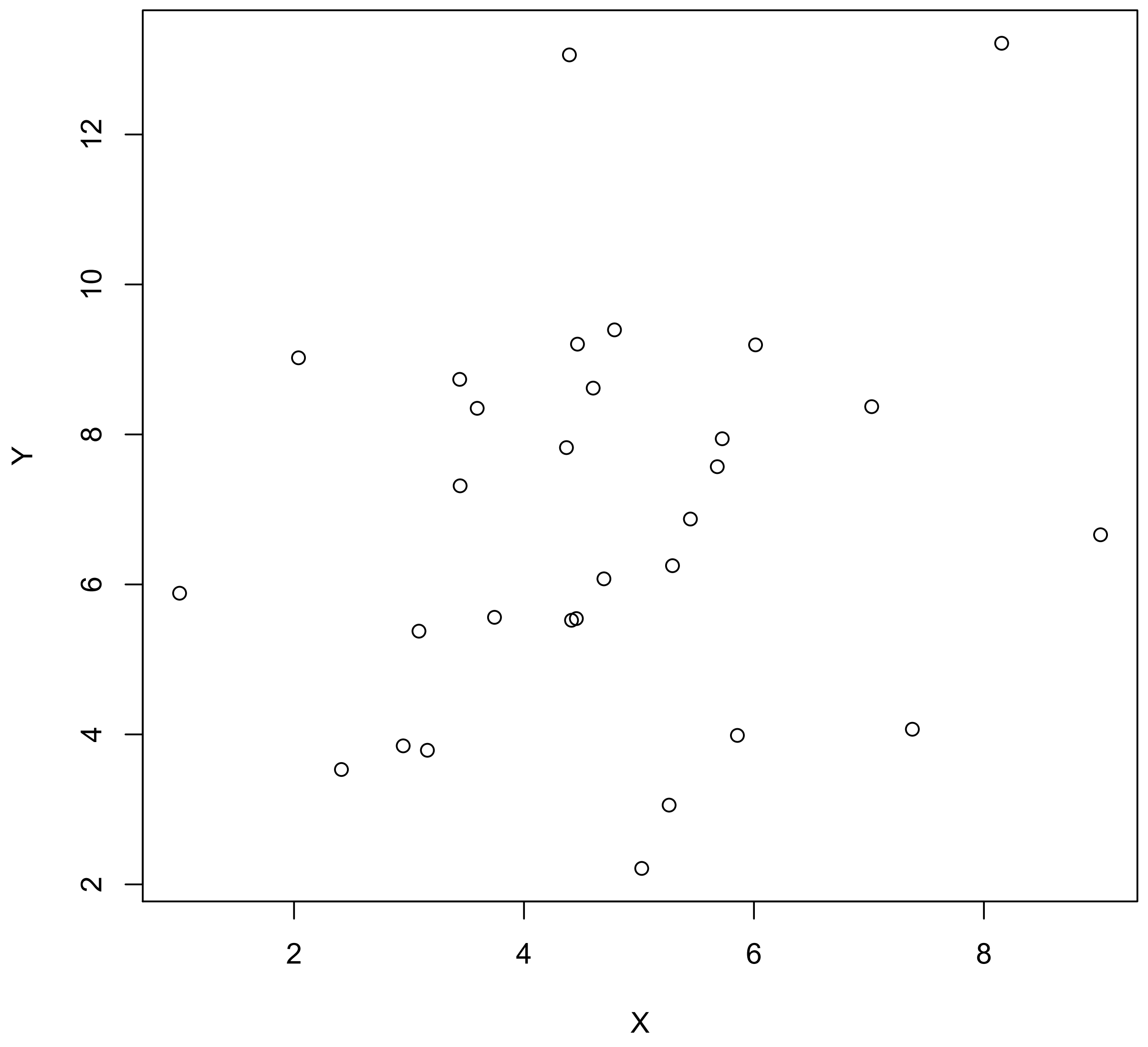












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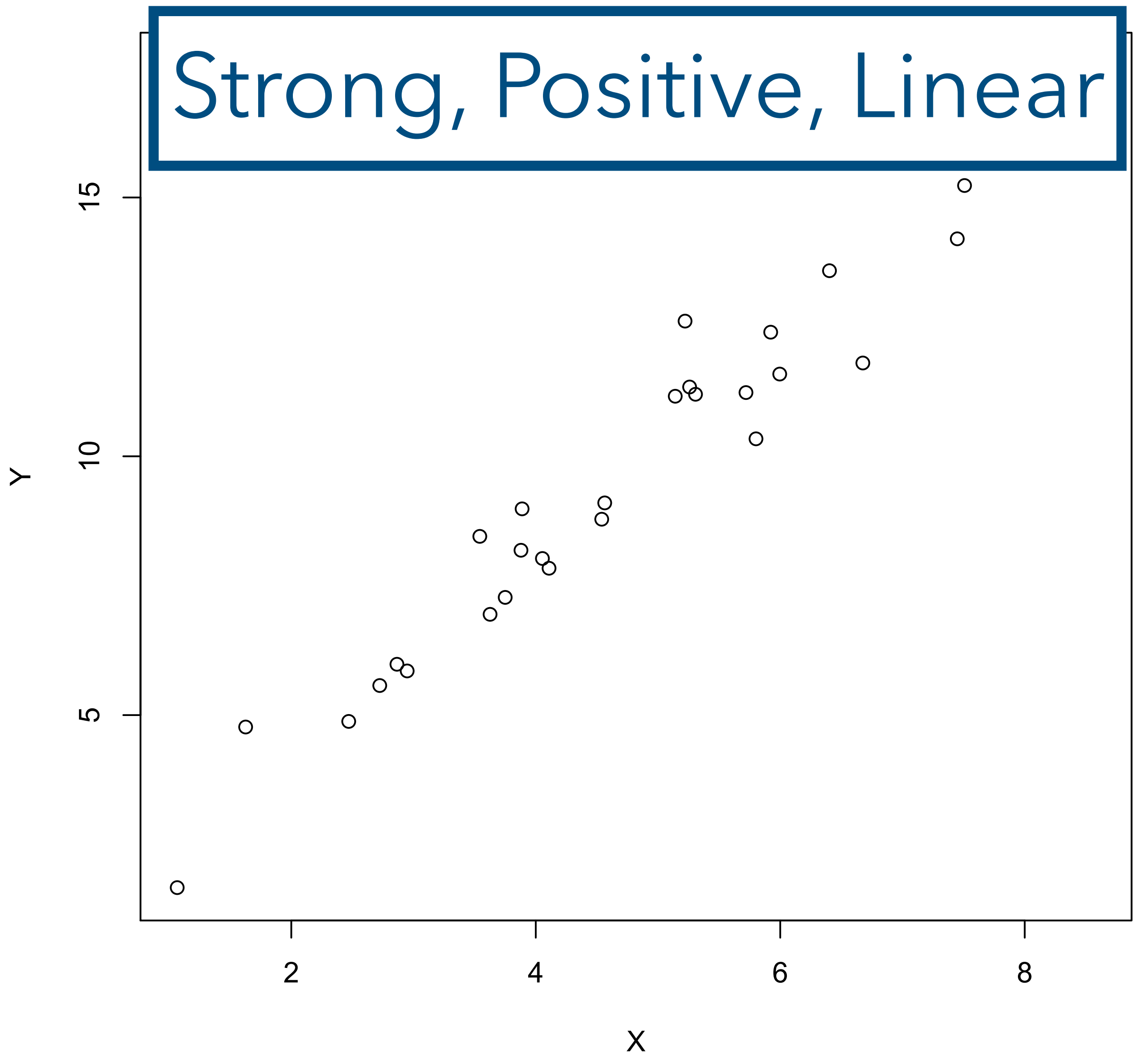

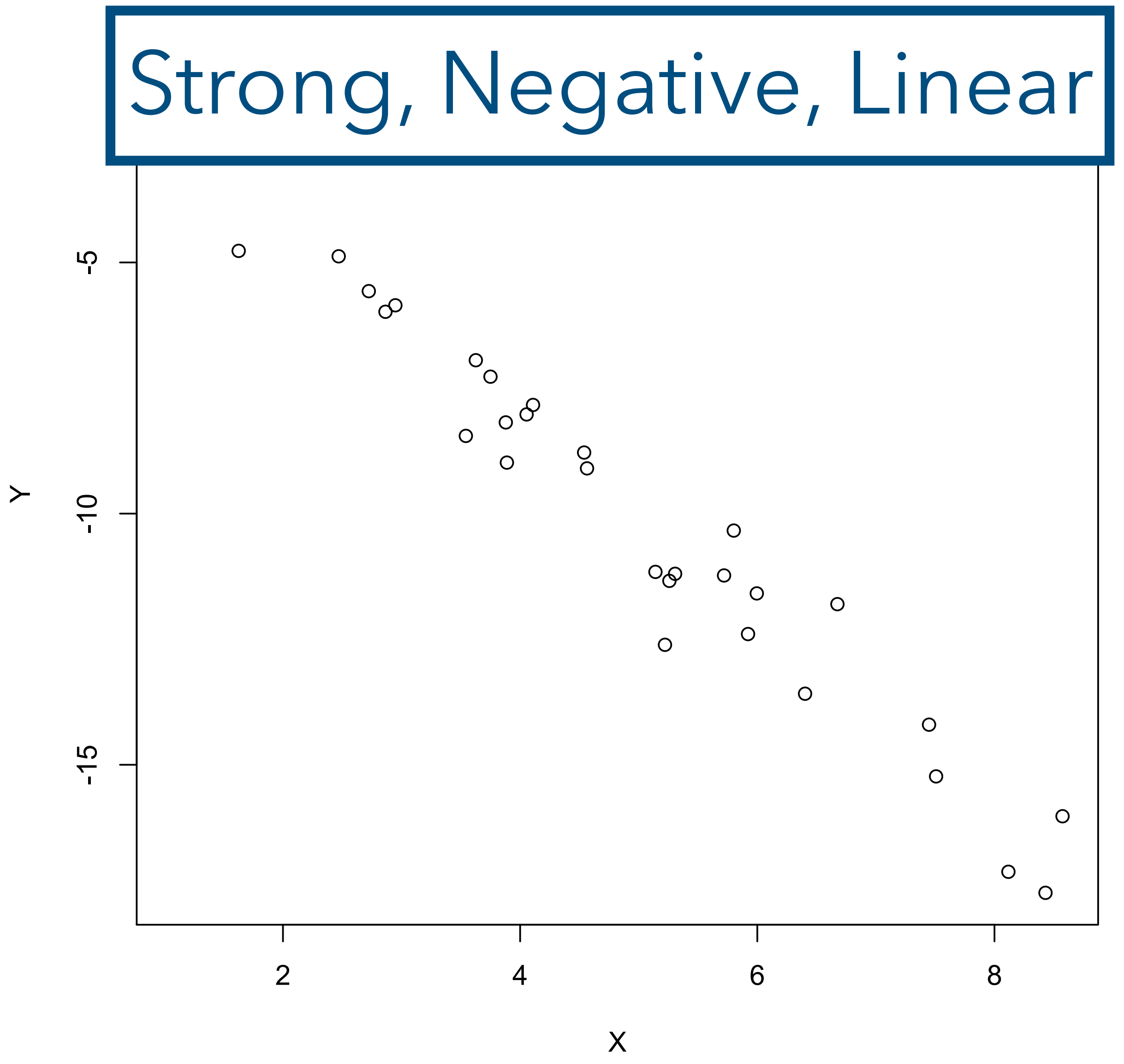

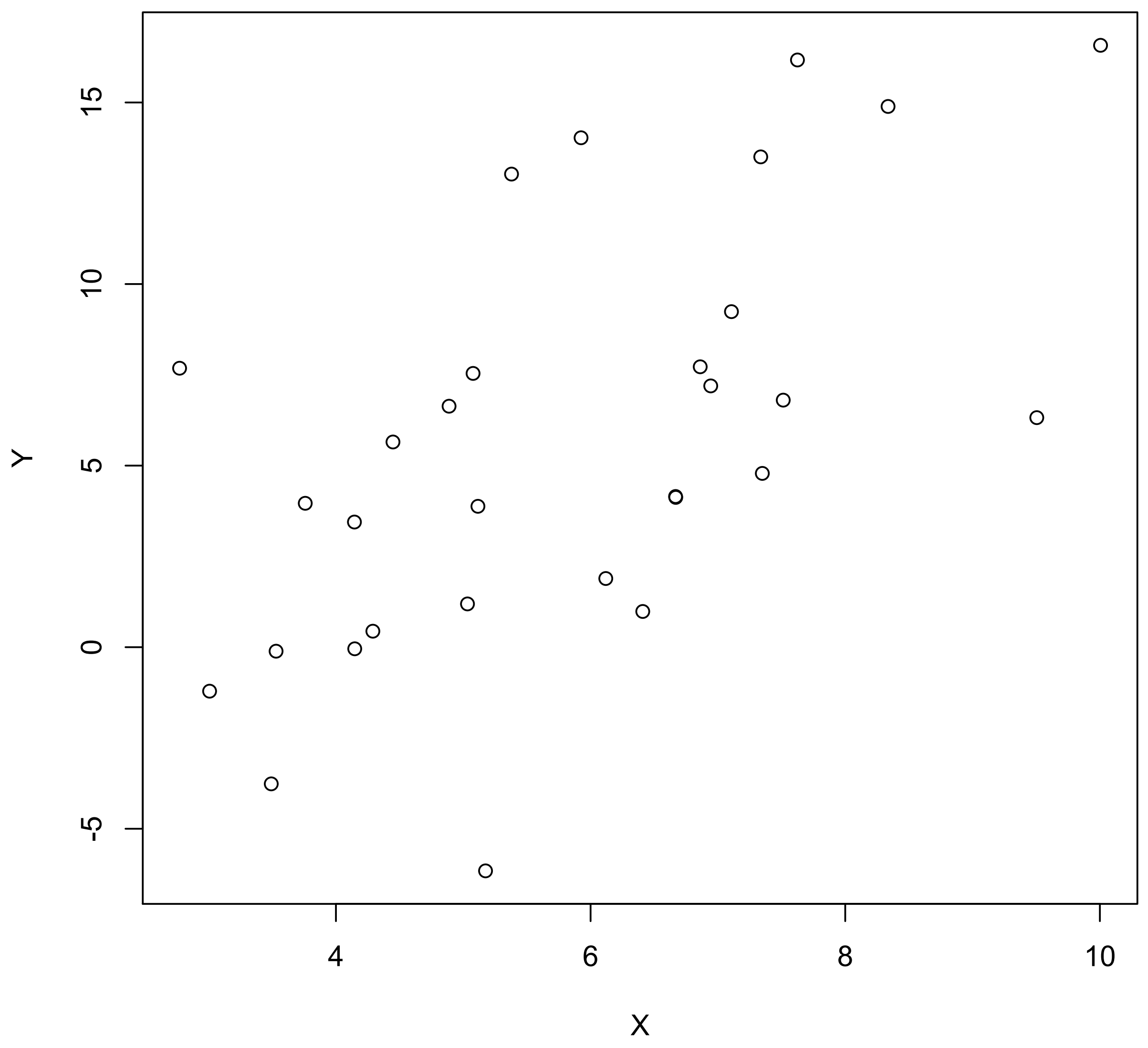

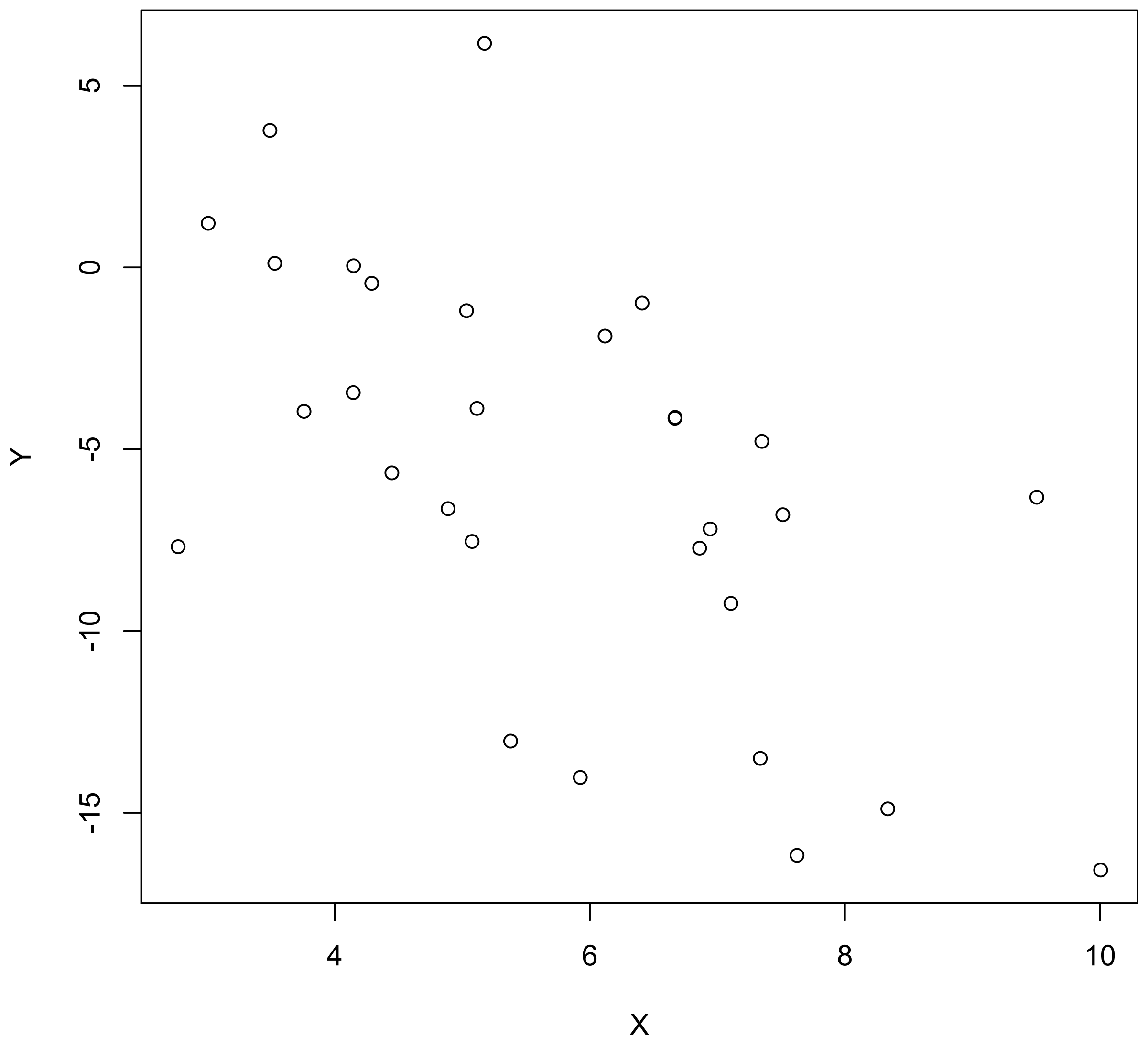

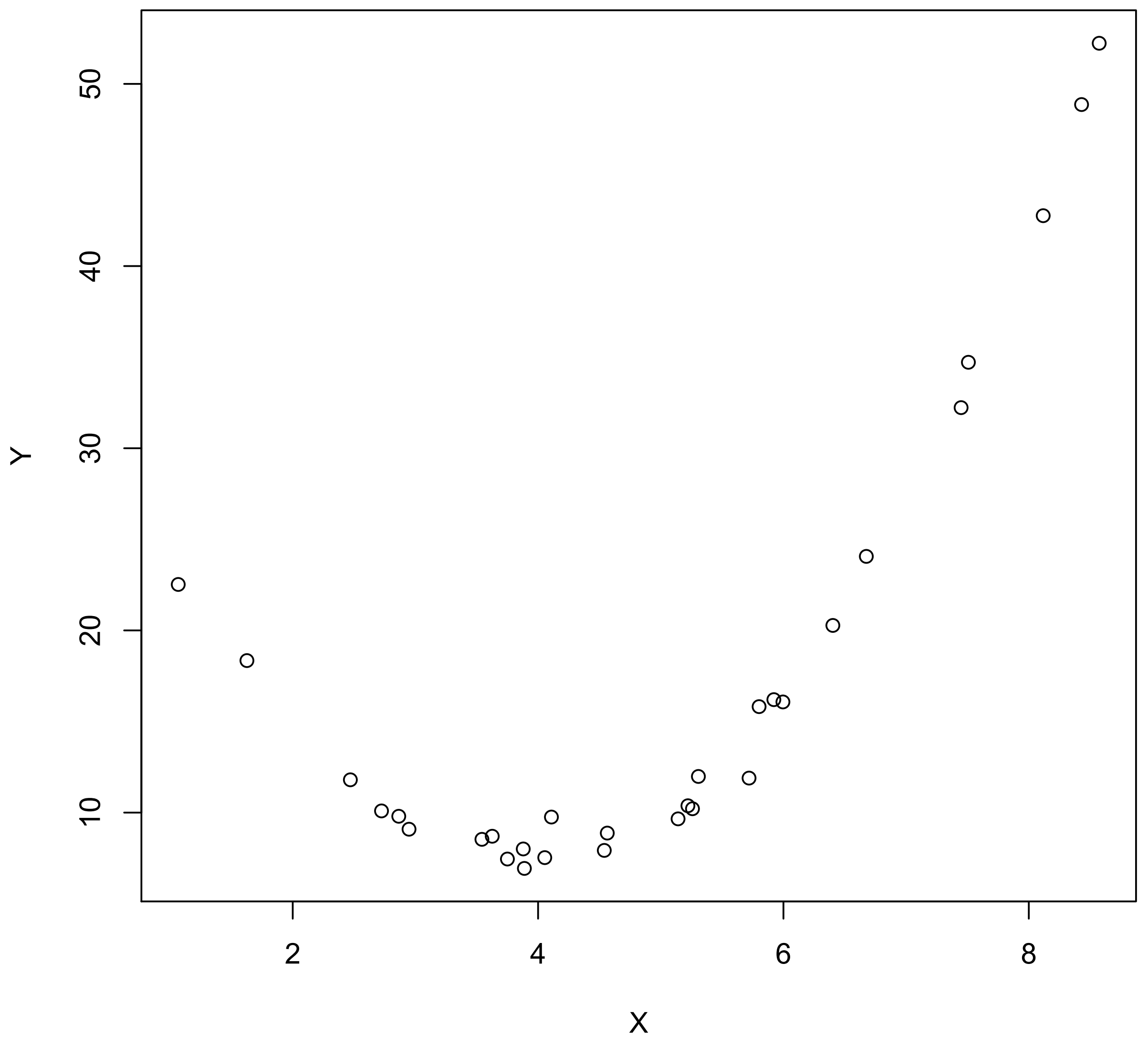

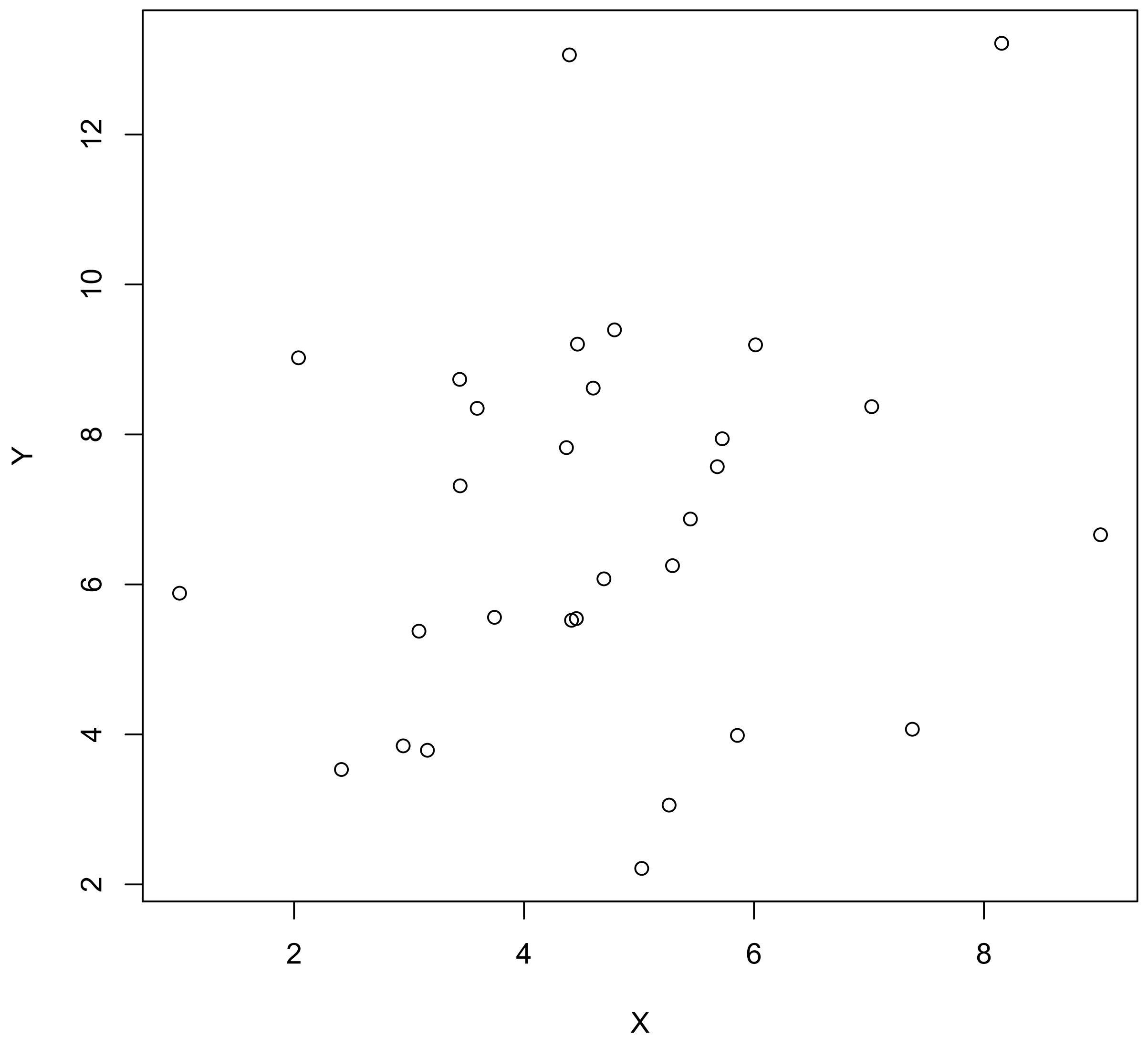

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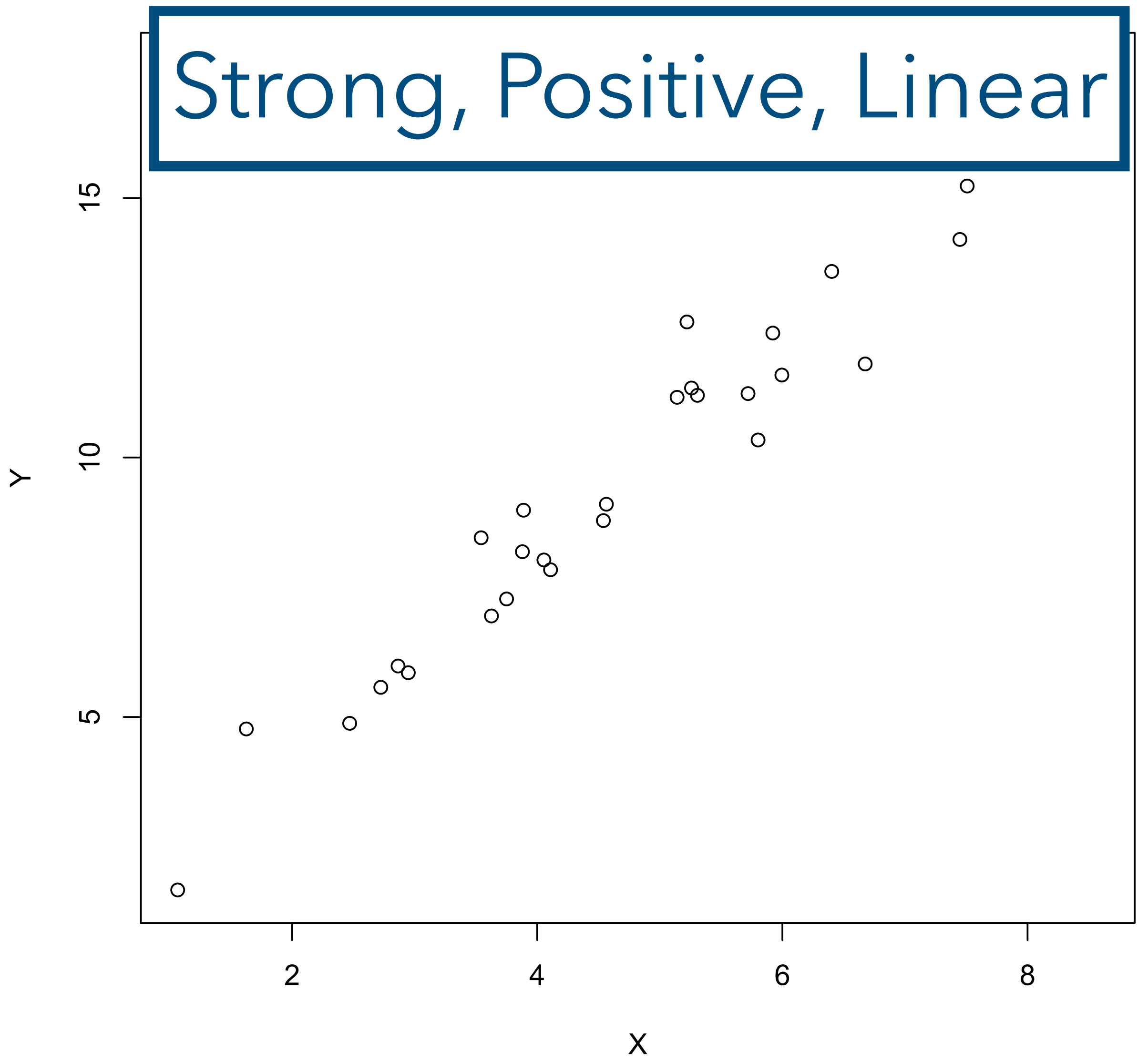

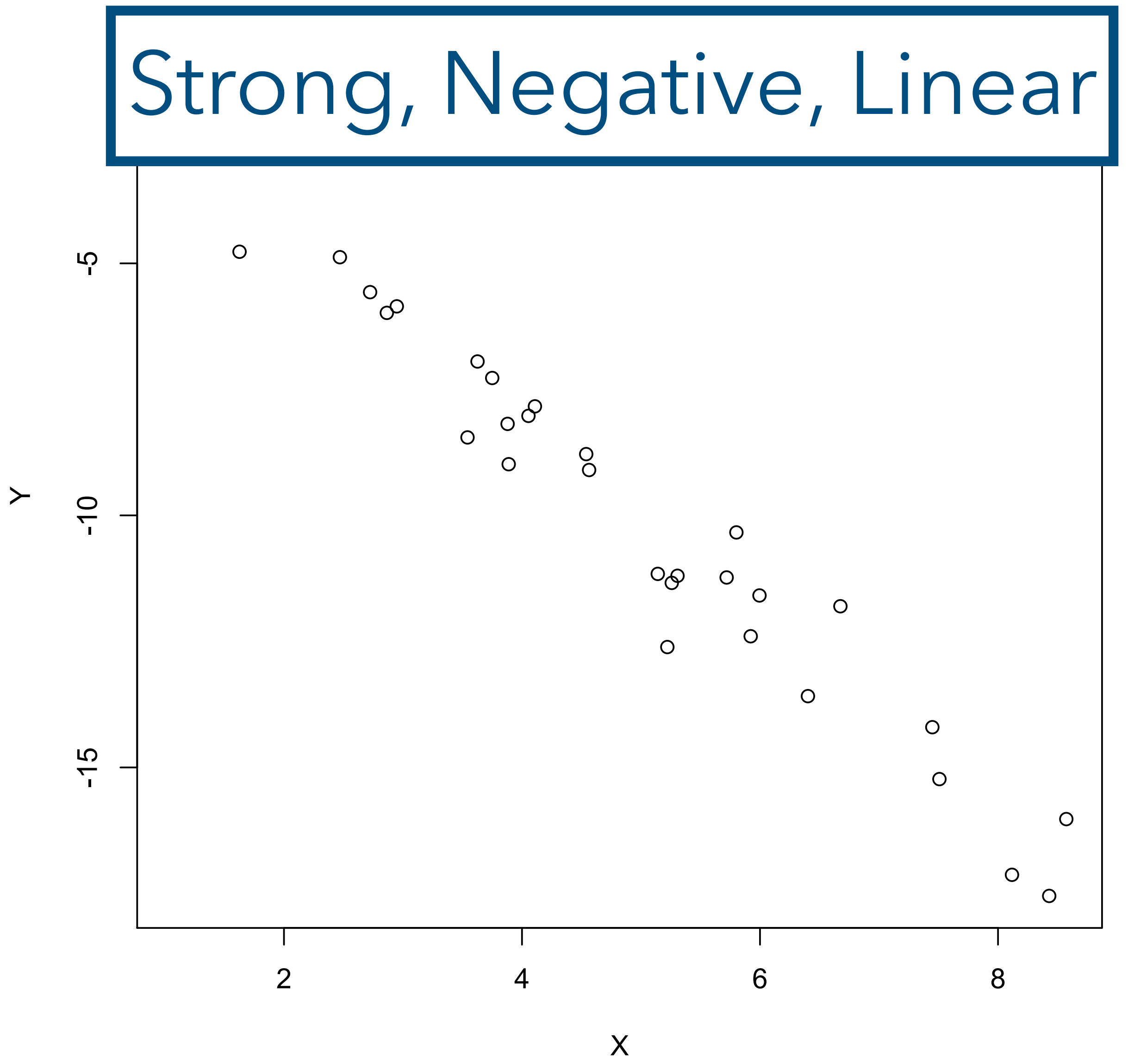

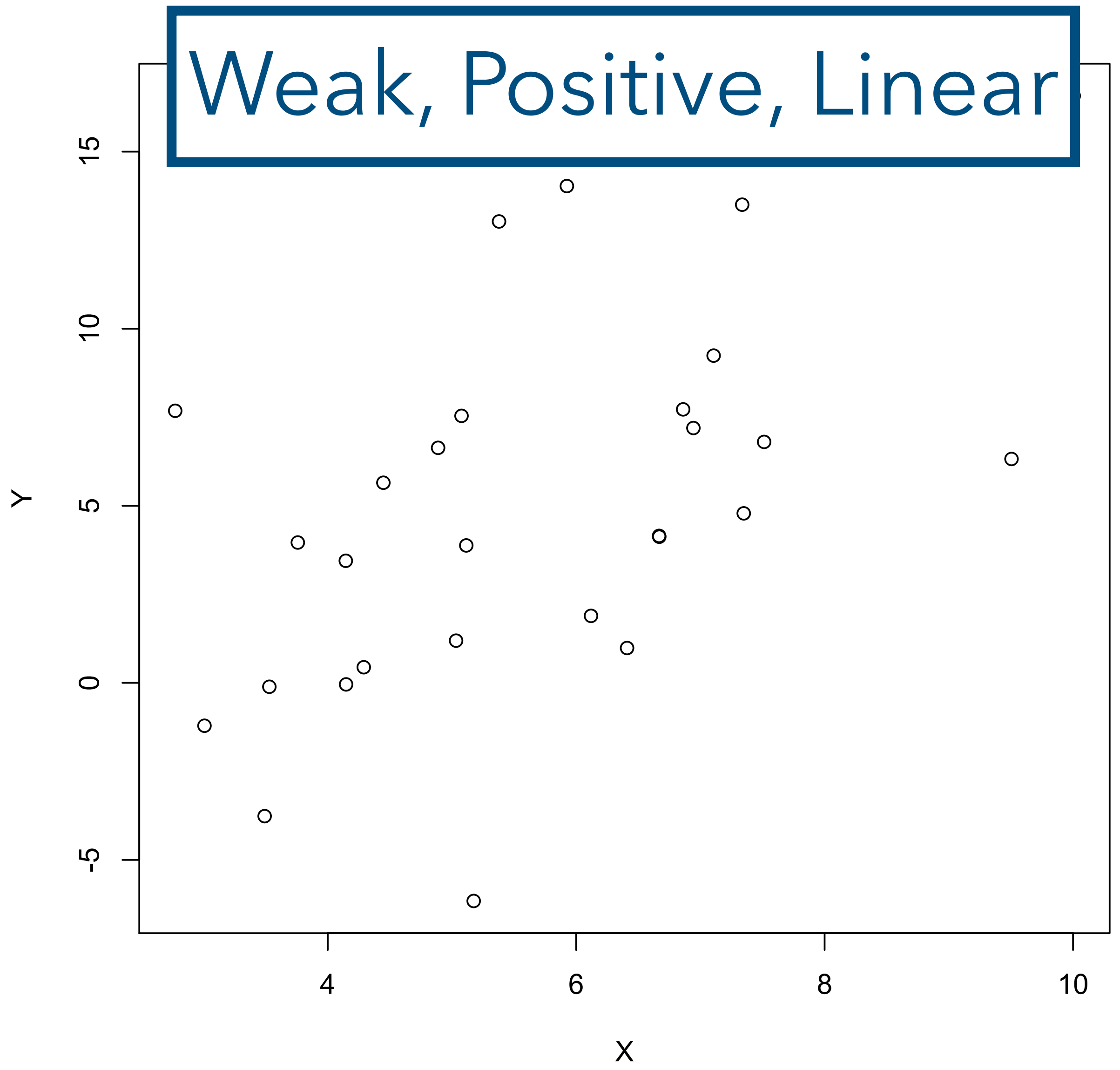

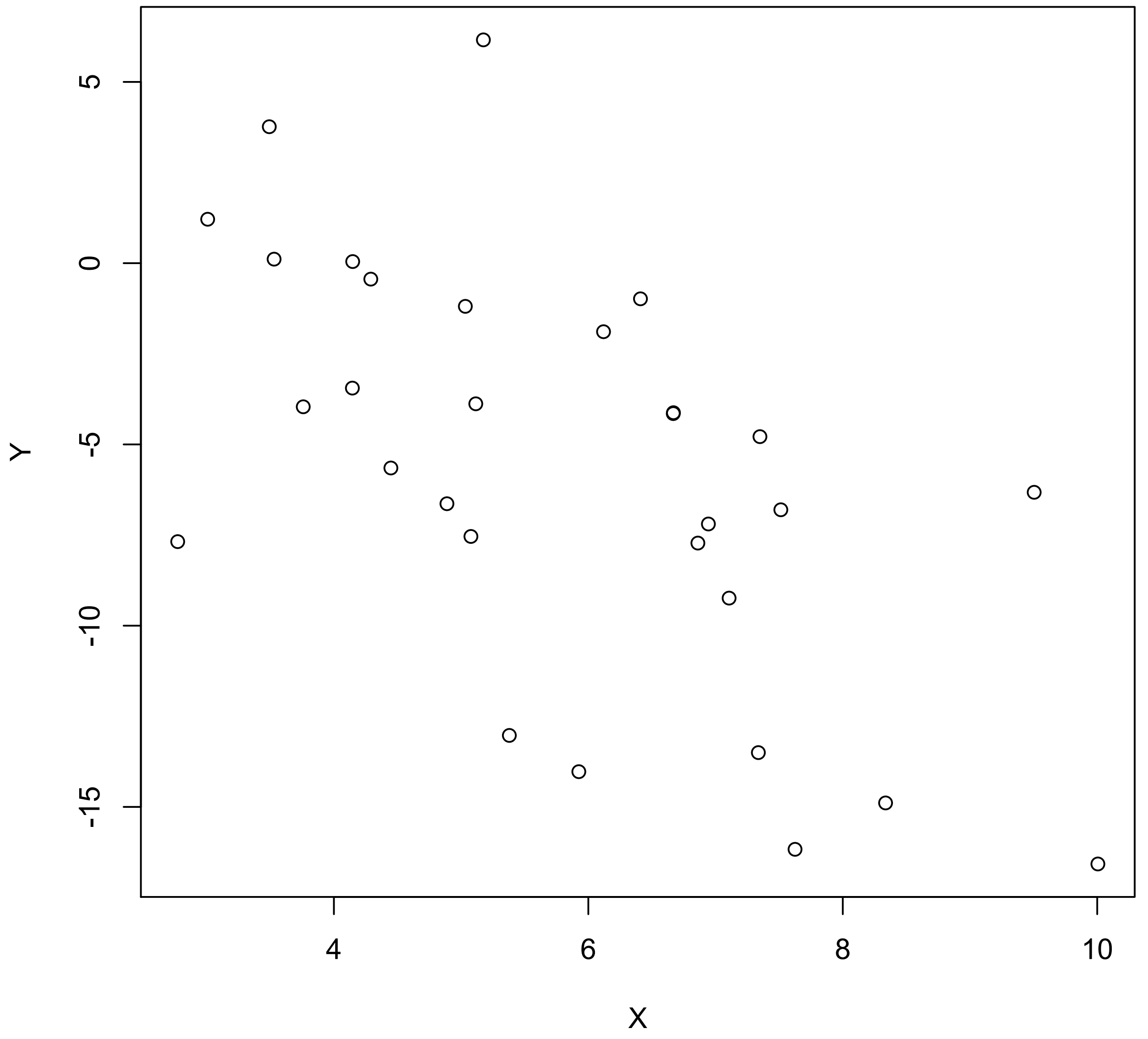

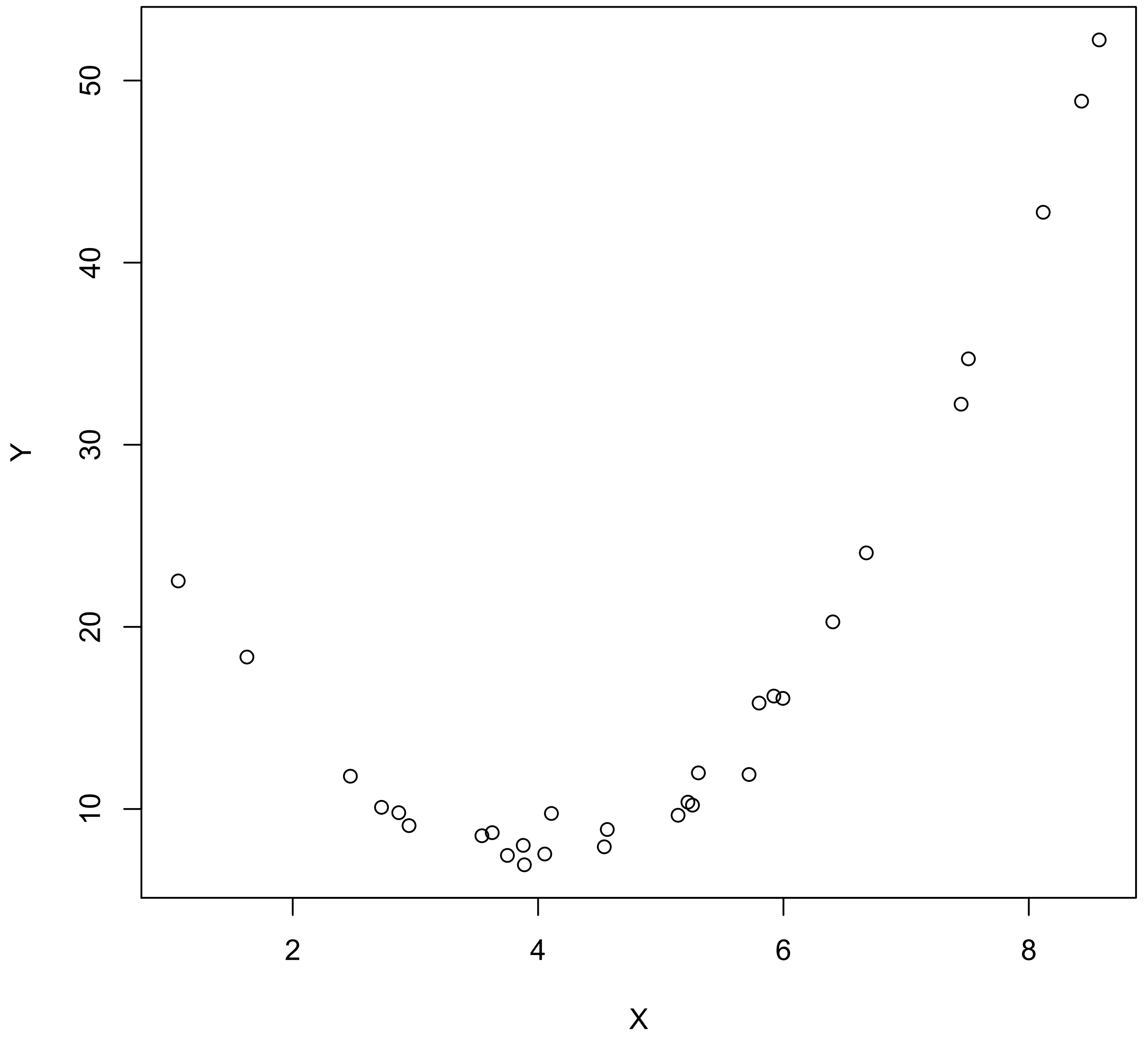

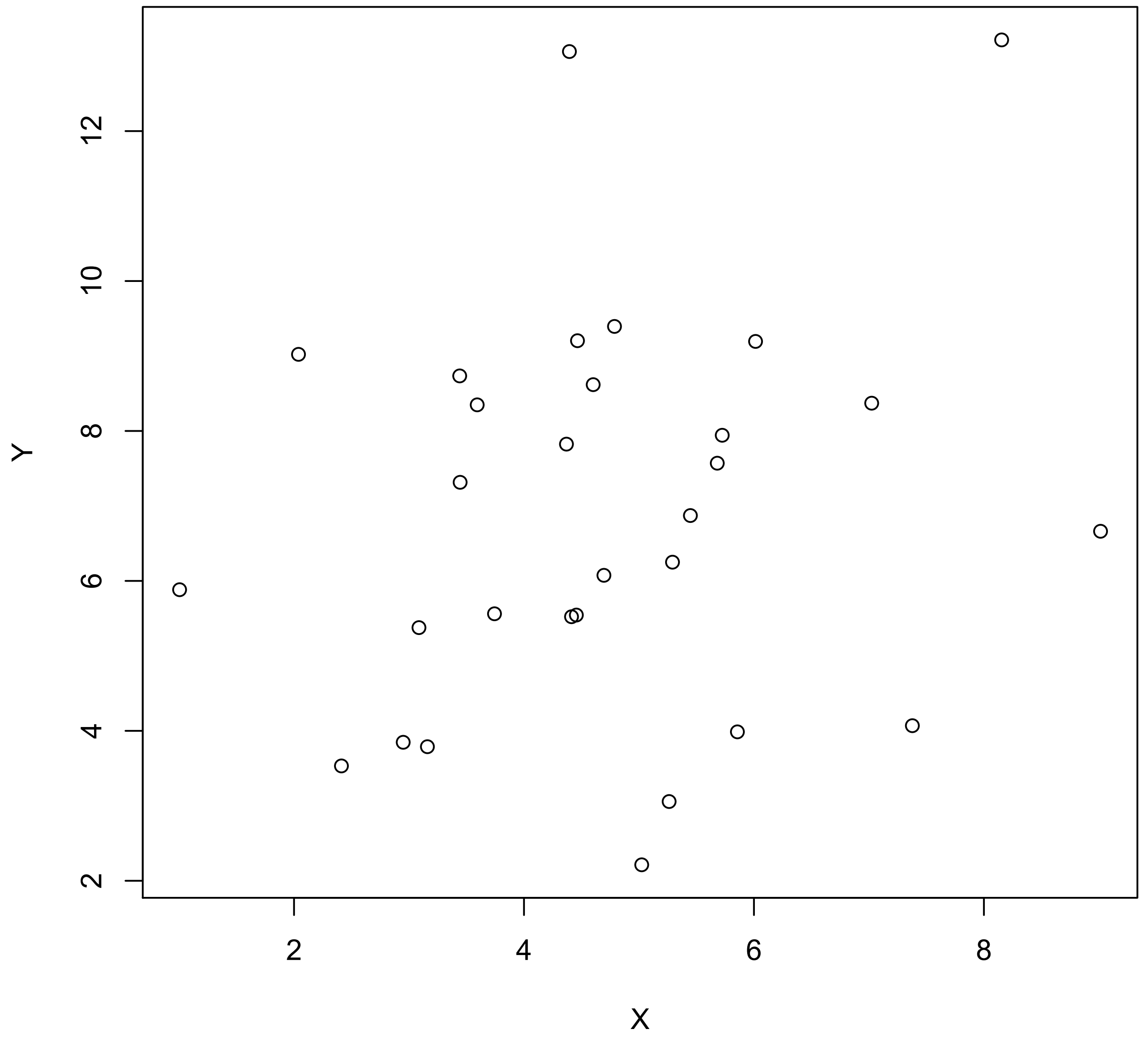

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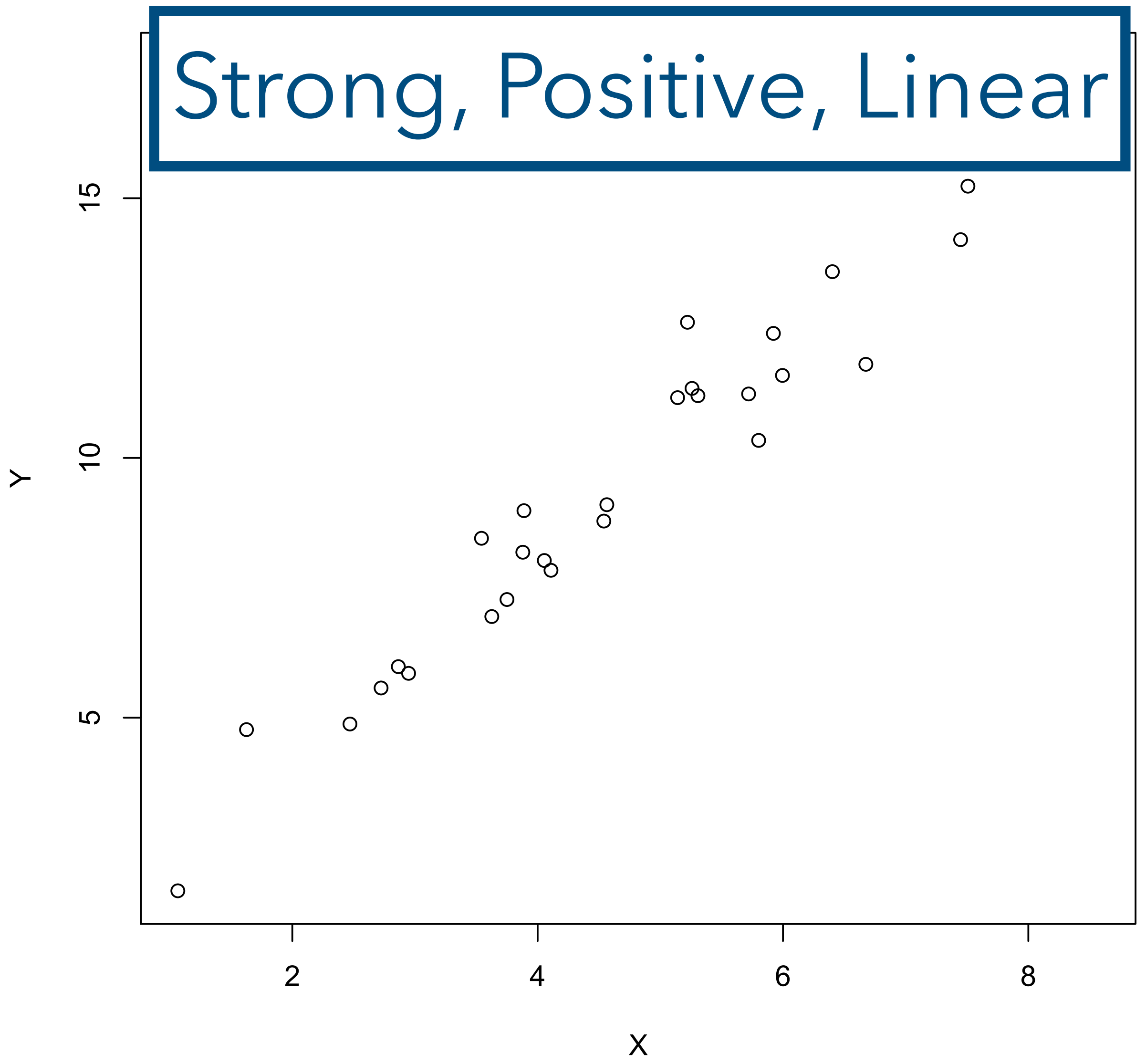

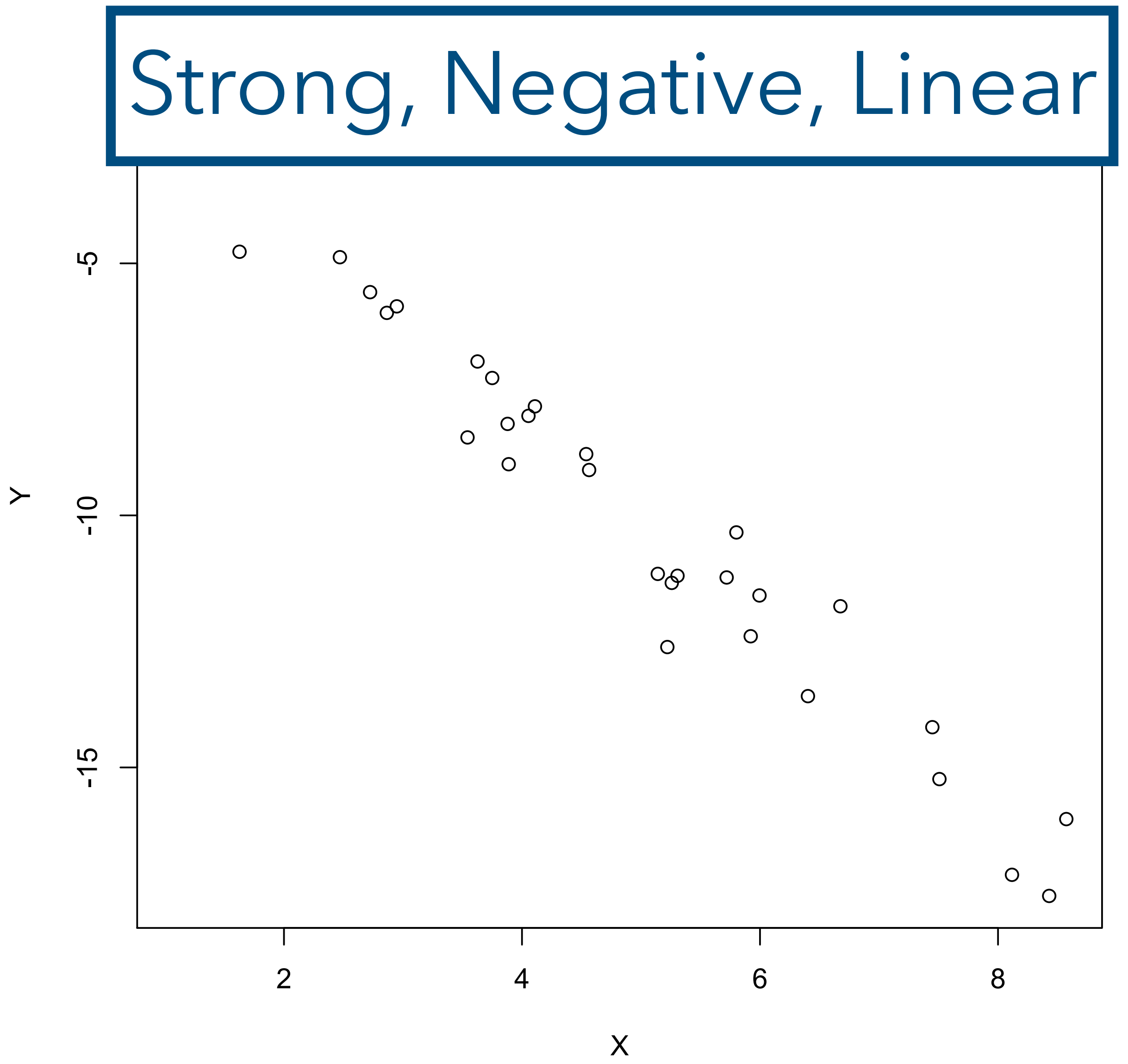

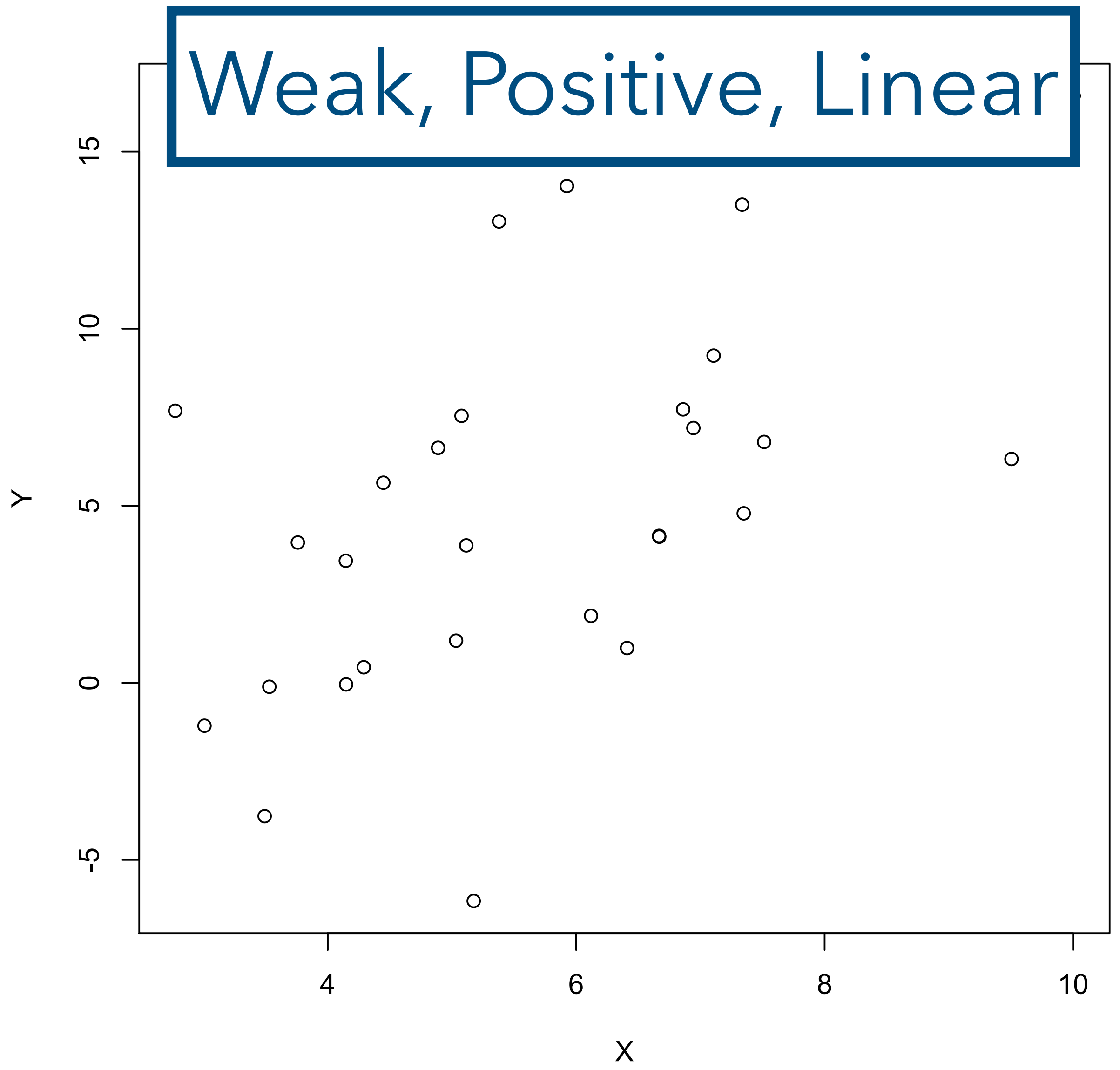

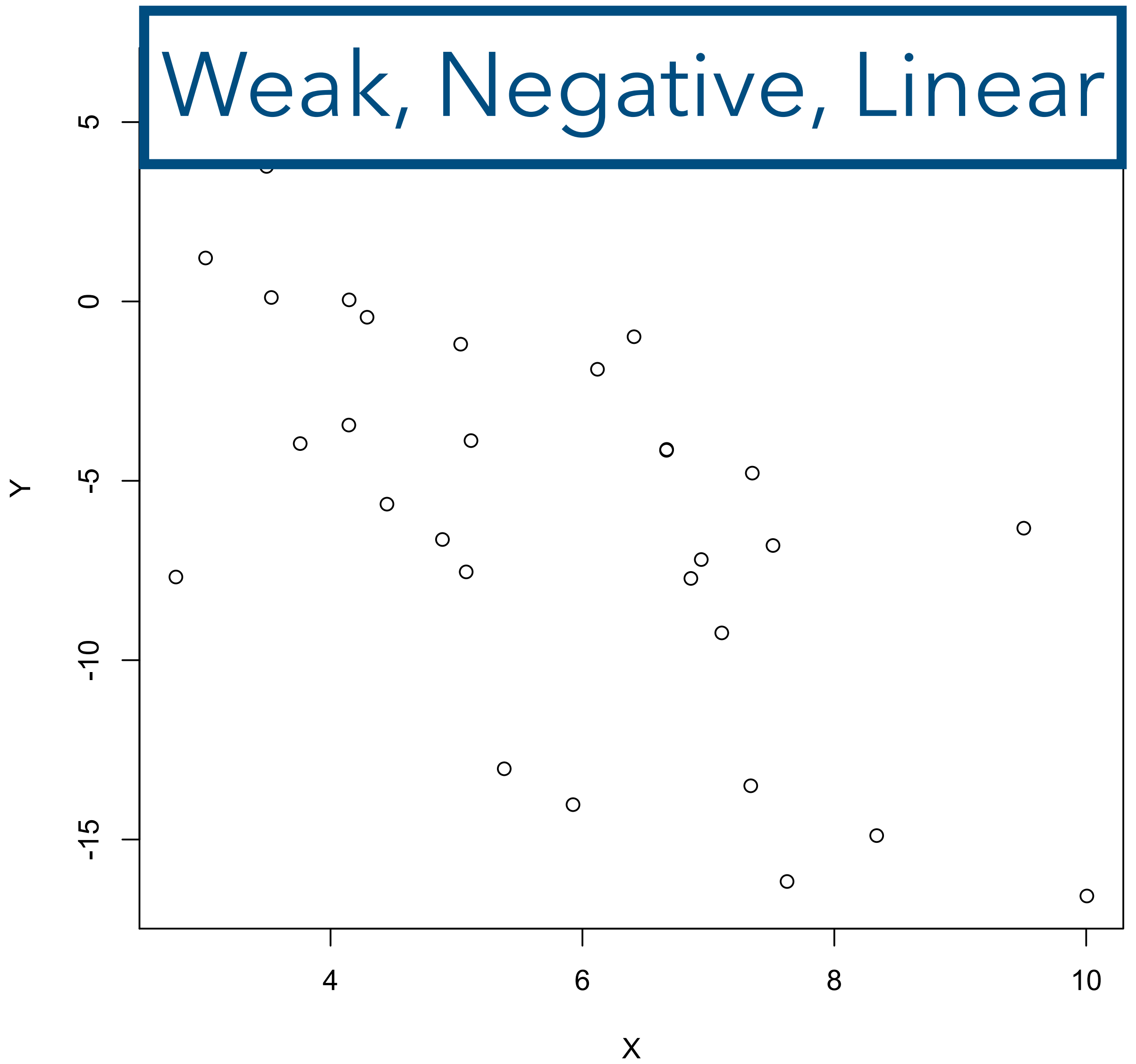

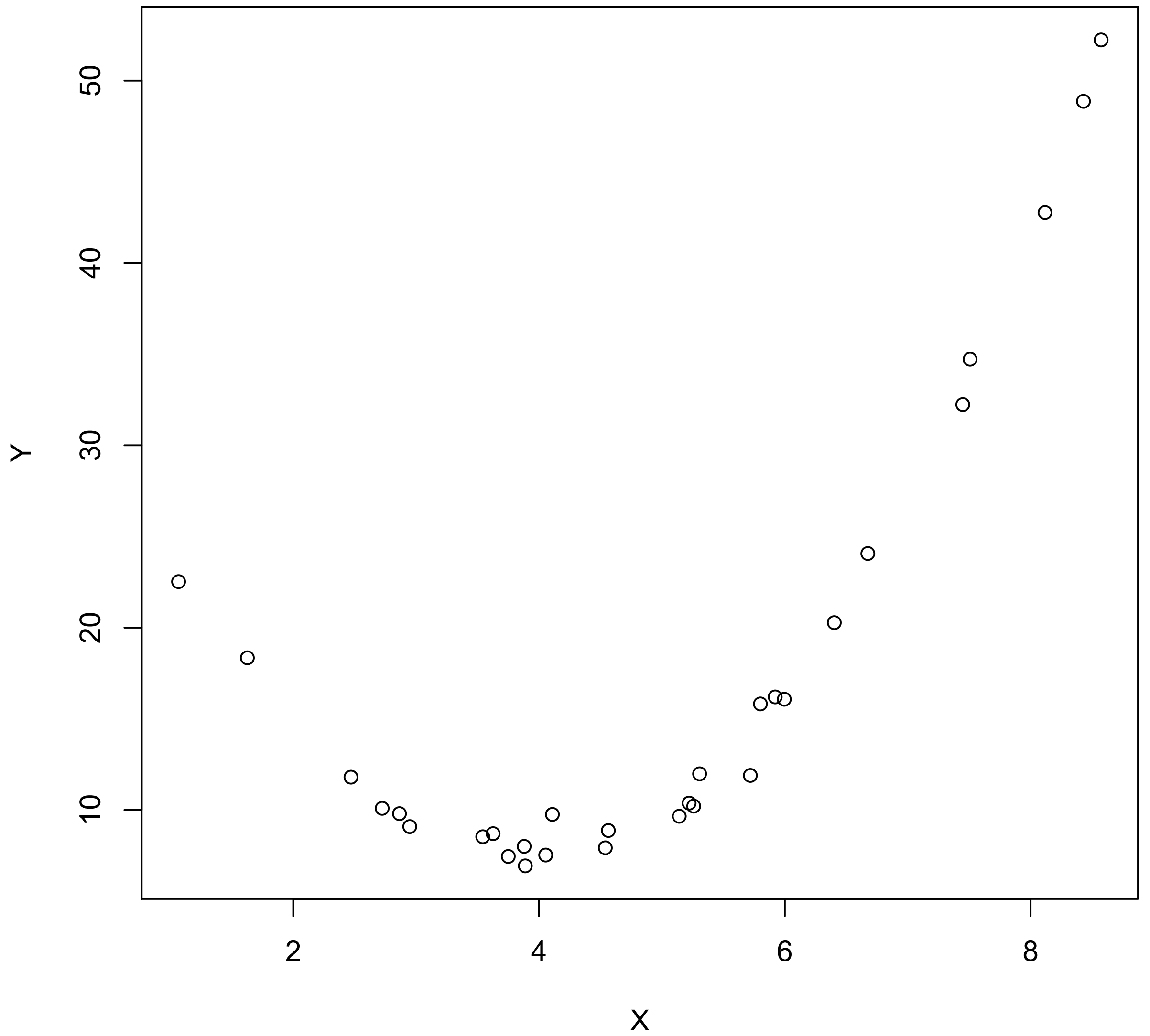

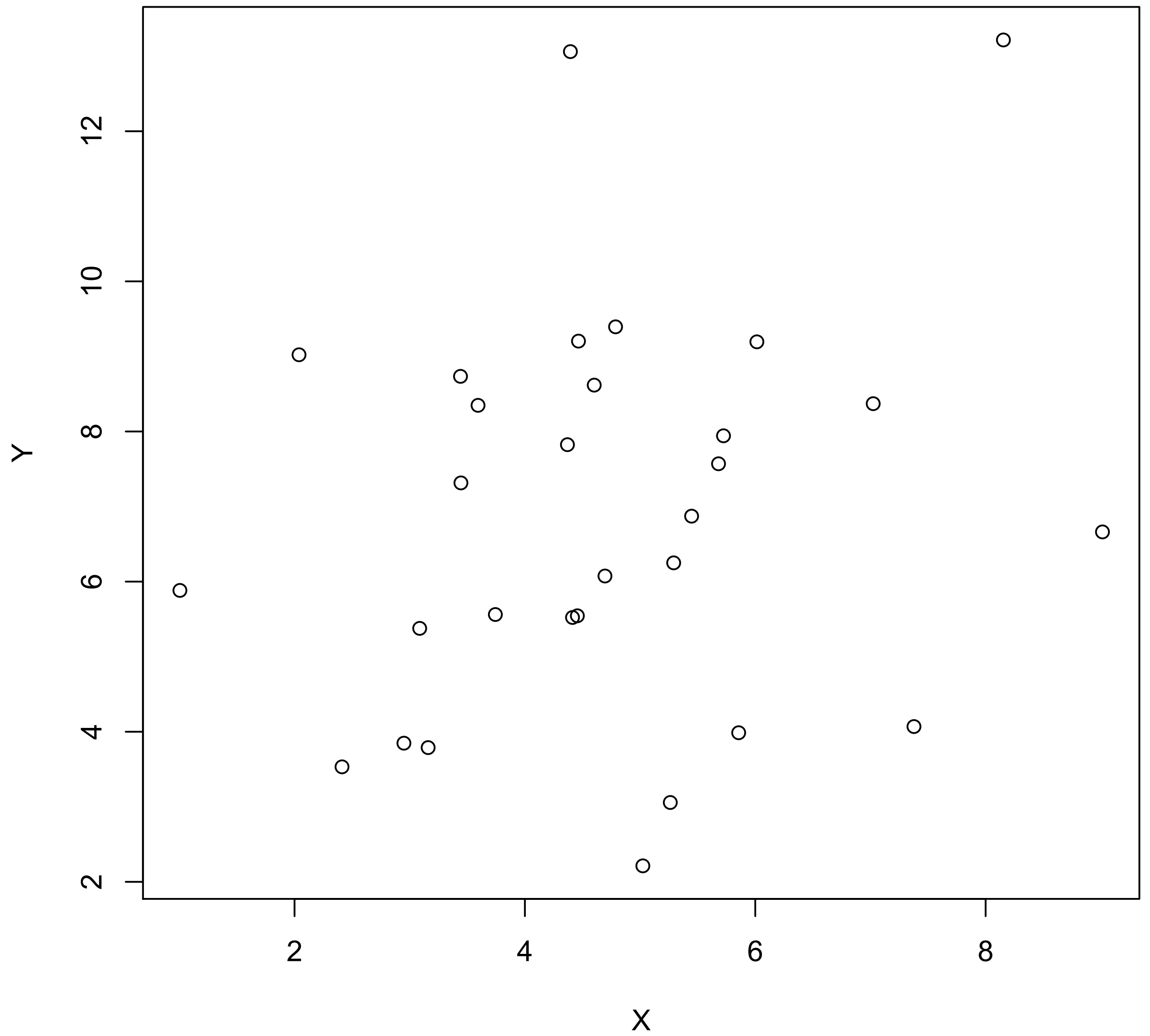

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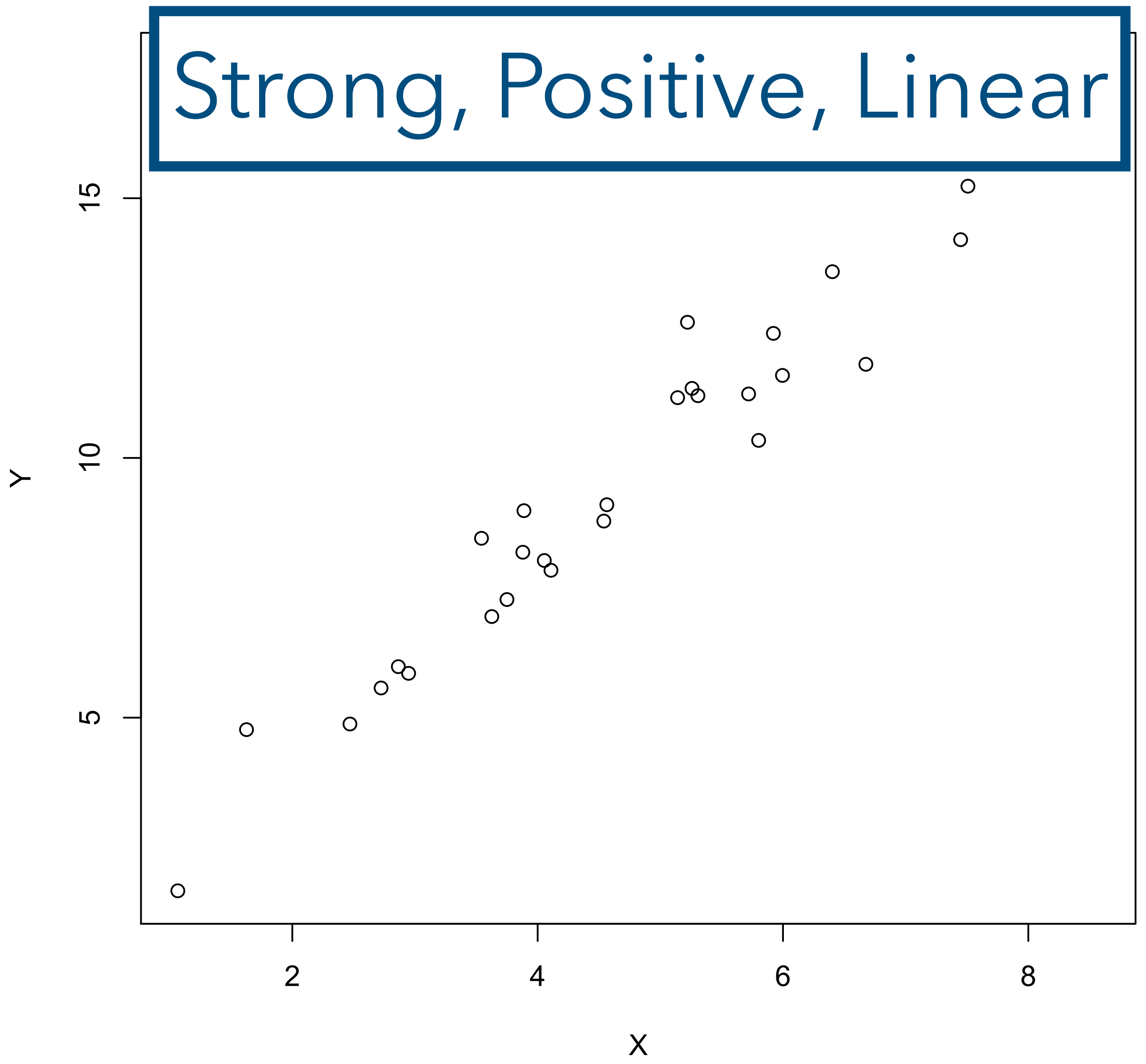

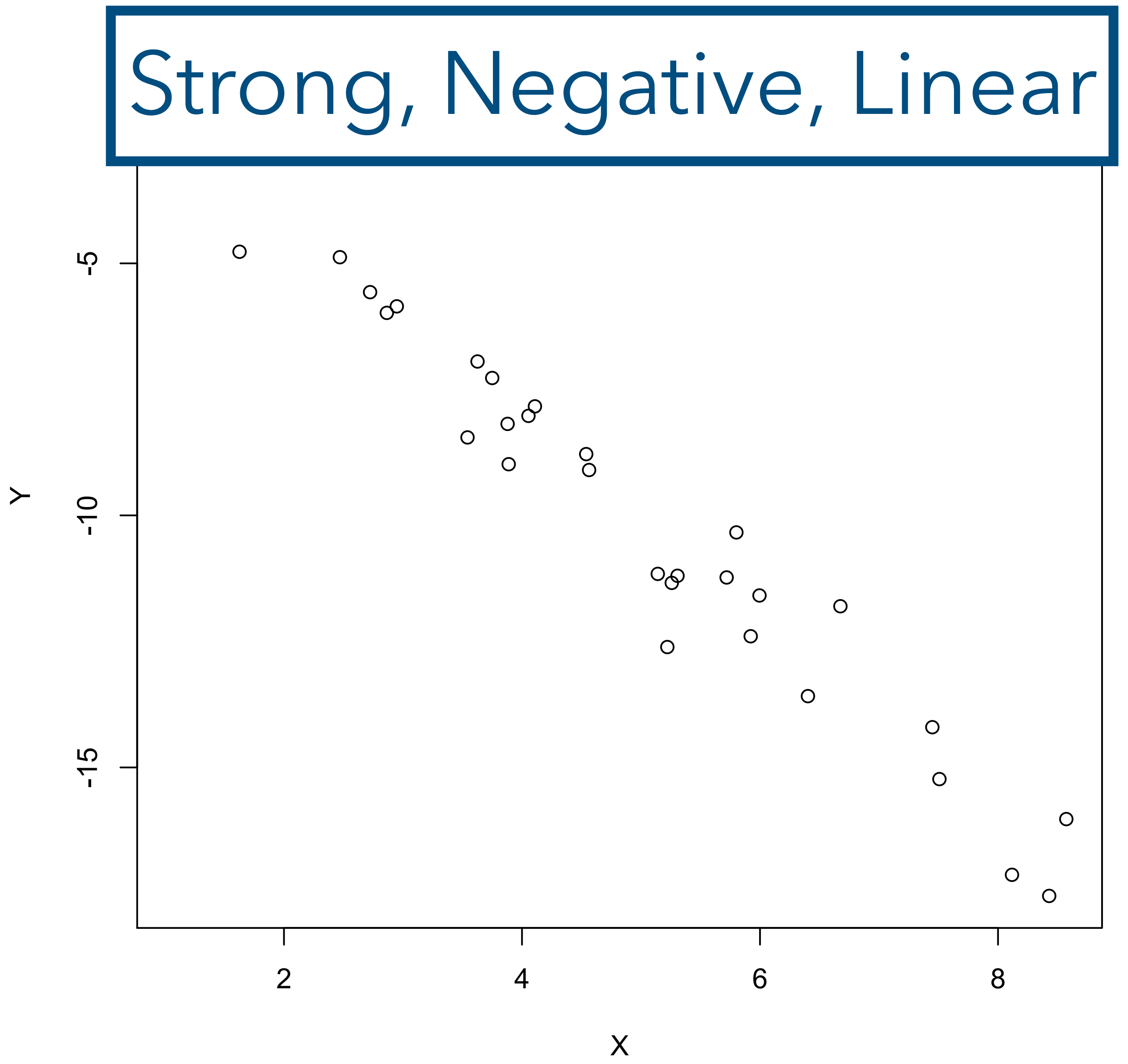

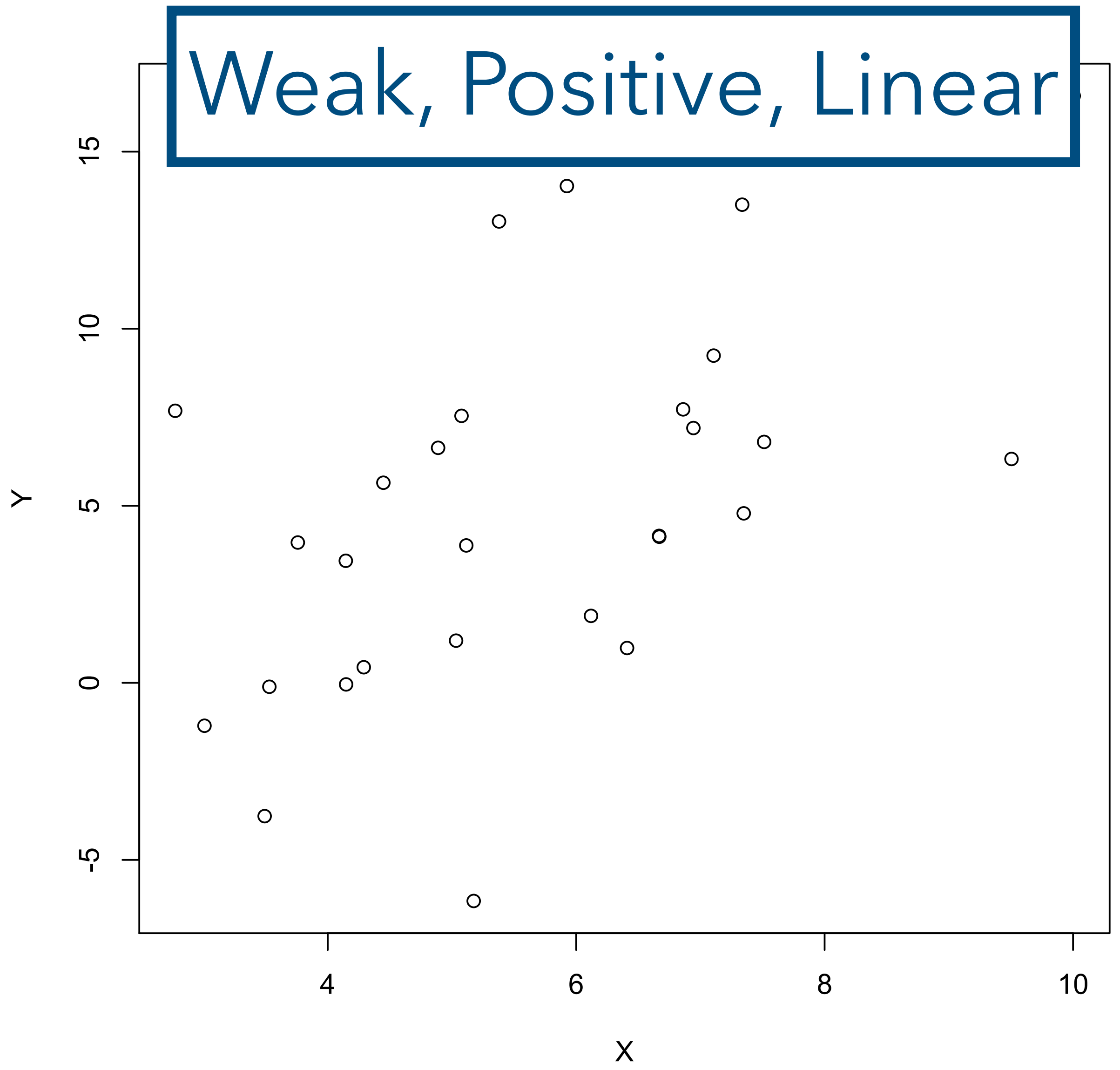

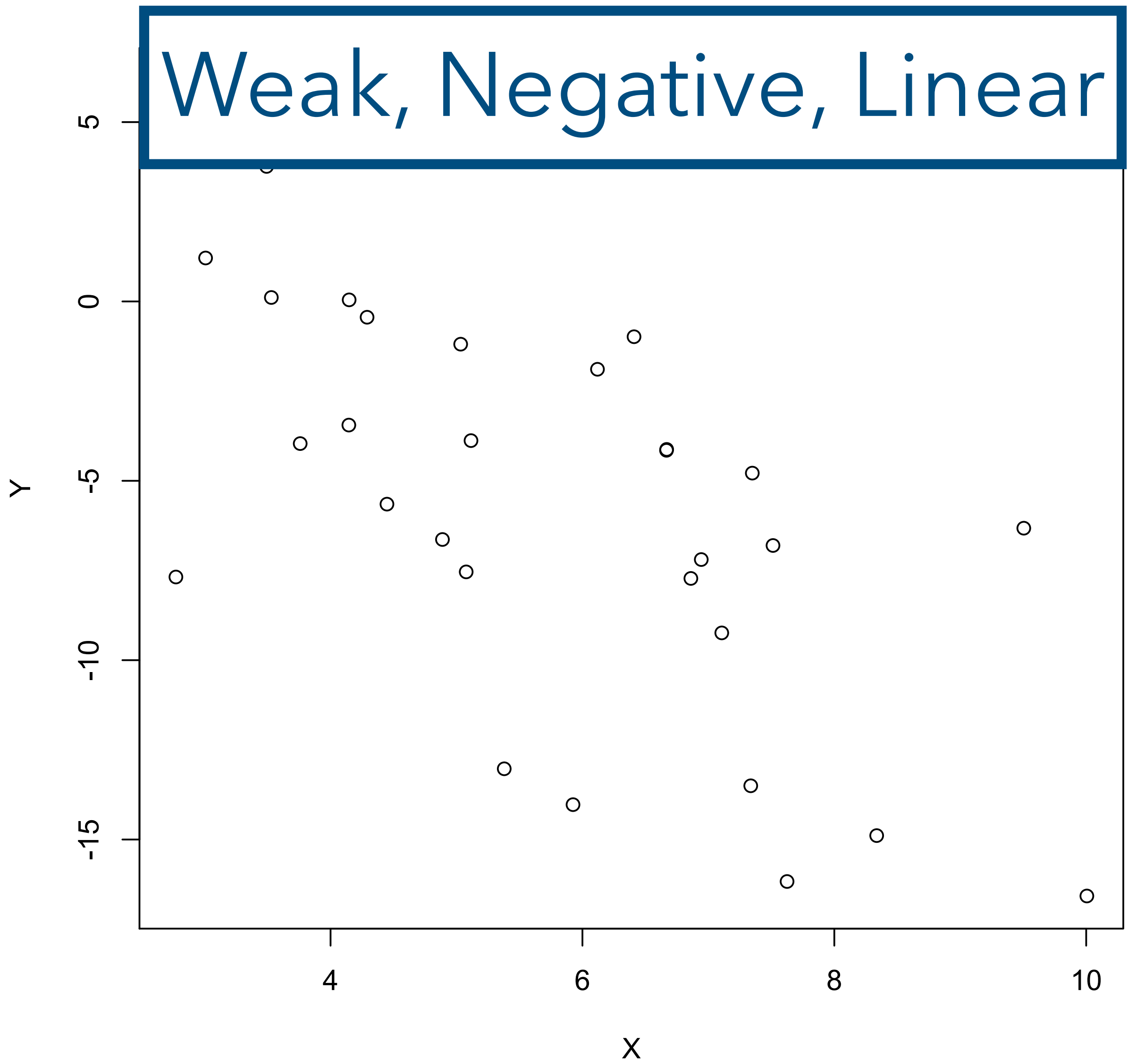

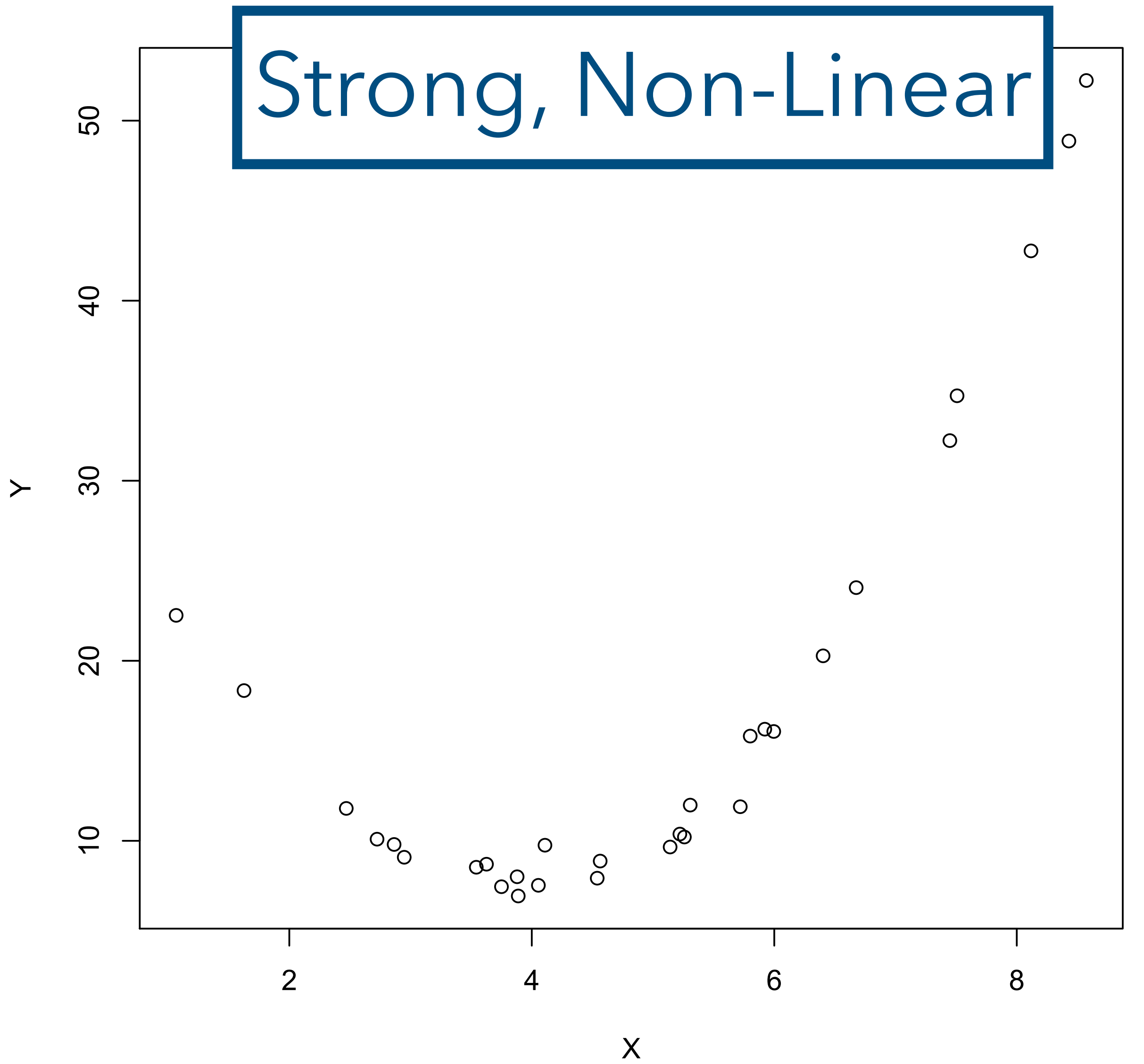

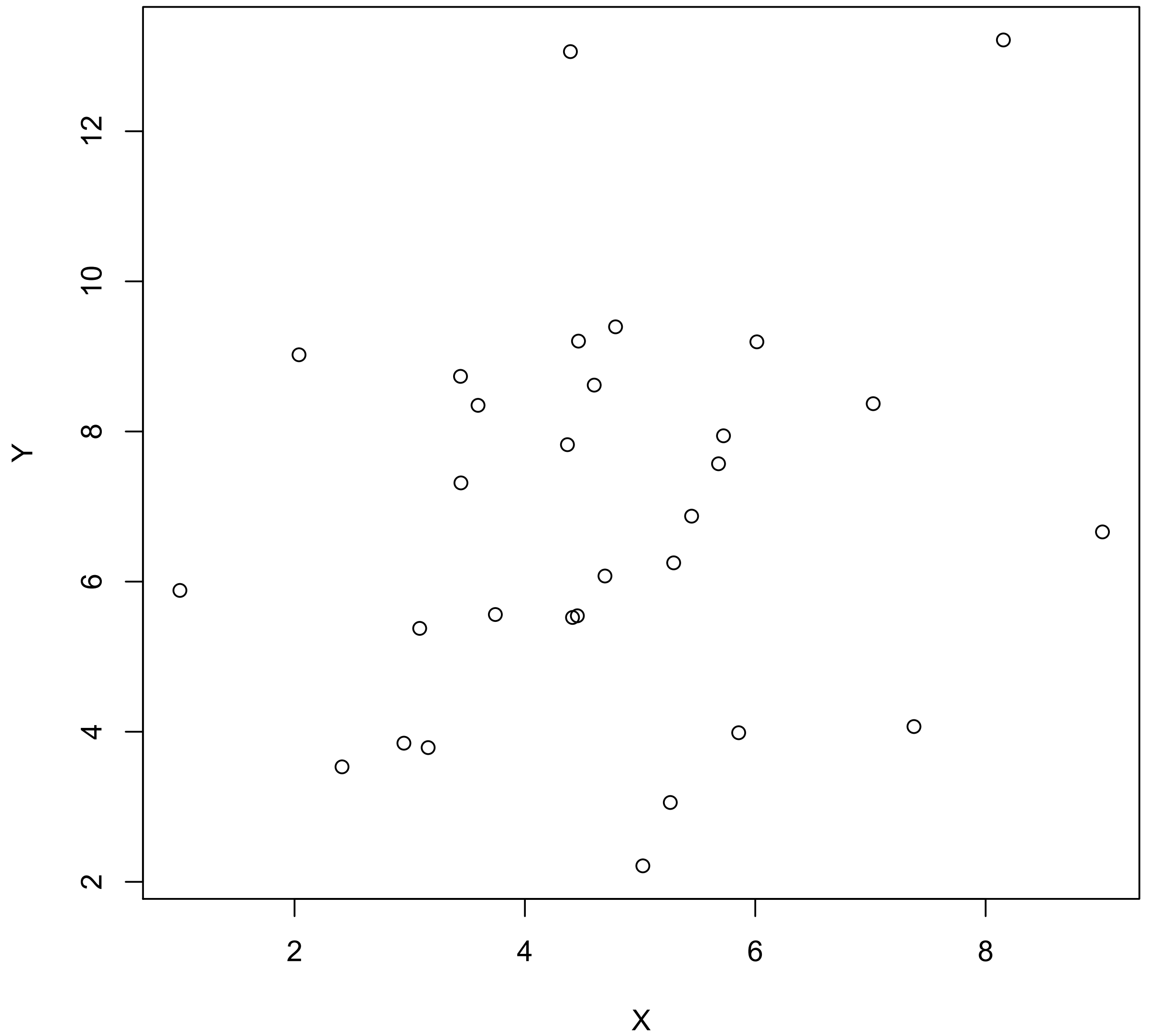

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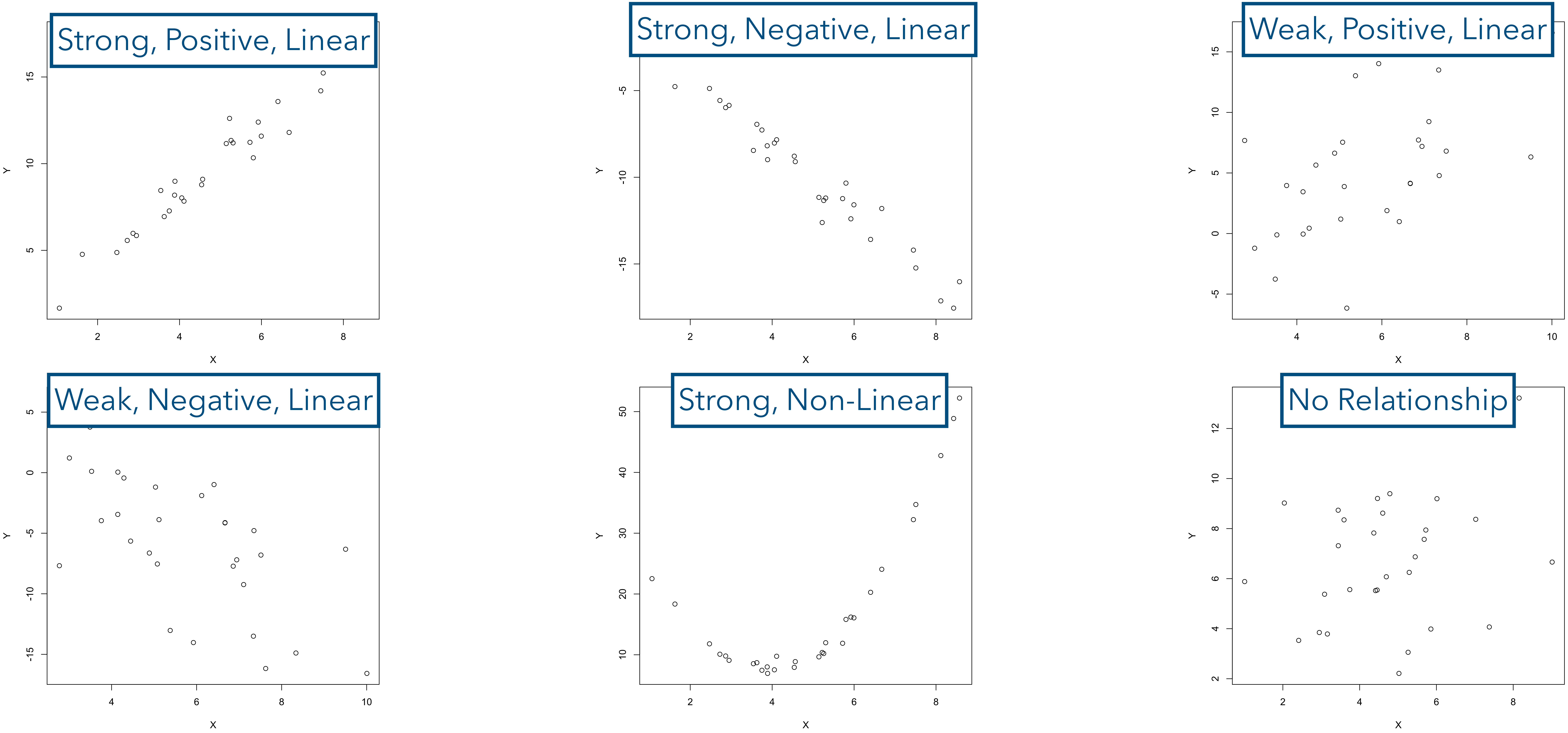

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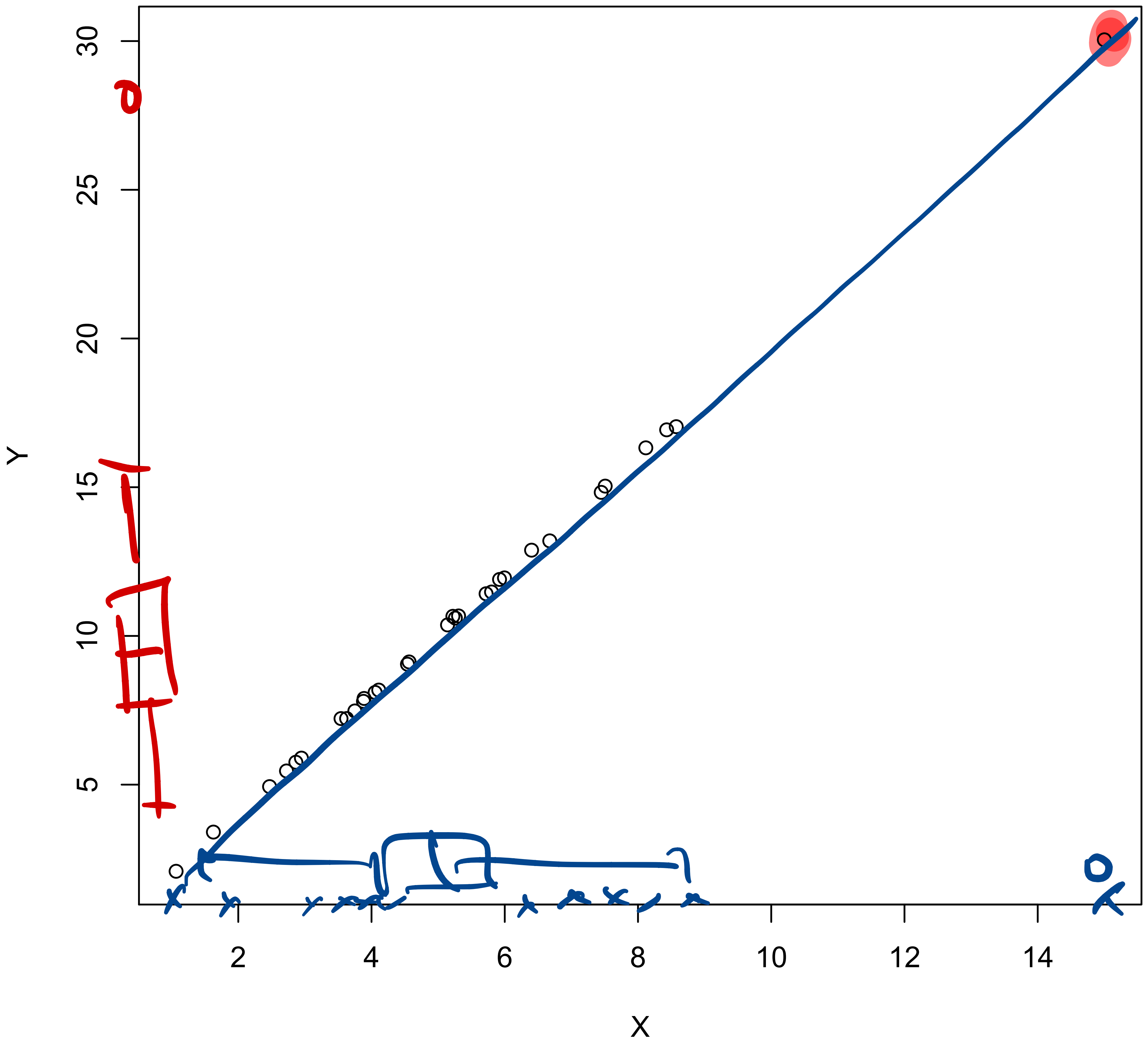
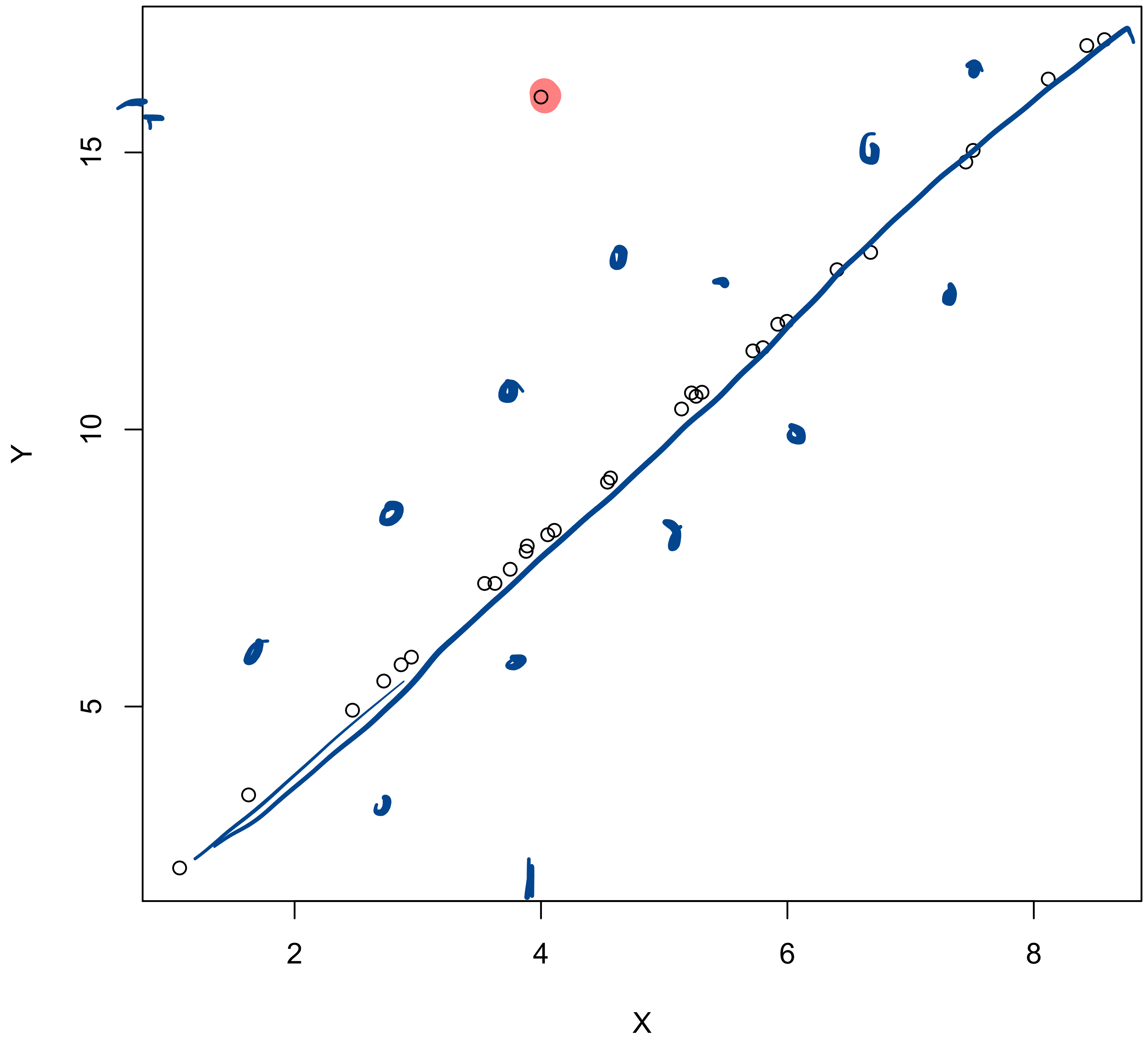
  - These points are outliers

Outliers: Examples





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- *Correlation* tells us the degree to which two random variables are (linearly) associated or related
- Setup: two quantitative variables, X and Y; X is on the horizontal axis of the scatterplot and Y is plotted on the vertical axis

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$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\boxed{\left[\sum_{i=1}^{n} (x_i - \bar{x})^2\right]} \boxed{\left[\sum_{i=1}^{n} (y_i - \bar{y})^2\right]}}}$$

$\underline{S_{xy}}$

$\sim Var(X)$ $\quad$ $\sim Var(Y)$

S_{xx} $\quad$ S_{yy}

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$$r = \frac{\sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\left[\sum_{i=1}^{n} (x_i - \overline{x})^2\right] \left[\sum_{i=1}^{n} (y_i - \overline{y})^2\right]}}$$

- A quantity related to the correlation is the *sample covariance:*

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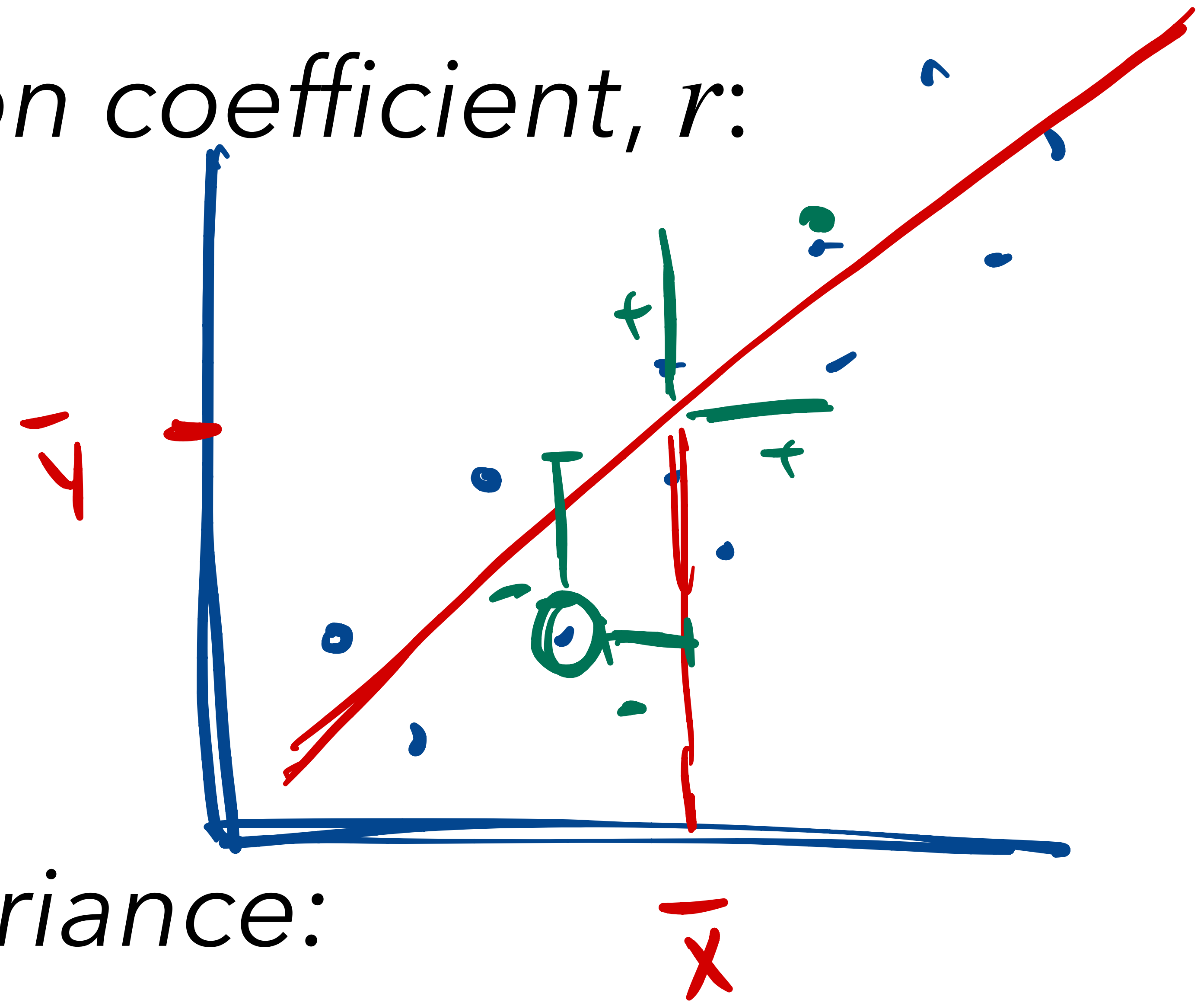

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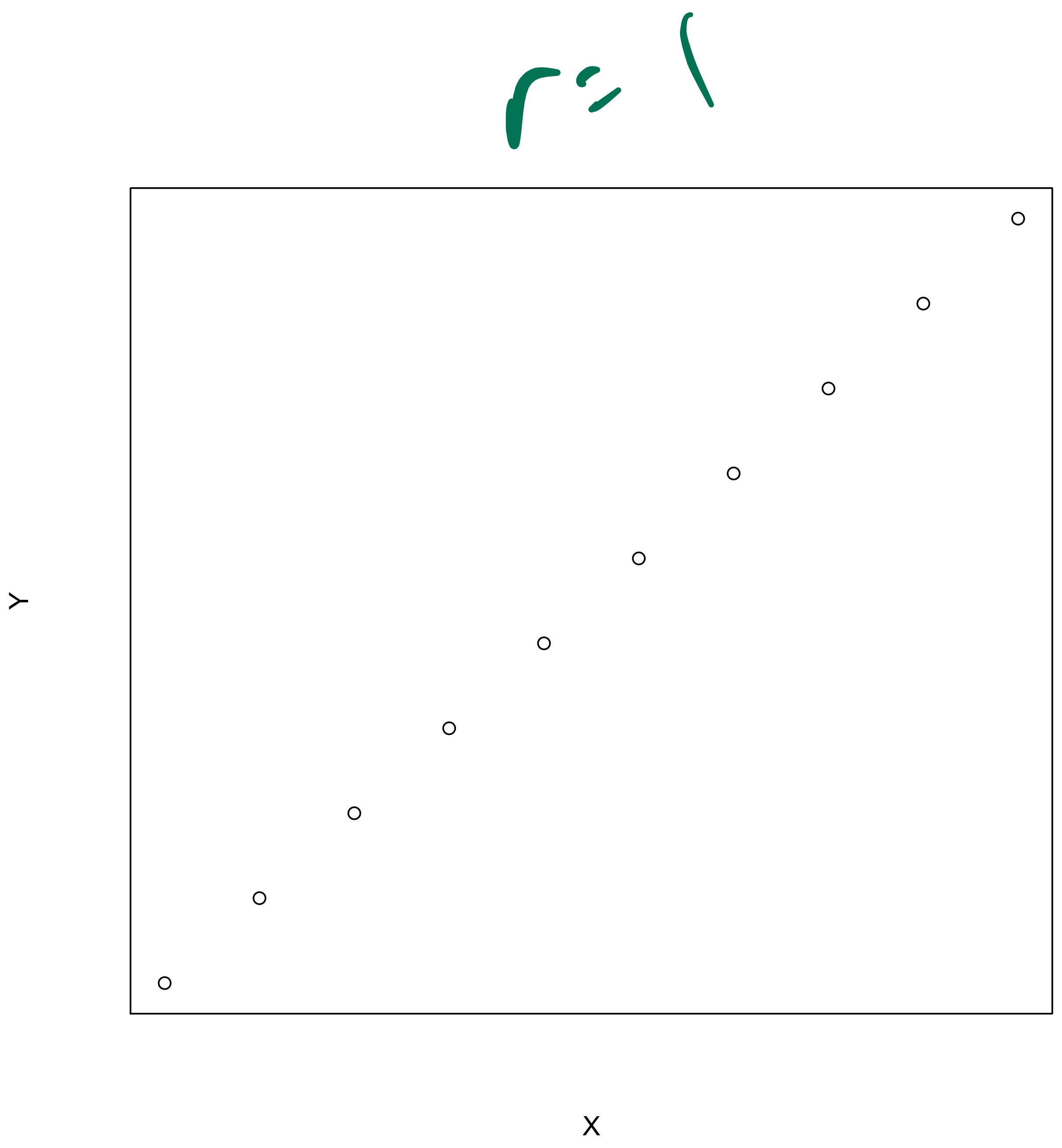
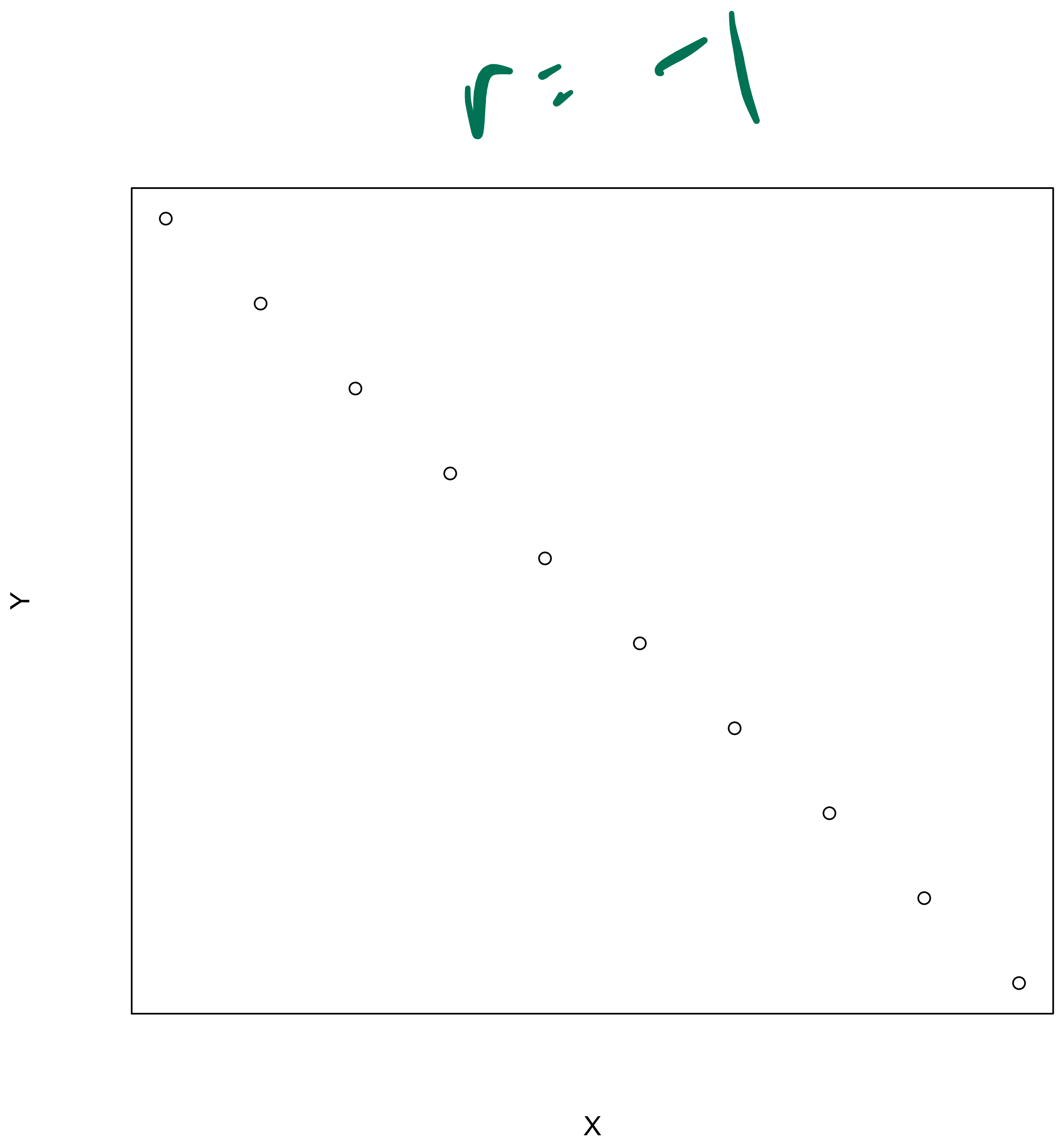
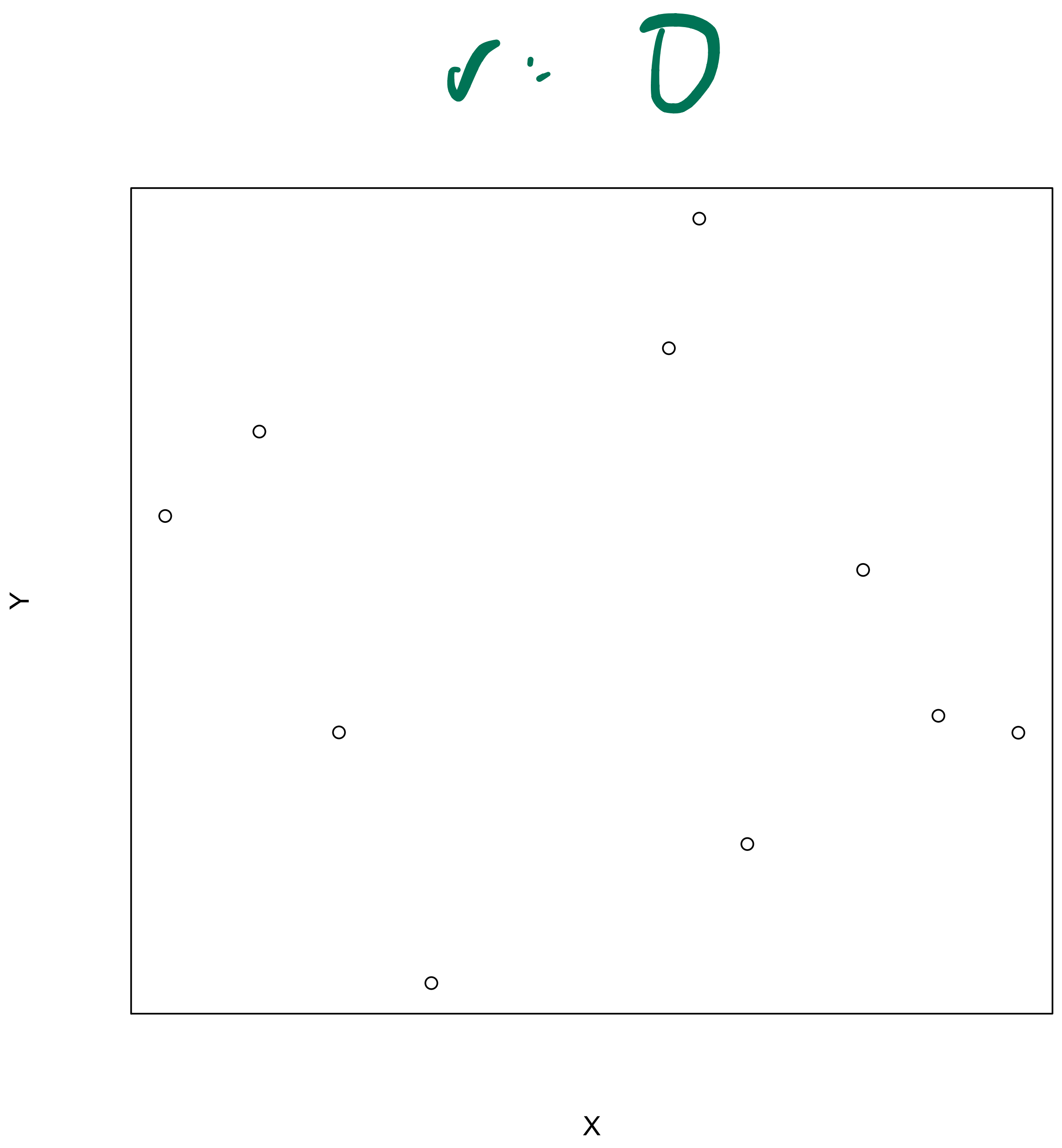
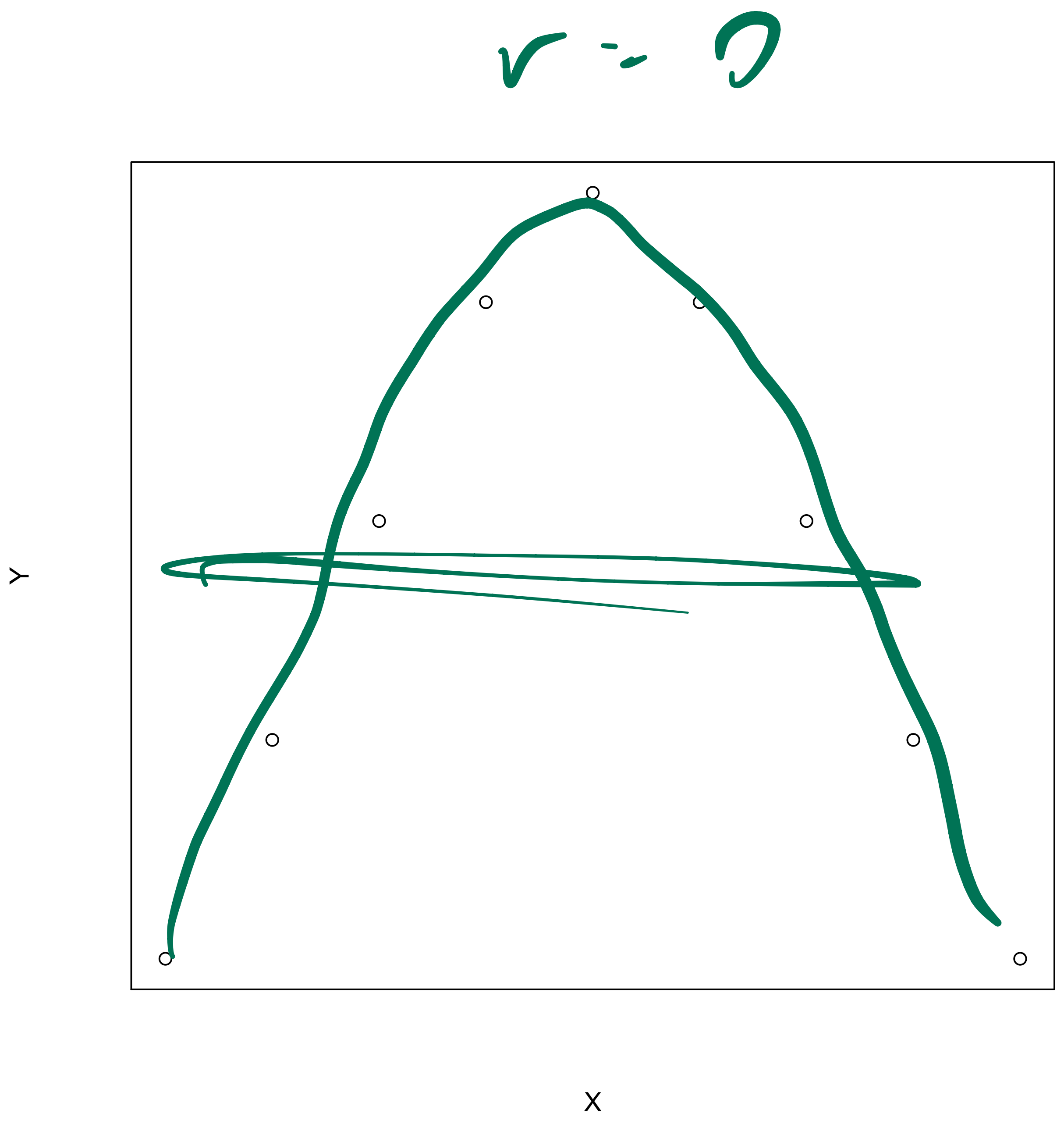
Pearson's Correlation Coefficient: Examples









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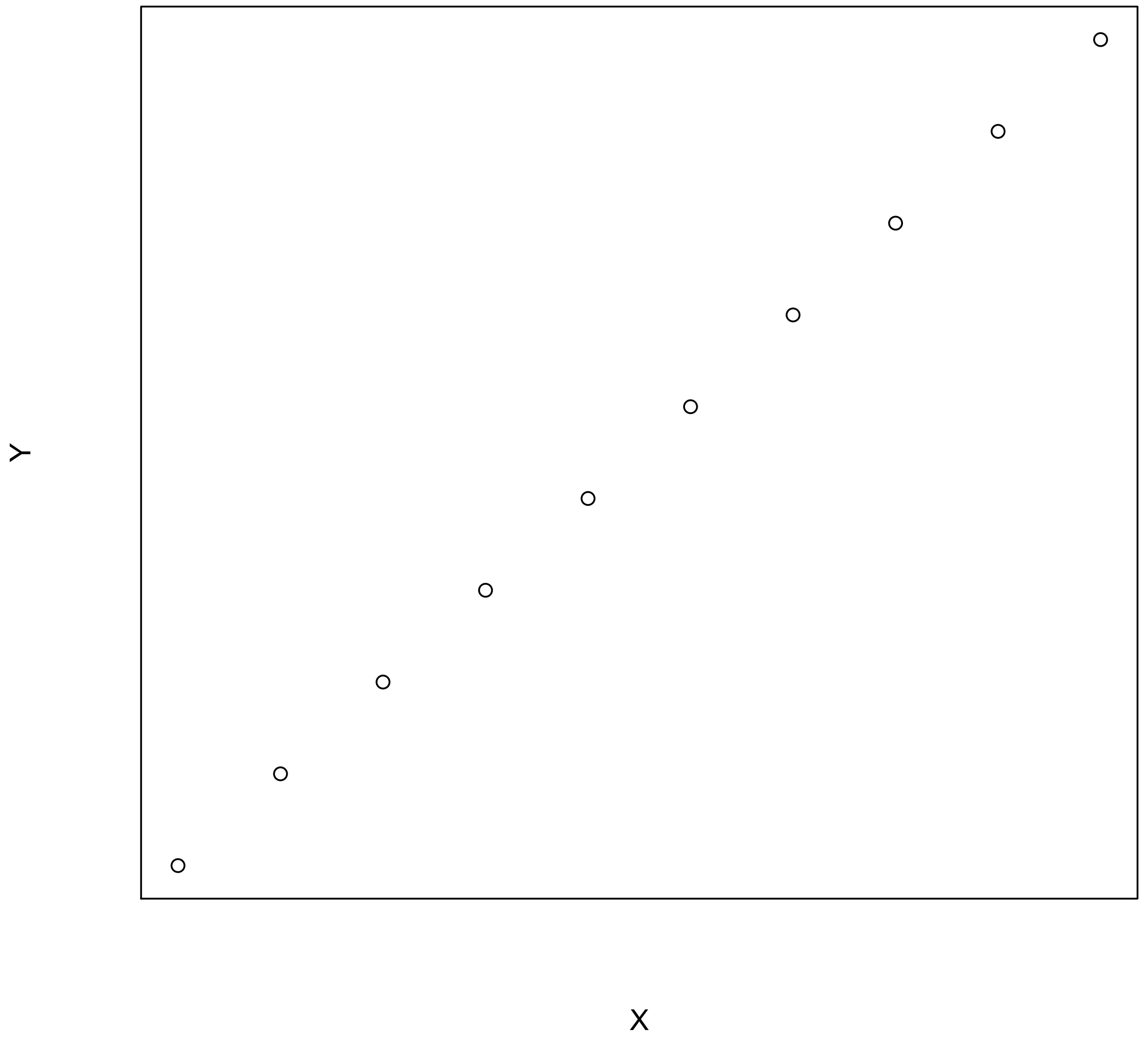

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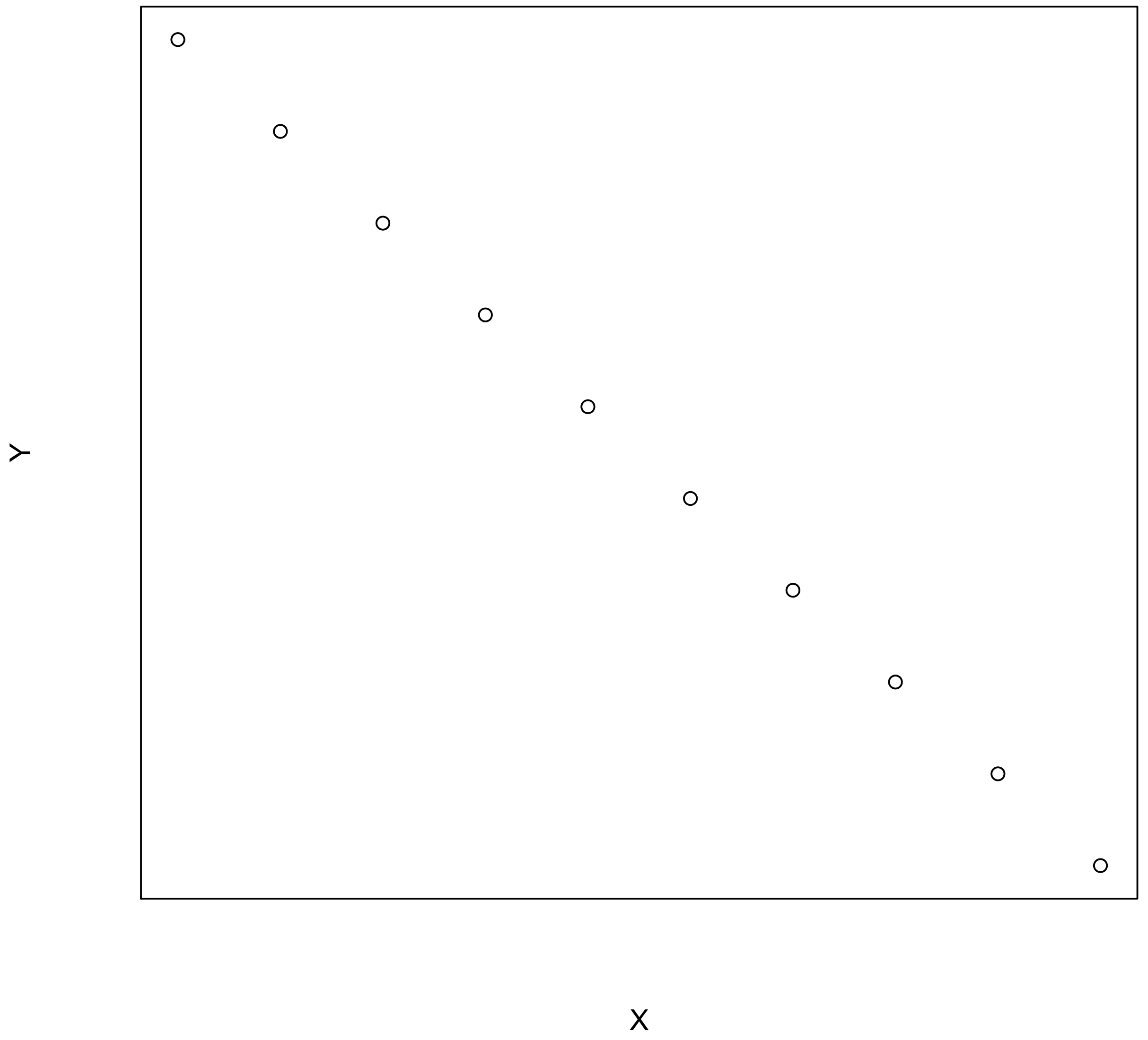

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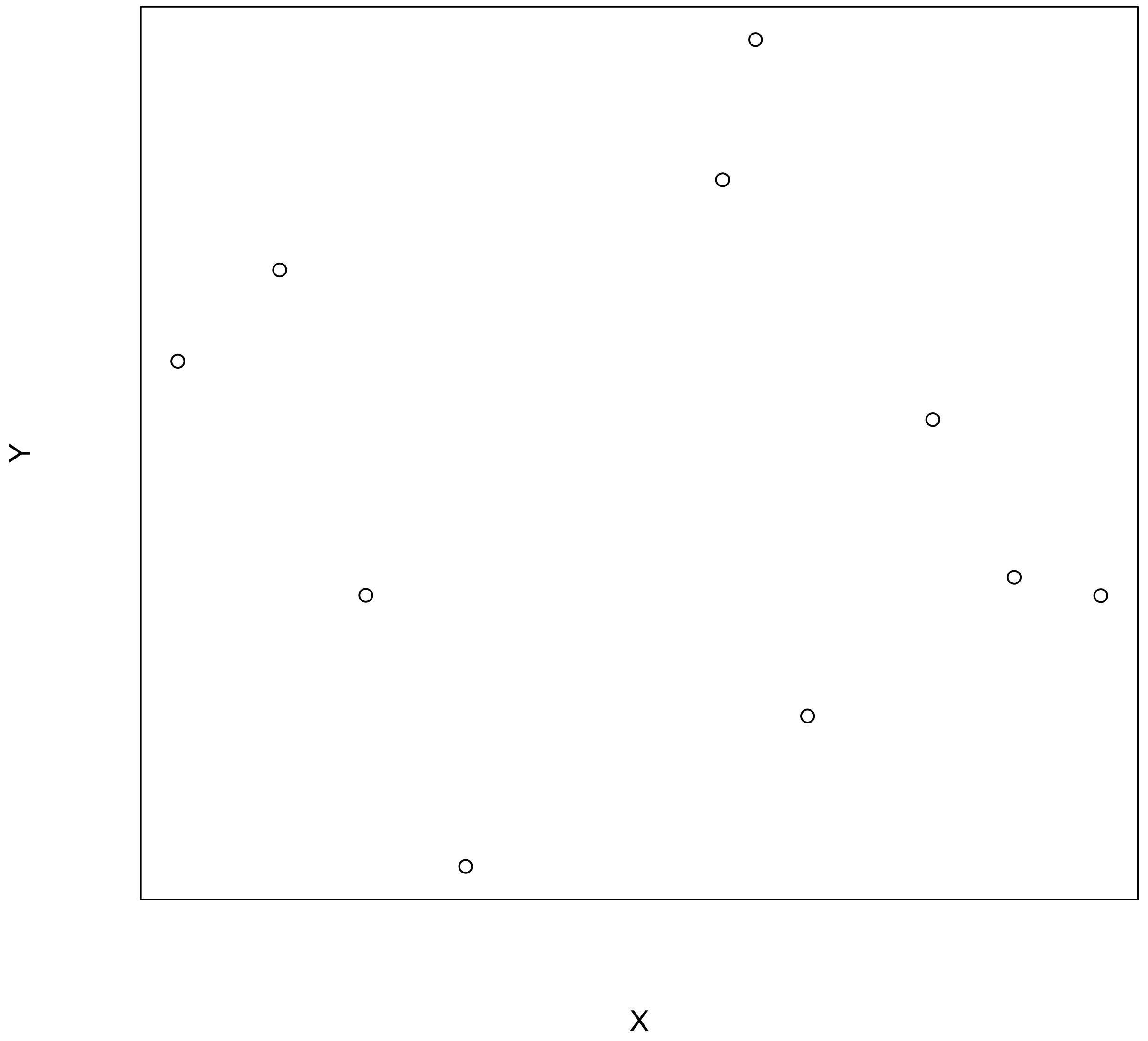

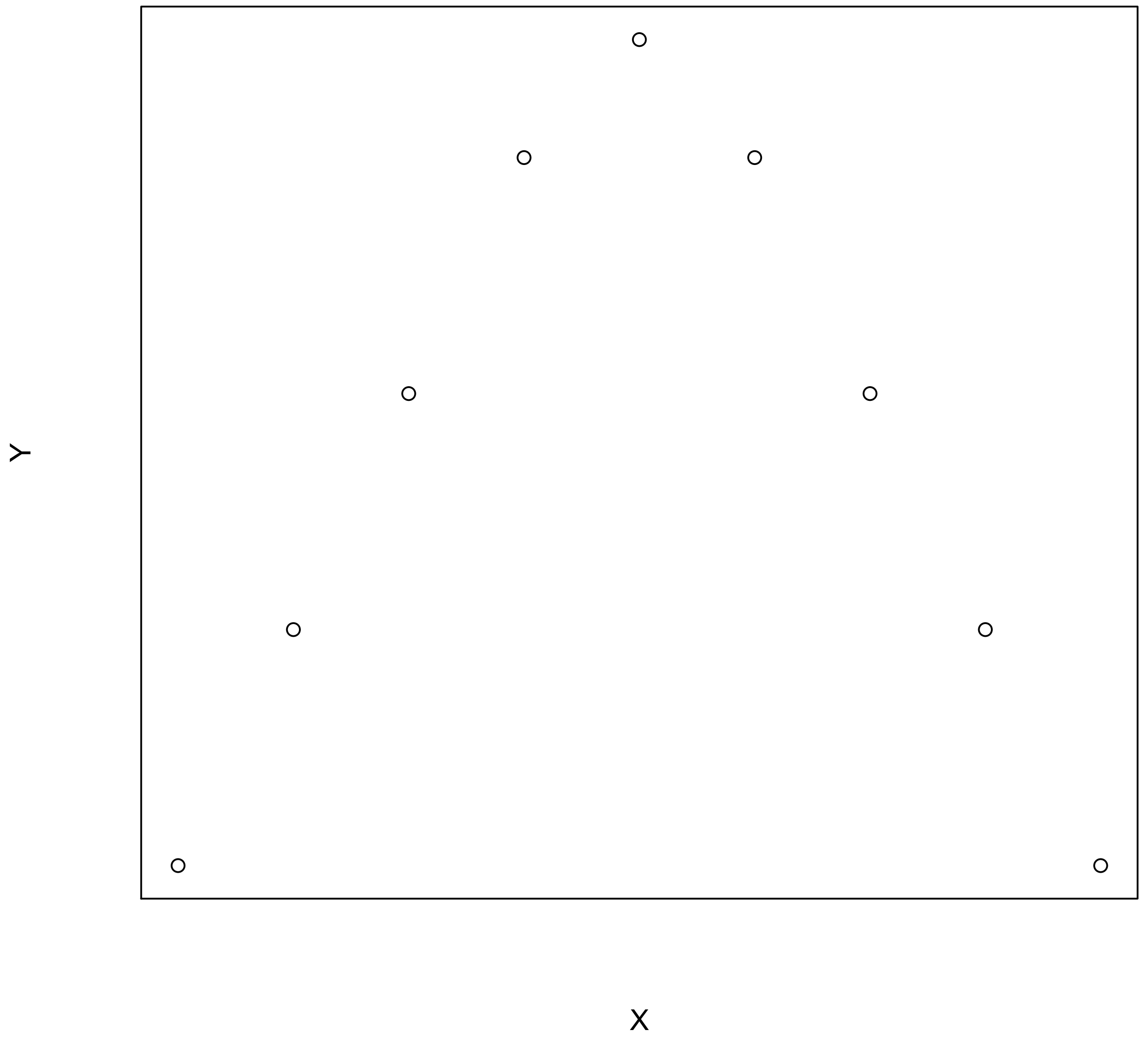

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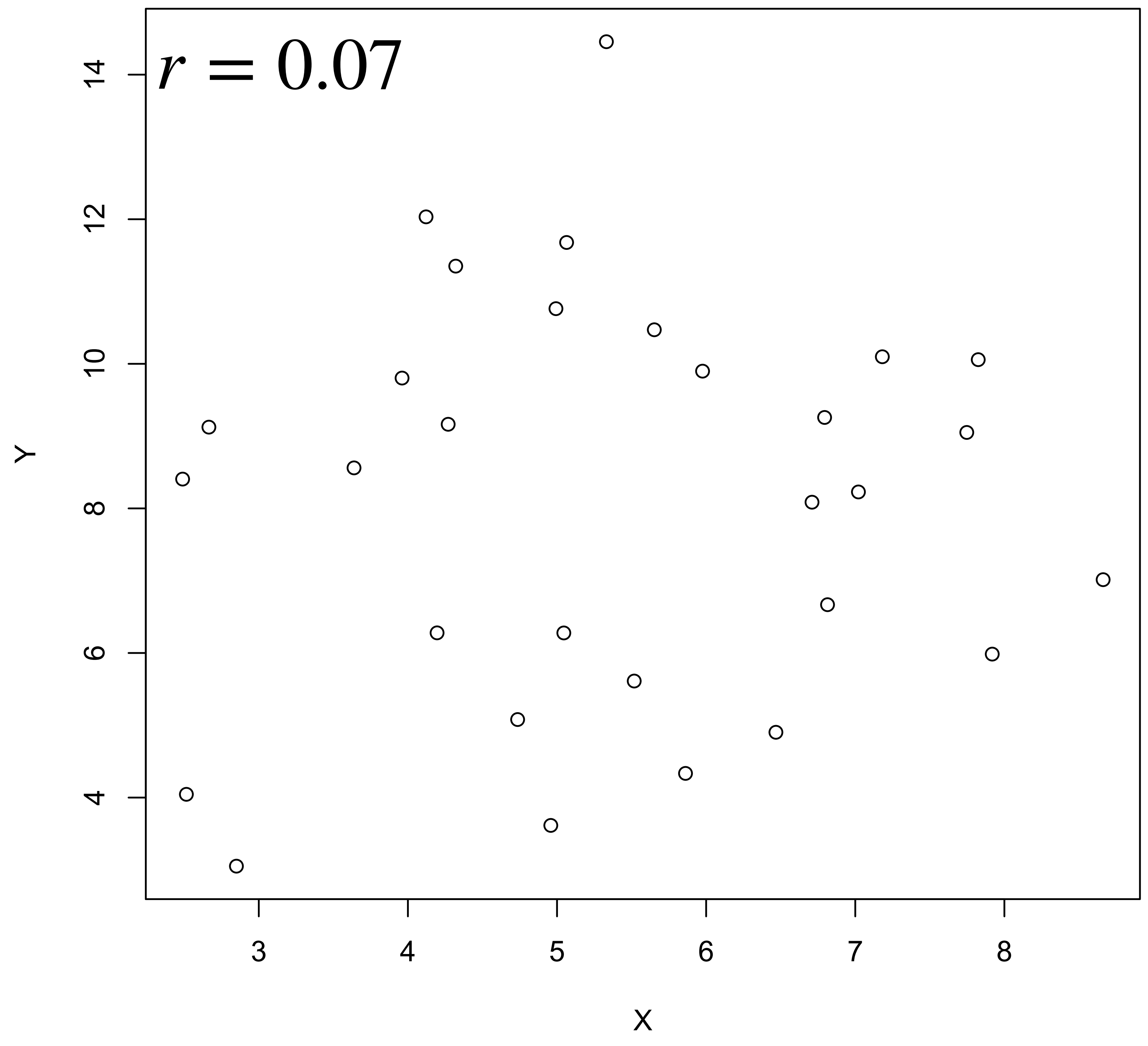

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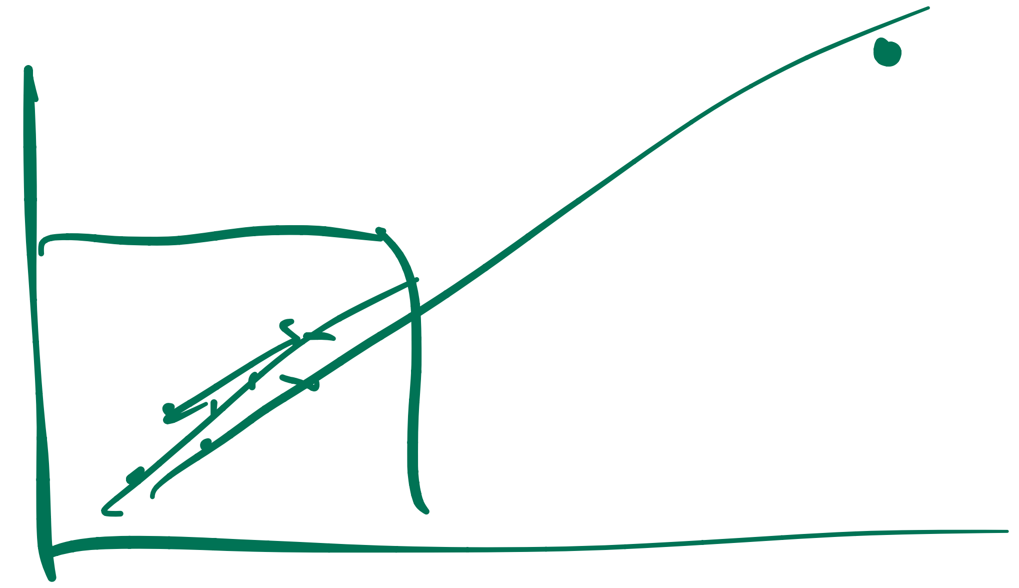

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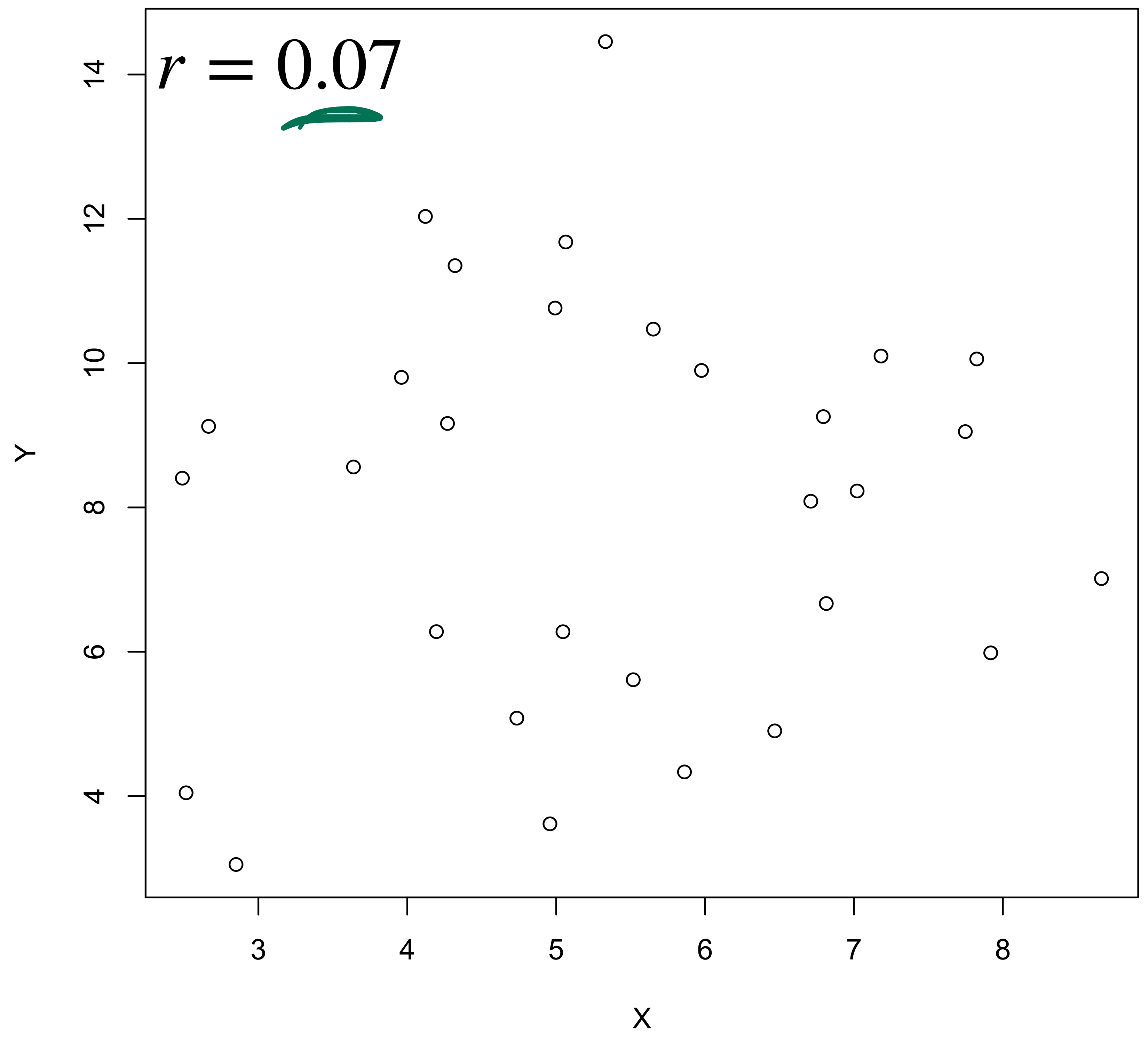

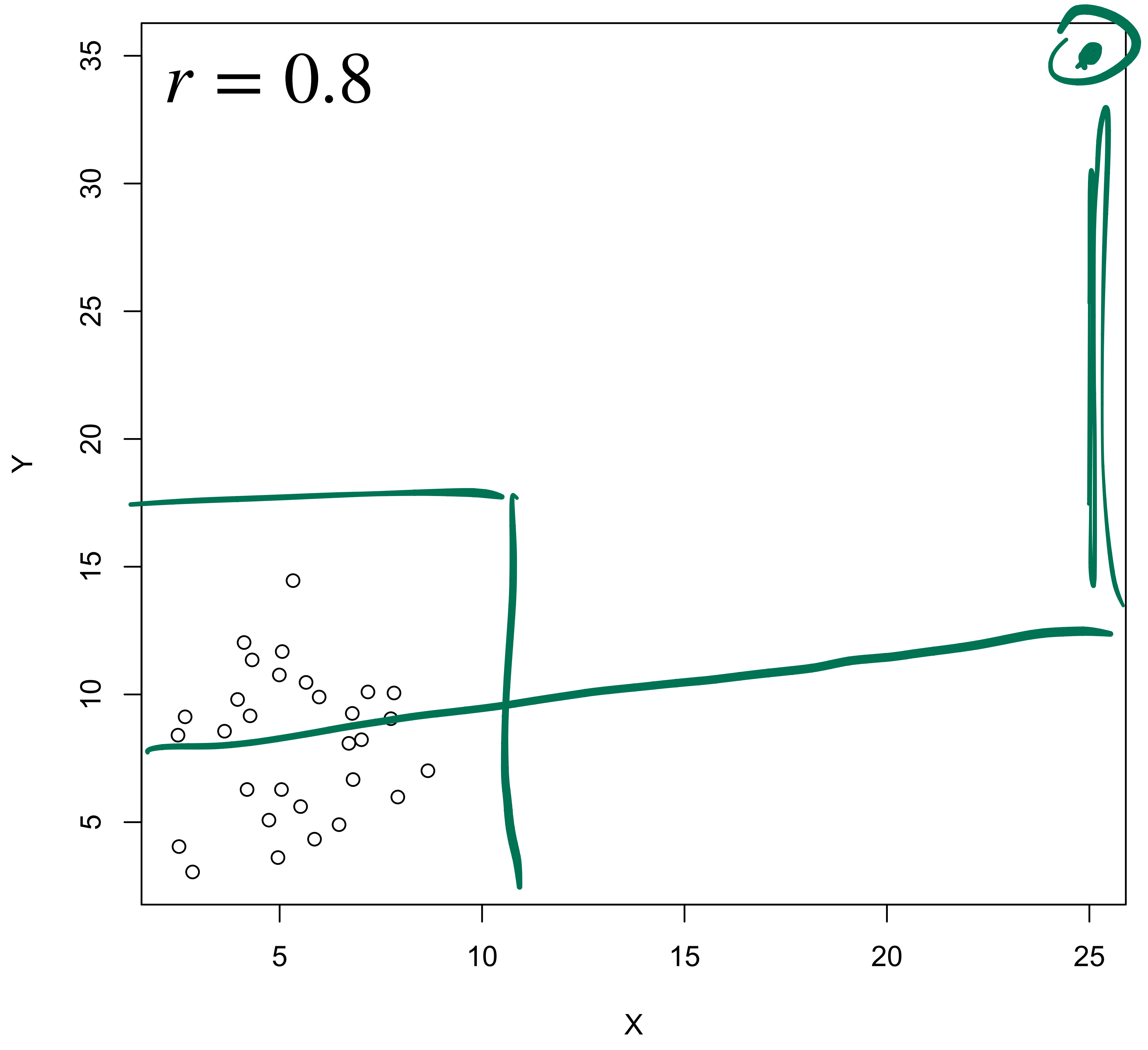

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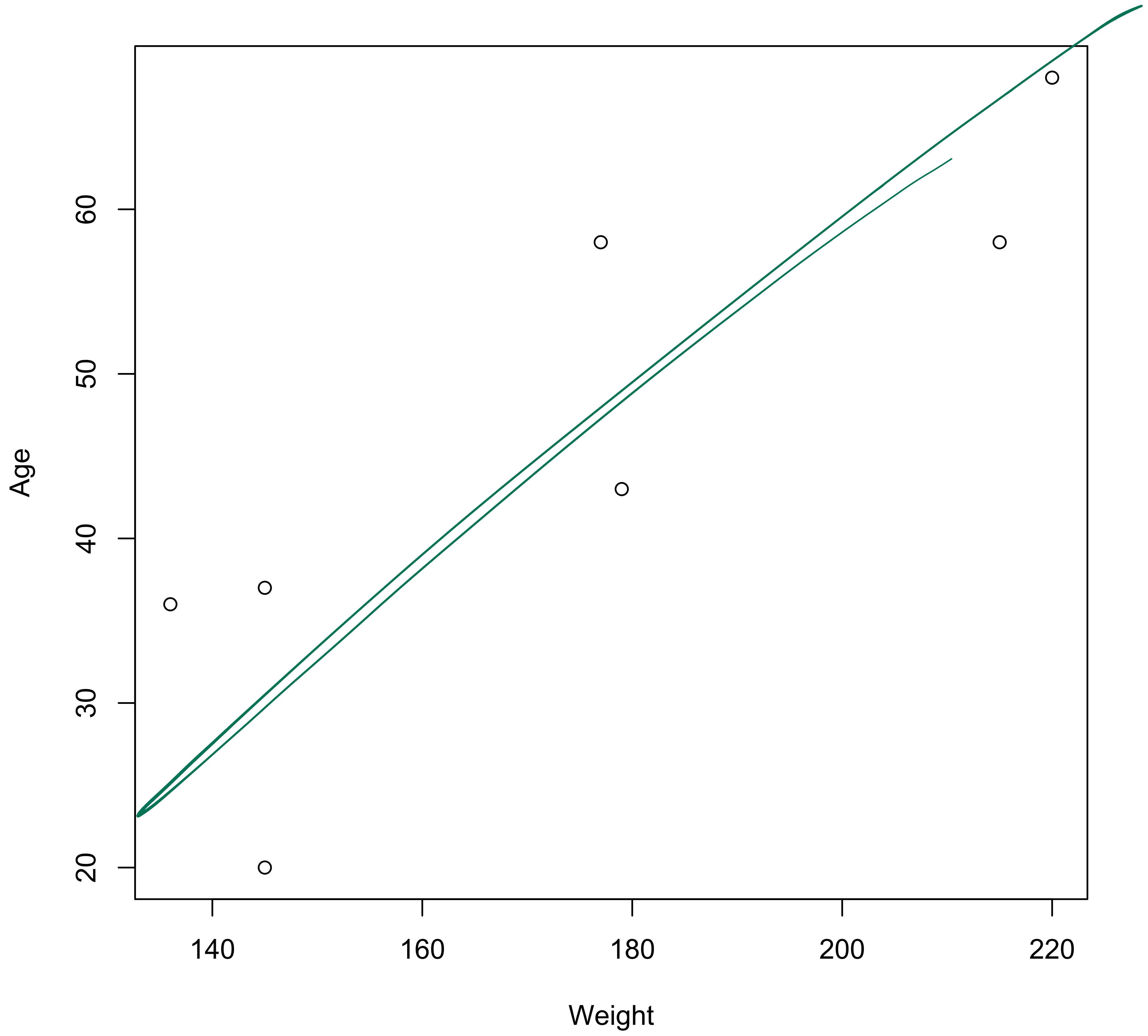

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https://clauswilke.com/dataviz/visualizing-associations.html

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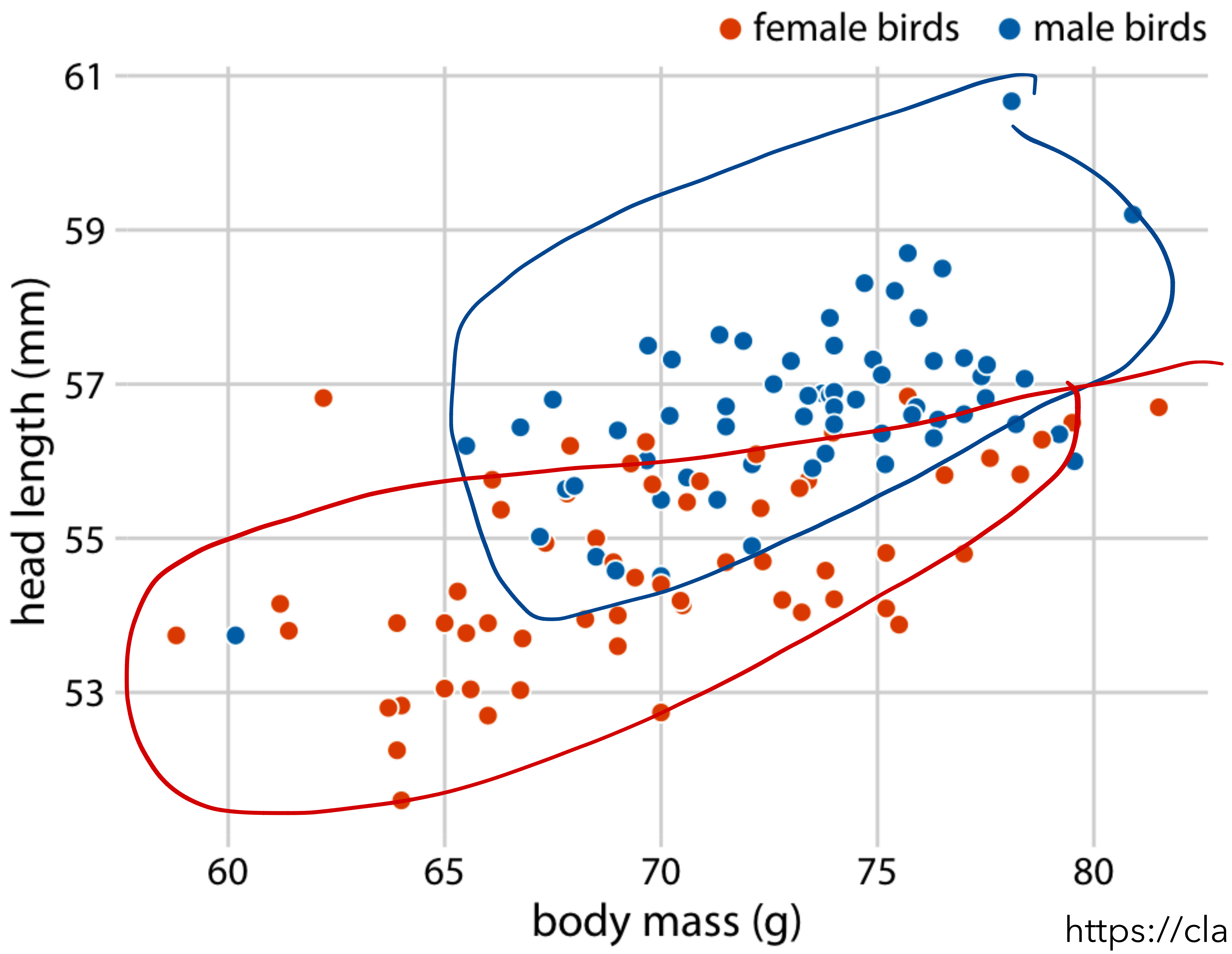

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$\in [-1, 1]$

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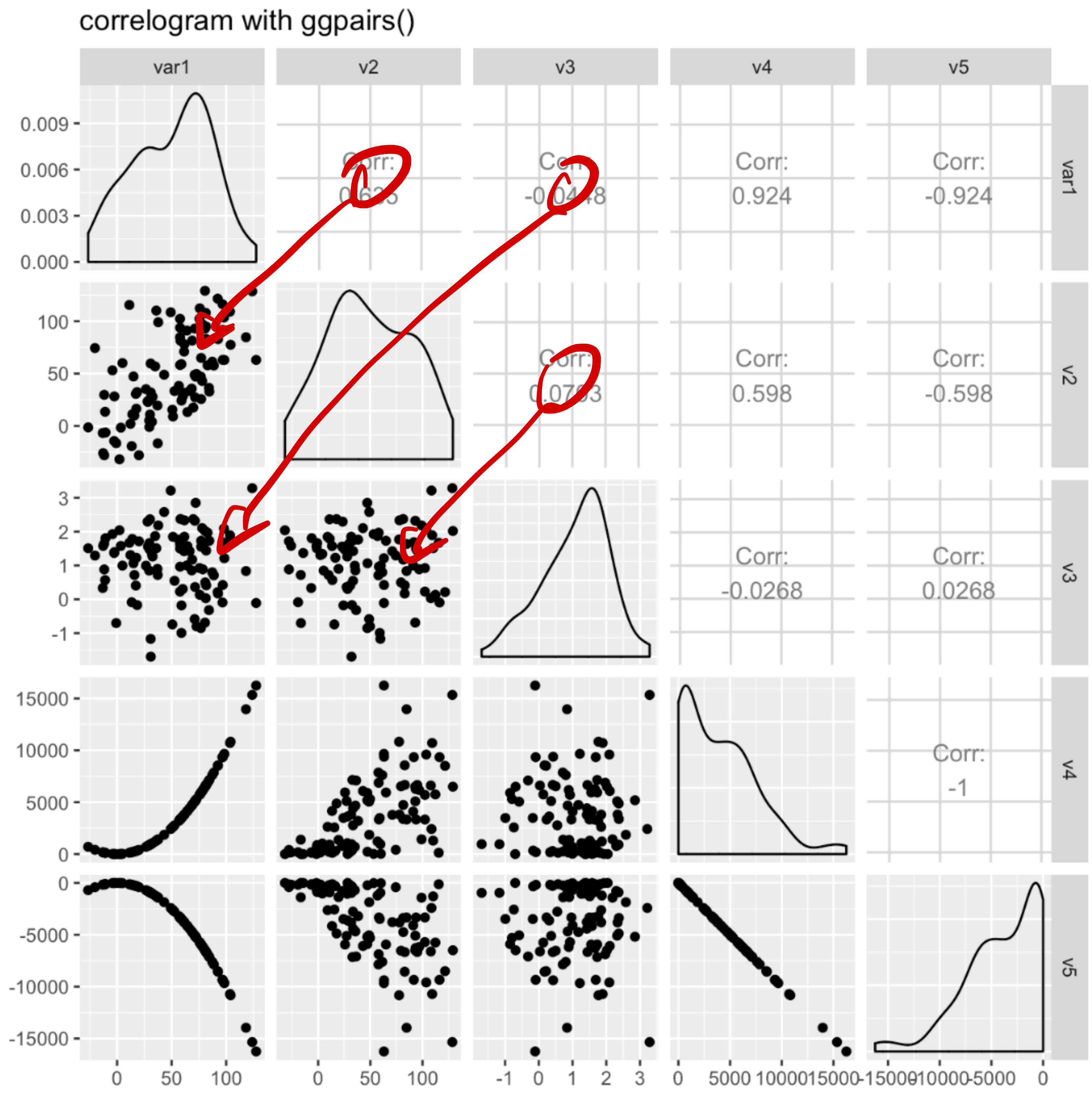

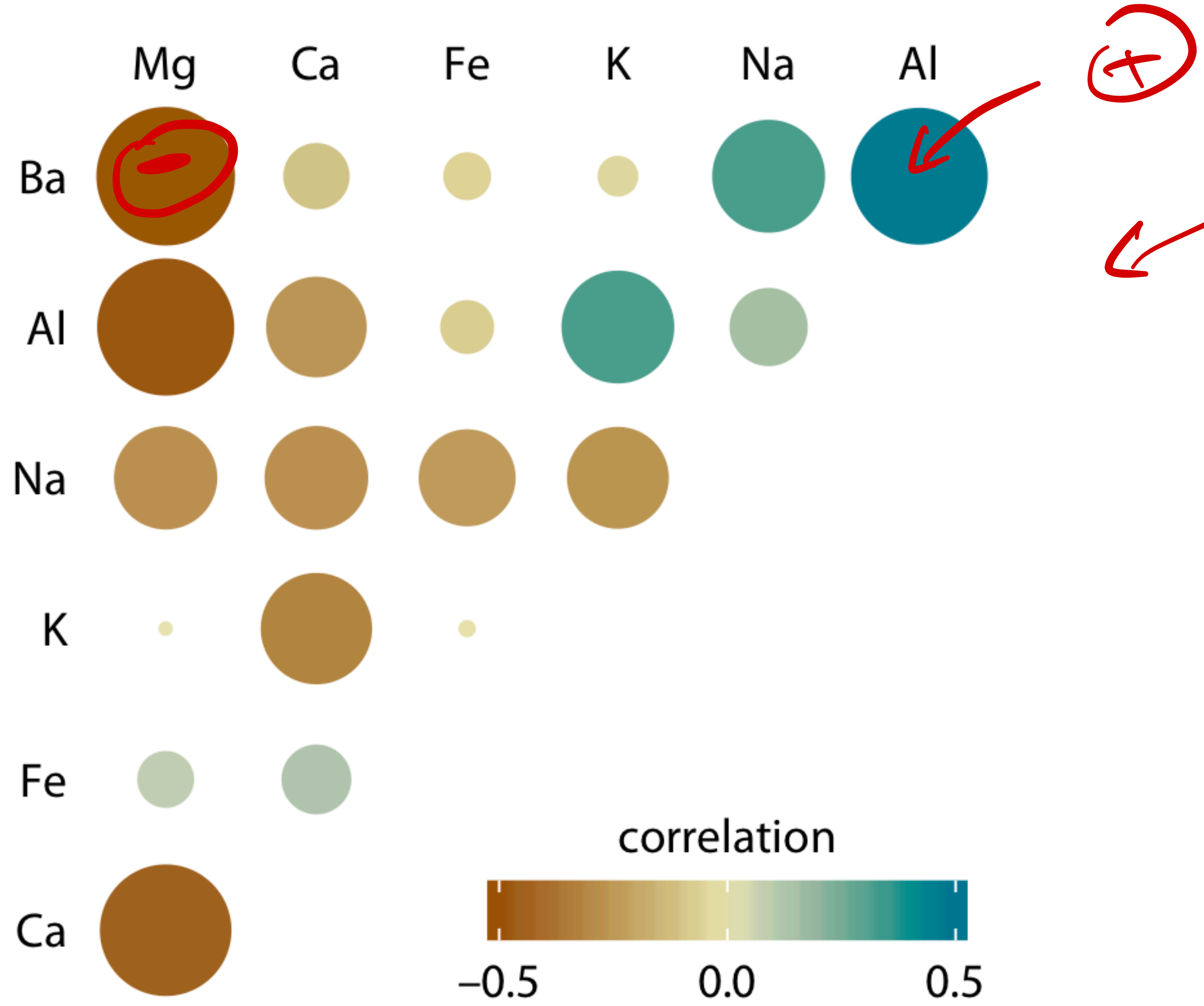

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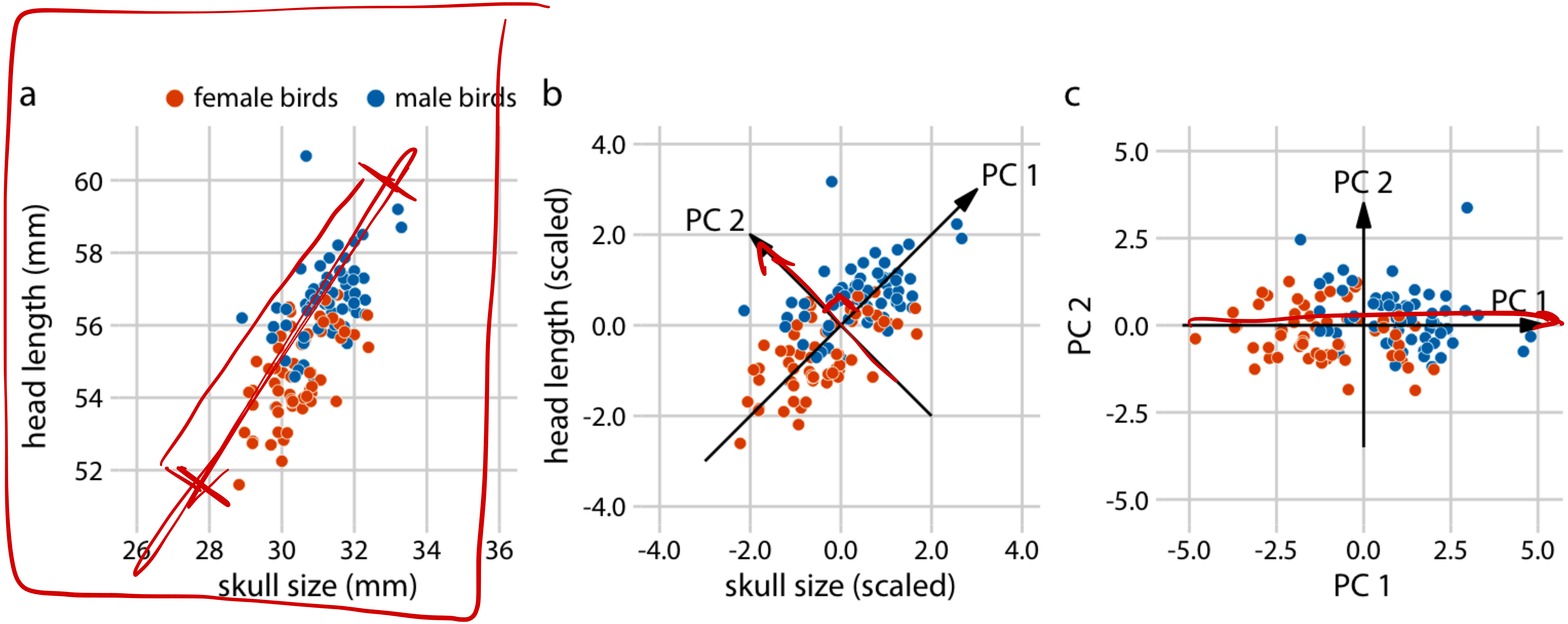

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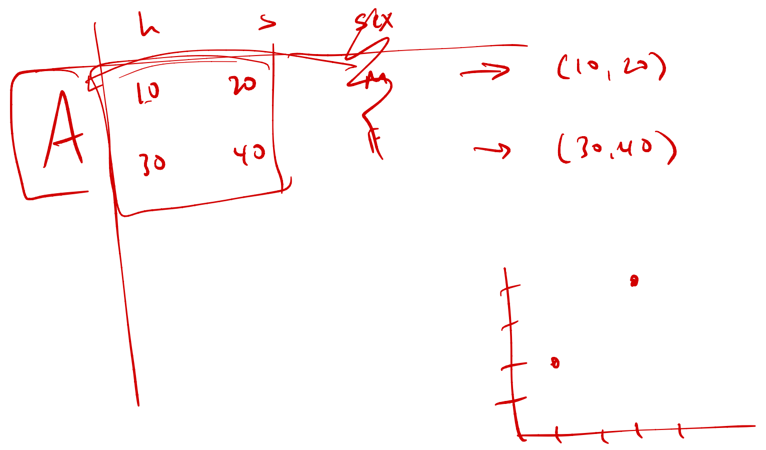

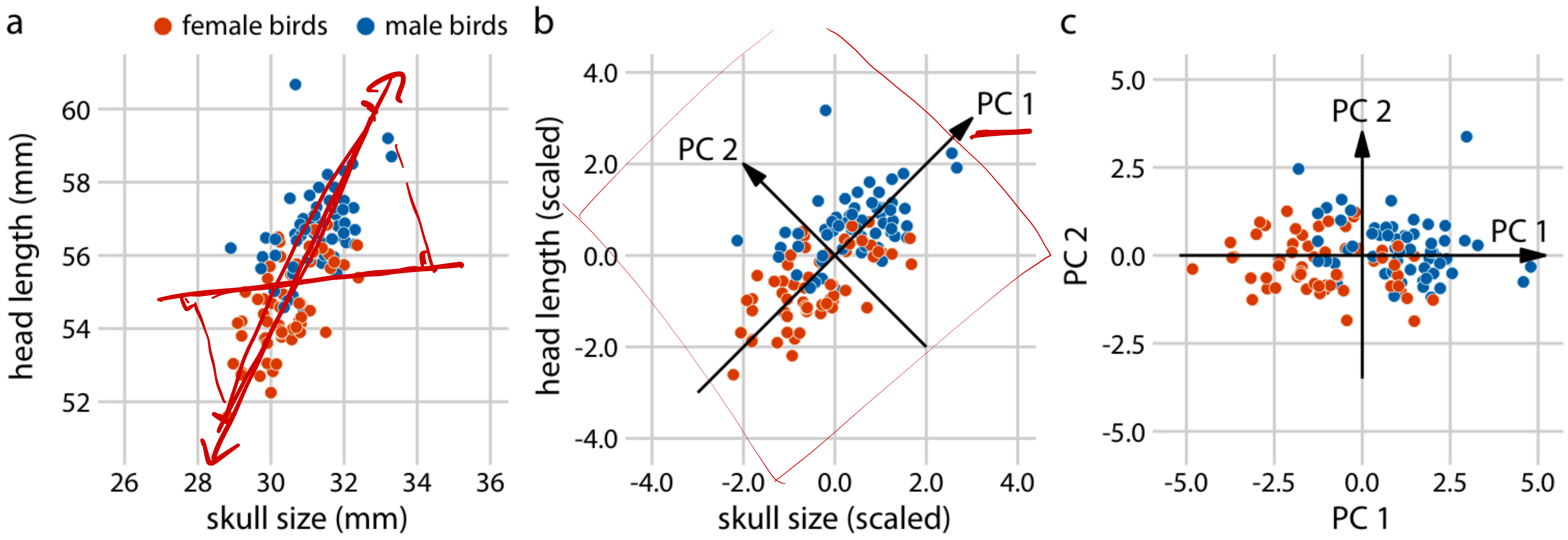

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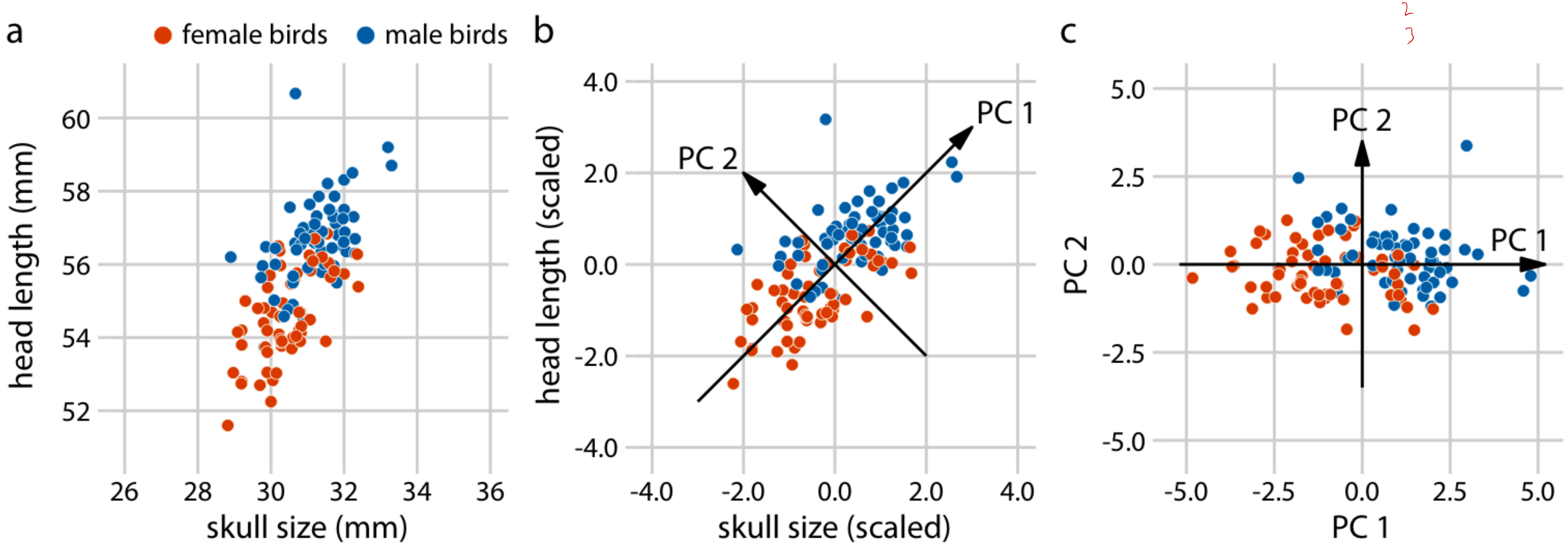
mat[,1]			mat[,4]
1			4
1			68
2			95
3			99.7

1
2
3



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