Chapter 5: Distributions

DSCC 462

Computational Introduction to Statistics

Anson Kahng

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Random Variables

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- For any $E \subseteq S_X$, we can define $p_X(E) = \Pr(X \in E)$

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- The support S_X consists of all x for which $p_X(x) > 0$ (all achievable outcomes)

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- Probability distribution tables resemble relative frequency distribution tables: probability of each outcome is the relative frequency distribution of each outcome in a large number of trials

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- *Normalization constant*: 1/denominator

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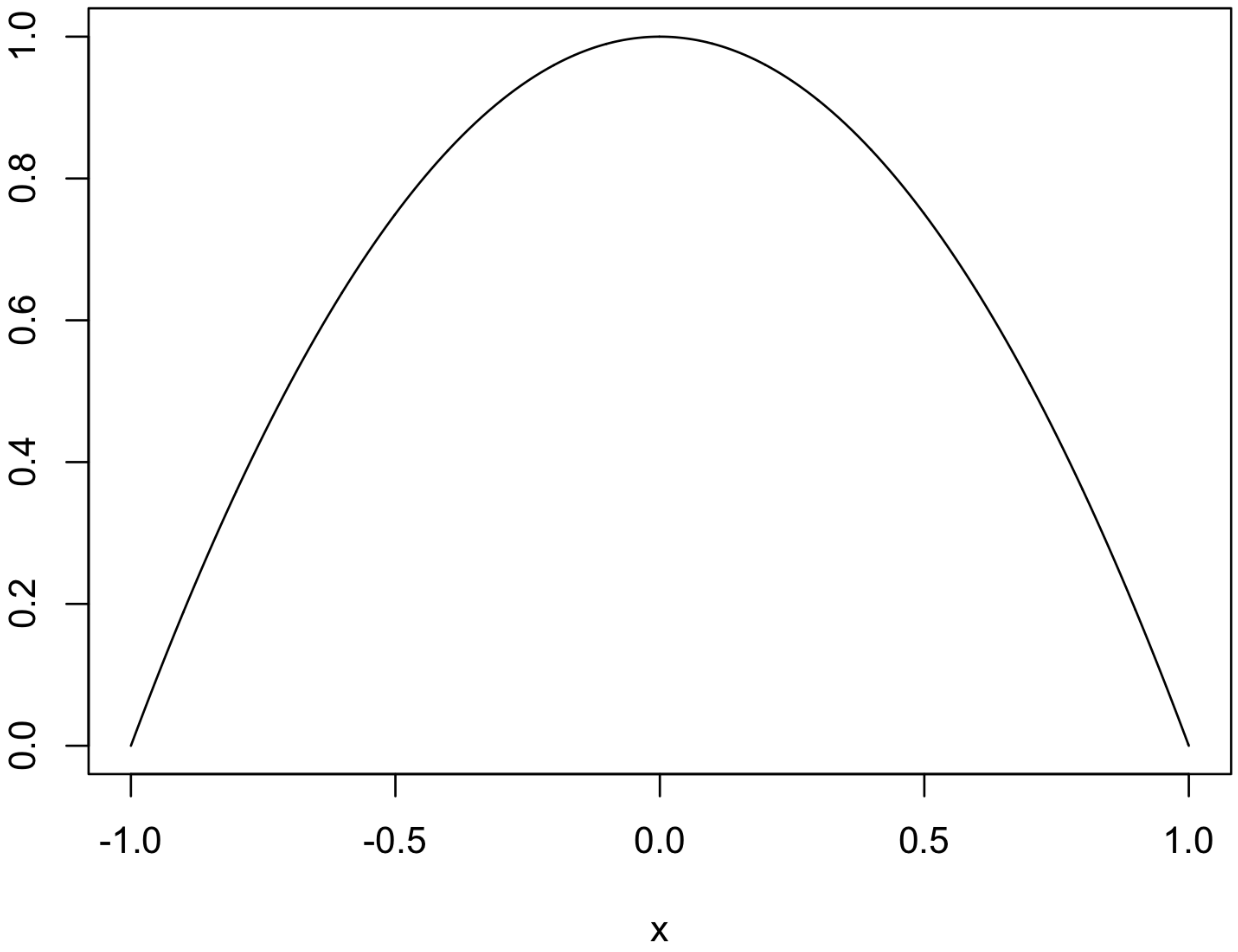
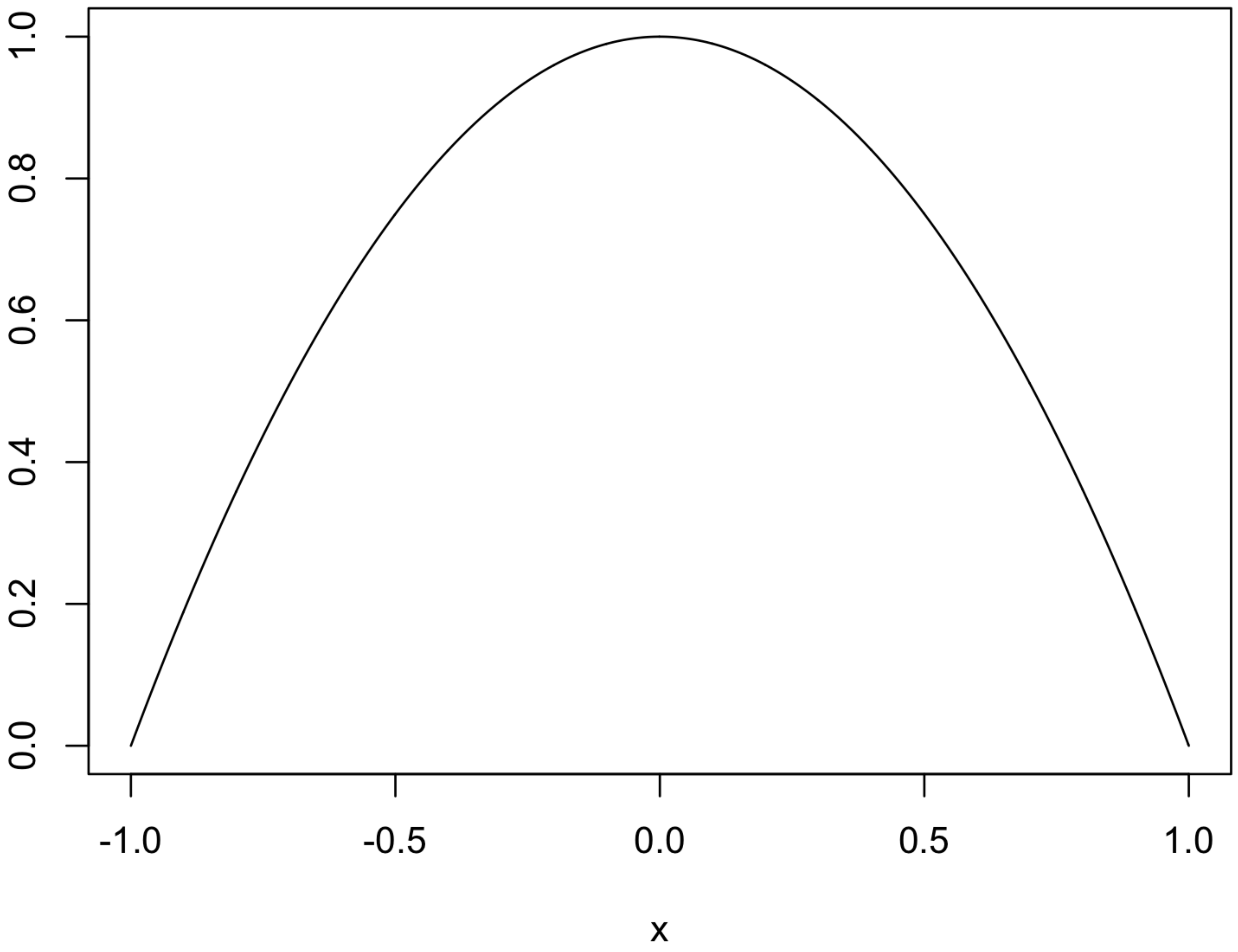


Normalization: Example

- Suppose that we generally know that probability is distributed according to the following curve:



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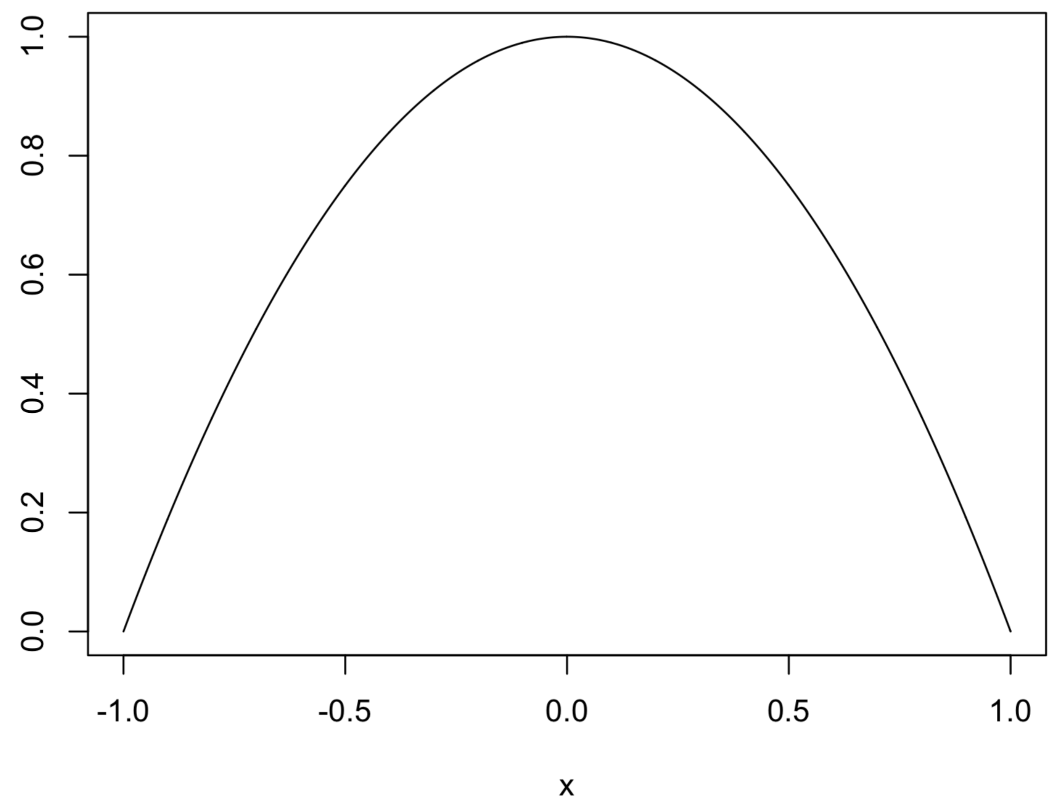

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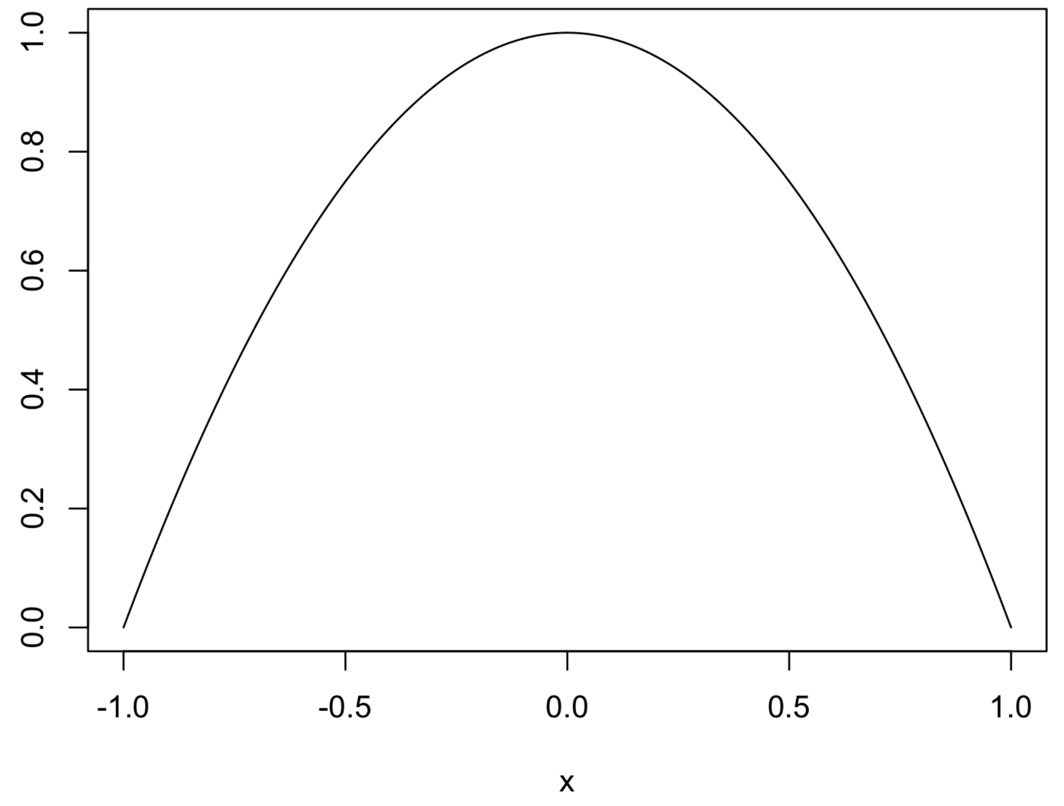
"  "

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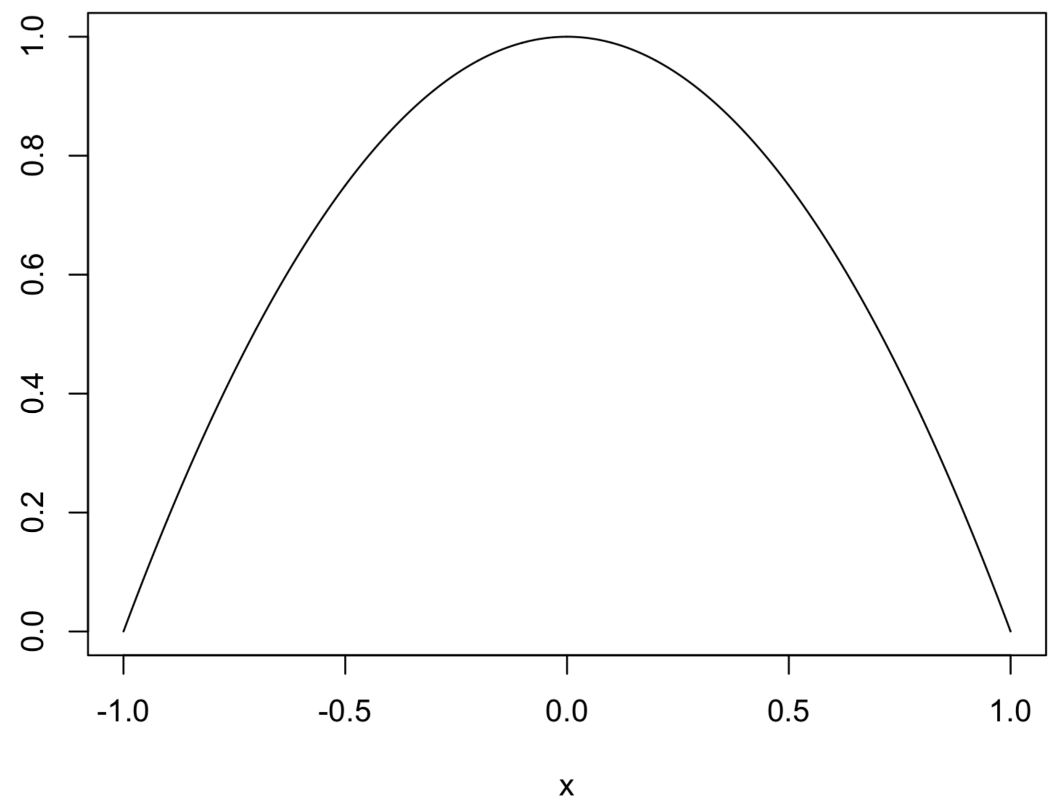
""

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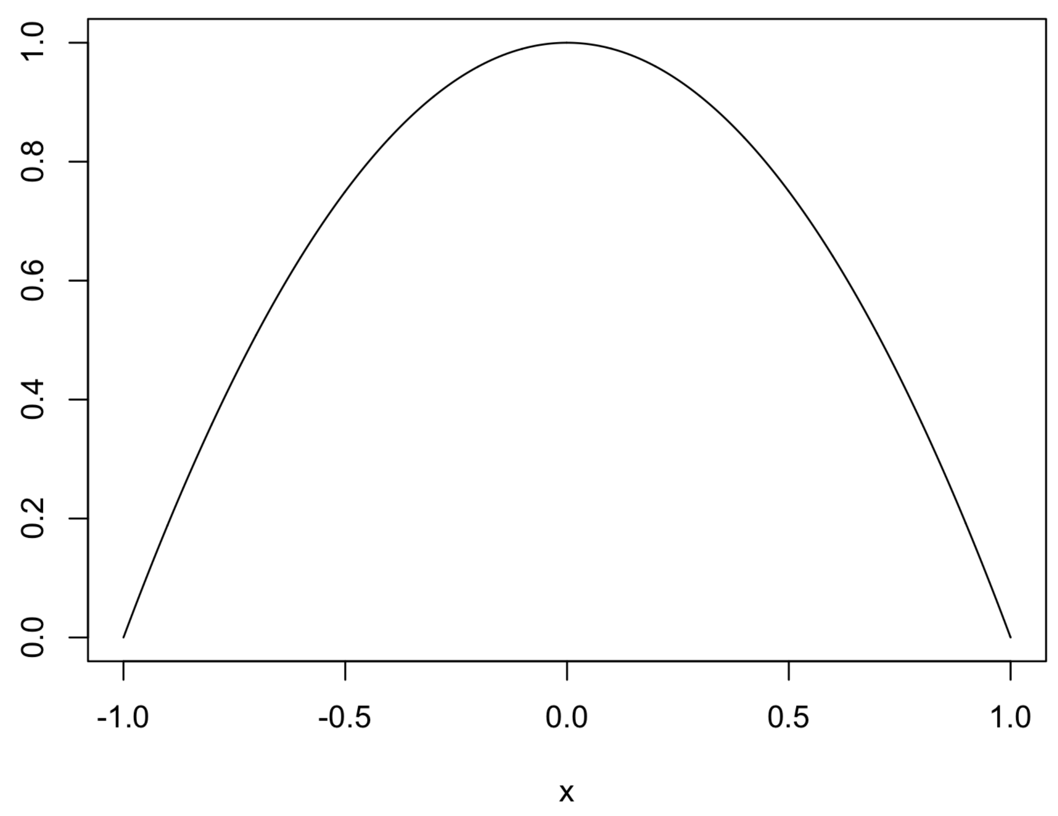

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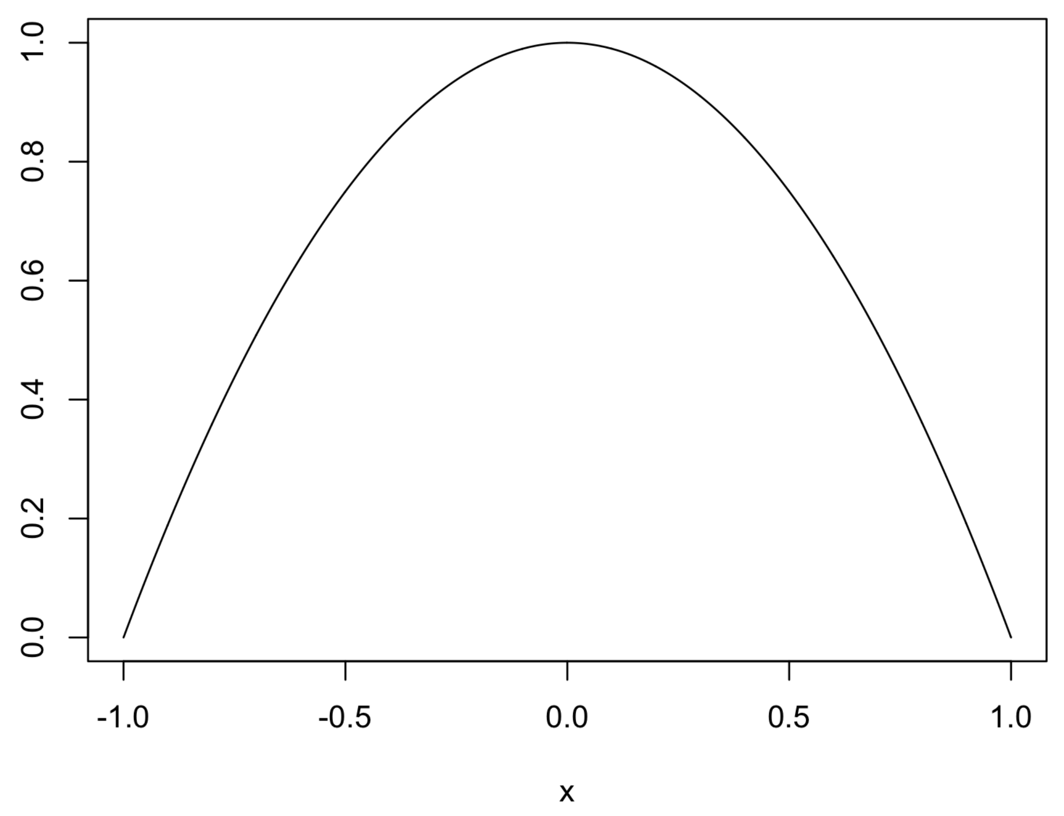

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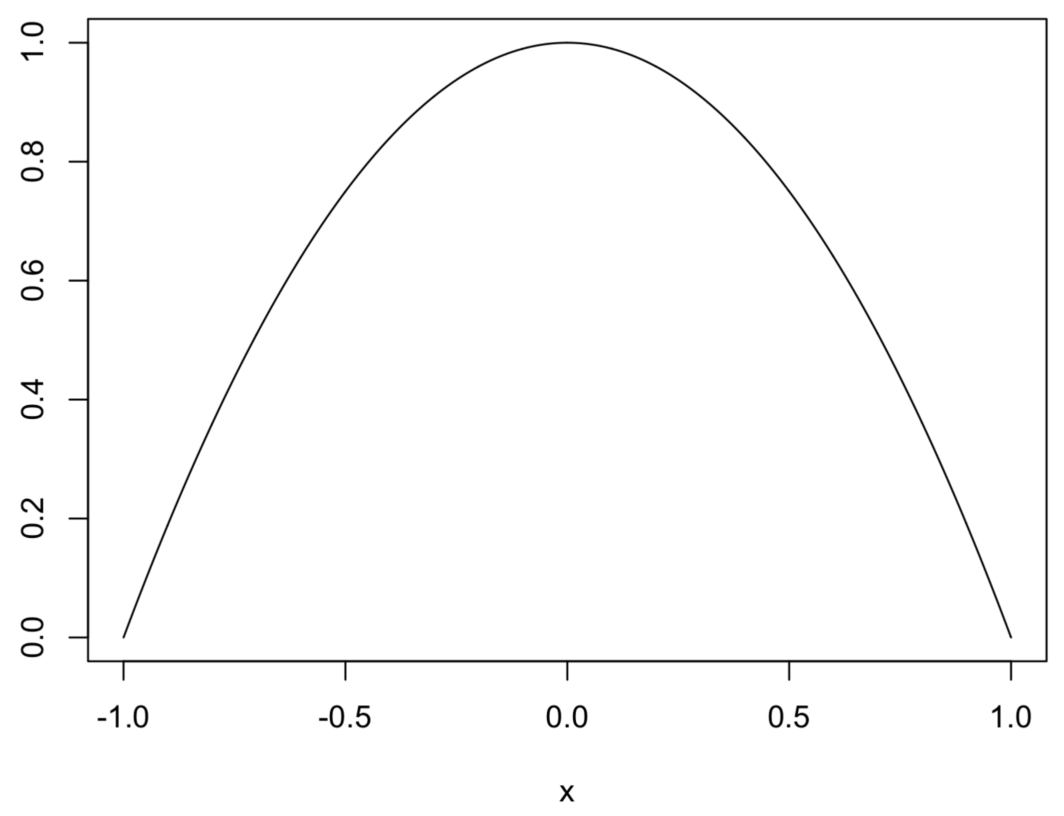

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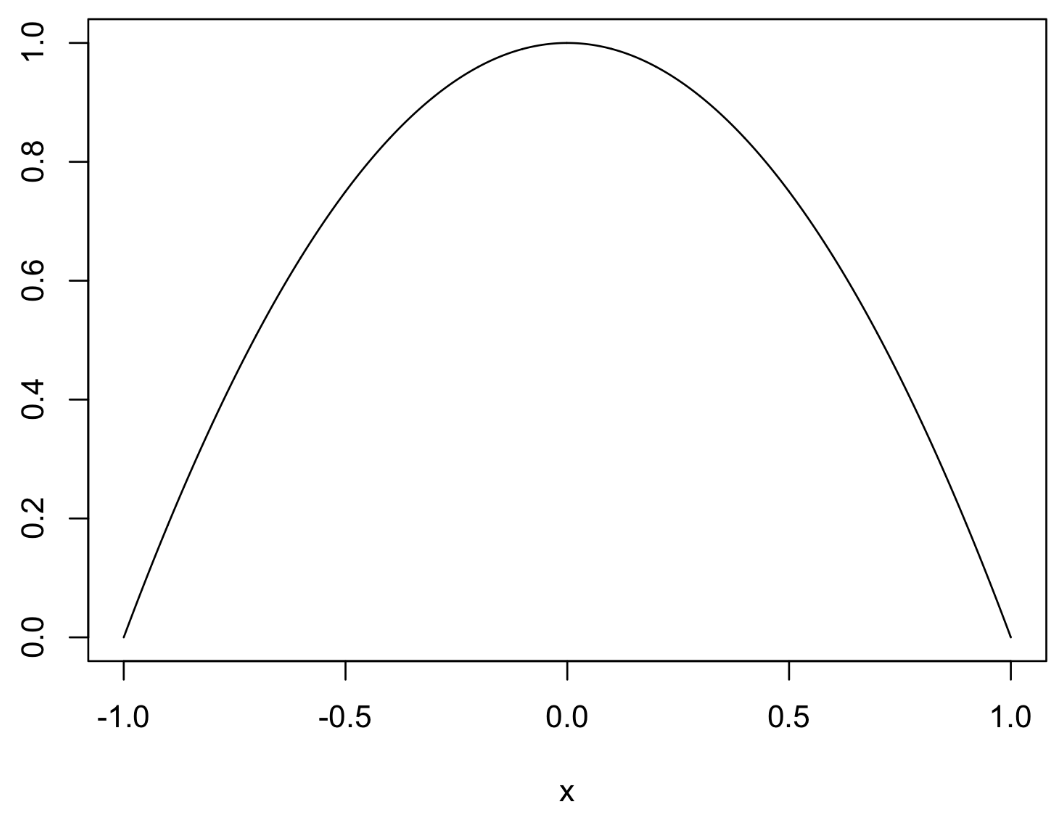

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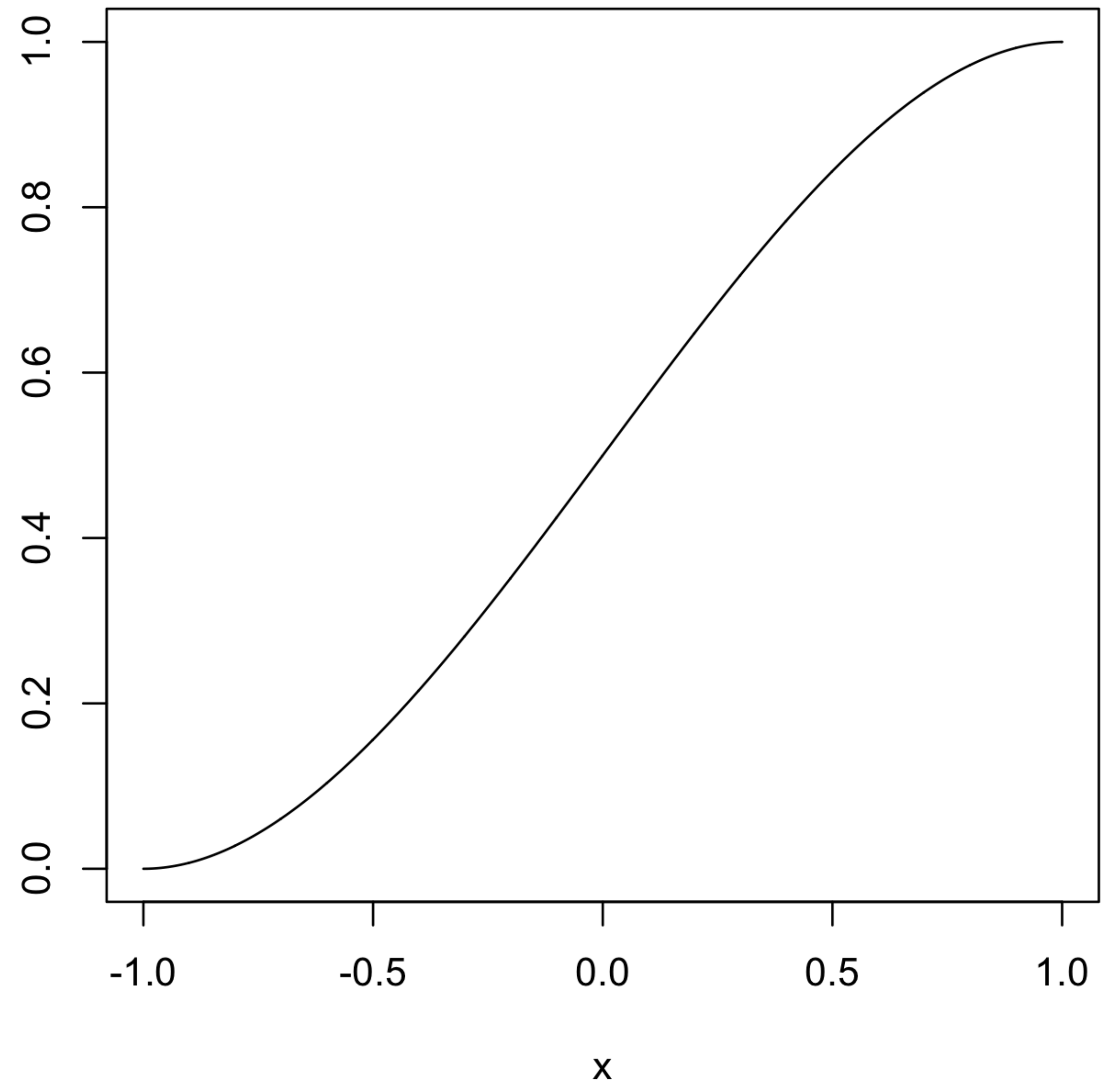

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- We say that $q = 85$ is the $p = 0.72$ quantile of this distribution (also called the 72nd percentile)

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- Although $Q(p)$ is uniquely defined, the p-quantile may not be unique

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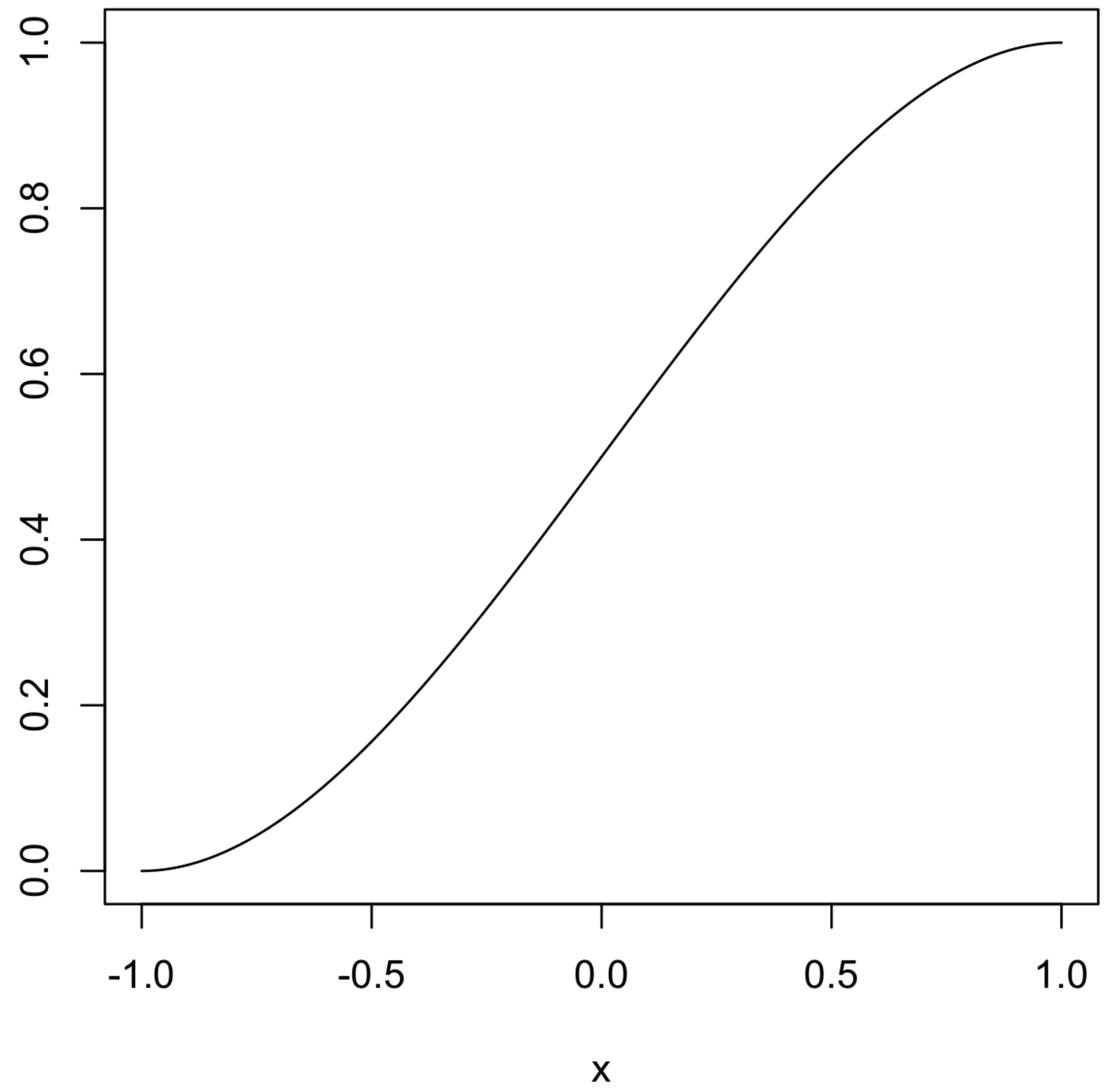

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- *Population standard deviation*: The square root of the population variance

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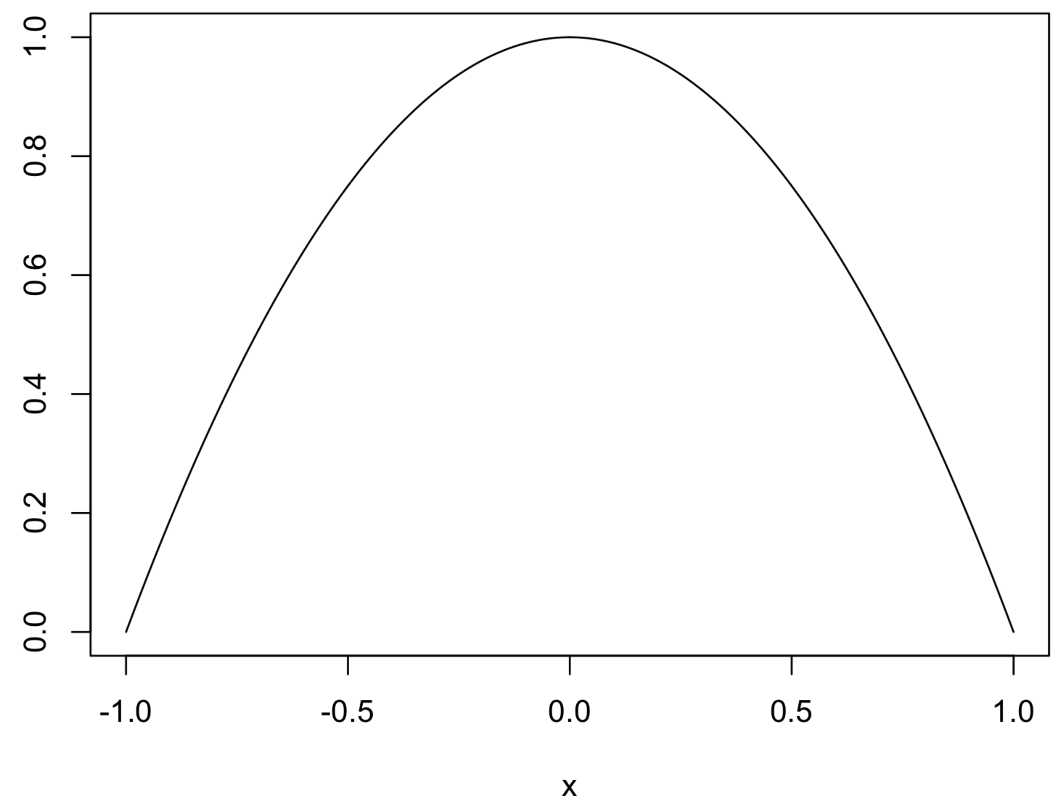

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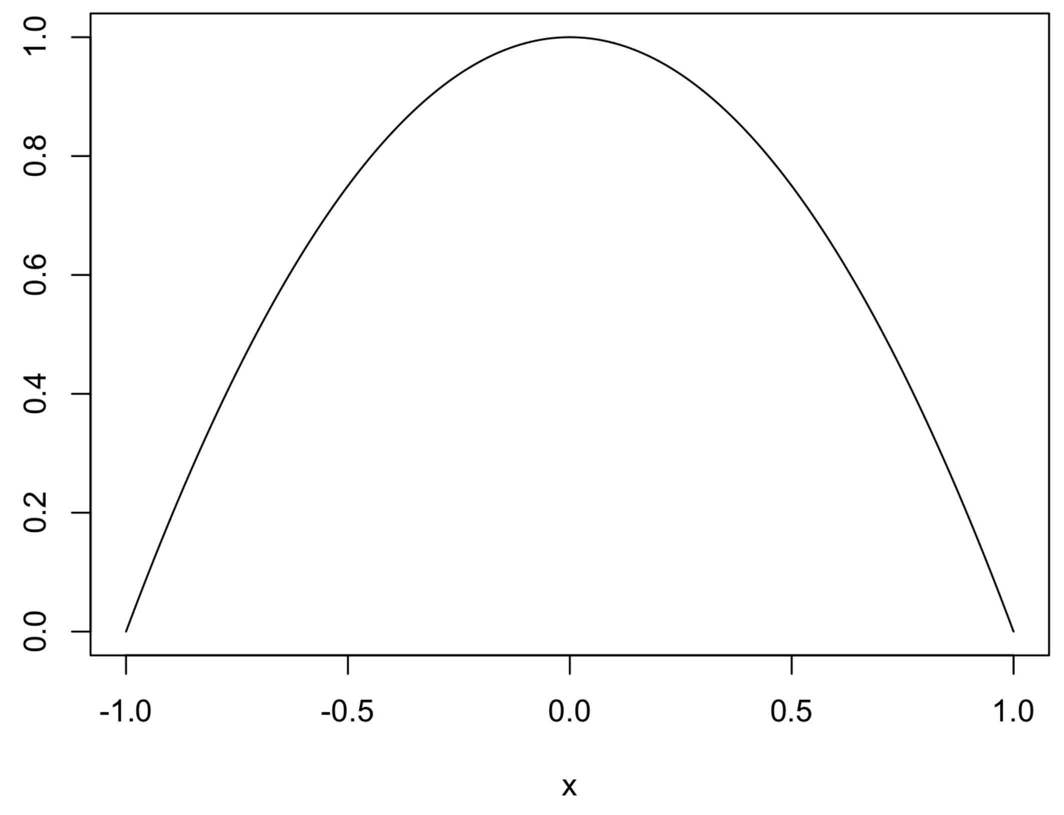

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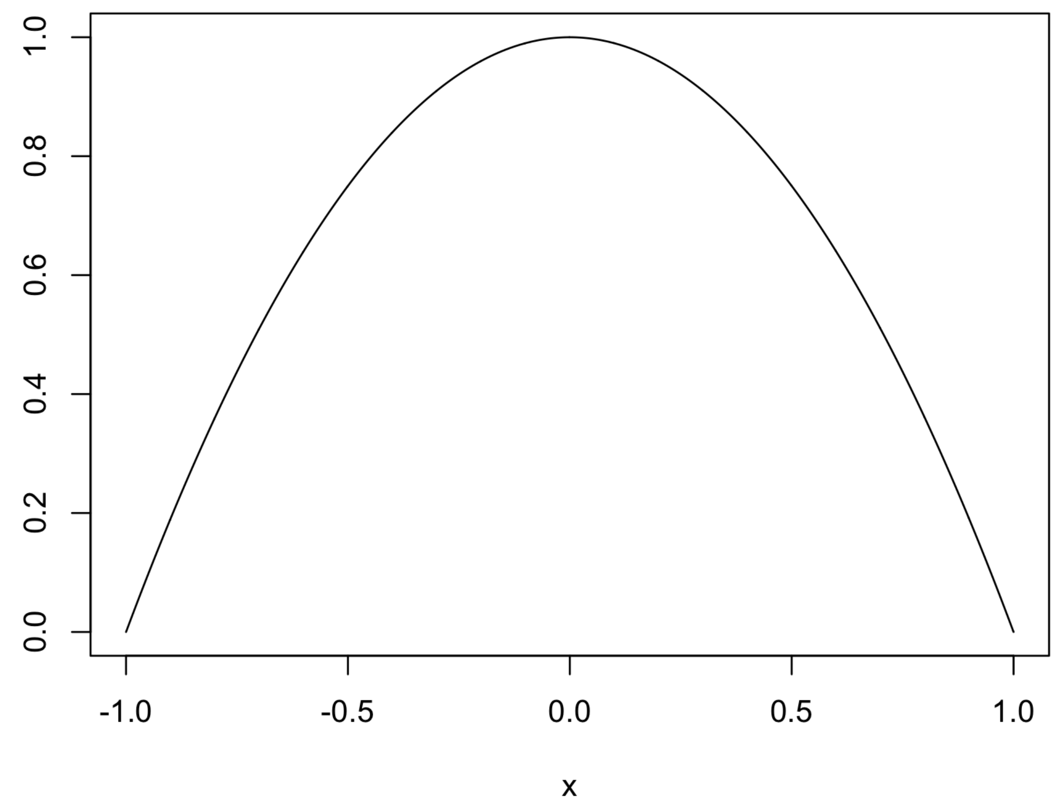

- What is $E(X)$?

- Intuition: By symmetry, $E(X) = 0$

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- In general, to get the distribution of Y, we have that for any event $E \subseteq S_Y$, we have $p_Y(E) = p_X(g^{-1}(E))$

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&= a^2 \cdot var(X) = a^2 \cdot \sigma_X^2
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$$g(E(X)) = \left(\frac{1 + 2 + 3 + 4 + 5 + 6}{6}\right)^2 = (3.5)^2 = 12.25$$

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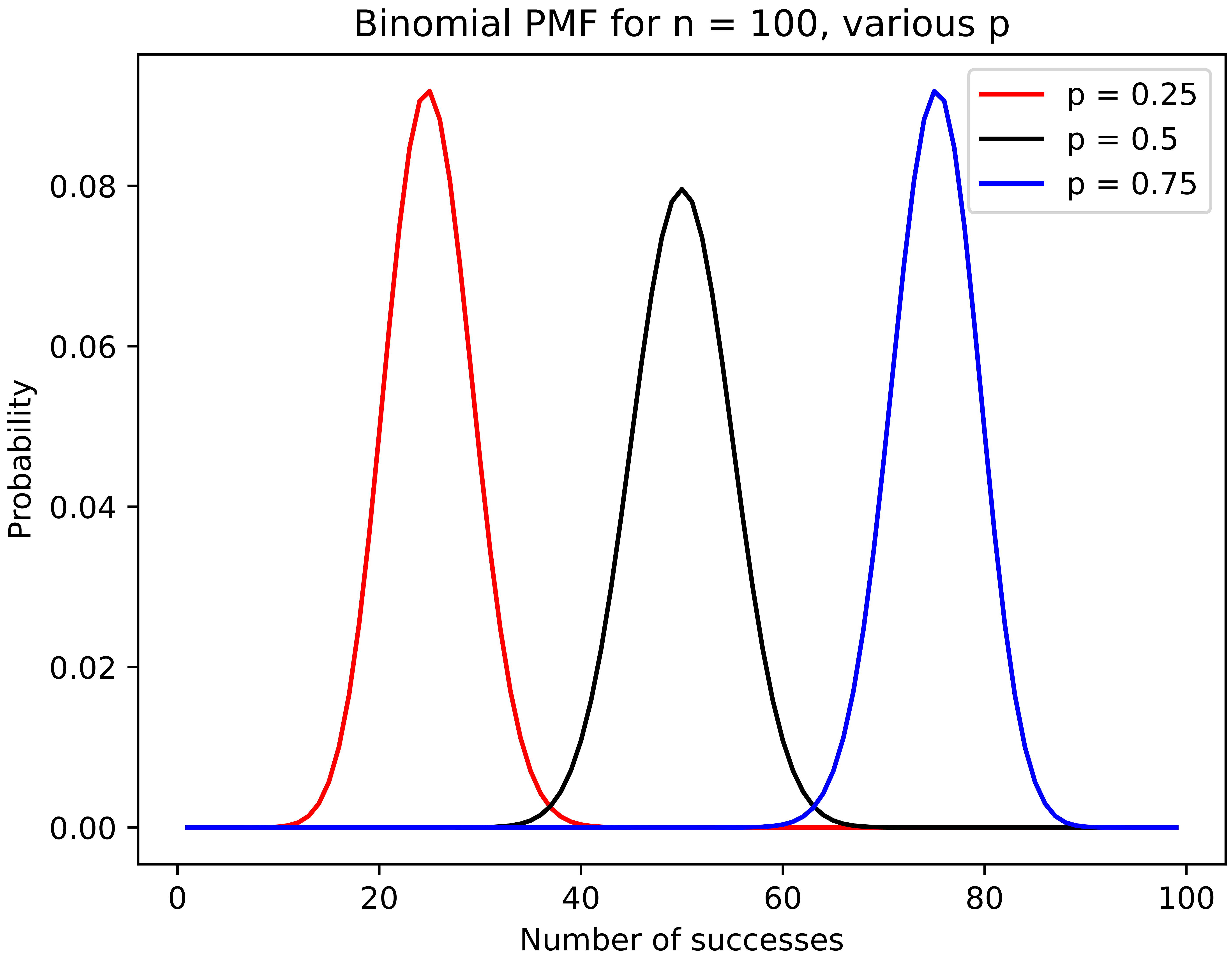

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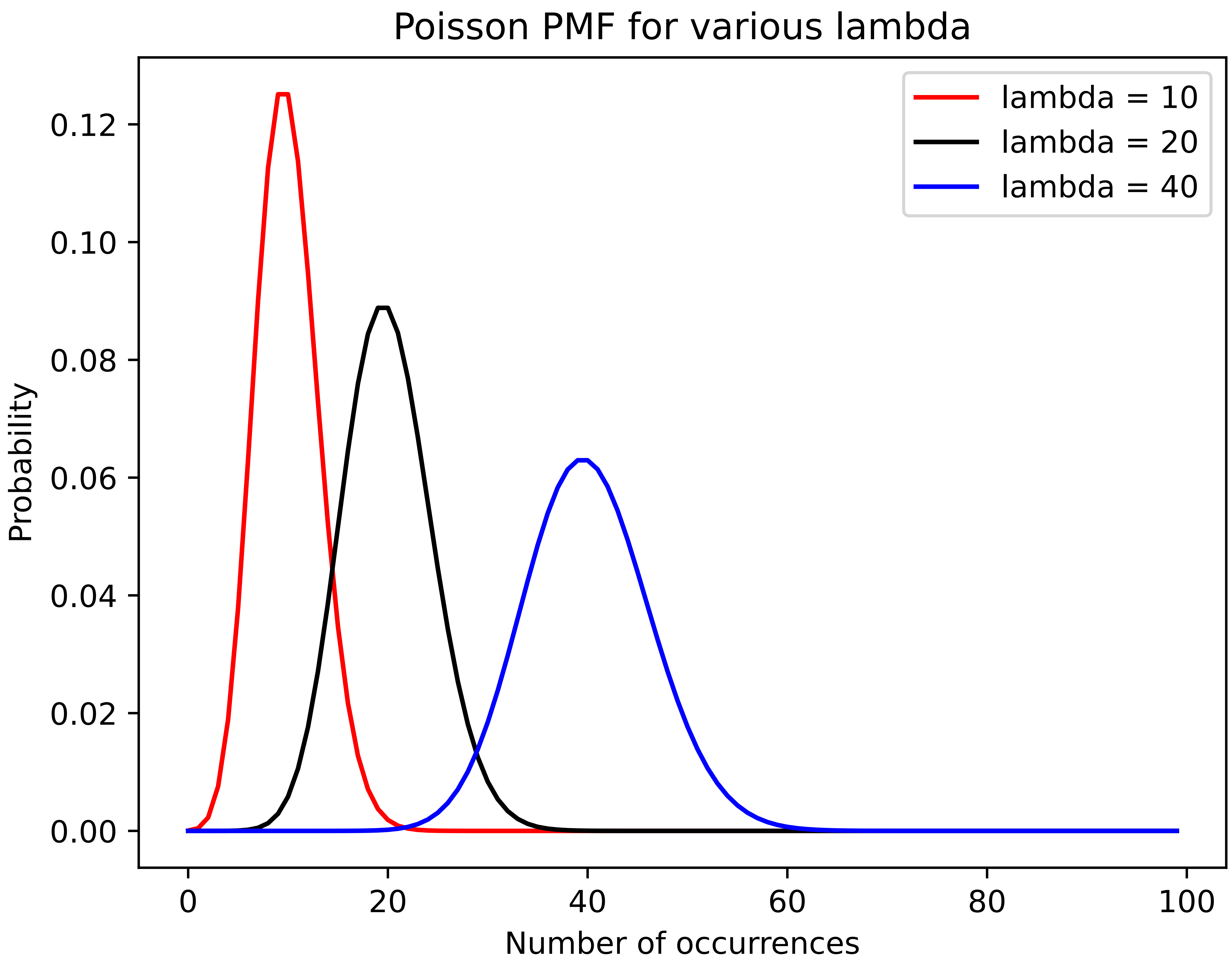

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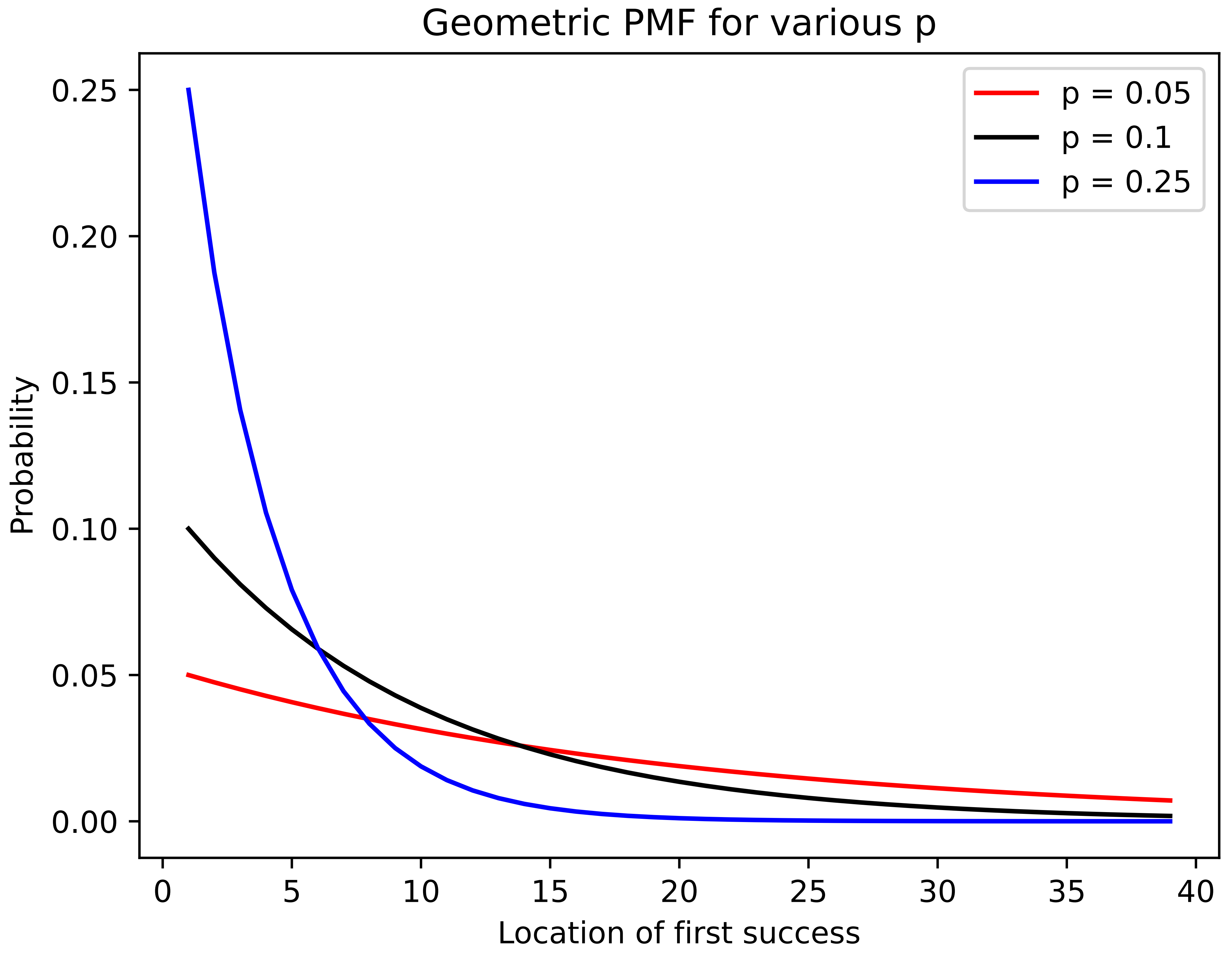

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$E((X - E(X))^2)$

$Var(X) = E(X^2) - E(X)^2$

$\int_a^b x^2 f(x)\,dx \qquad \left(\frac{a+b}{2}\right)^2$

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$\left(\frac{1}{b-a}\right) \frac{x^3}{3}\Big|_a^b \qquad \frac{b^3 - a^3}{3} \cdot \frac{1}{(b-a)} - \left(\frac{a+b}{2}\right)^2$

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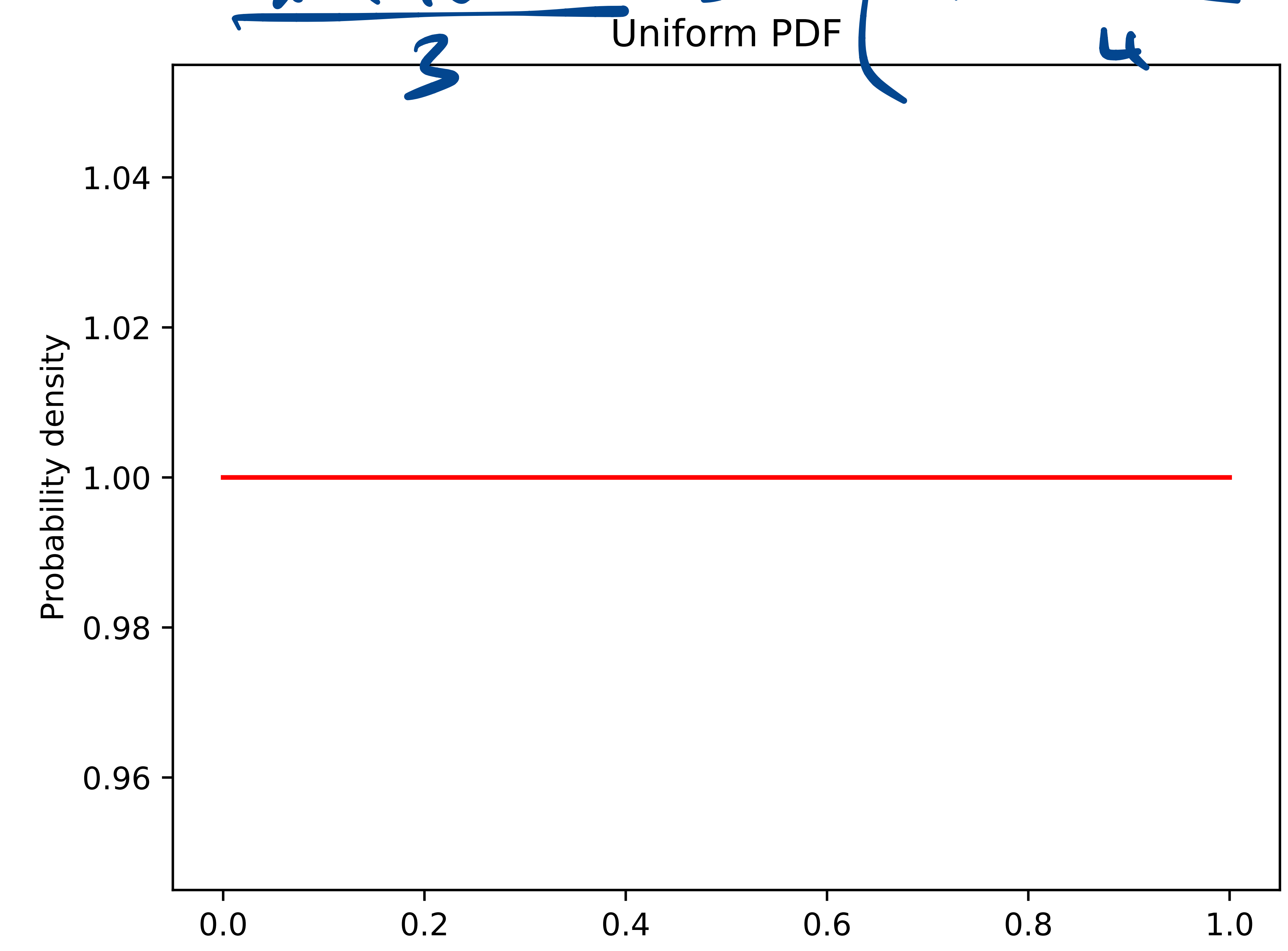

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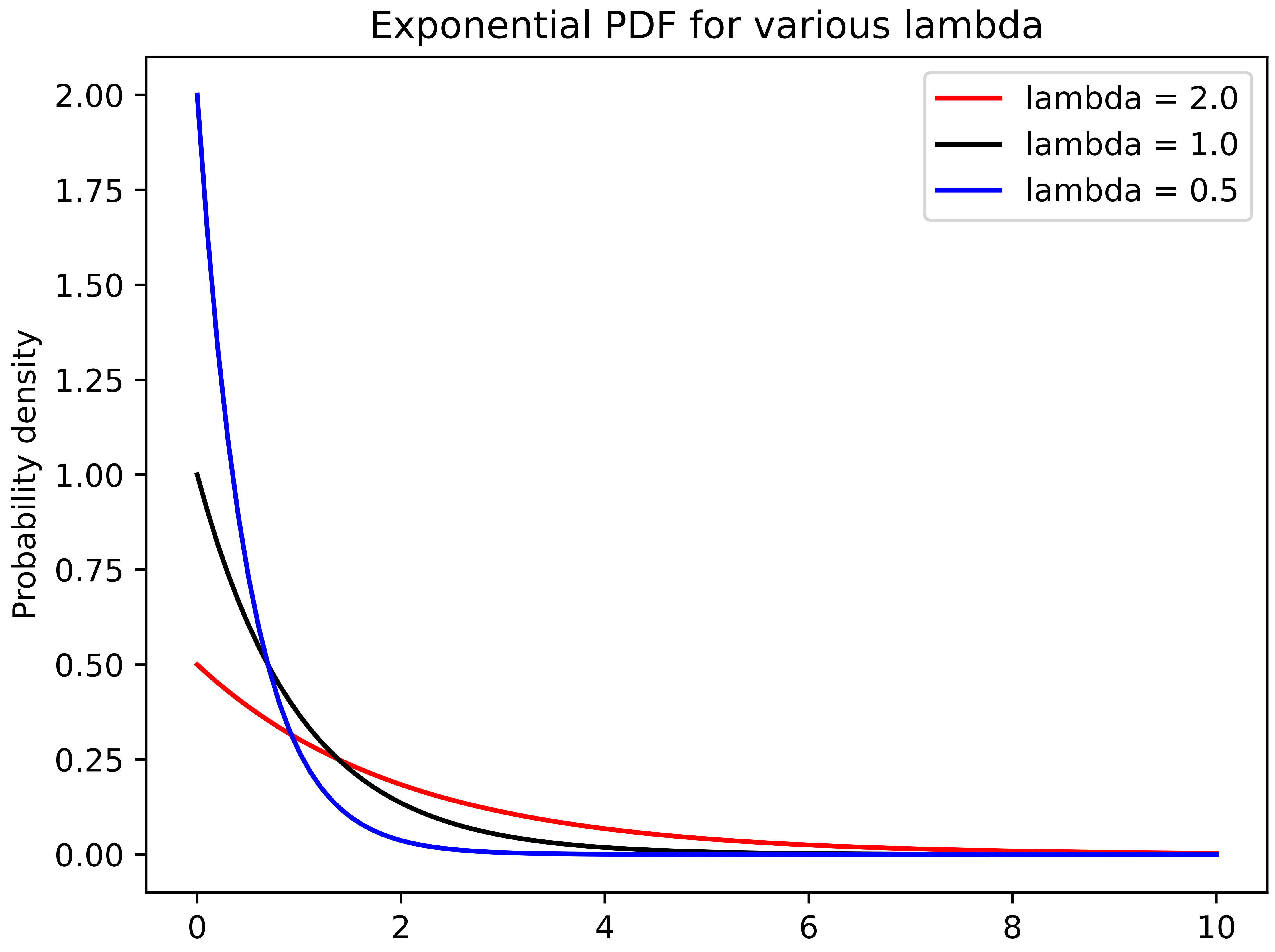

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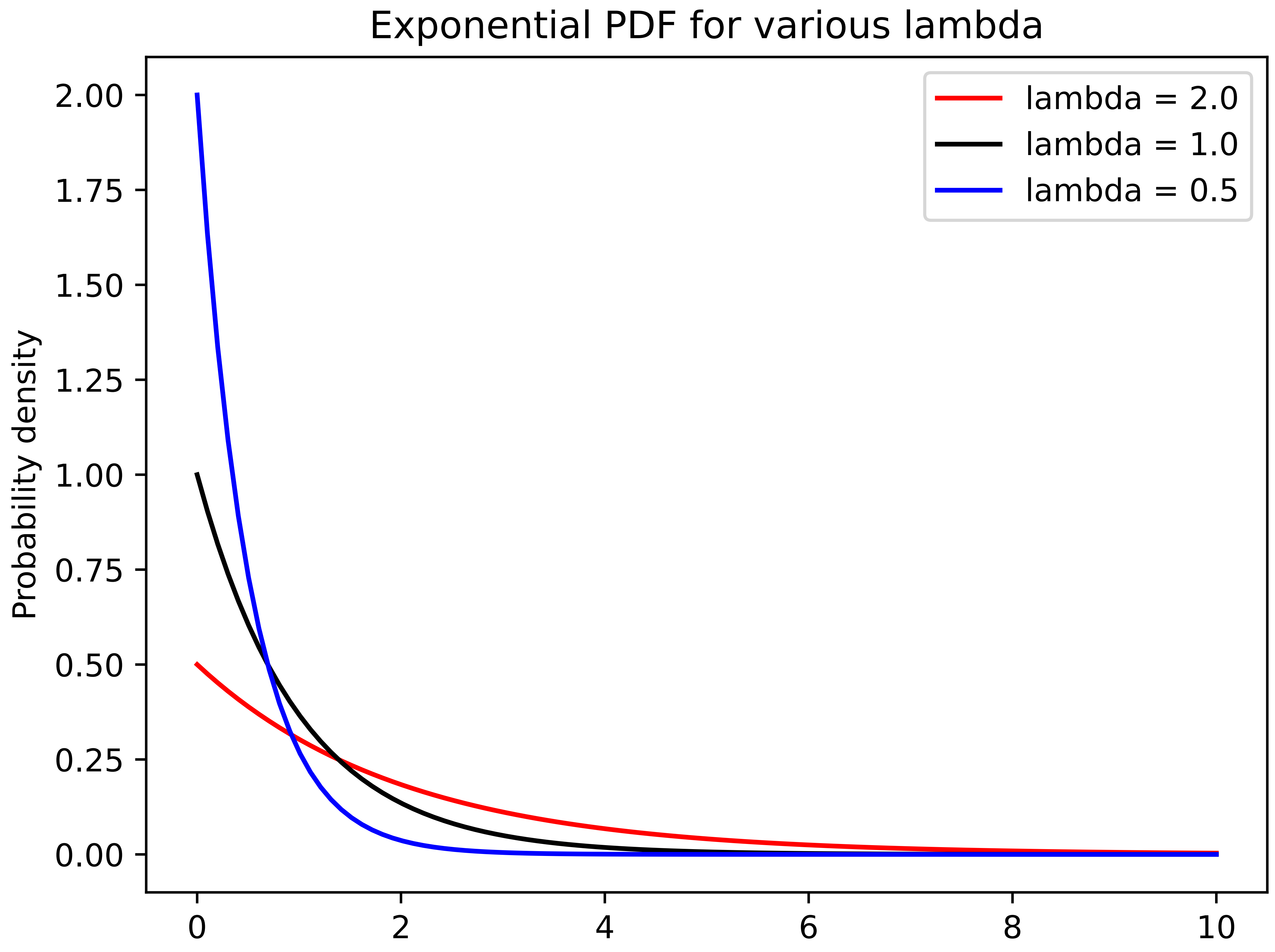

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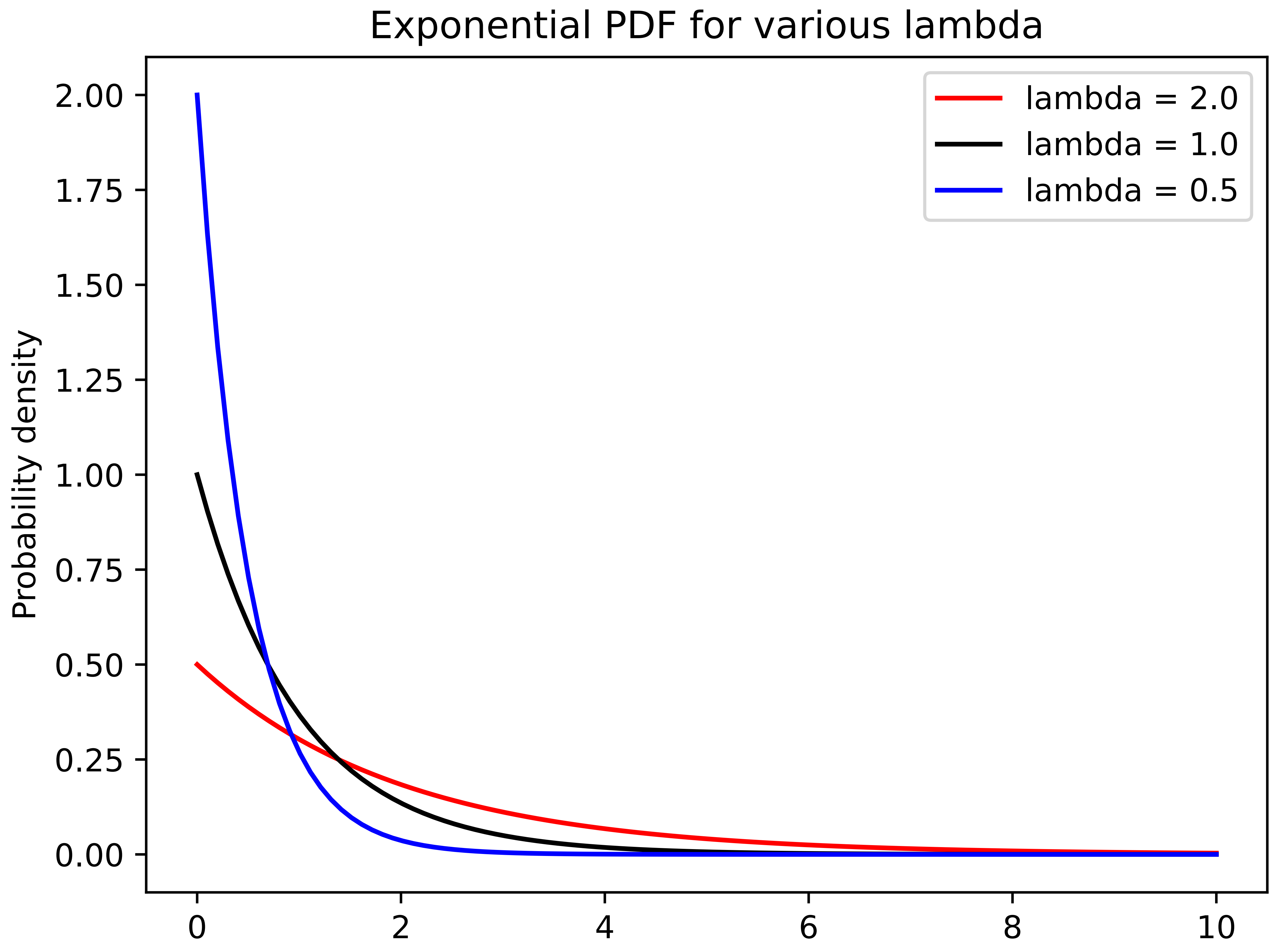

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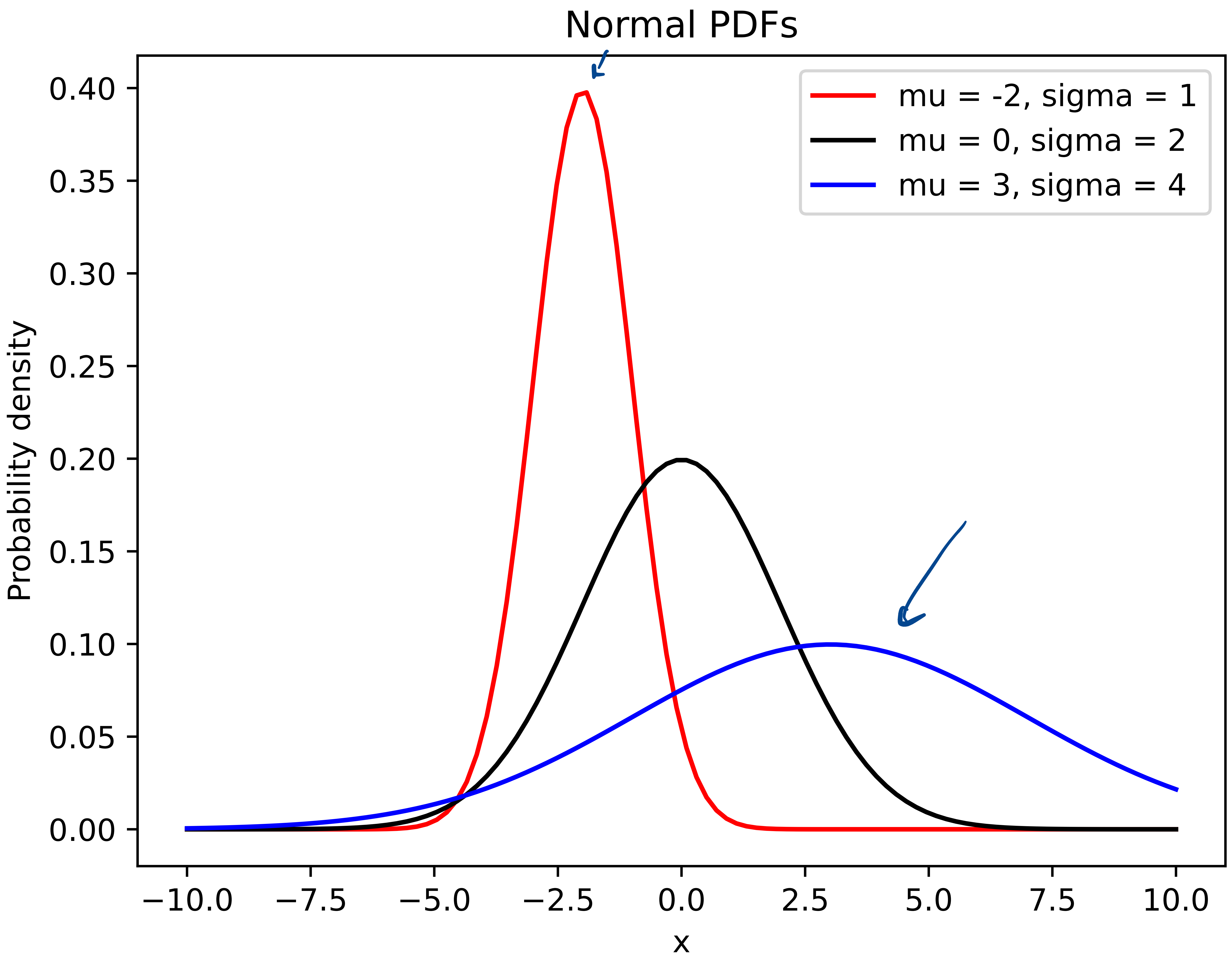
Normal Distribution: Visualization



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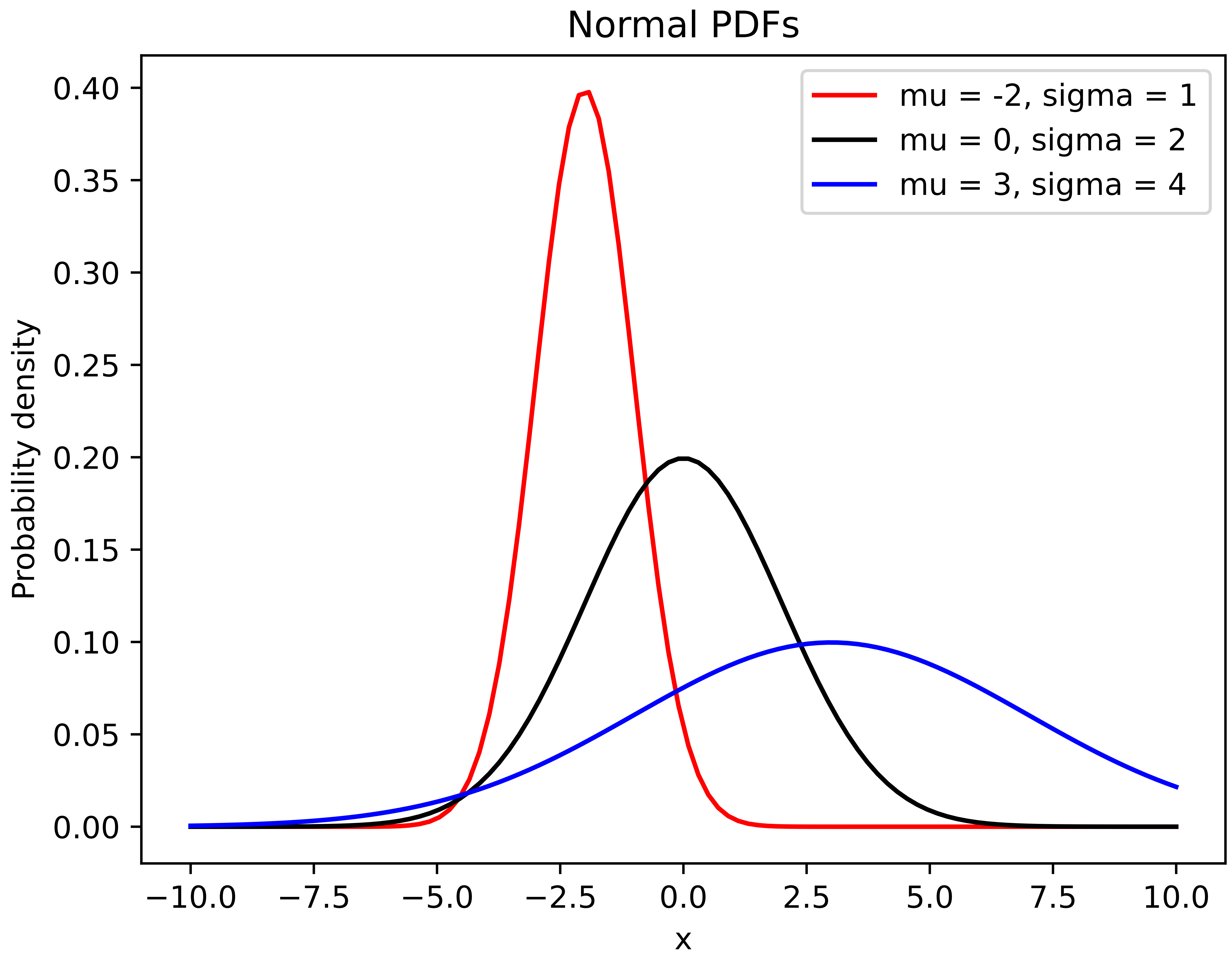

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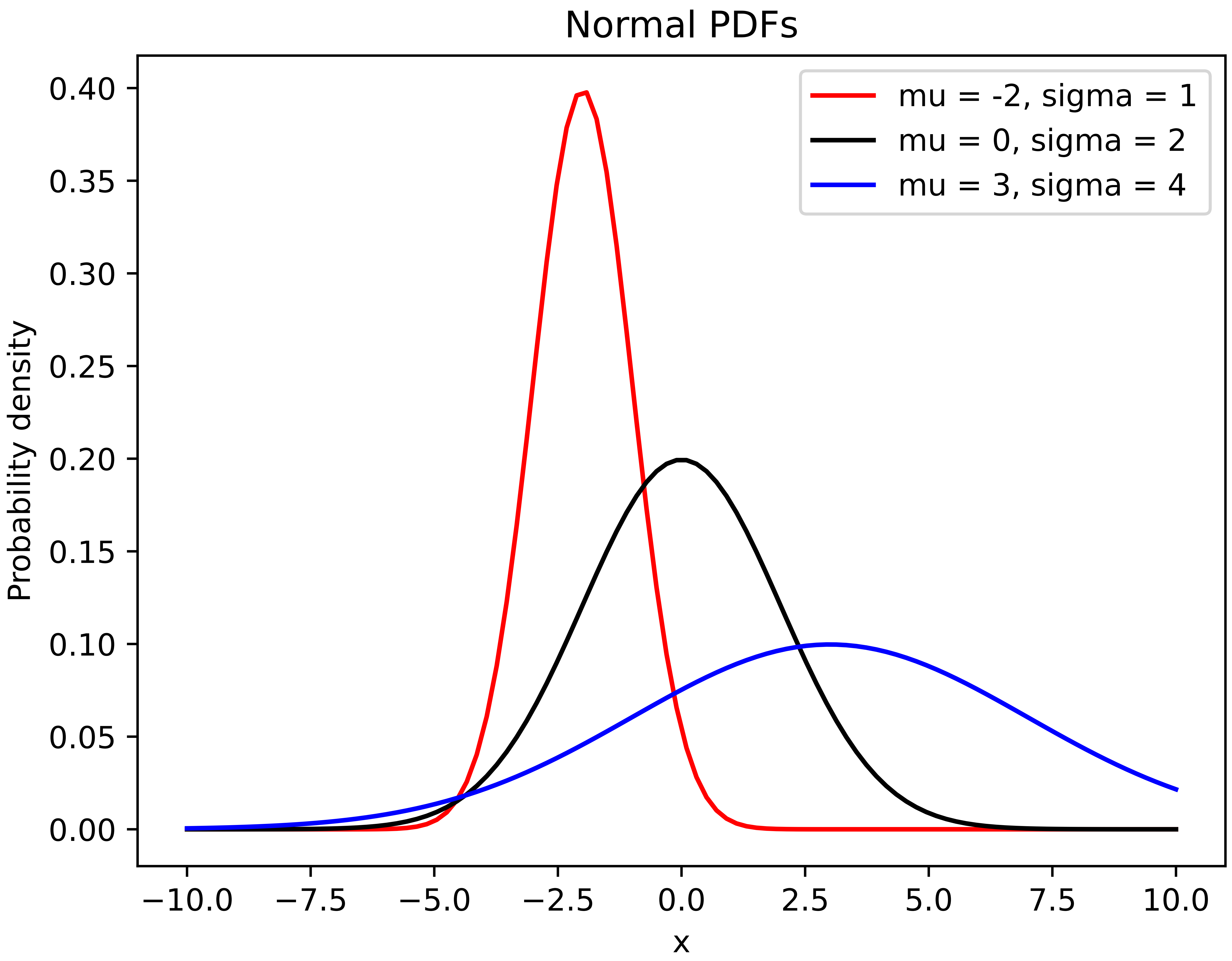
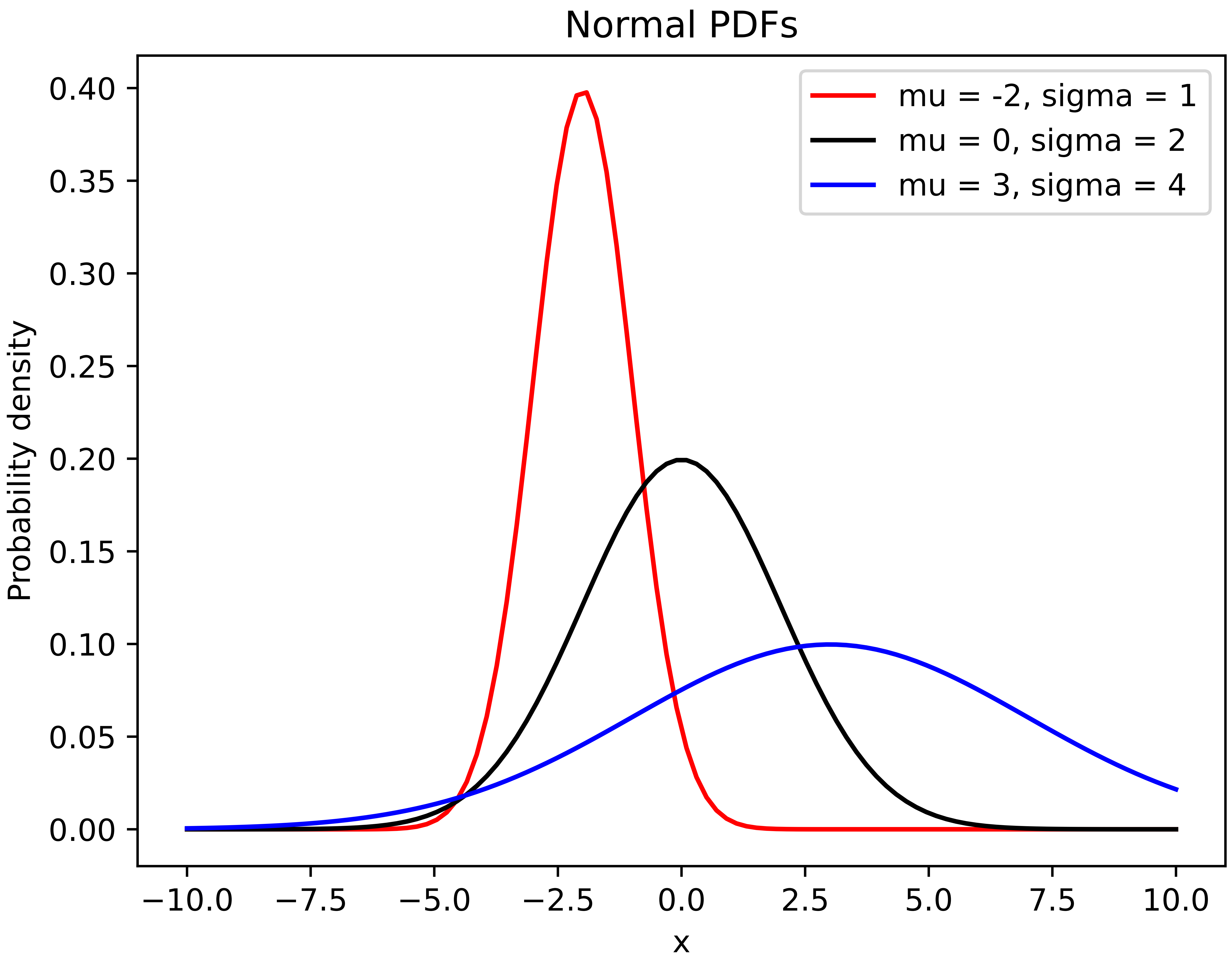


Normal Distribution: Visualization

- μ (center) and σ^2 (spread) fully define the normal distribution
- Always symmetric
- When $\mu = 0$ and $\sigma^2 = 1$, we have the *standard normal distribution*



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- Q: What is the probability that a person scored below 60?

$$Z = \frac{x - \mu}{\sigma} = \frac{60 - 78}{9} = -2 \qquad \approx 0.0228$$

Normal Distribution: Example

pnorm (90)

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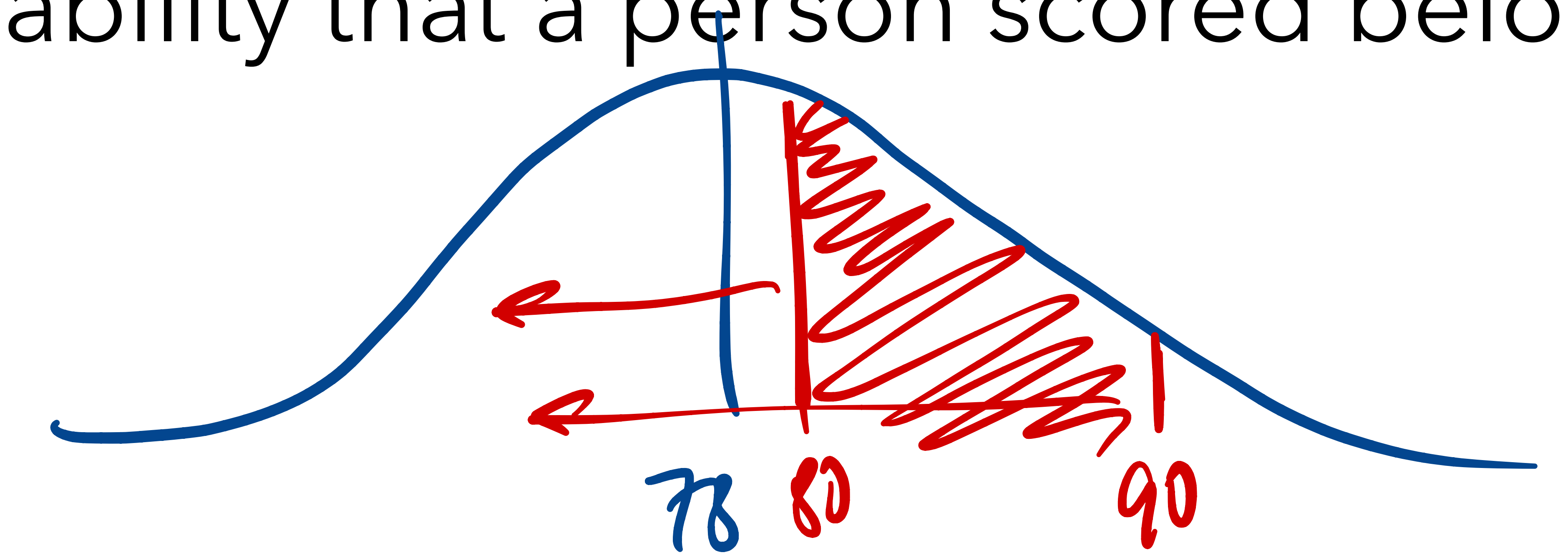

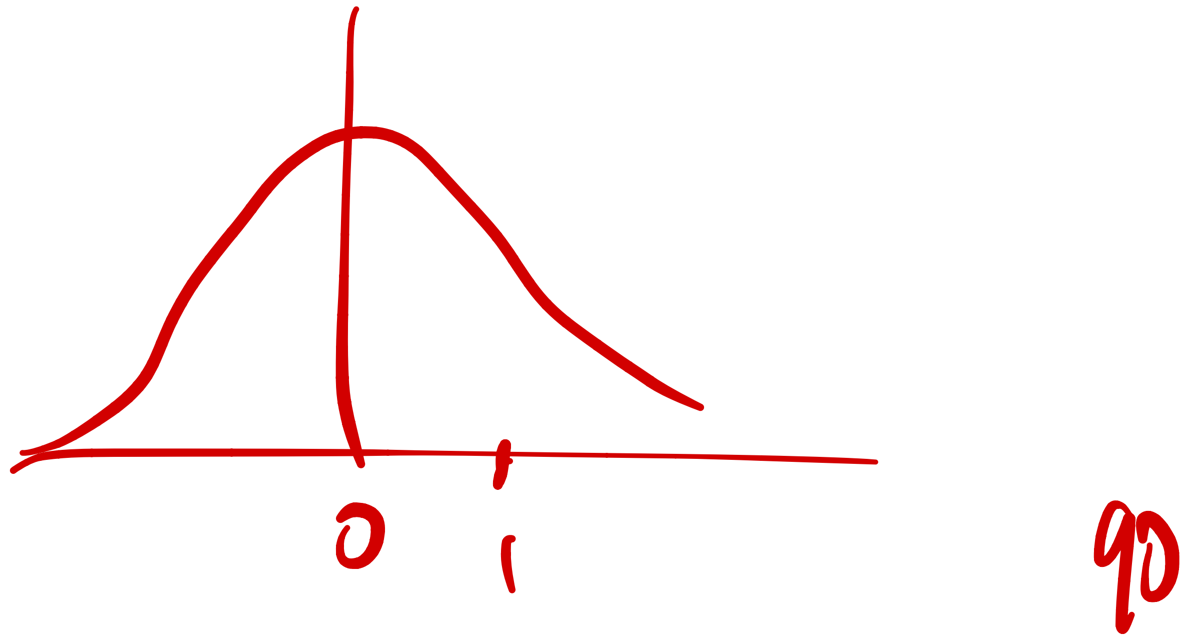

- Q: What is the probability that a person scored between 80 and 90?

$z_2 = \frac{90-78}{9} = \frac{4}{3}$ || $z_1 = \frac{80-78}{9} = \frac{2}{9}$

pnorm $(4/3)$ - pnorm $(2/9)$

$= .32$

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$\Pr(X > x)$

$\Pr(X \geq x)$

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- What is the probability that a person has a BMI in the range (18.5, 24.9)?

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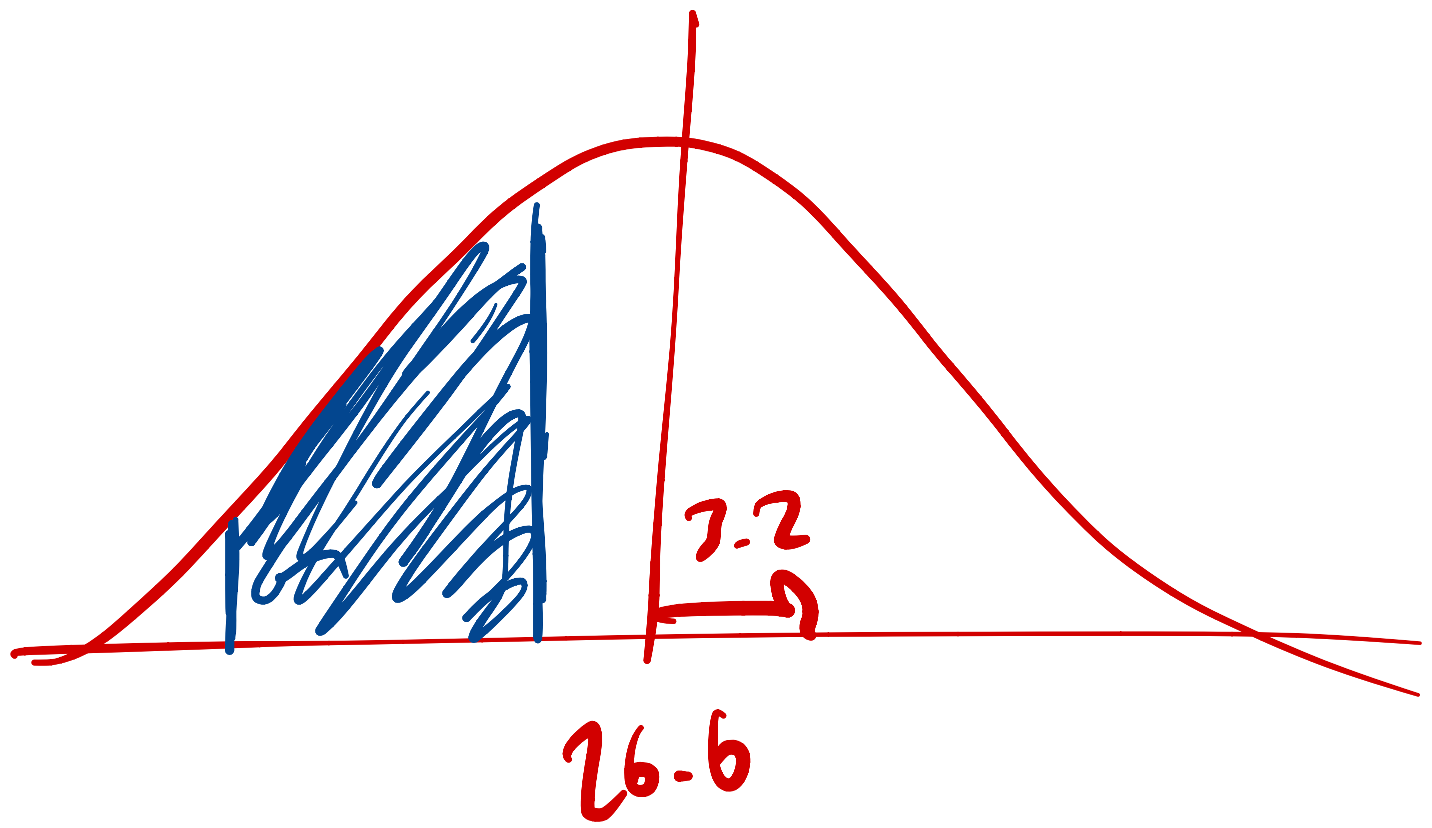

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- Empirical rule (68%, 95%, 99.7%) appears to be quite good

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- Directly in R: `qnorm(p, mean, sd)`; `qnorm(p)` for z value

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- Q1: Find the probability that a randomly selected patient in the ED weighs between 140 pounds and 210 pounds

x_1

x_2

$Pr(x_1 \le X \le x_2)$

$z_1 = \frac{140-160}{15} = -\frac{4}{3}$

$z_2 = \frac{210-160}{15} = \frac{10}{3}$

$\approx .9$

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- Q1: Find the probability that a randomly selected patient in the ED weighs between 140 pounds and 210 pounds

- Q2: Find the value that cuts off the upper 10% of the curve in American ED patient weights

$z =$ qnorm(.90, 160, 15)

$z = \underline{1.282} = \frac{x - \mu}{\sigma} \rightarrow x = 15 \times 1.282 + 160 \approx \underline{179.2}$

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- Standard deviation of $\bar{X}$ is $\dfrac{\sigma}{\sqrt{n}}$; this is known as the **standard error**

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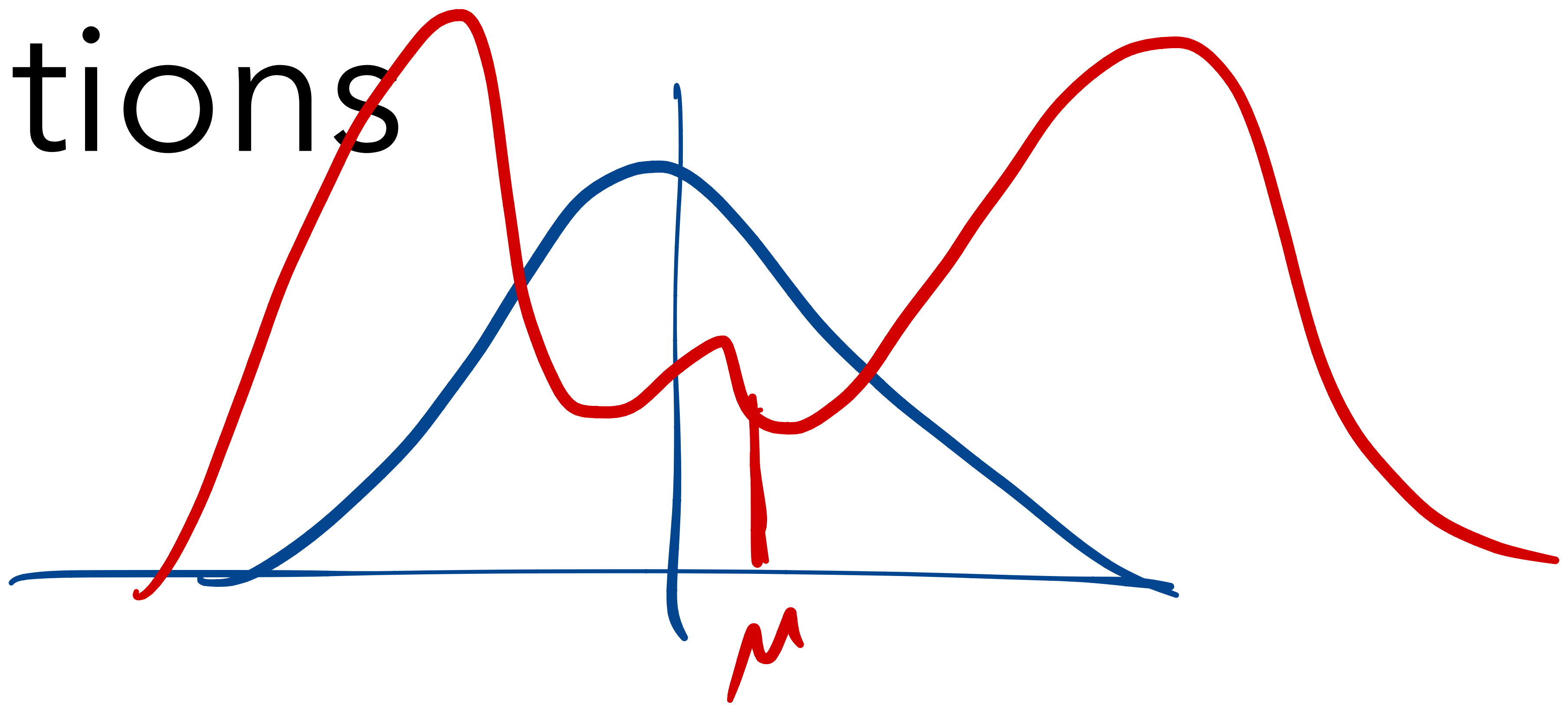

- If the population we are sampling from is normal, then the distribution of $\bar{X}$ will also be normal

- If $X \sim N(\mu, \sigma)$, then $\bar{X} \sim N\left(\mu, \dfrac{\sigma}{\sqrt{n}}\right)$

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- If the population we are sampling from is not normal, then we can use the *Central Limit Theorem (CLT)* to get the distribution of $\overline{X}$
- **Central Limit Theorem**: If the population we are sampling from is not normal, then the shape of the distribution of $\overline{X}$ will be normal as long as n is sufficiently large (typically $\boxed{n \geq 30}$ suffices)

Berry - Esseen

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- In notation: $\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$

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$N(\mu, \sigma)$

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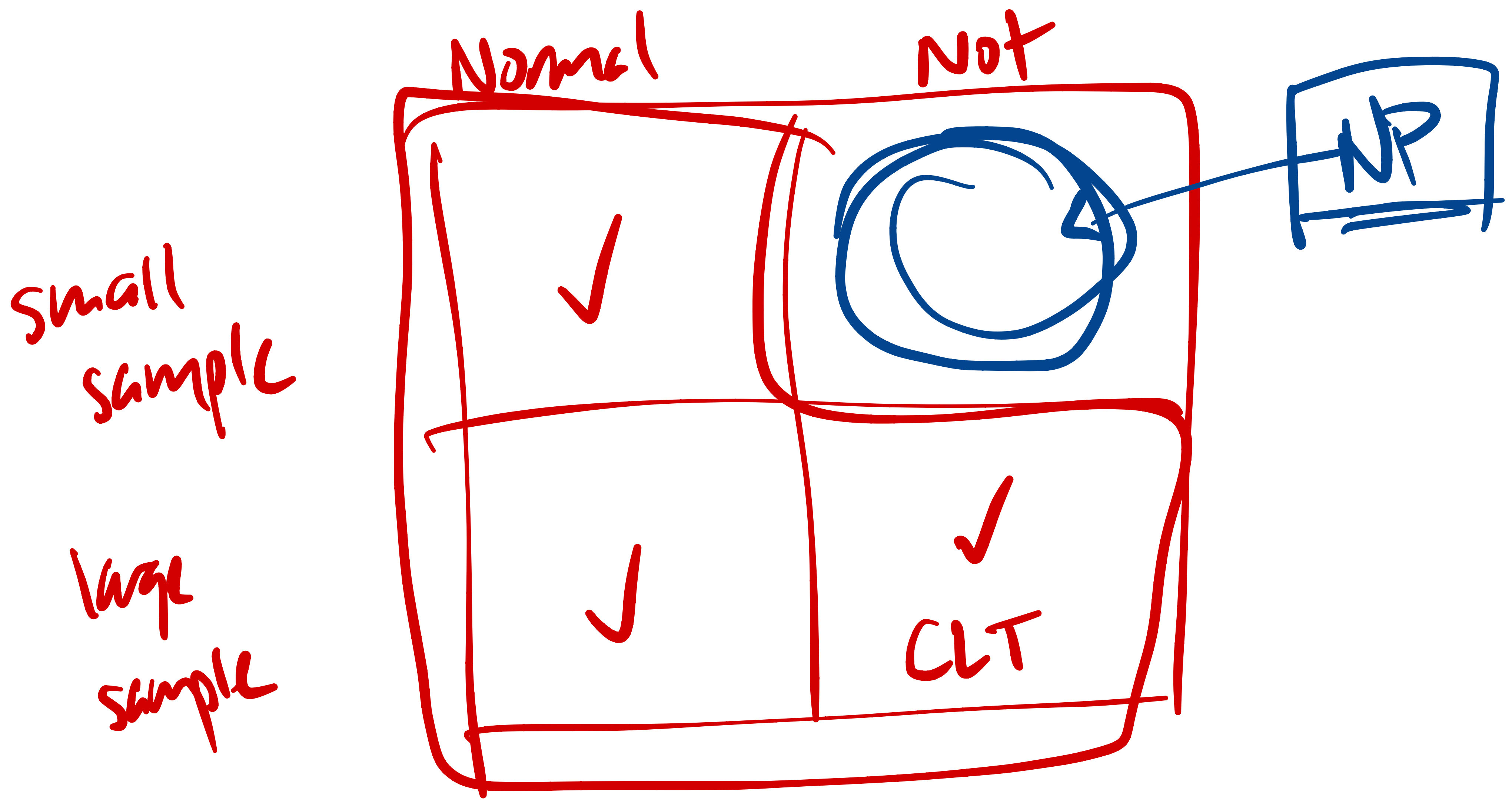

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- Since $\bar{X} \sim N\left(\mu, \sigma/\sqrt{n}\right)$, we can standardize $\bar{X}$ to a standard normal distribution as follows:

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Sampling Distributions Example

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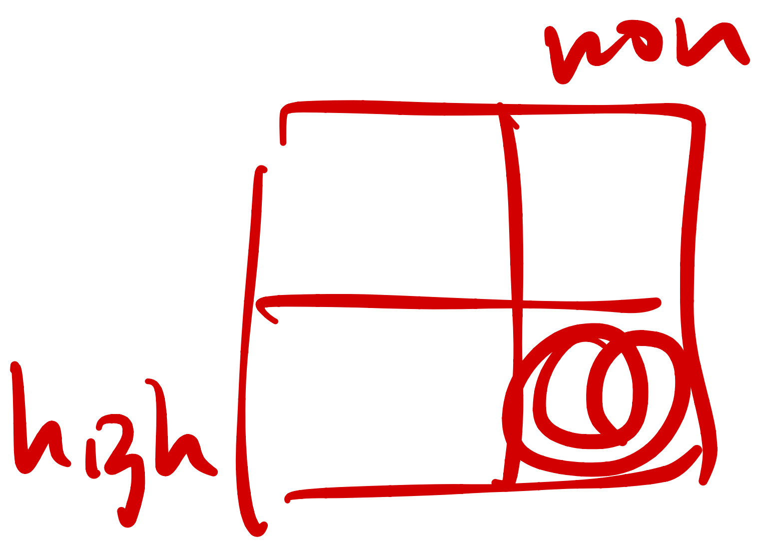
$E[\bar{x}] = \mu$

Sampling Distributions Example



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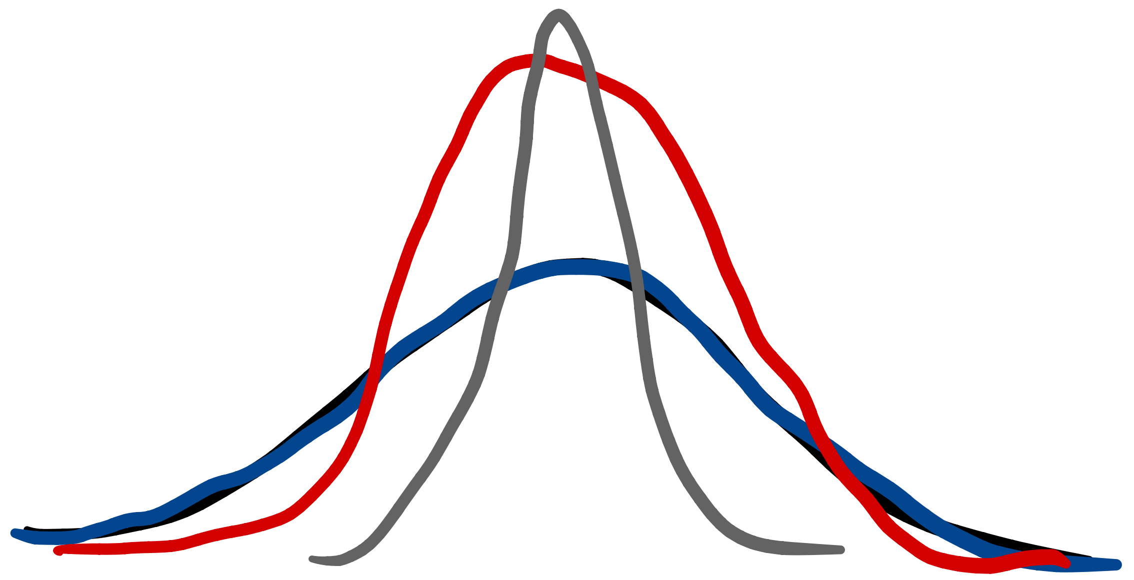
- Q1: What is the mean of the distribution of sample means? $\mu = 450K$

- Q2: What is the standard error of the sample mean?

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{100K}{8} = 12.5K$$

Sampling Distributions Example



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- Q1: What is the mean of the distribution of sample means? 450K

- Q2: What is the standard error of the sample mean? 12.5K

- Q3: What distribution does the sample mean follow?

$$\bar{X} \sim N(450K, 12.5K) \qquad \sigma' = \frac{\sigma}{\sqrt{n}}$$

Sampling Distributions Example

$1 - Pr(Z < 0.5)$

$\approx 40\%$

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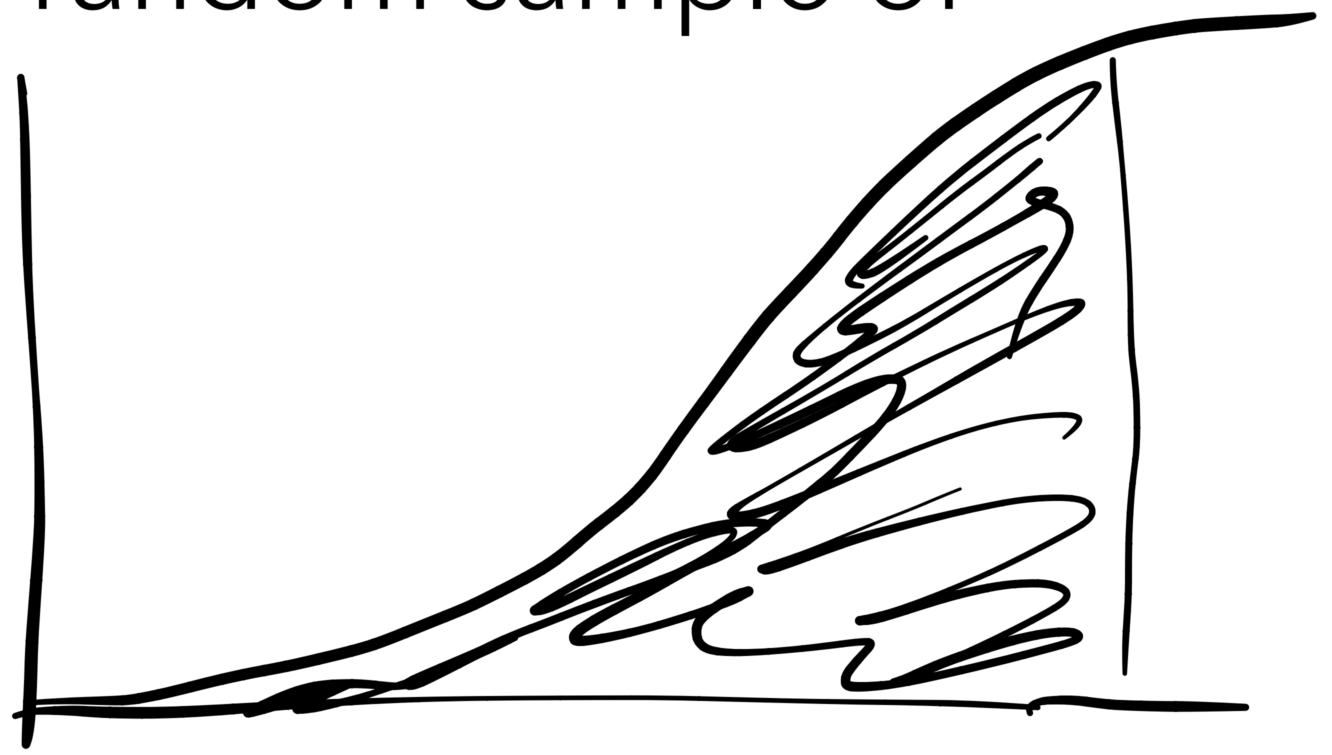


- Q1: What is the mean of the distribution of sample means?
- Q2: What is the standard error of the sample mean?
- Q3: What distribution does the sample mean follow? $N(450, 12.5)$

$Pr(Z \leq 4)$

- Q4: What is the probability that the sample mean of $n = 64$ house prices is greater than \$500,000?

$Pr(\bar{X} > 500) = 1 - Pr(\bar{X} \leq 500) = 3.5 \times 10^{-5}$

pnorm$(500, 450, 12.5)$

Sampling Distribution of a Proportion

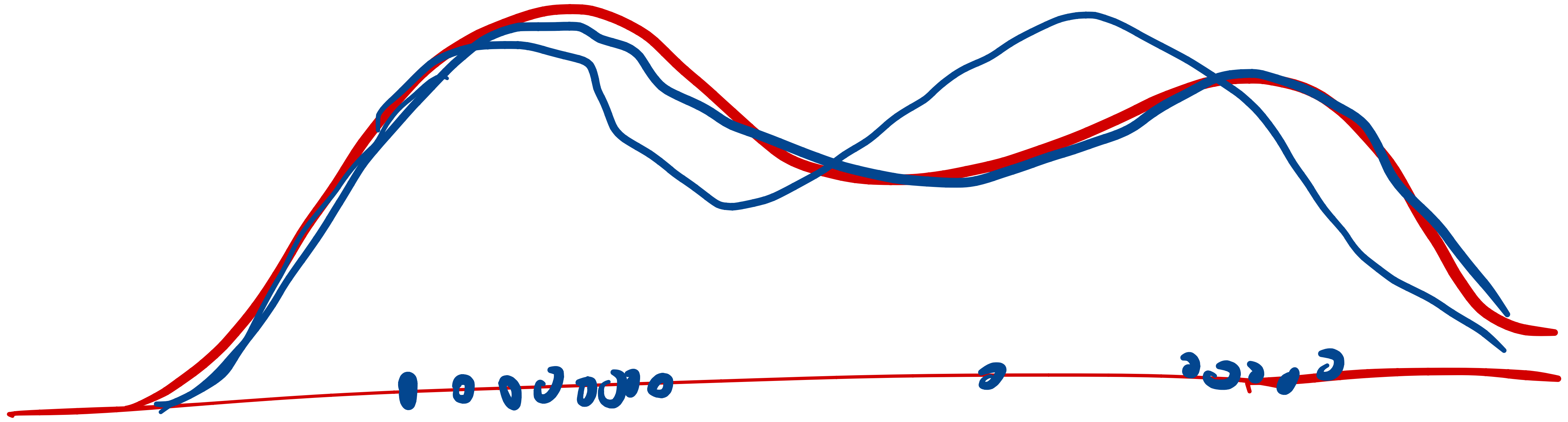
$m = 1$

$\bar{X} \rightarrow$ samples of size $\boxed{n}$

repeat $\underline{m}$ times

	N	not N
n small	✓	?
n big	✓	✓

$\bar{X} \sim D(n)$



Sampling Distribution of a Proportion

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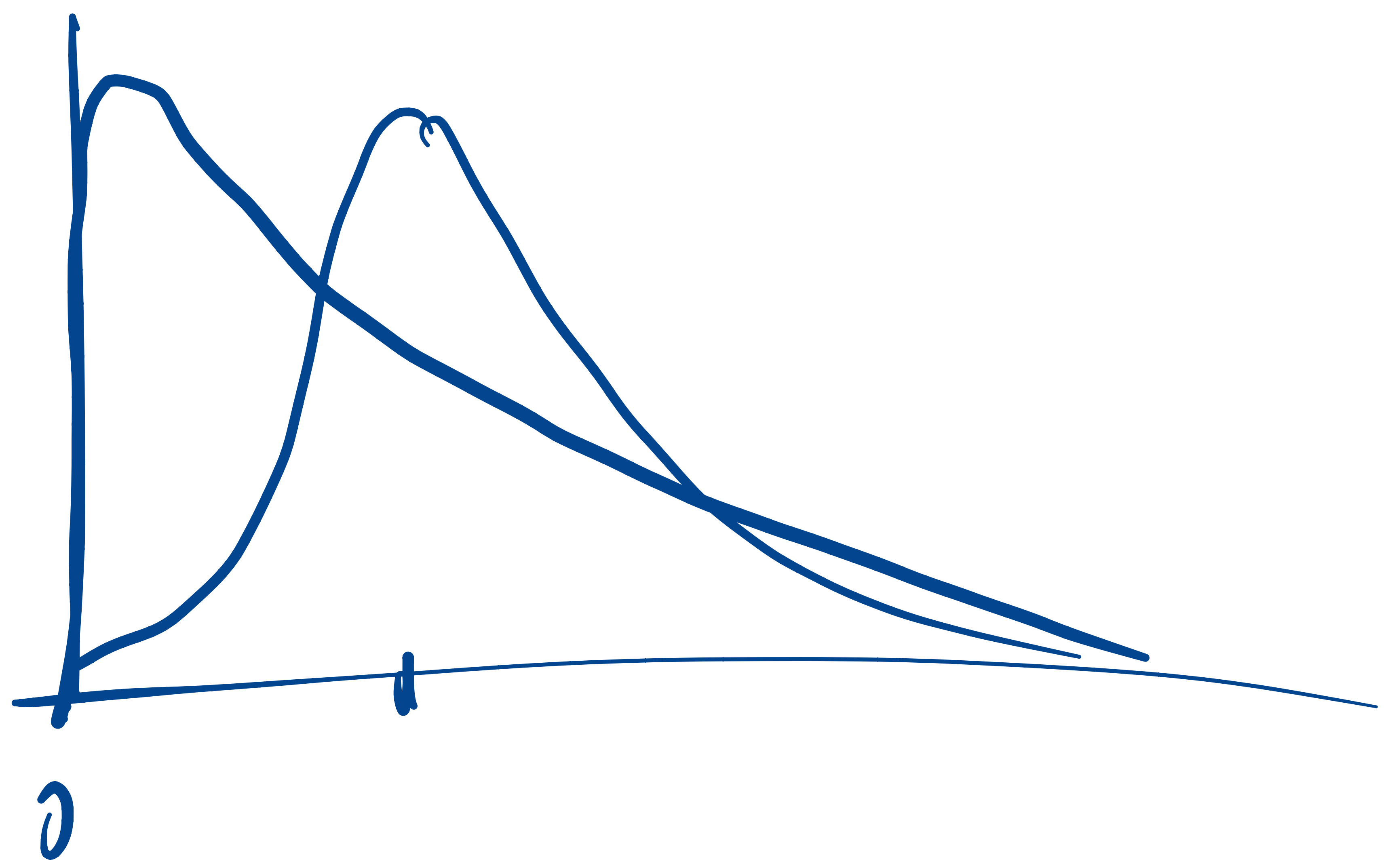

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