Chapter 2: Descriptive Statistics and Displays

DSCC 462
Computational Introduction to Statistics

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Fall 2022

Plan for Today

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- Visualize datasets using graphs

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- Describe important aspects of data using measures of **center** and **spread**

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- Describe important aspects of data using measures of **center** and **spread**
- Transform data to allow for statistical tests

Describing Data

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- The type of summary statistics you use will depend on the type of data

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- Make plots and construct tables to see what the distribution looks like

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Cause of Death	Number of Deaths
Cancer	12
Heart Attack	30
Stroke	10
Car Accident	53
Other	37

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Cause of Death	Number of Deaths	Relative Frequency (%)	
Cancer	12	8.45 ←	12/142
Heart Attack	30	21.13	30/142
Stroke	10	7.04	⋮
Car Accident	53	37.32	
Other	37	26.06	
	/142		

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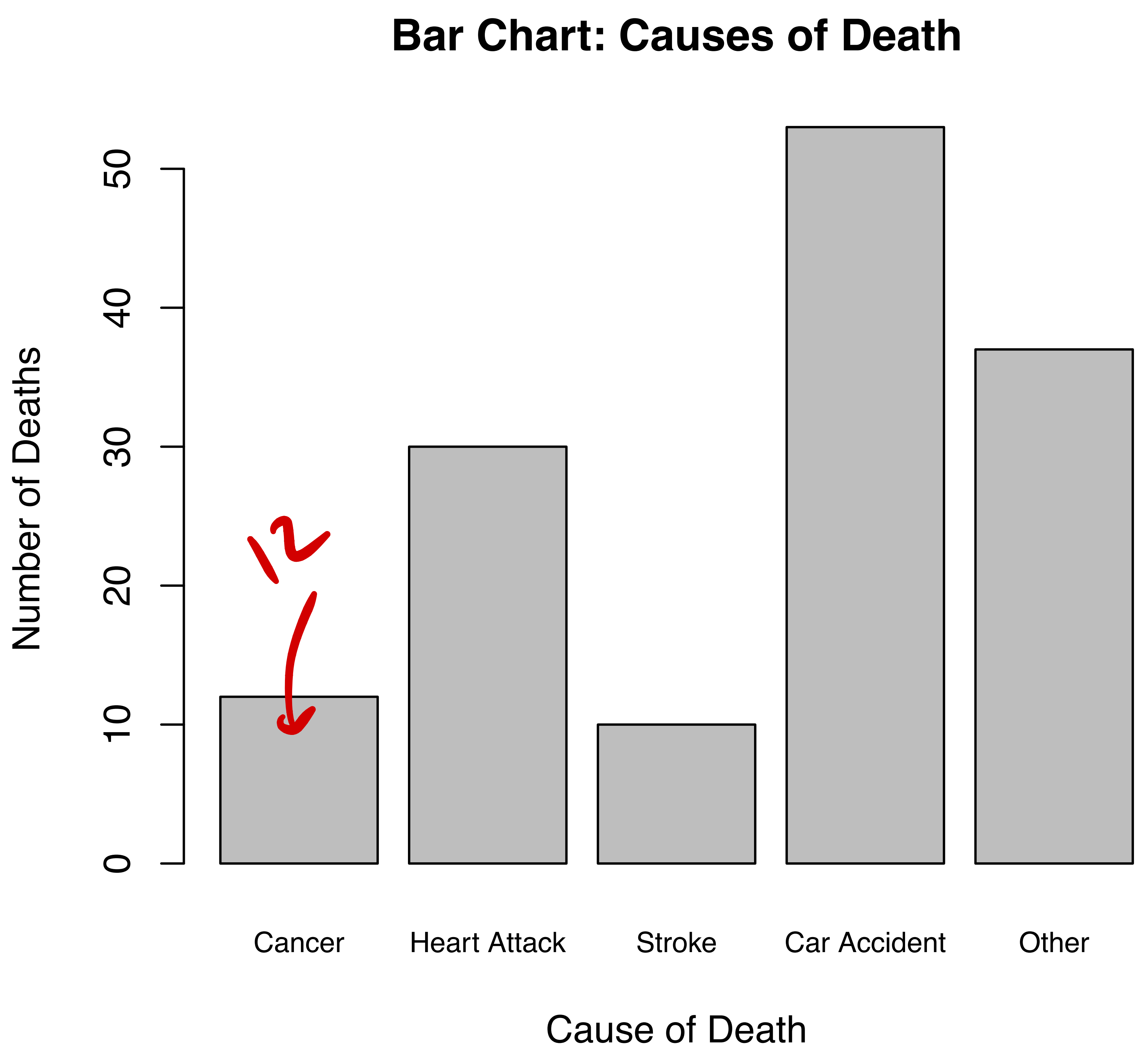
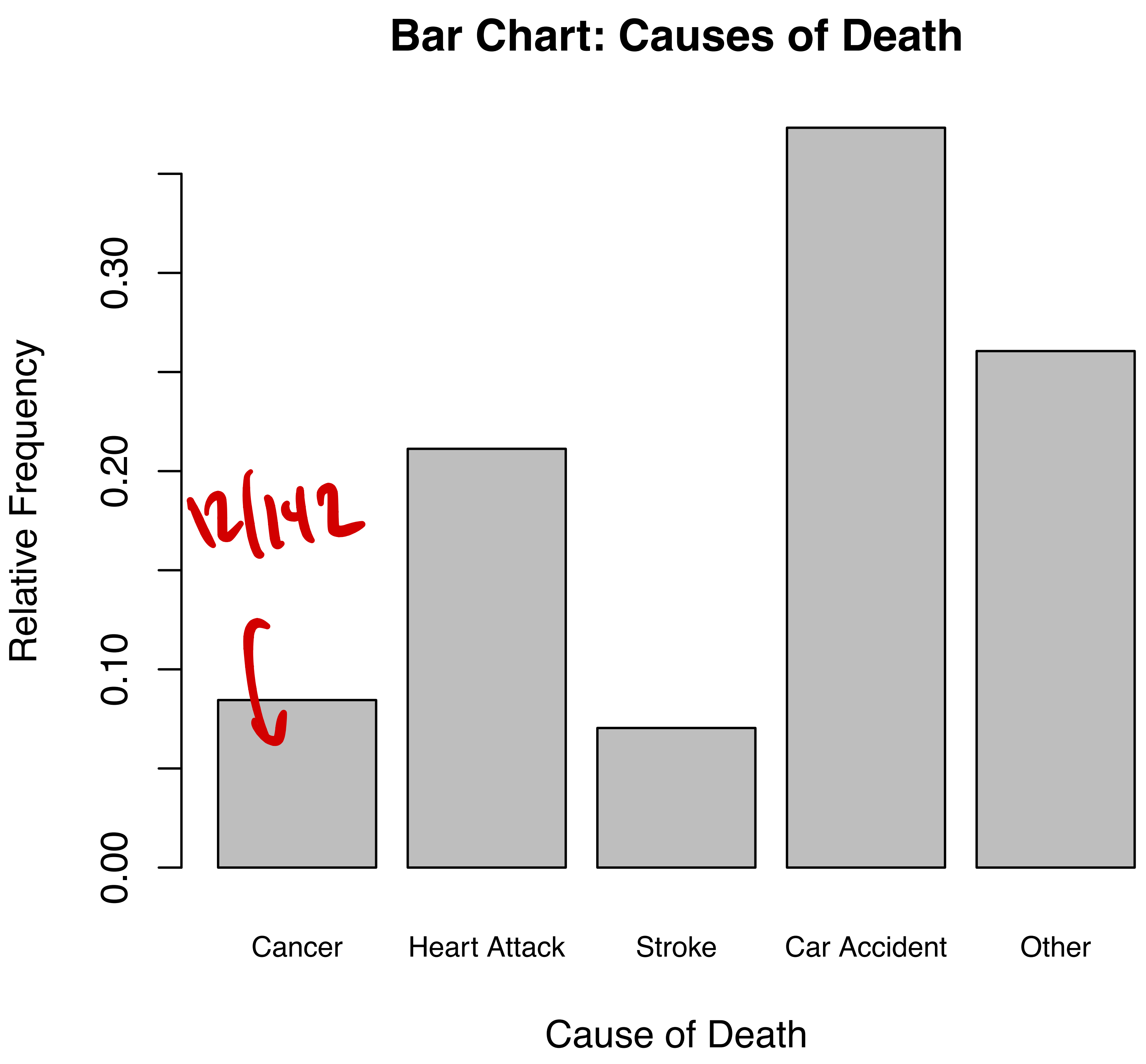
- Bars do not touch and are all of the same width

Bar Charts

Abs



Rel



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- Typically, we are interested in how much of the data falls within a given range
    - Generally, ranges should be of equal length

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$\lceil x \rceil$ $\lceil 1.2 \rceil = 2$ $\lceil 3 \rceil = 3$

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- By default, R uses Sturges' formula: number of bins $k = \lceil \log_2(n) \rceil + 1$

  # datapoints

Histogram

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- Since the horizontal axis represents a continuous number line, there should *not* be breaks between bars of the histogram

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- The only time that there should be space between histogram bars is when 0 entries are observed in a given interval

Histogram Workflow

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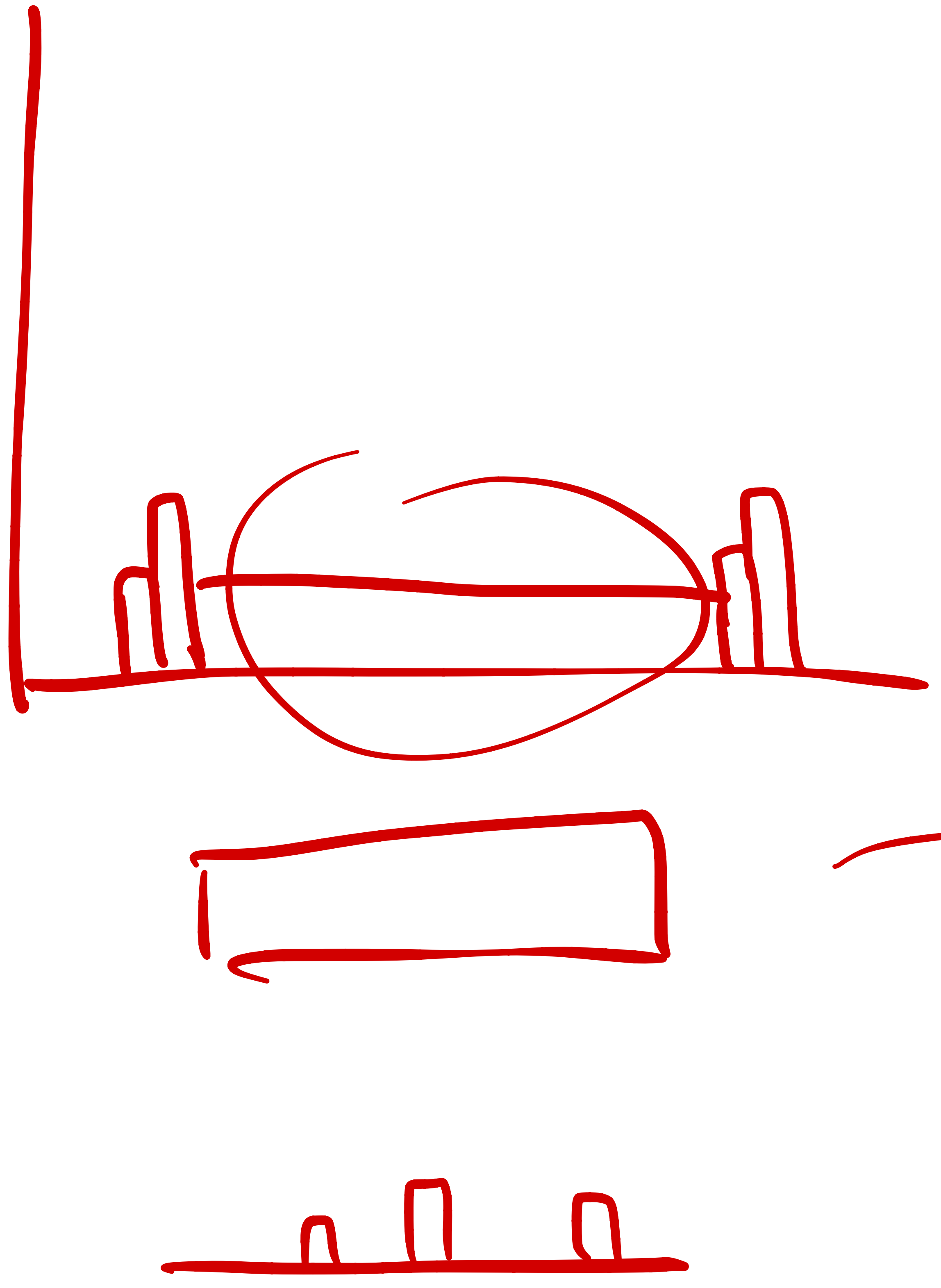
4. Determine how many observations fall within each bin
5. Indicate position of the class interval on the horizontal axis
6. At each class interval, draw a vertical bar equal in height to the number of observations in that class

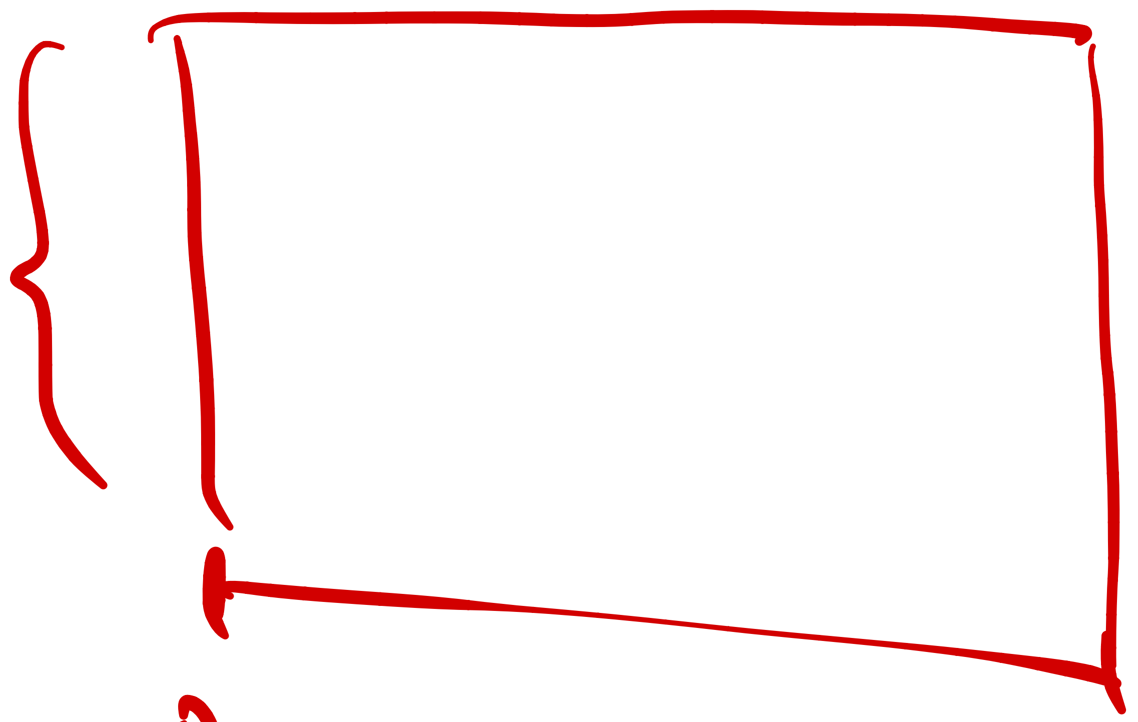
Histogram Example



165.5	158.1	154.7	178.9	180.9	194.5	174.1	160.5
202.0	259.8	125.5	160.0	160.9	179.0	195.0	149.3
194.8	148.0	193.9	162.2	159.1	206.3	178.7	164.5
204.4	179.5	126.9	196.6	231.1	202.8	174.0	181.9
186.4	198.0	150.8	172.2	187.1	152.8	169.6	206.6
177.3	148.0	196.7	179.7	180.6	152.6	185.3	158.7
211.3	185.1	177.3	184.6	165.7	170.1	178.4	184.2

Table 1: Weights (lbs) of 56 patients

56



0

400

Histogram Example

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- Range: 120 - 260, so each bin has length 20
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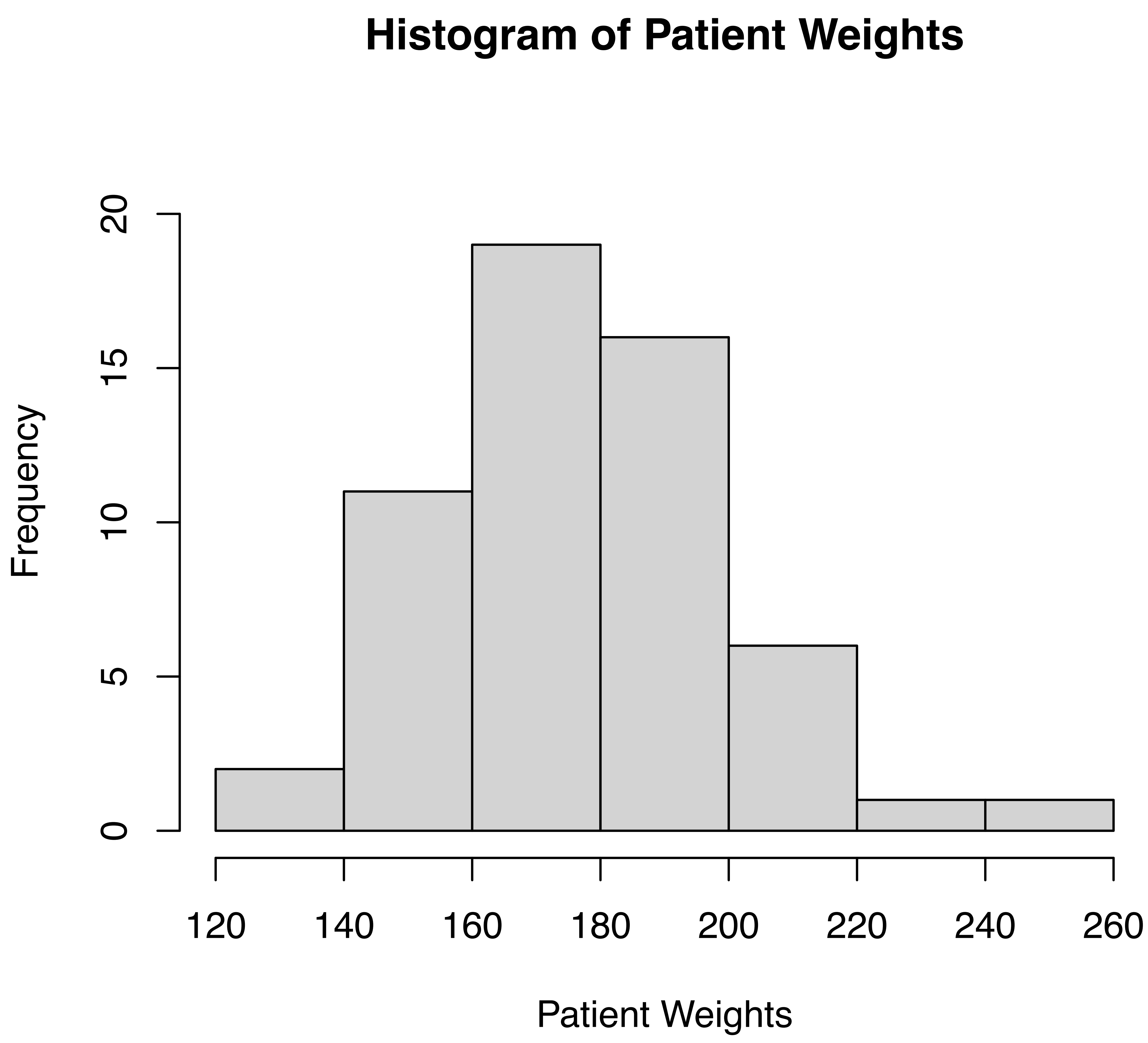
Weight	Frequency
(120, 140]	2
(140, 160]	10
(160, 180]	20
(180, 200]	16
(200, 220]	6
(220, 240]	1
(240, 260]	1

Histogram Example: Count

Abs

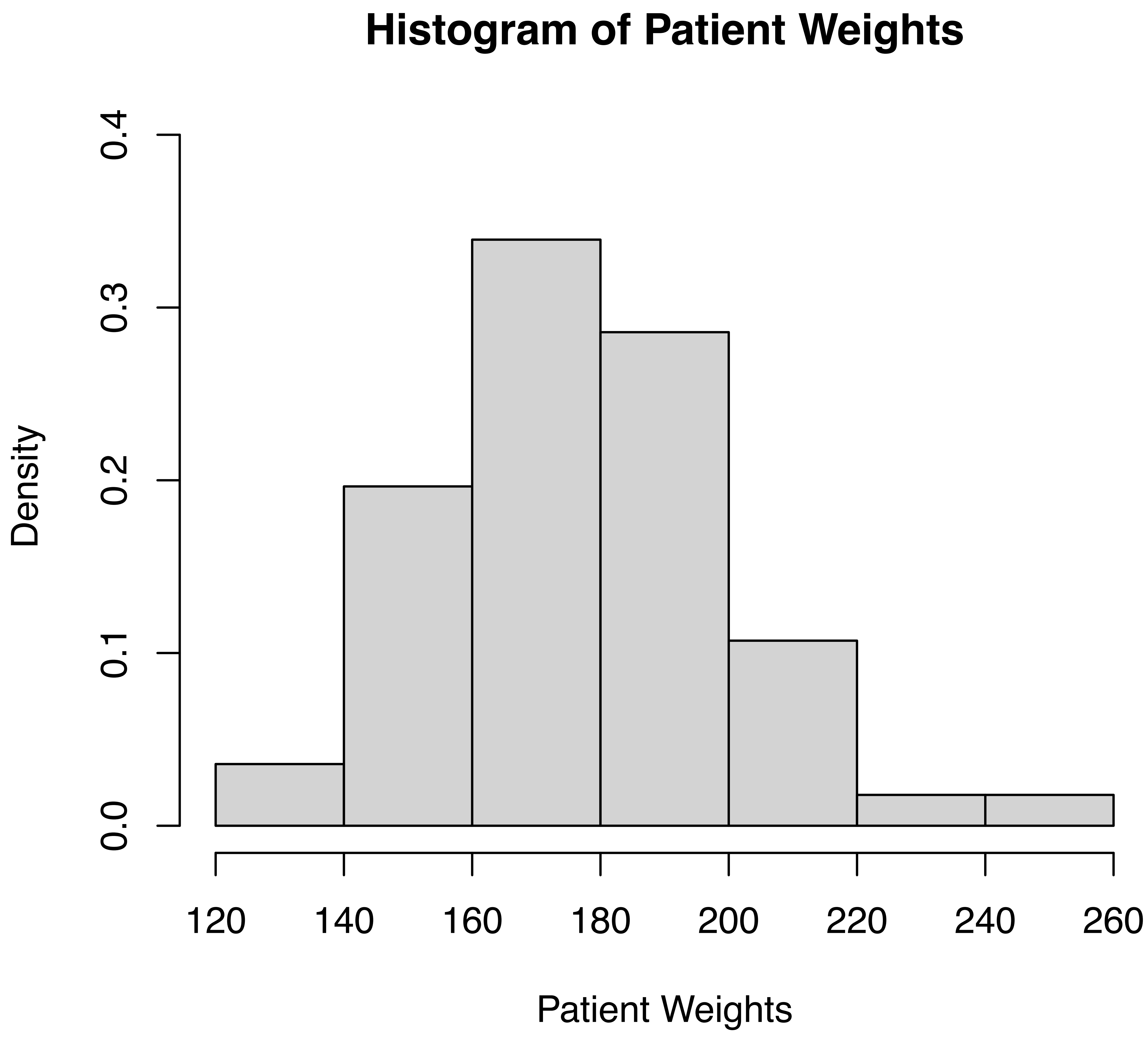
Weight	Frequency
(120, 140]	2
(140, 160]	10
(160, 180]	20
(180, 200]	16
(200, 220]	6
(220, 240]	1
(240, 260]	1



Histogram Example: Density

Rel

Weight	Rel. Frequency
(120, 140]	0.04
(140, 160]	0.20
(160, 180]	0.34
(180, 200]	0.29
(200, 220]	0.11
(220, 240]	0.02
(240, 260]	0.02



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- Typically, we want to examine a histogram's:
  - Center
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  - Spread
- Be aware of deviations from the typical pattern / extreme points
- **Outliers**: Data that are not typical of the rest of the values in the dataset

Center, Shape, and Spread

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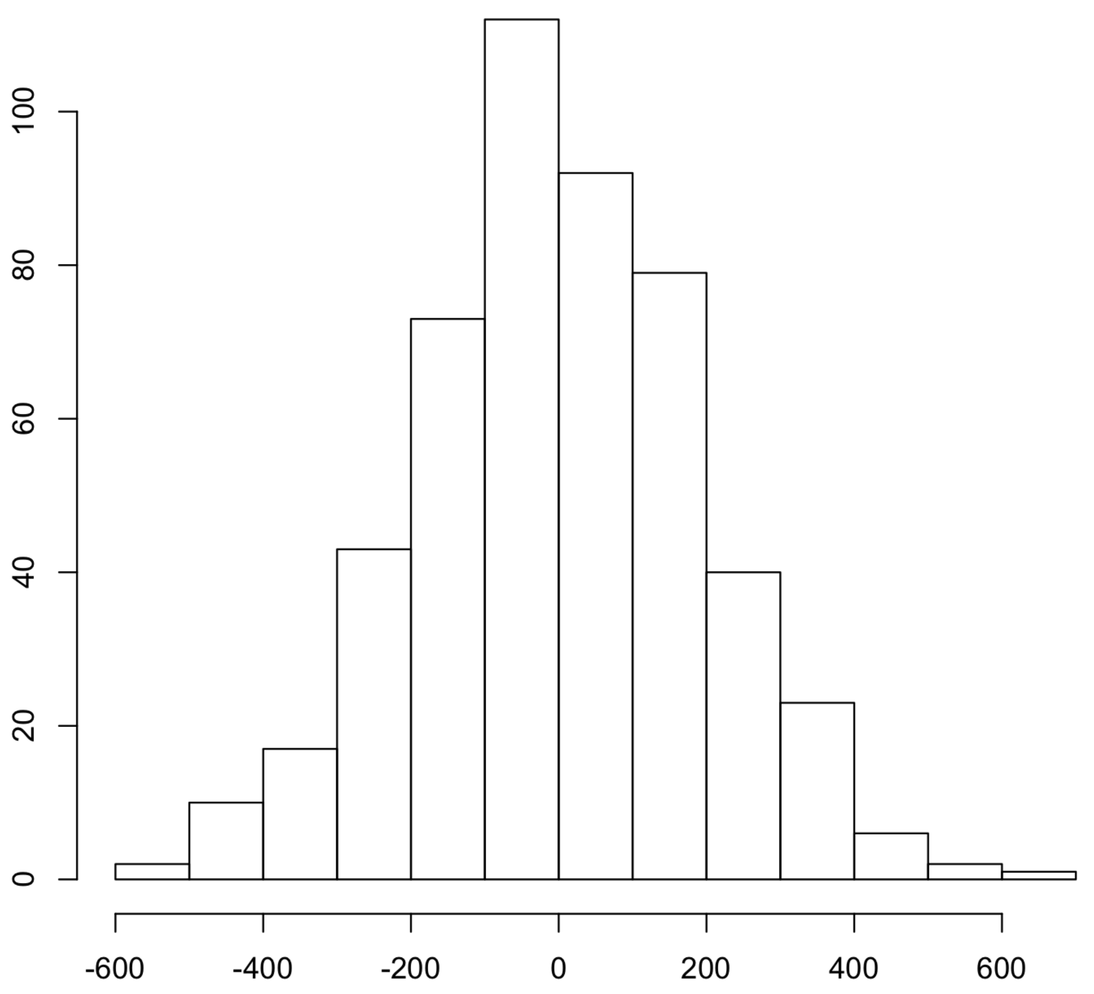
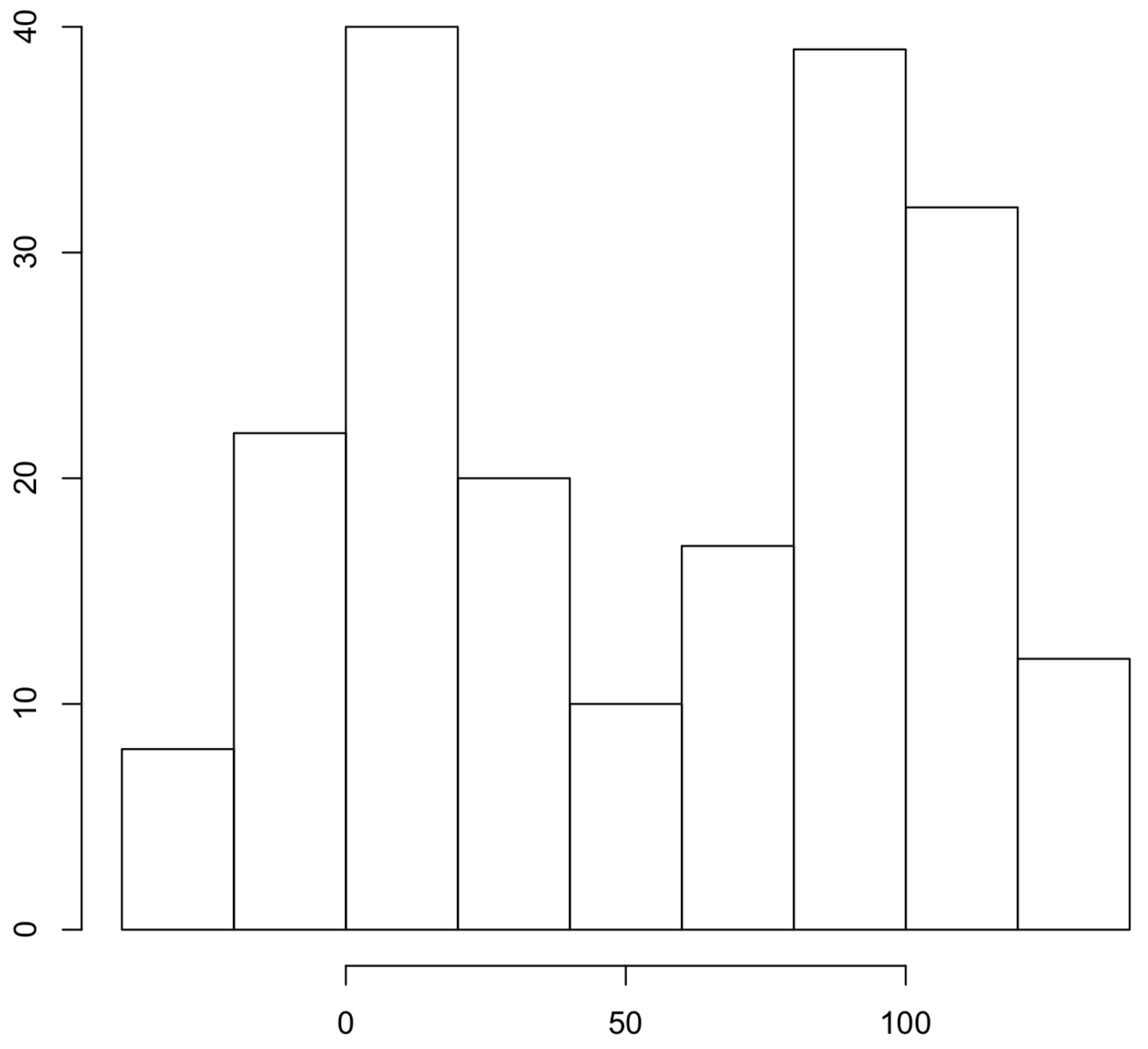
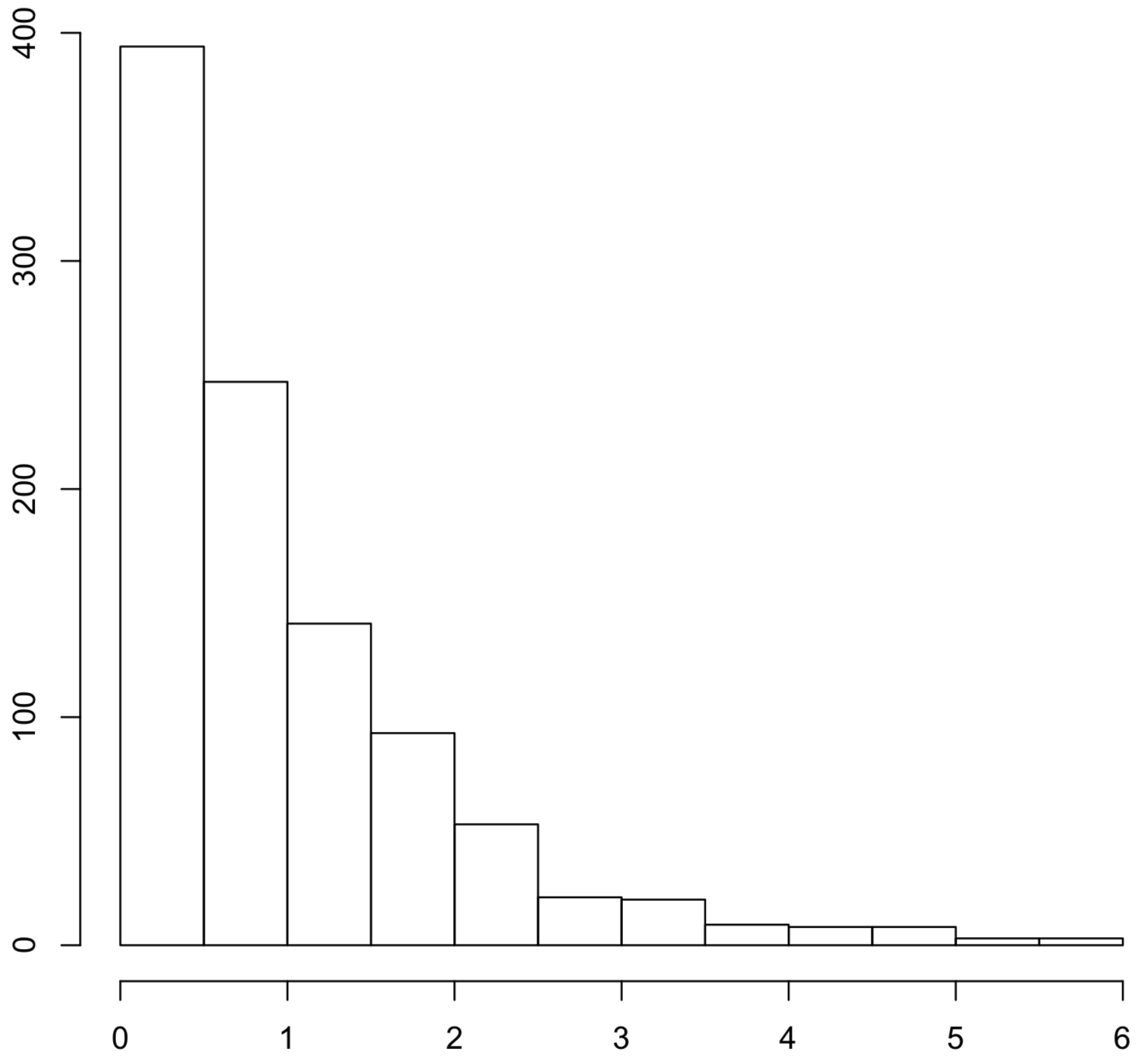
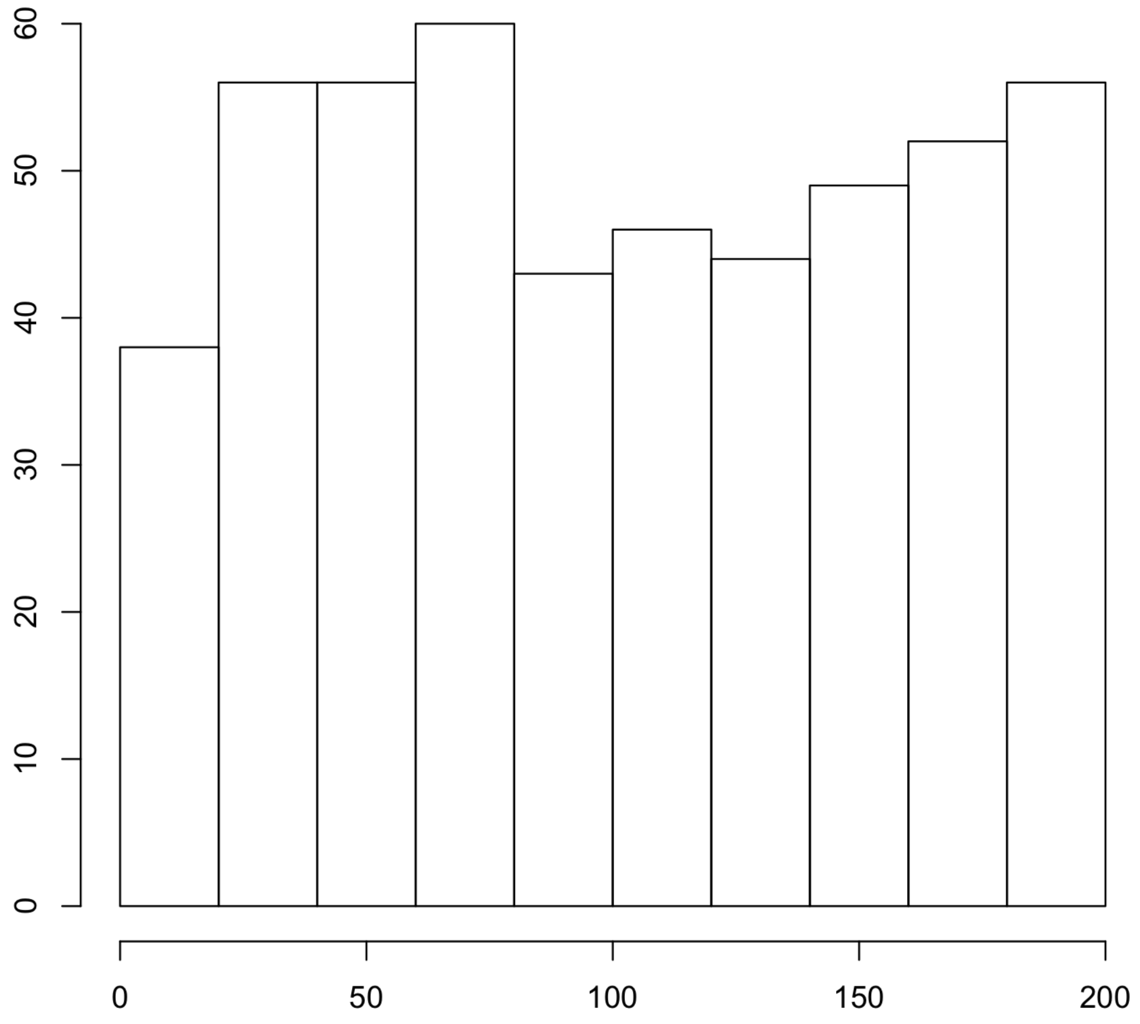
- Does the distribution appear to have one center, or are there multiple peaks?
- Is the distribution symmetric?
- Are the data all close together, or are they largely spread over the domain?
- Do any data points seem to deviate from typical patterns?

Center, Shape, and Spread: Examples









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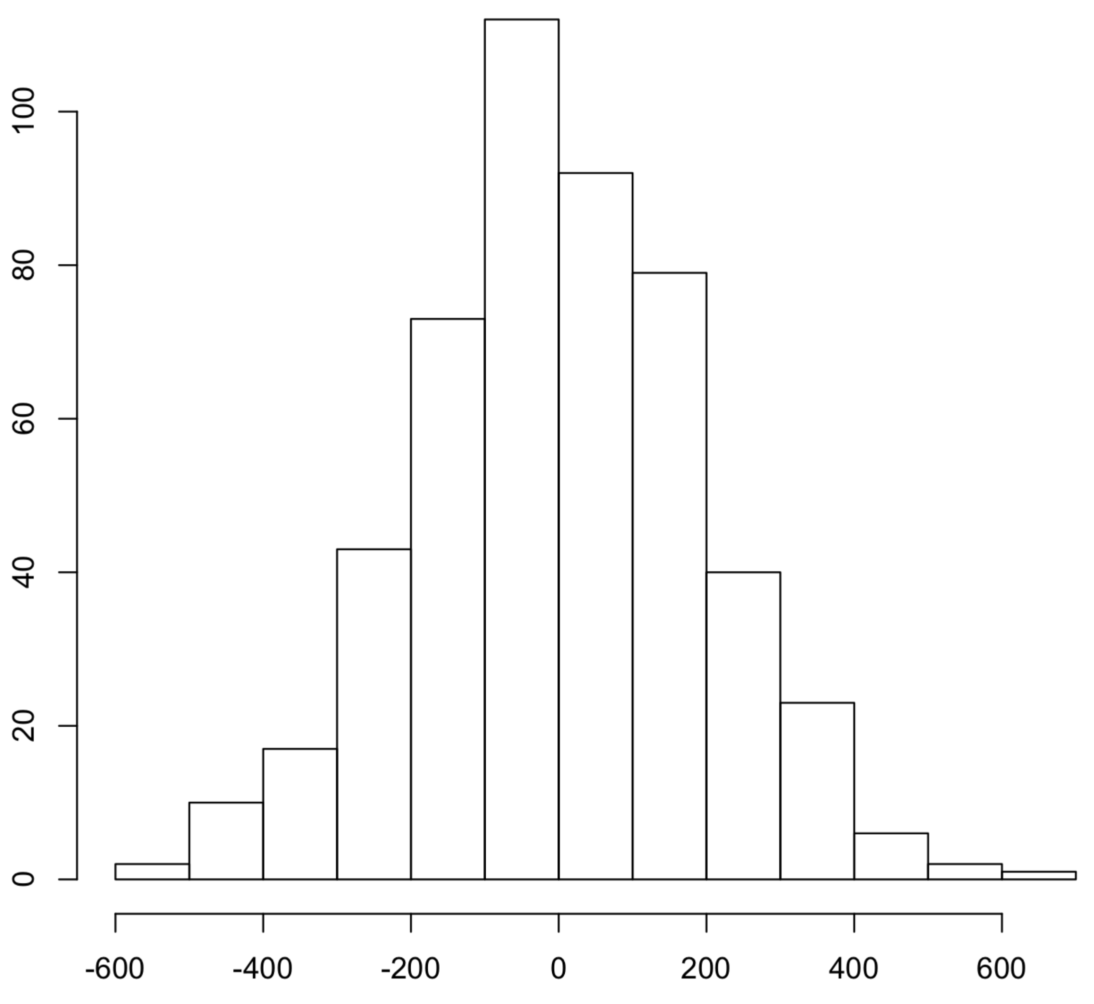

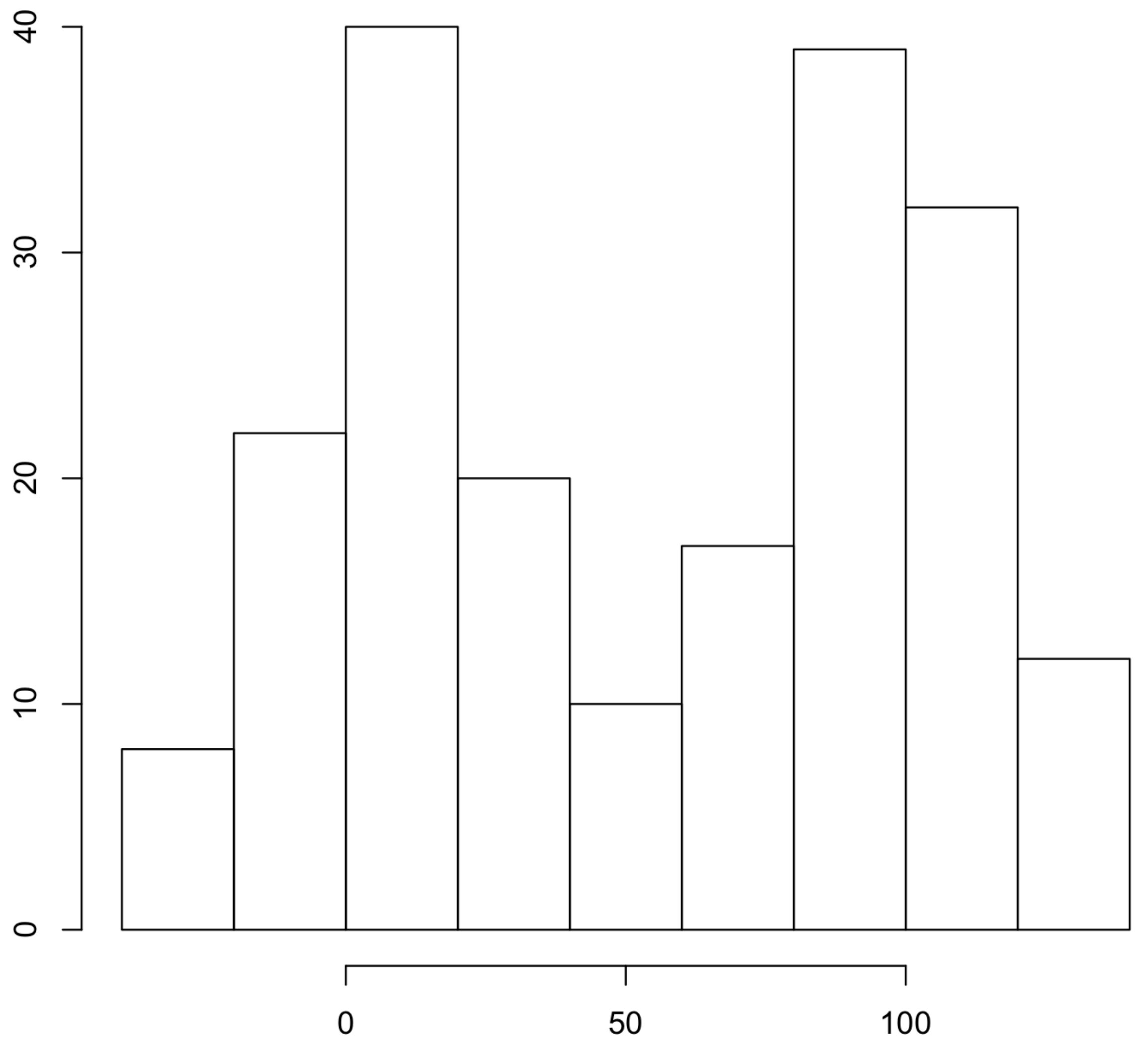

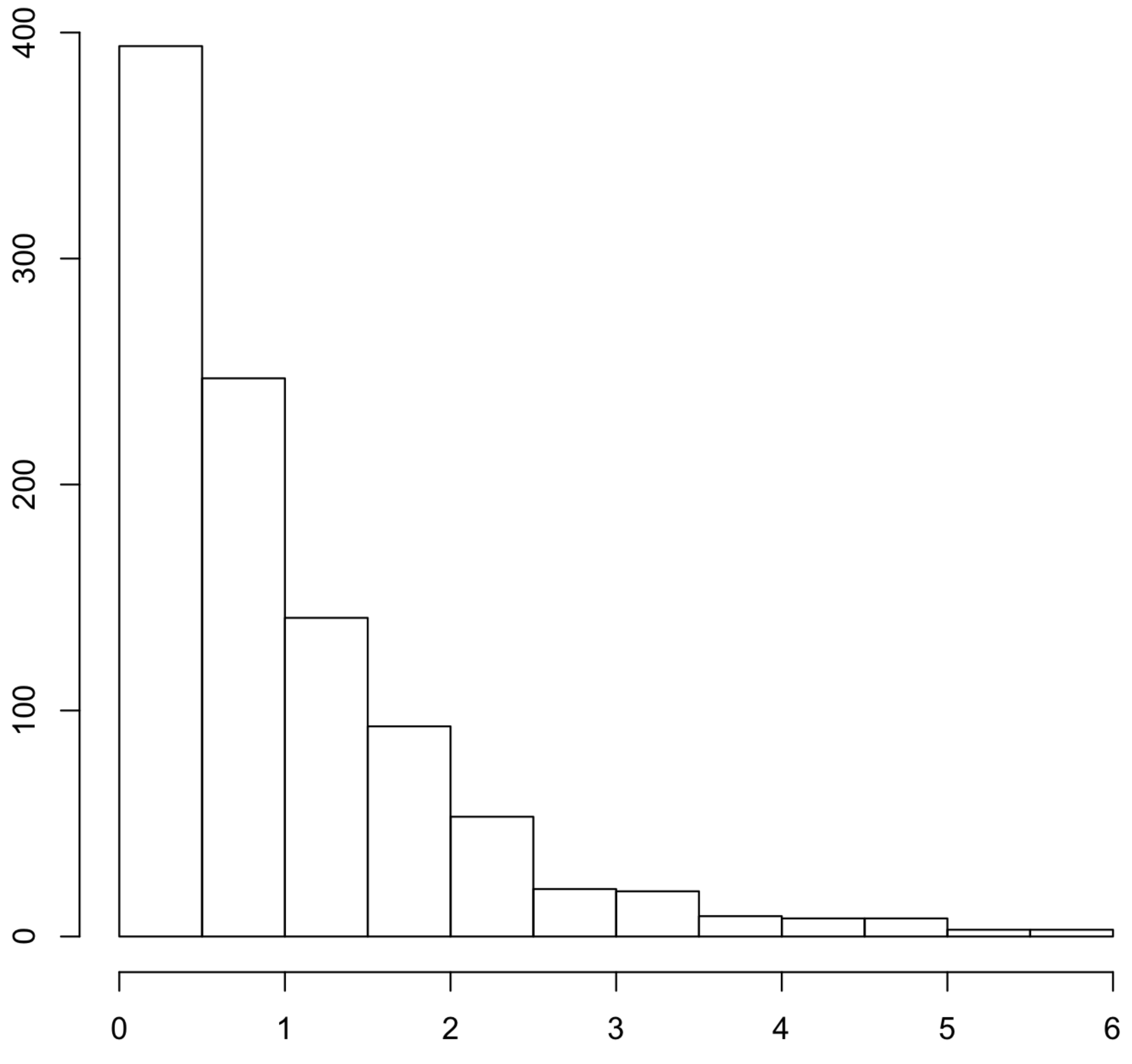

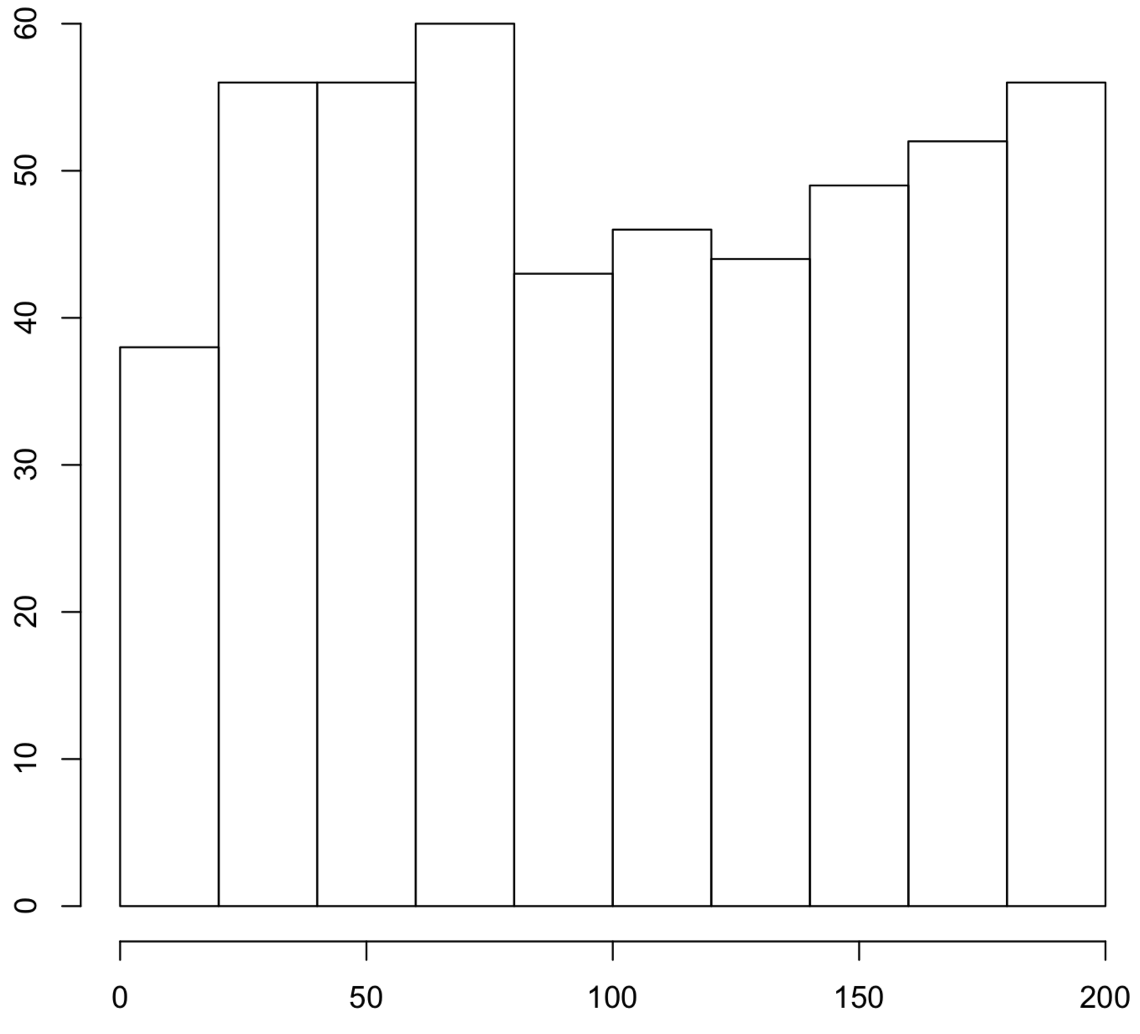

One center
Symmetric

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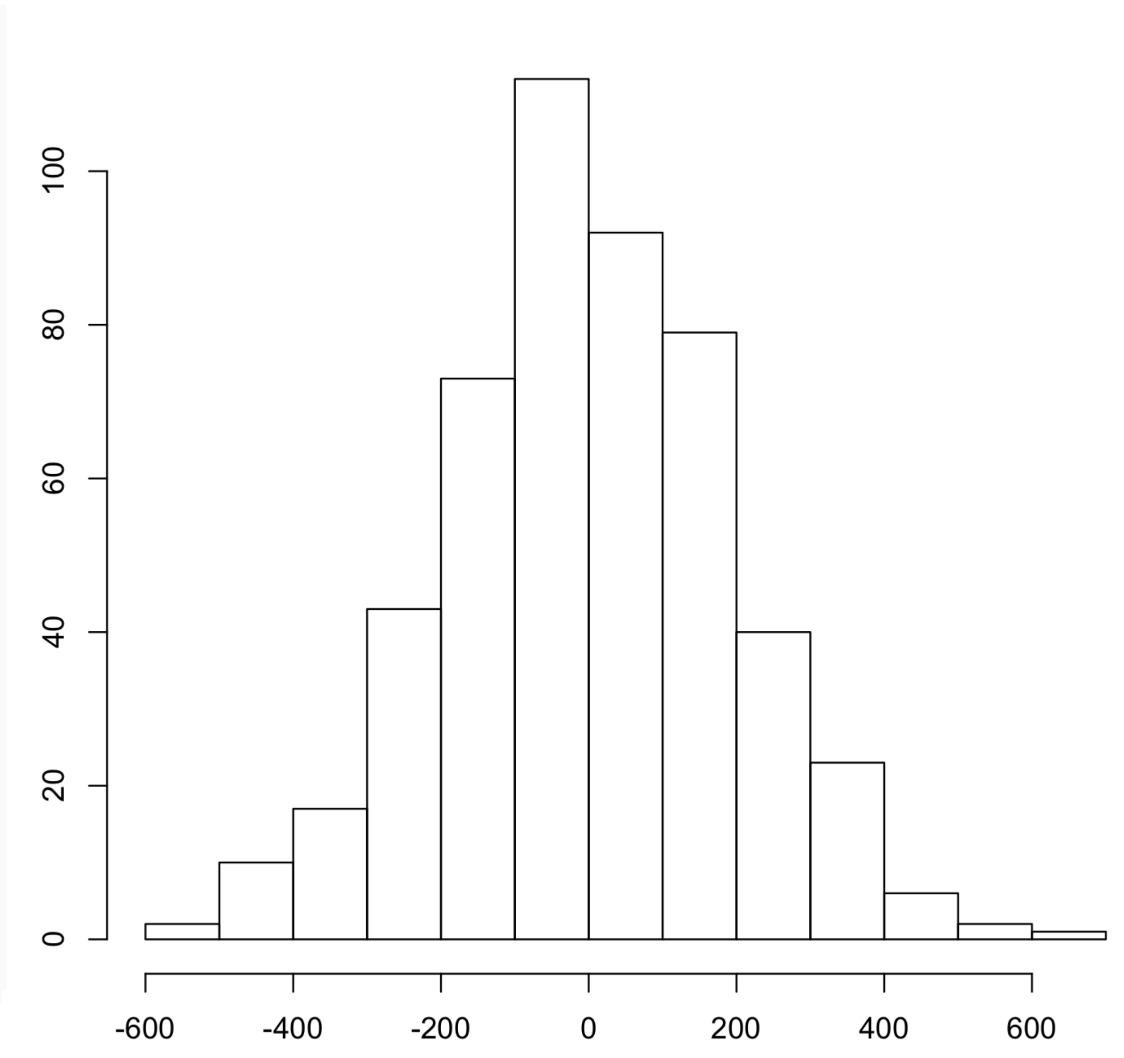

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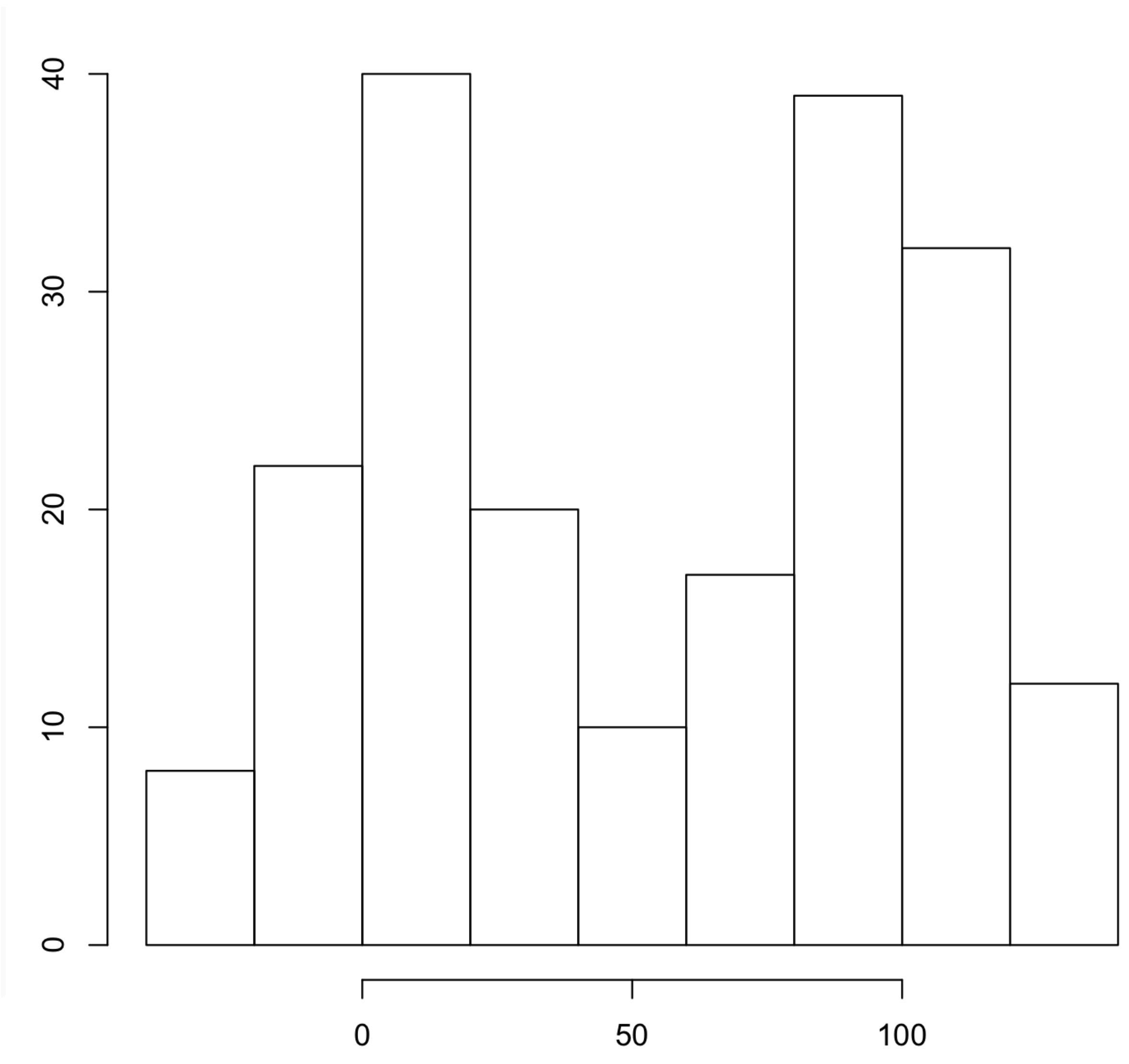

Two centers
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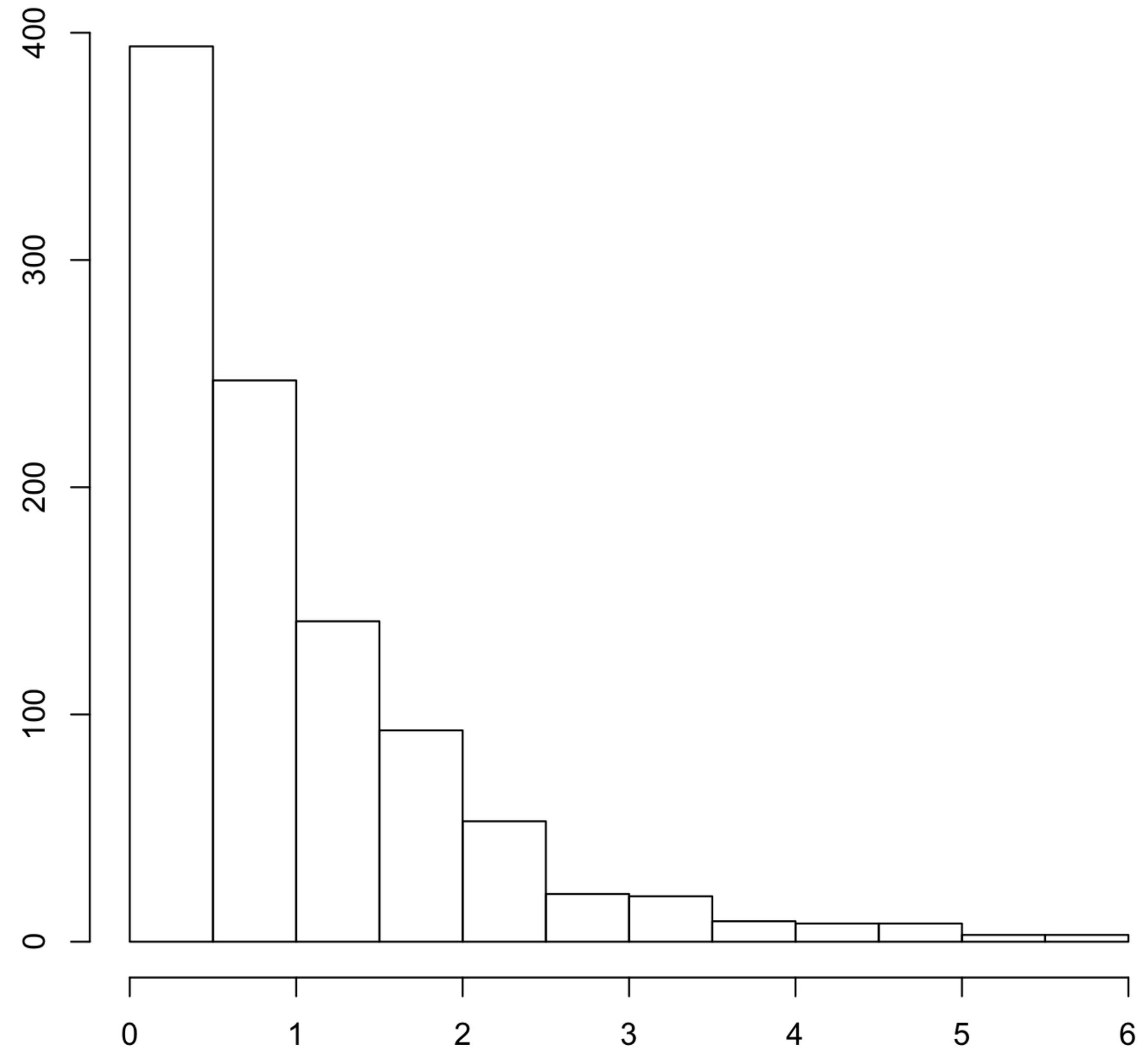

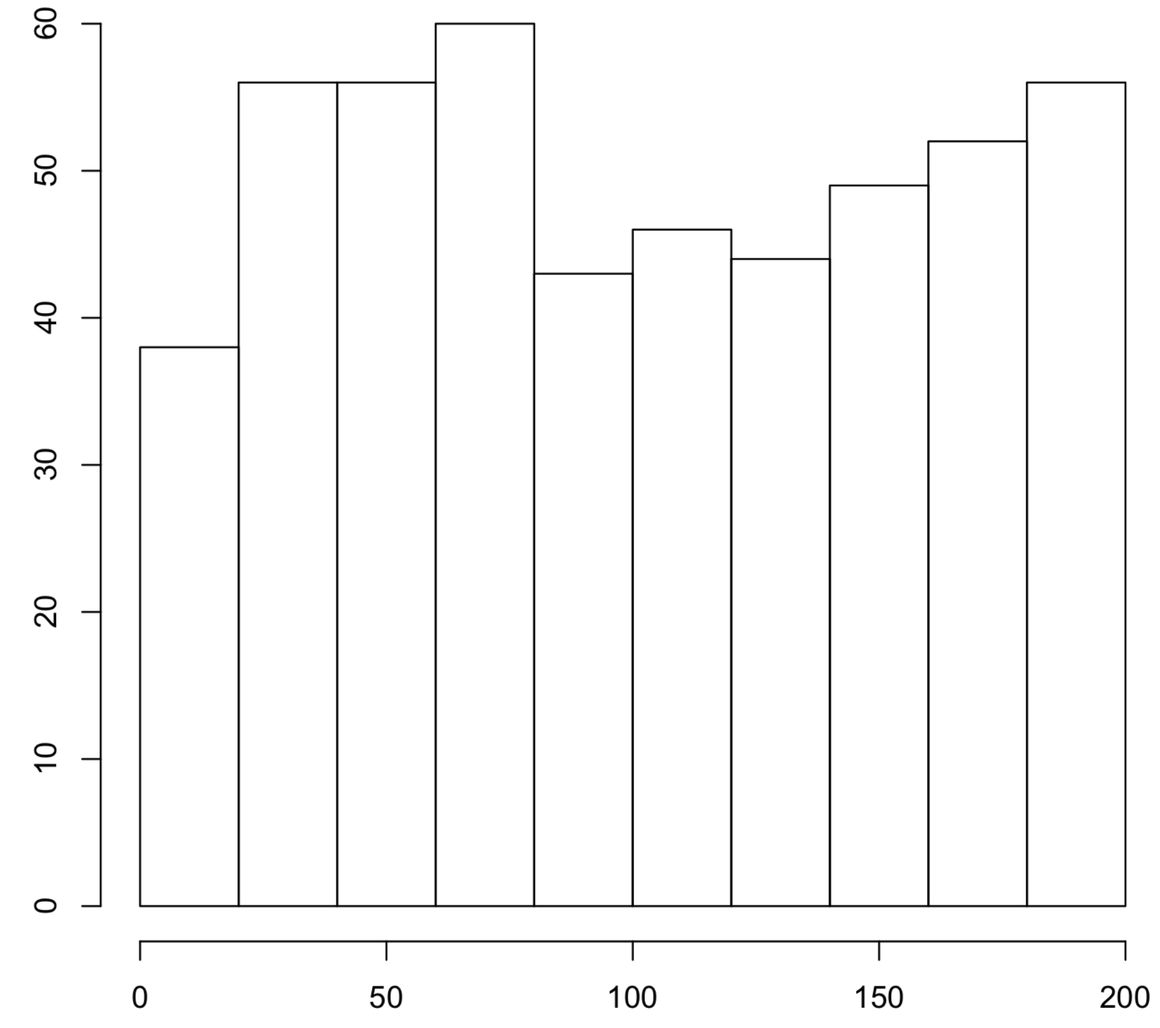

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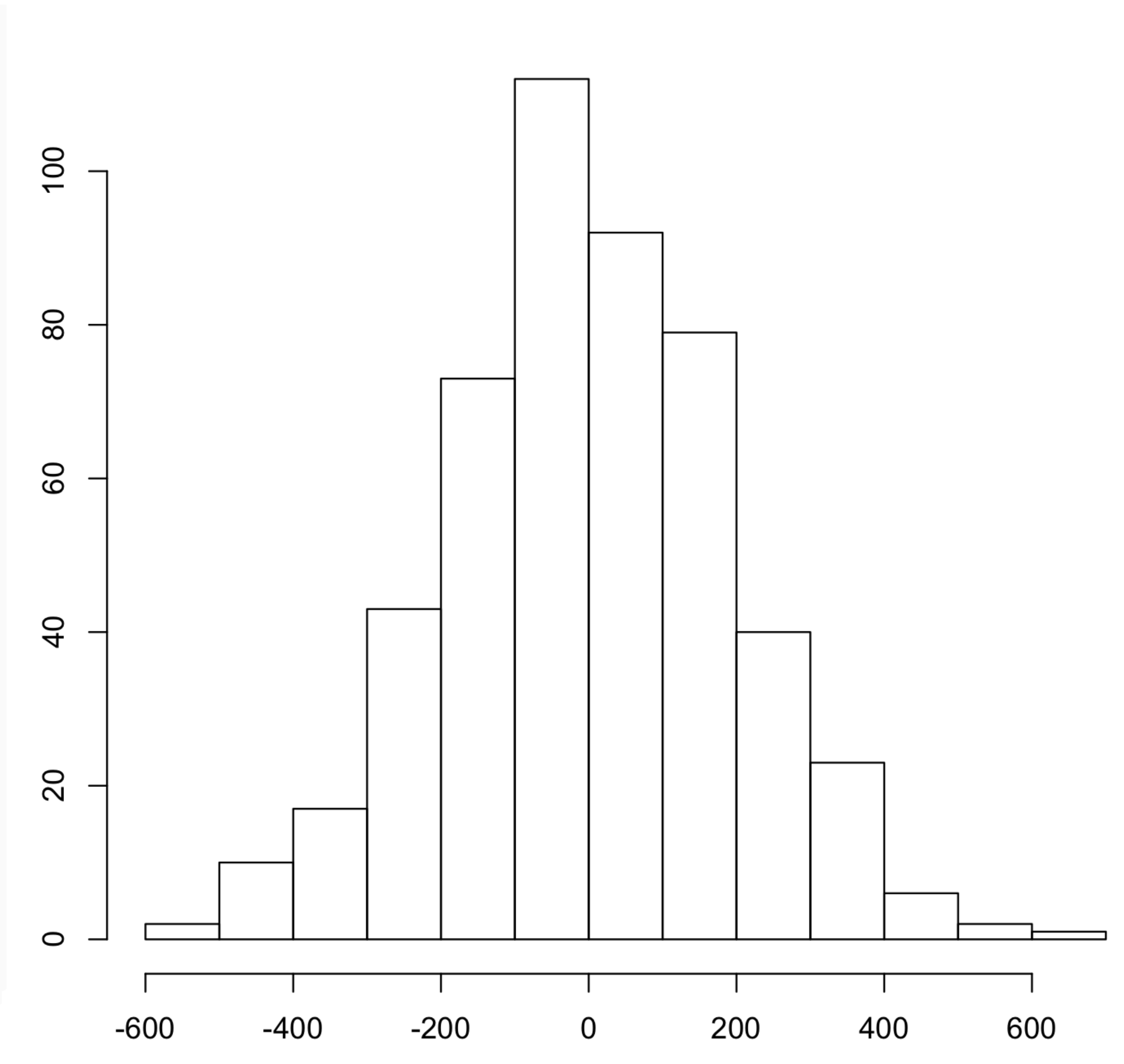

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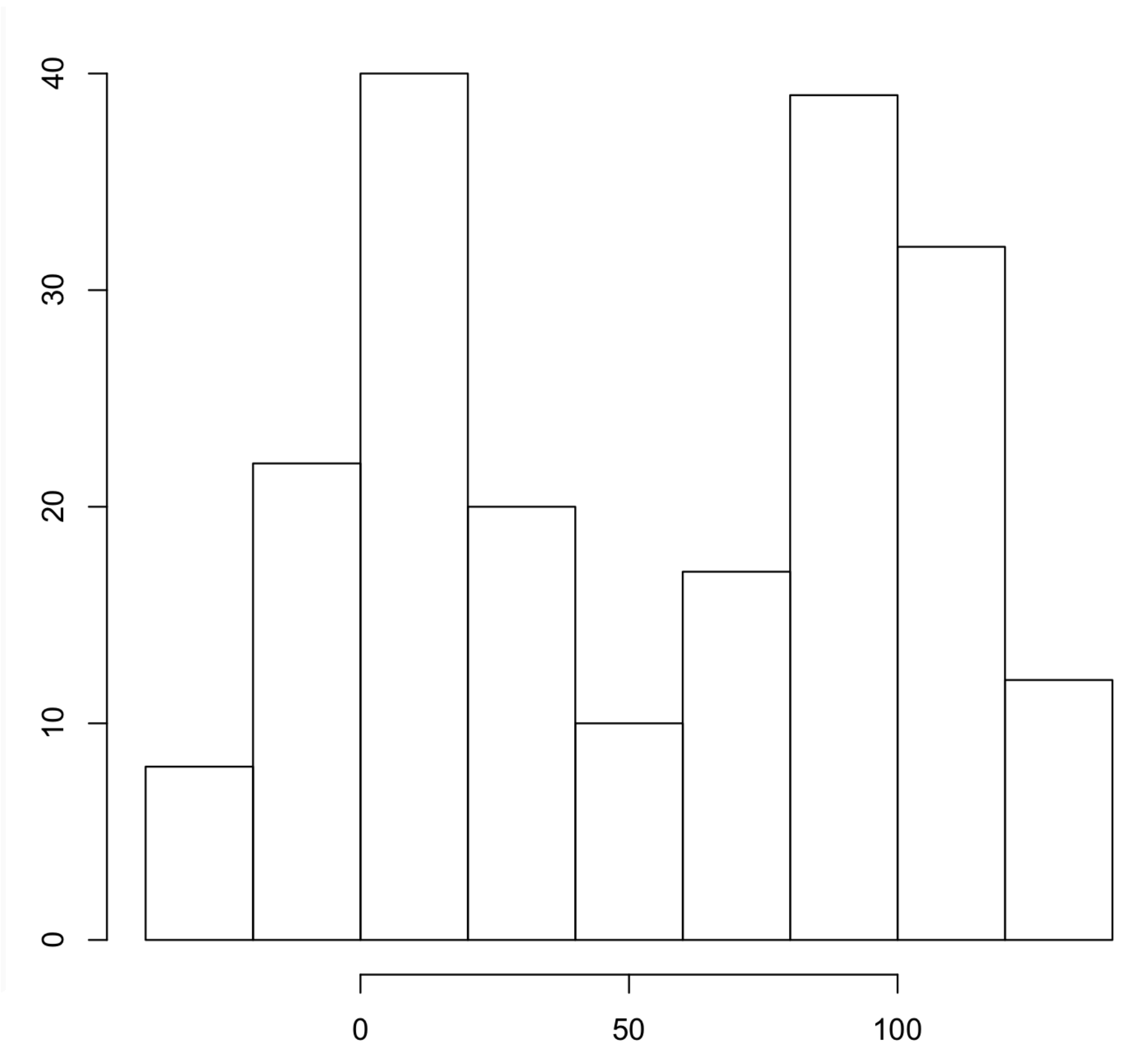

Two centers
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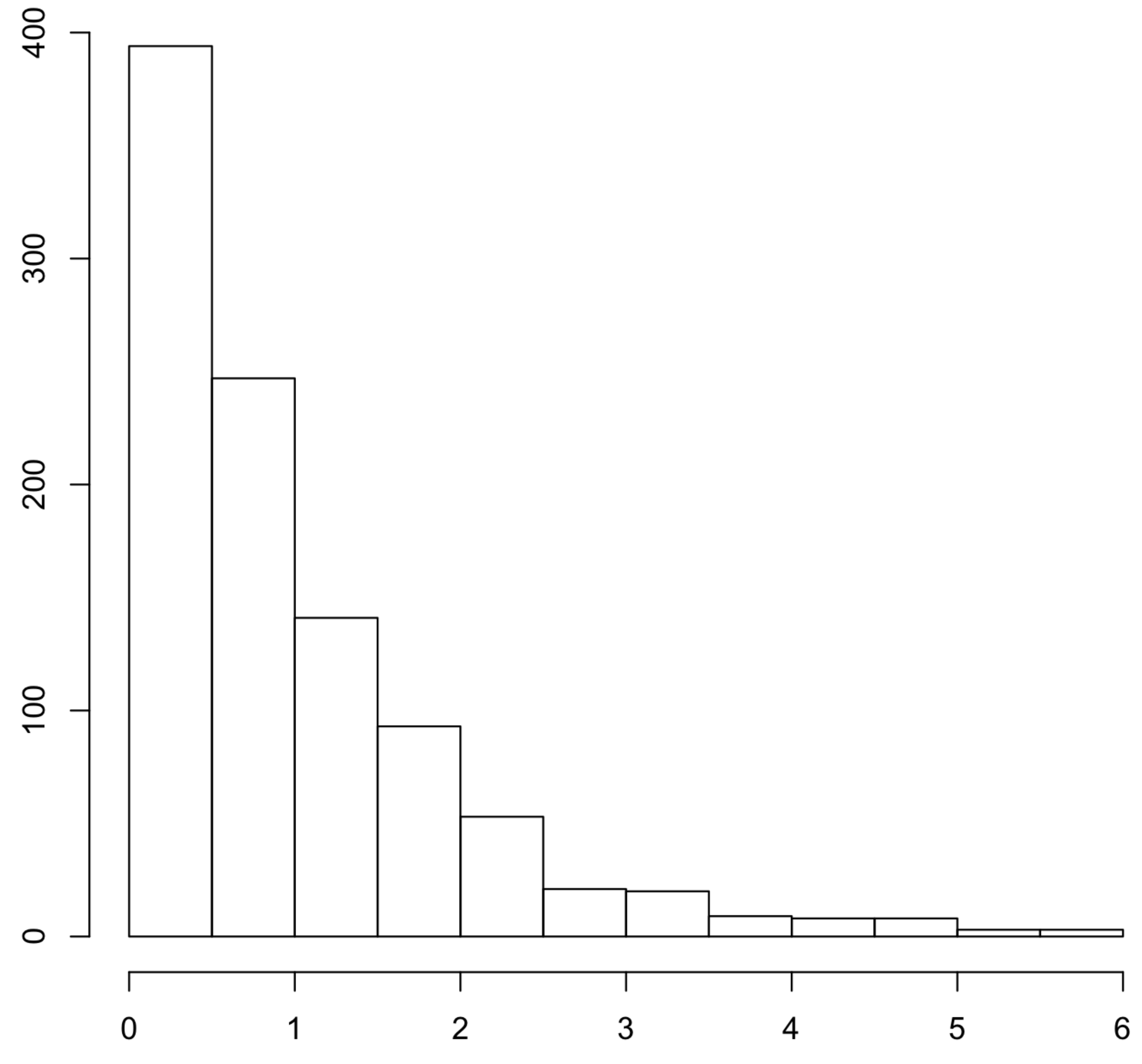
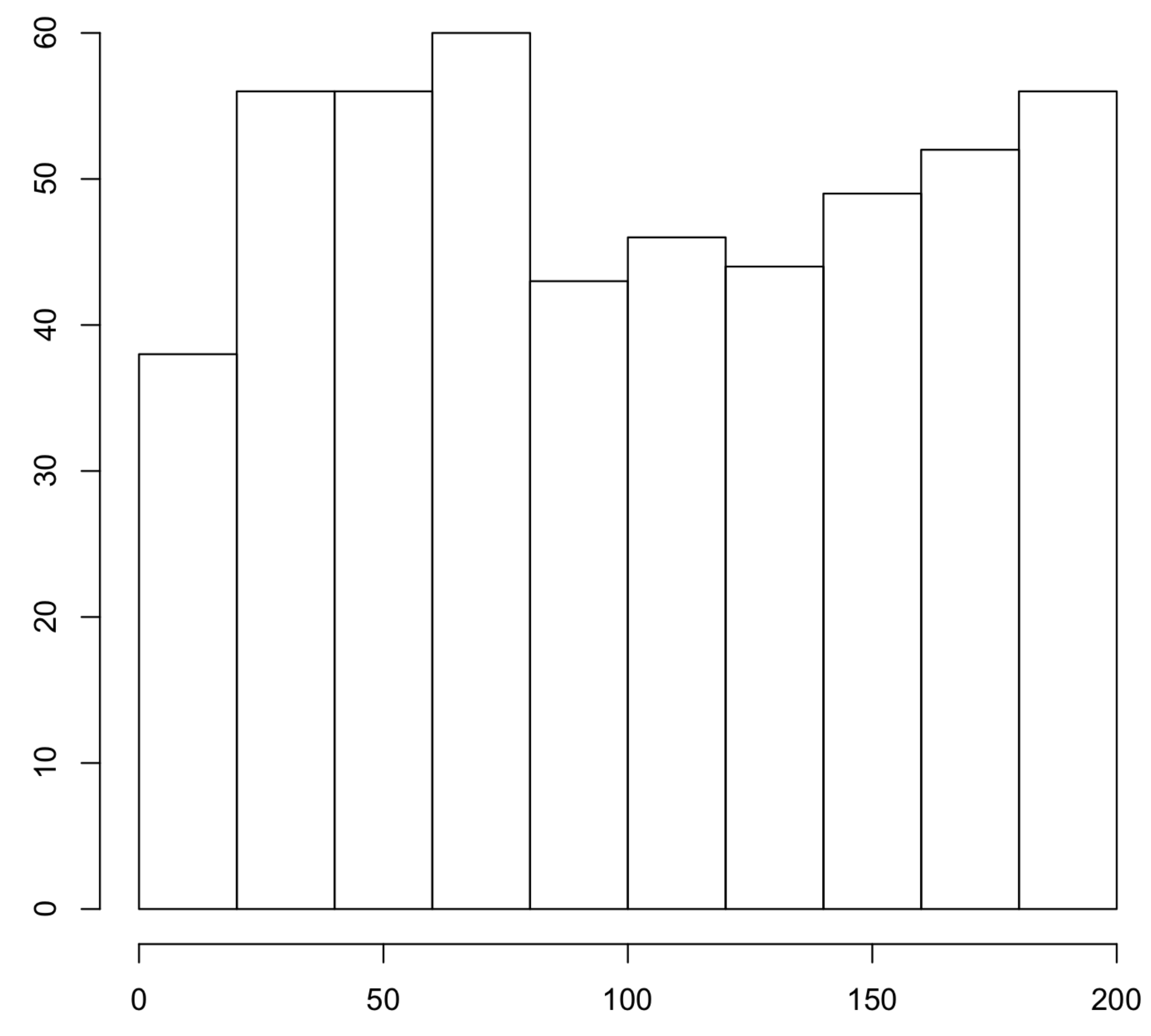


One peak
Asymmetric



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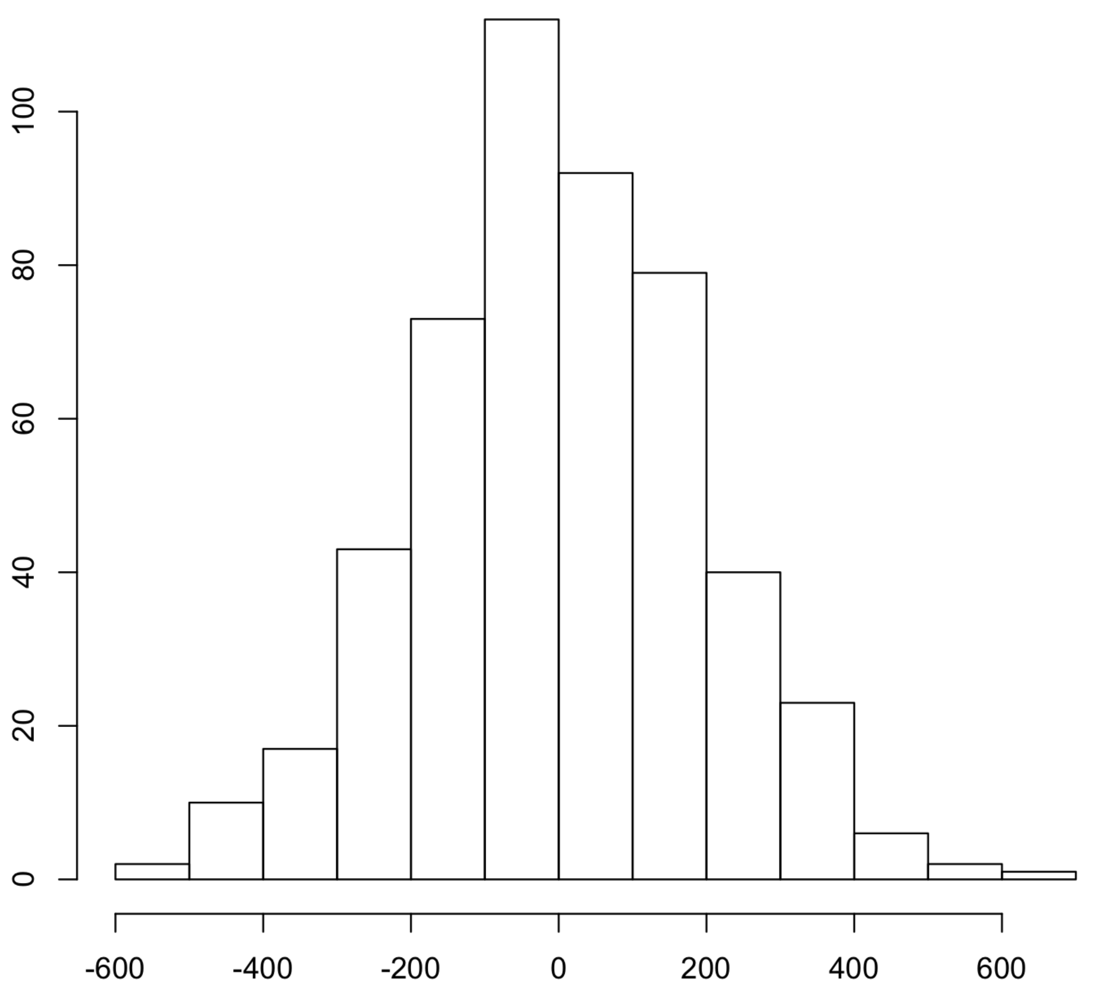

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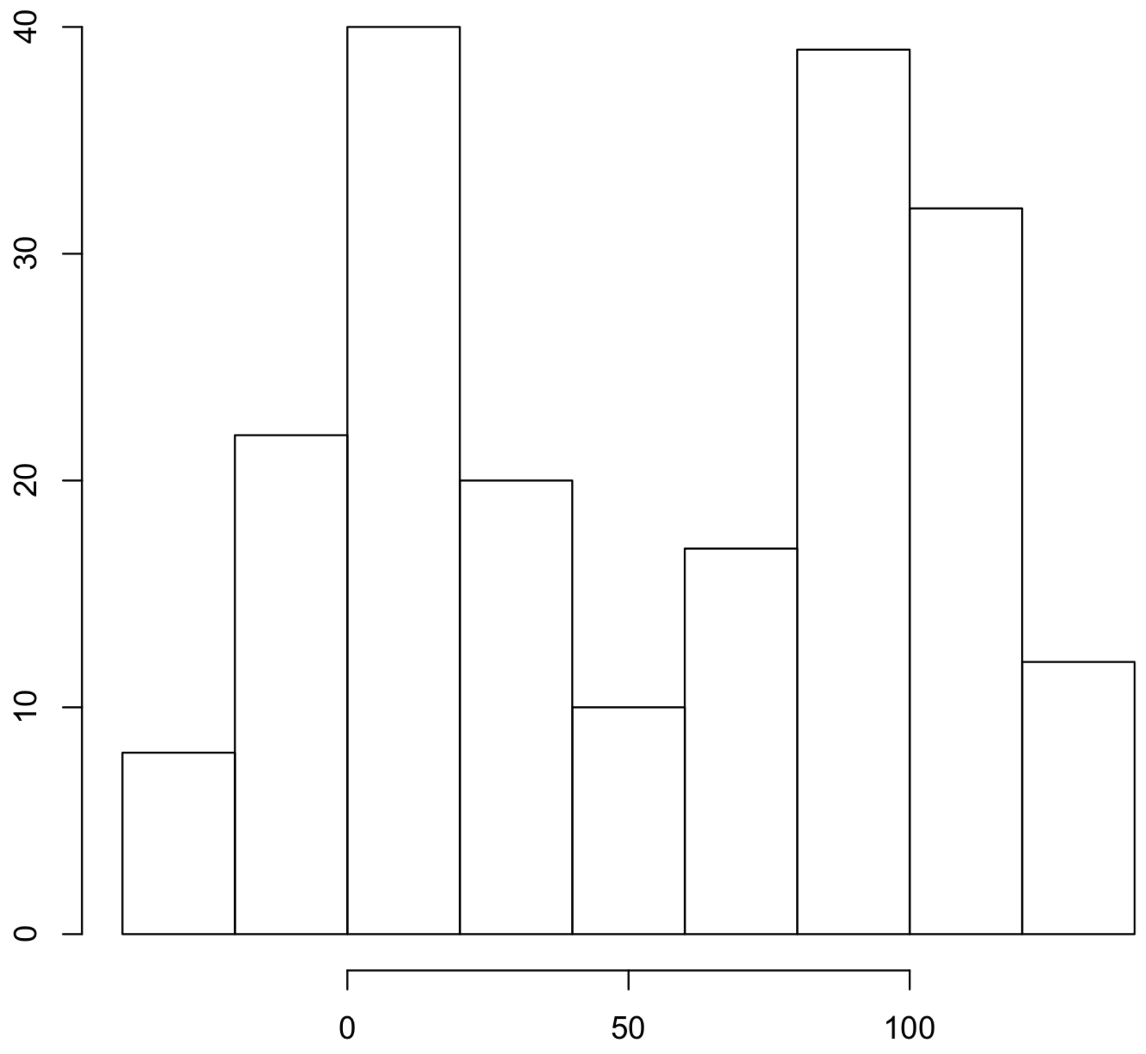

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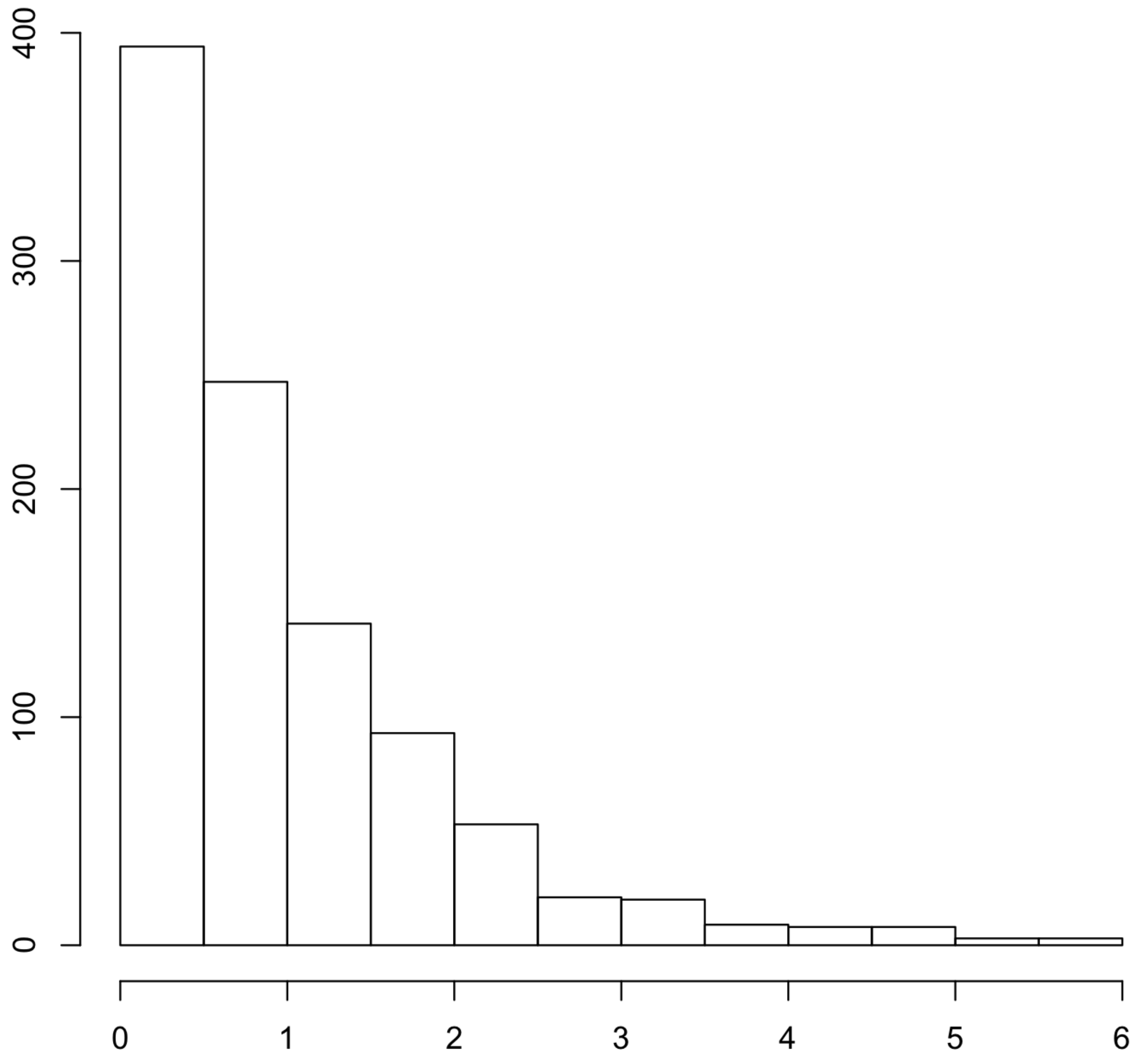

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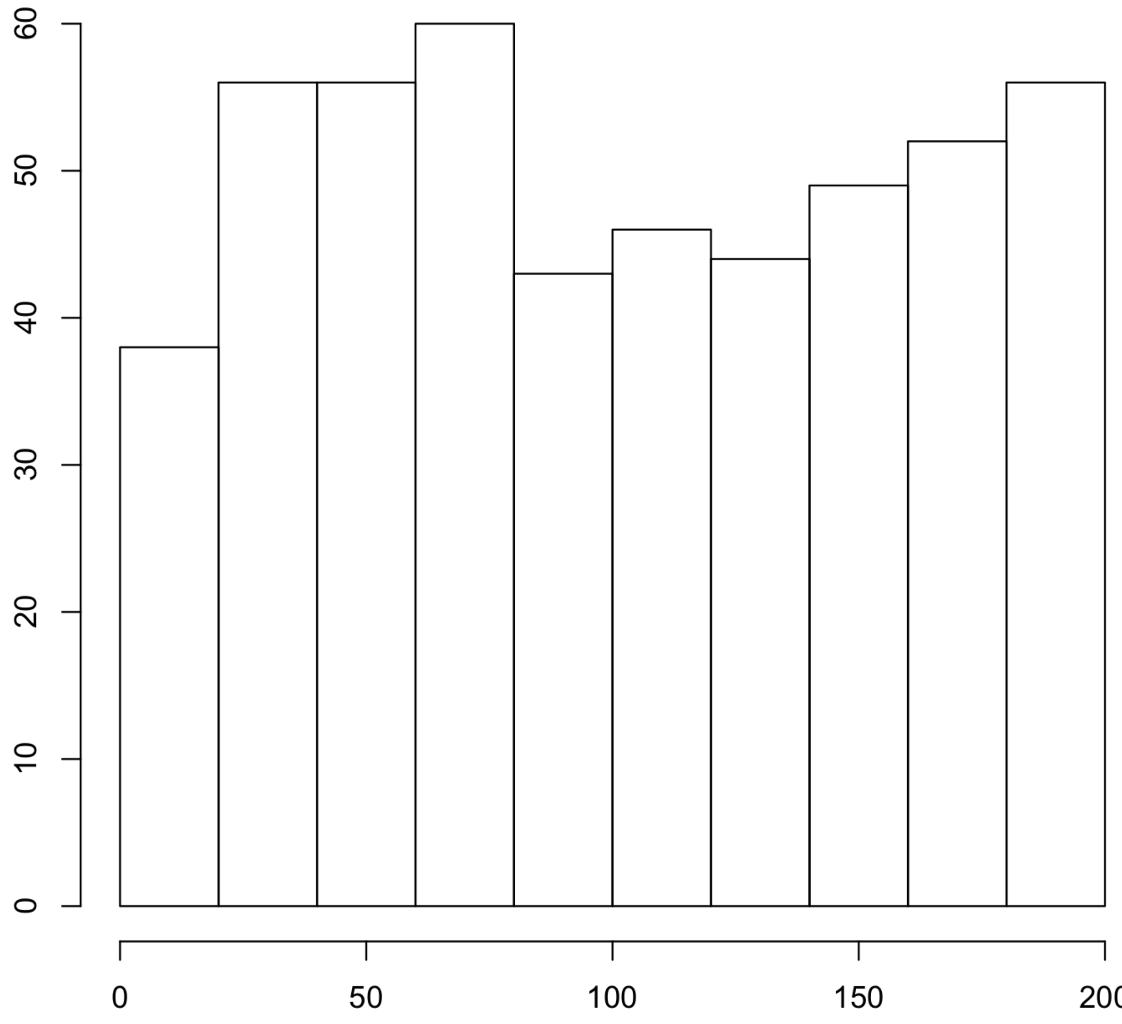

Large spread
Uniform

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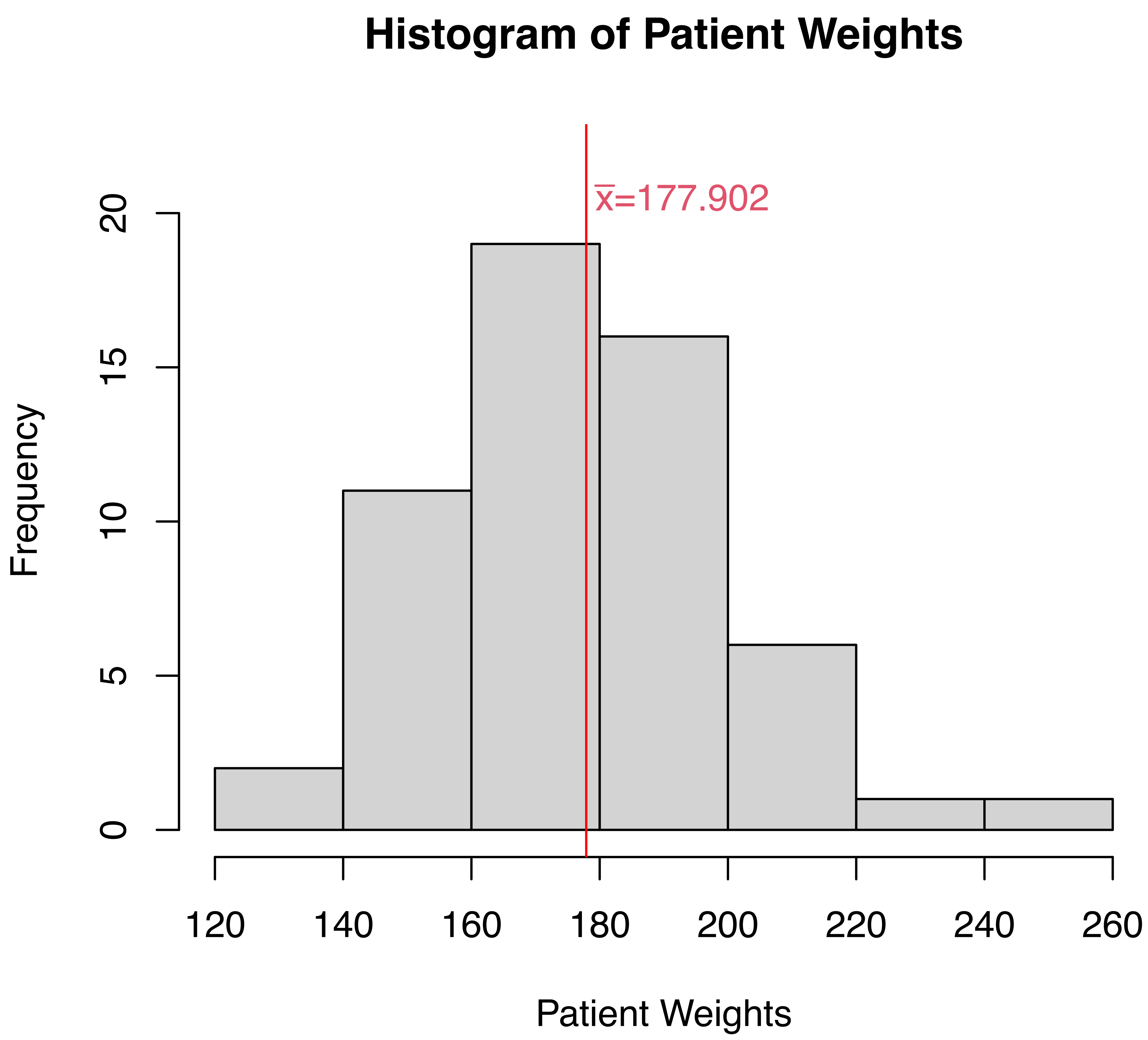
    - R code: `mean(c(66,46,68,71,72))`

Mean for Patient Weights




```
hist(weights, xlab="Patient Weights", main="Histogram of Patient Weights", ylim=c(0,22))
abline(v=mean(weights), col="red")
text(mean(weights)+15,20.5,substitute(paste(bar(x),"=",m),list(m=round(mean(weights),3))),col=2)
```

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- The order statistics are then $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$

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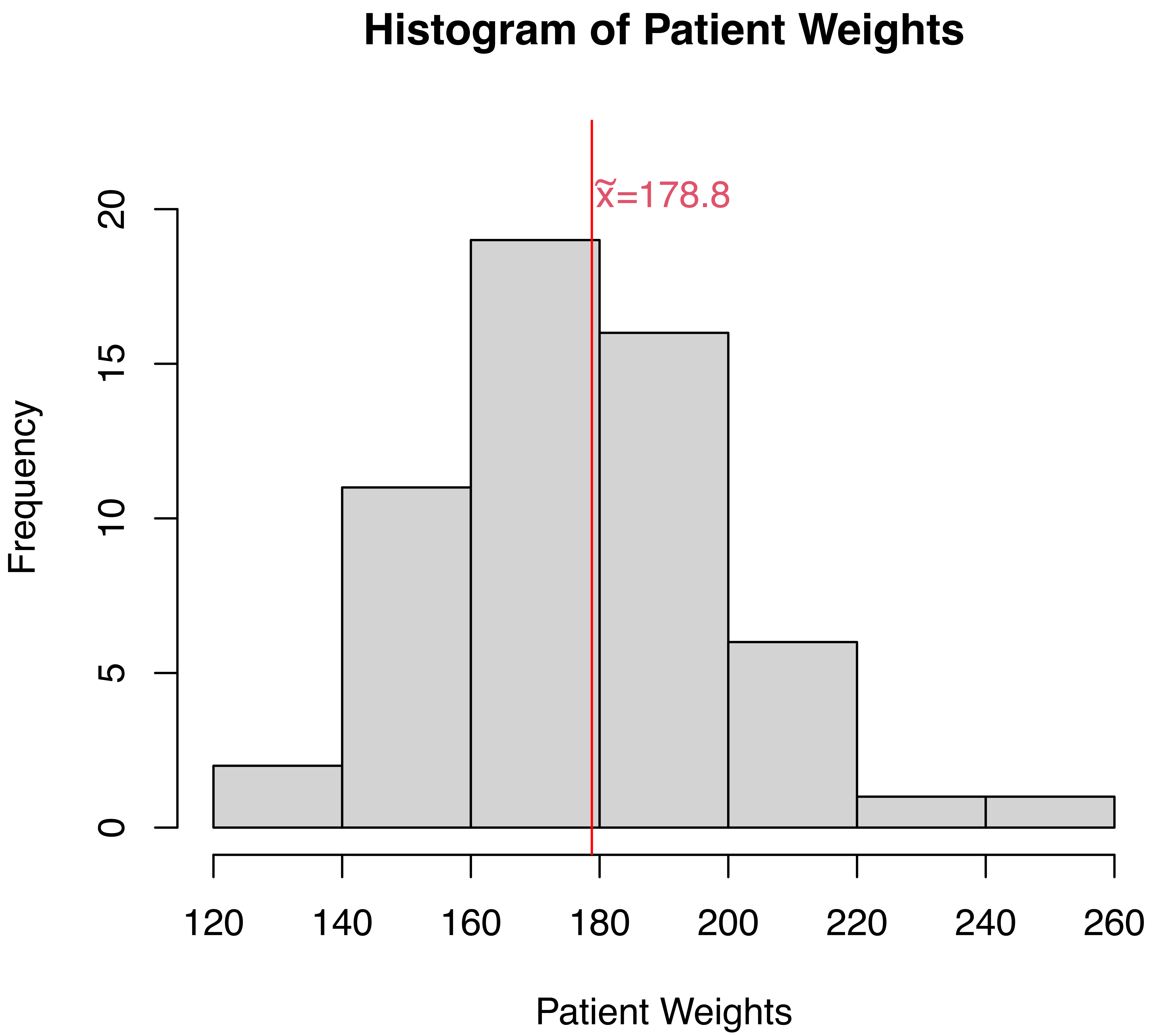
Median for Patient Weights




```
hist(weights, xlab="Patient Weights", main="Histogram of Patient Weights", ylim=c(0,22))
abline(v=median(weights), col="red")
text(median(weights)+11,20.5,substitute(paste(tilde(x),"=",m),list(m=round(median(weights),3))),col=2)
```

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- R code:

```
x <- c(5,15,18,2,17,10,23,20,17,16)
mean(x, trim=0.1)
```

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- Example:
  - Find the mode: 3.1, 3.2, 4.5, 5.1, 5.9, 6.0
  - Find the mode: 7.1, 7.8, 7.8, 9.1, 9.3, 9.4, 9.4, 9.4

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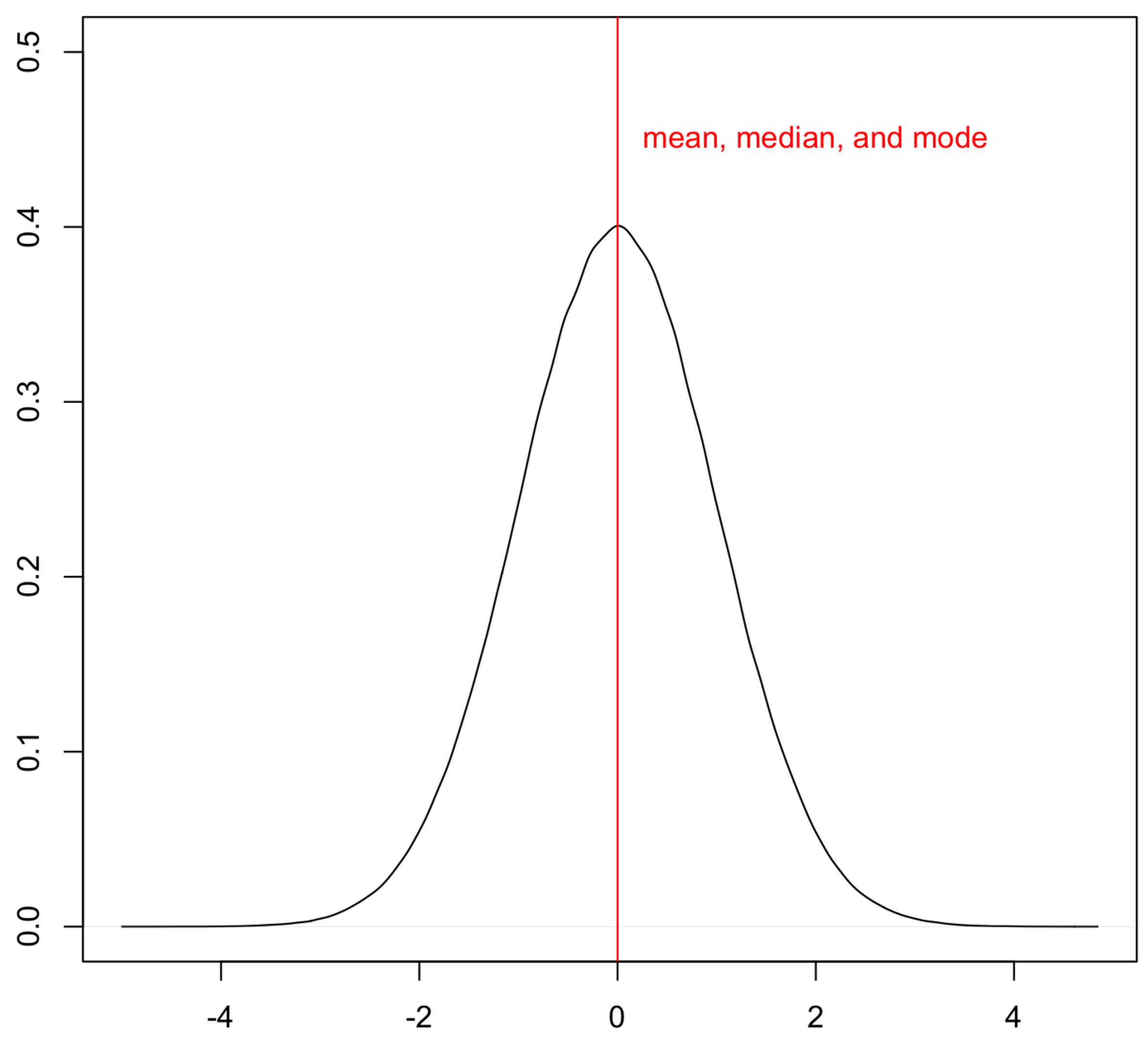
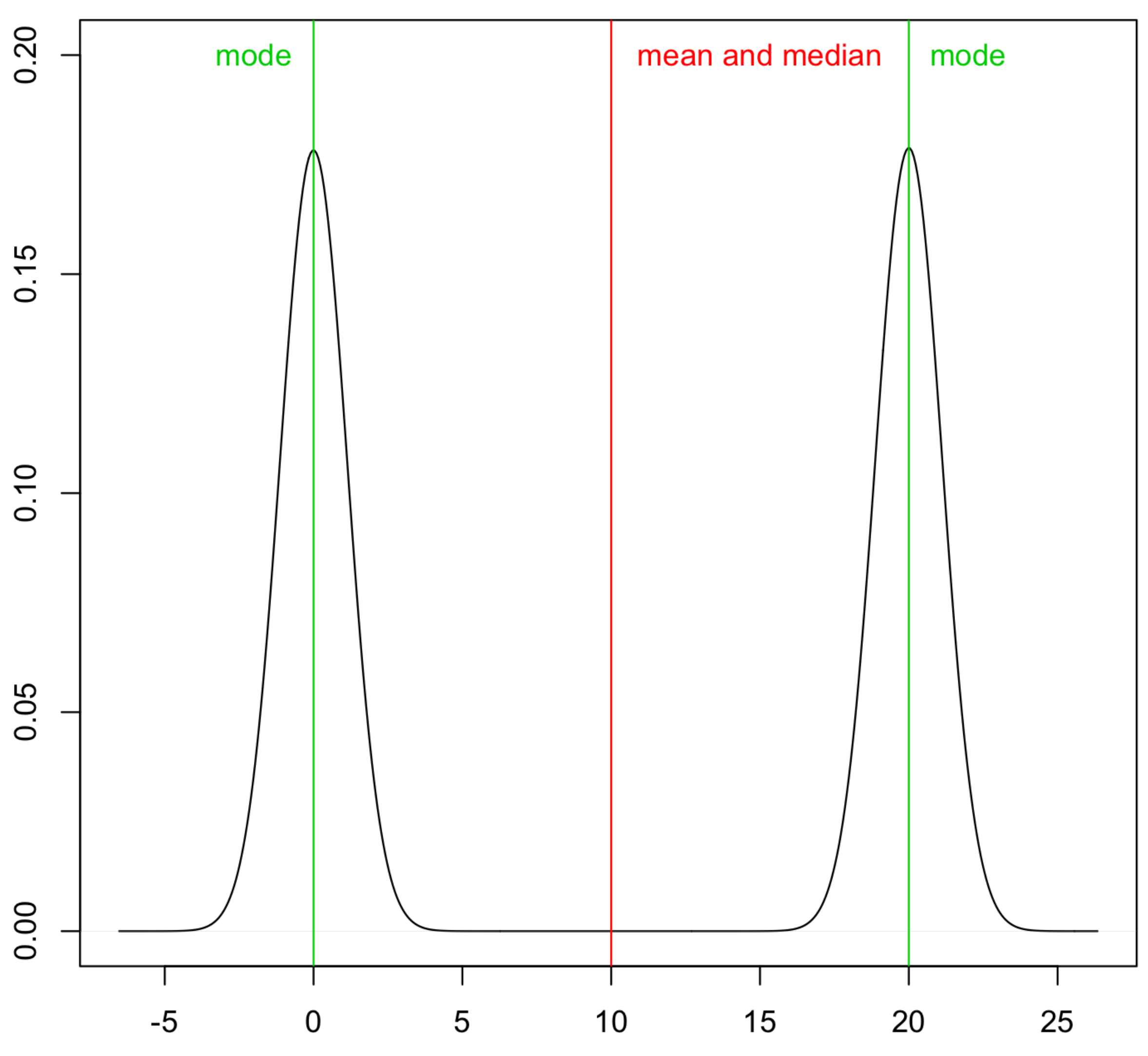
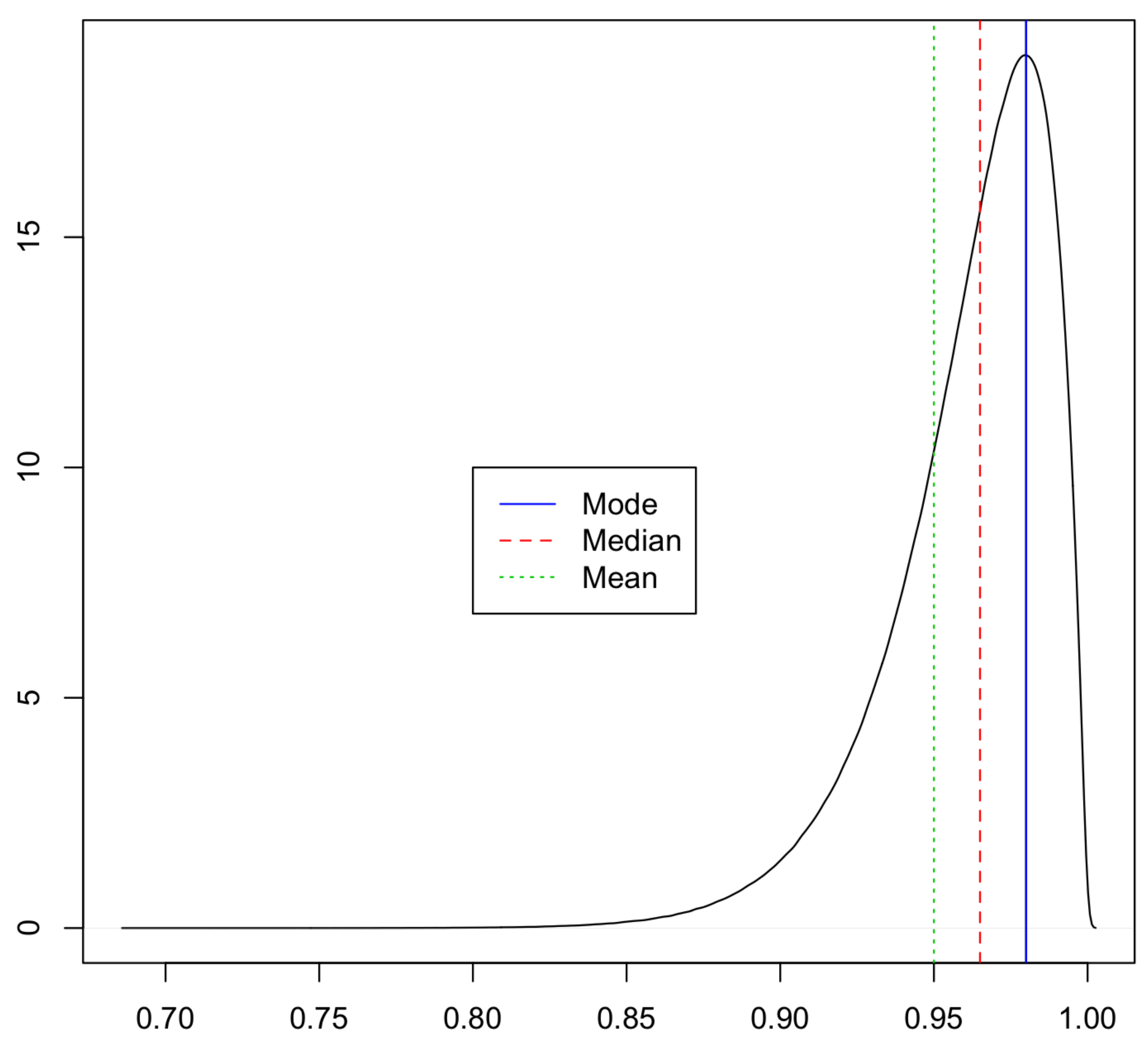
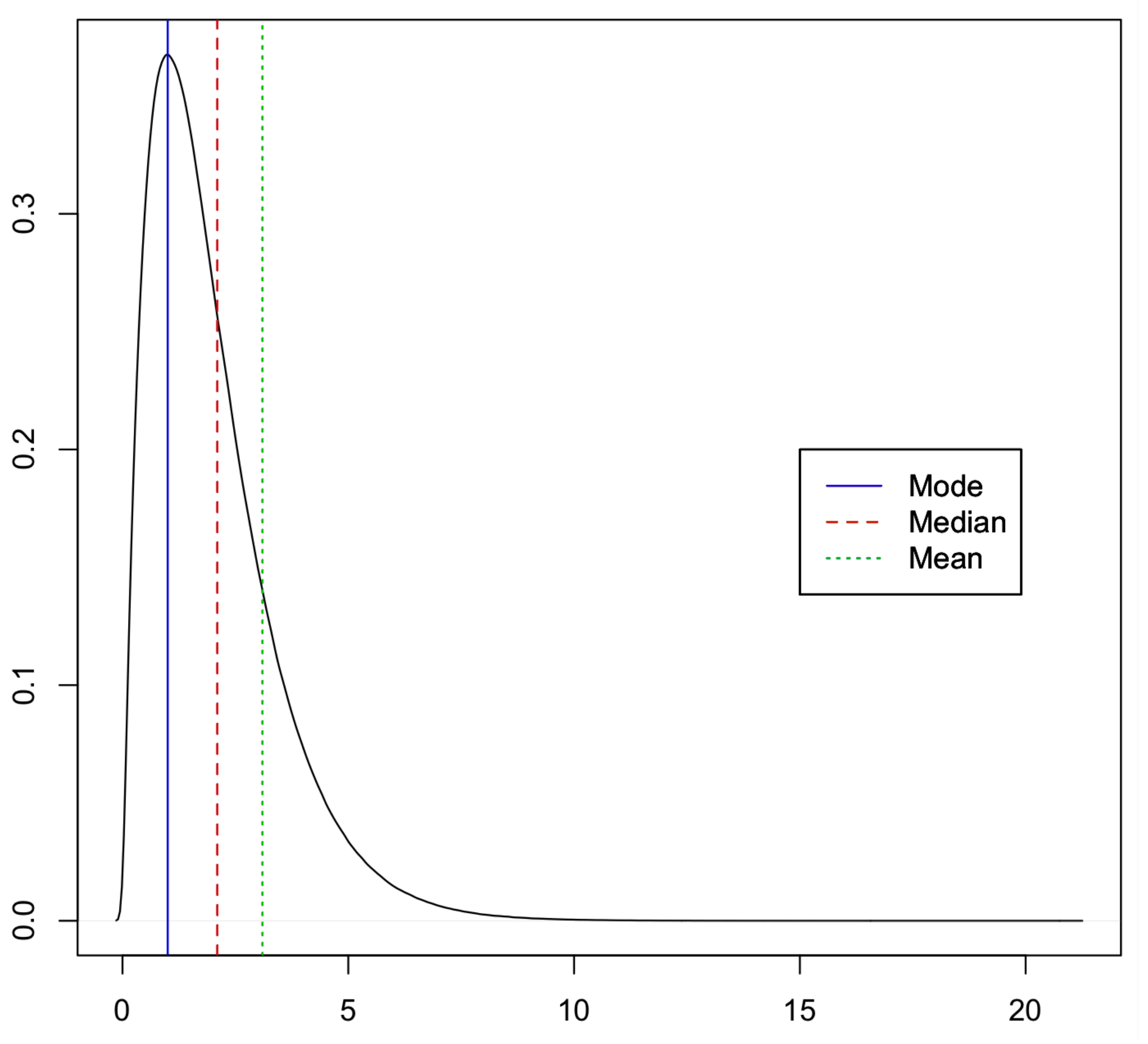


Symmetric
Unimodal







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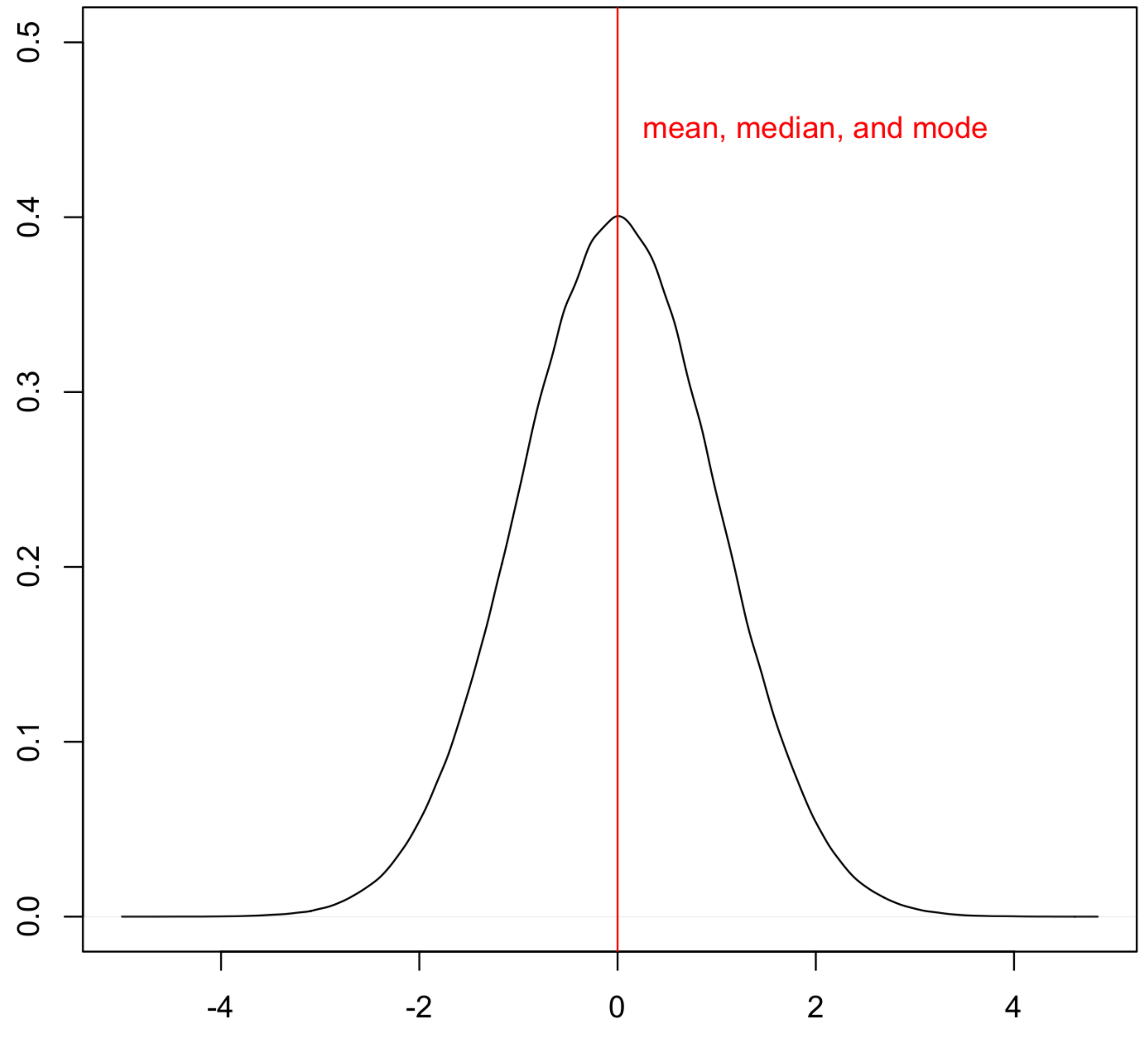

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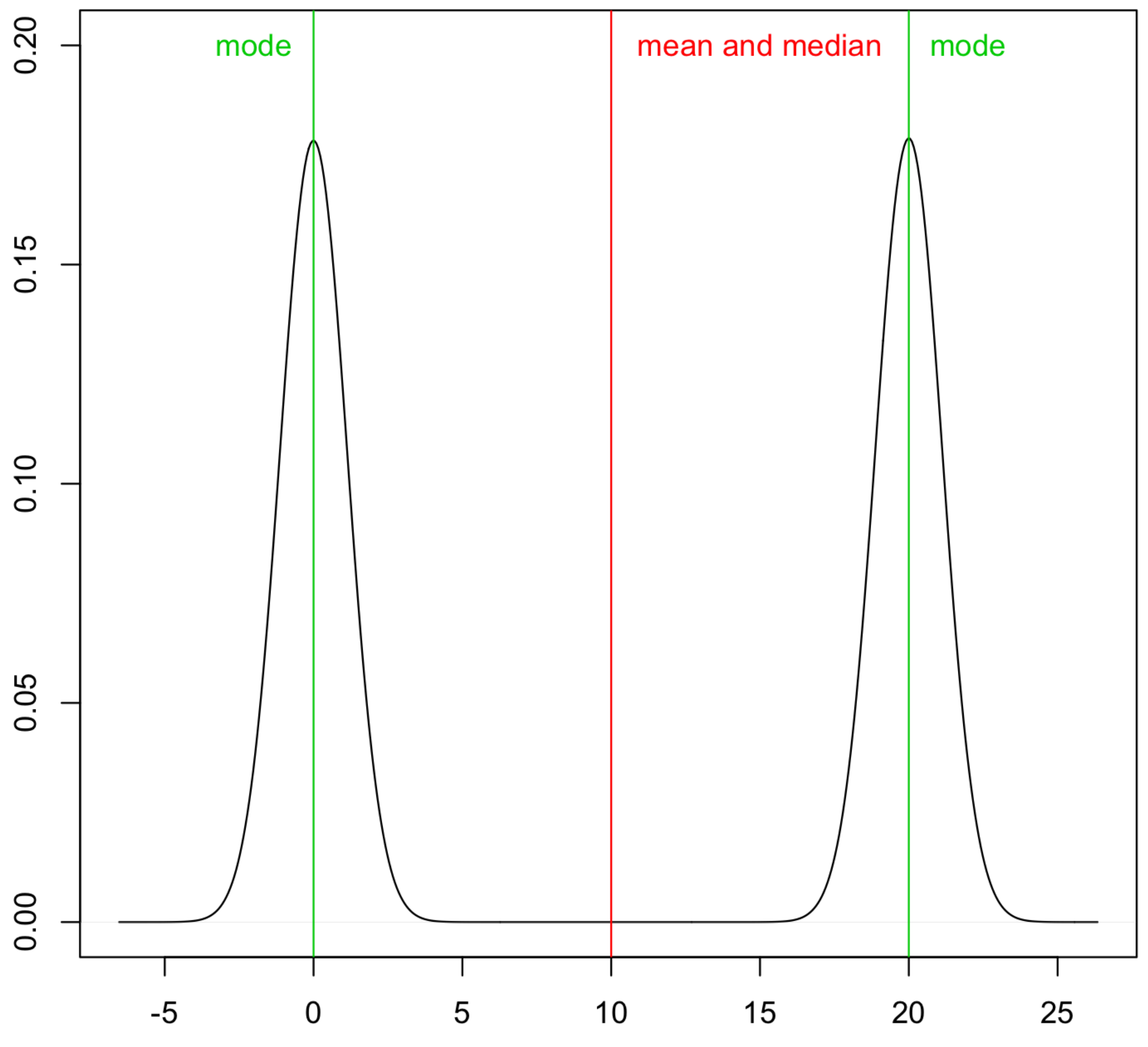

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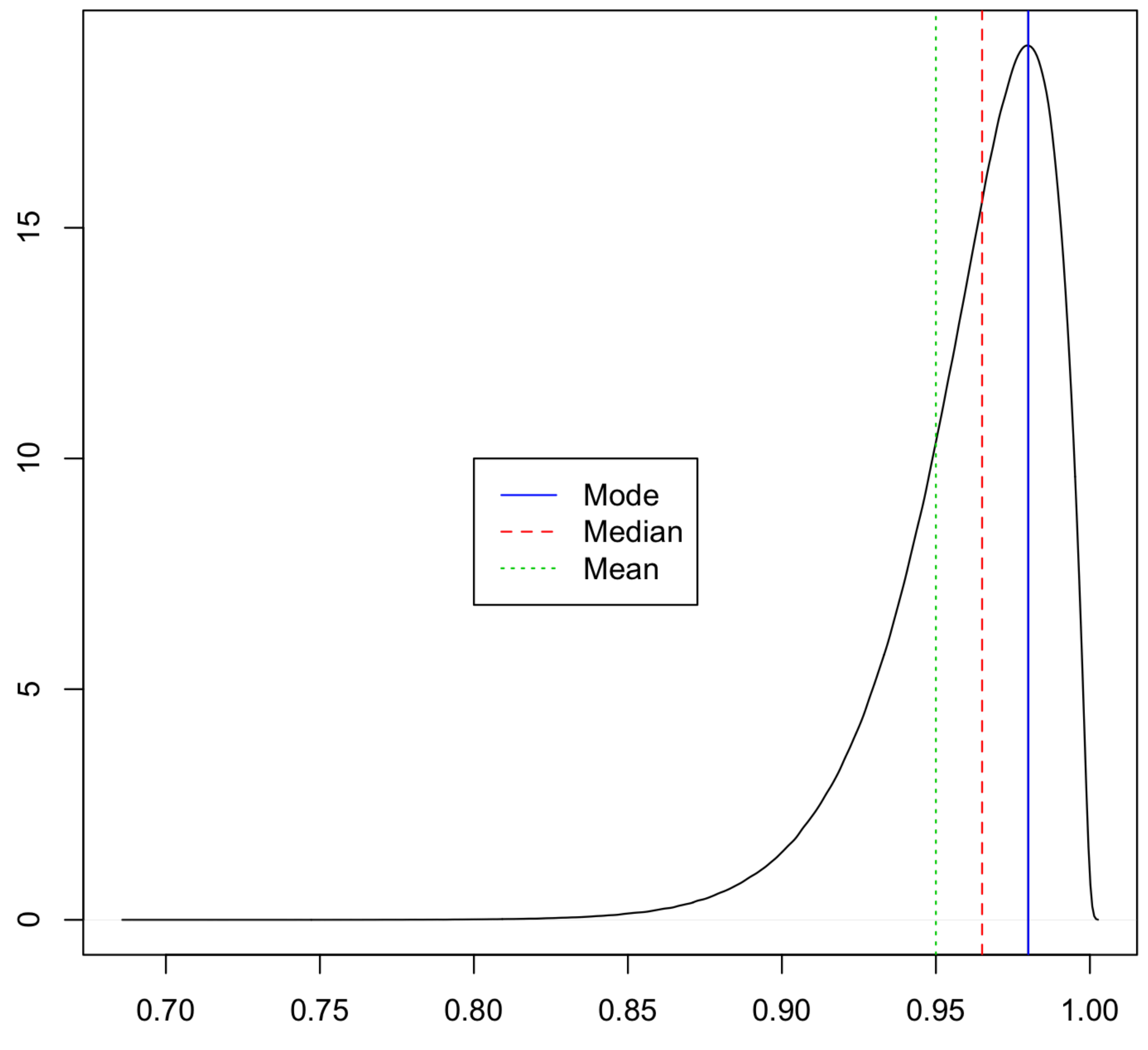

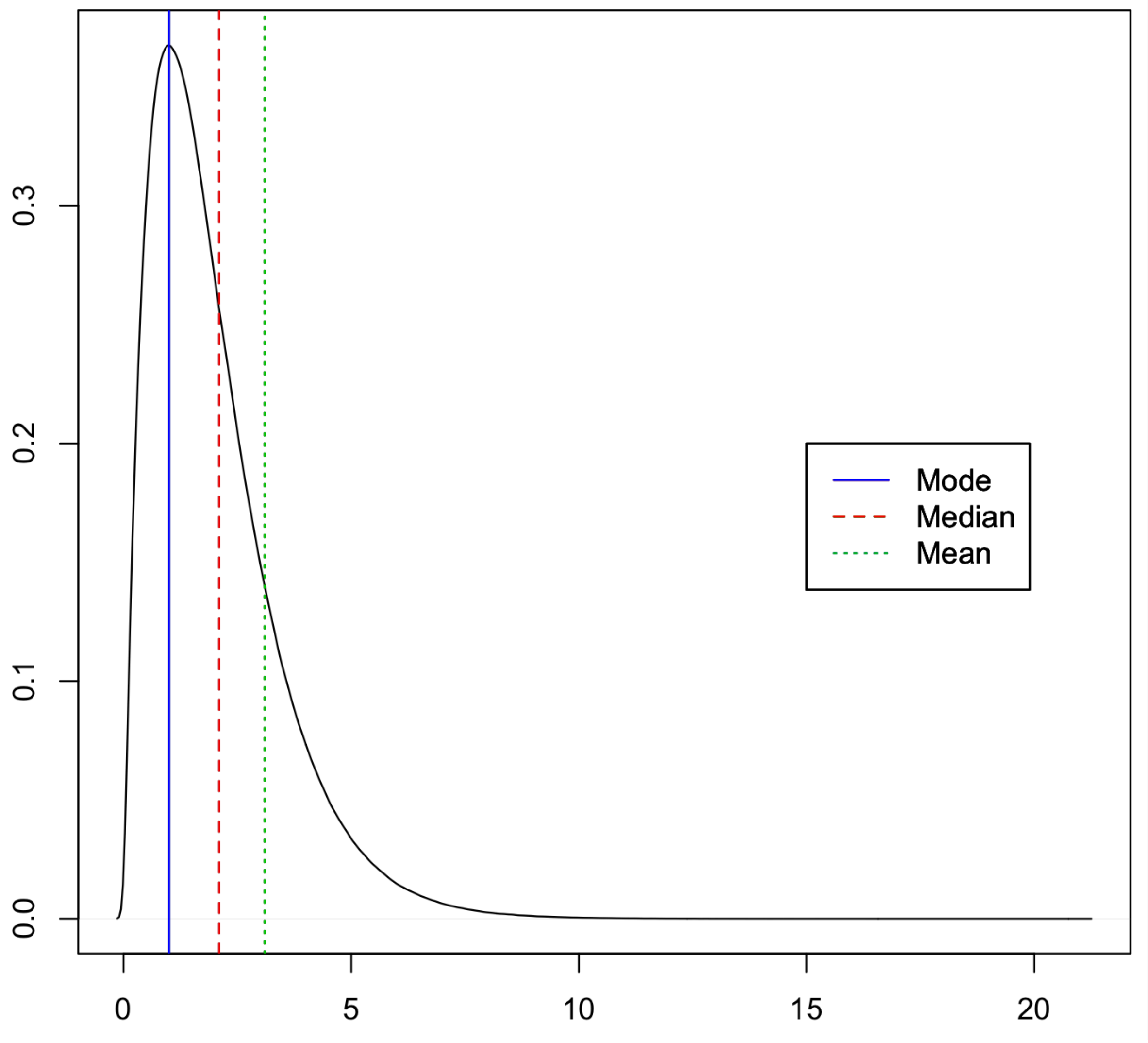

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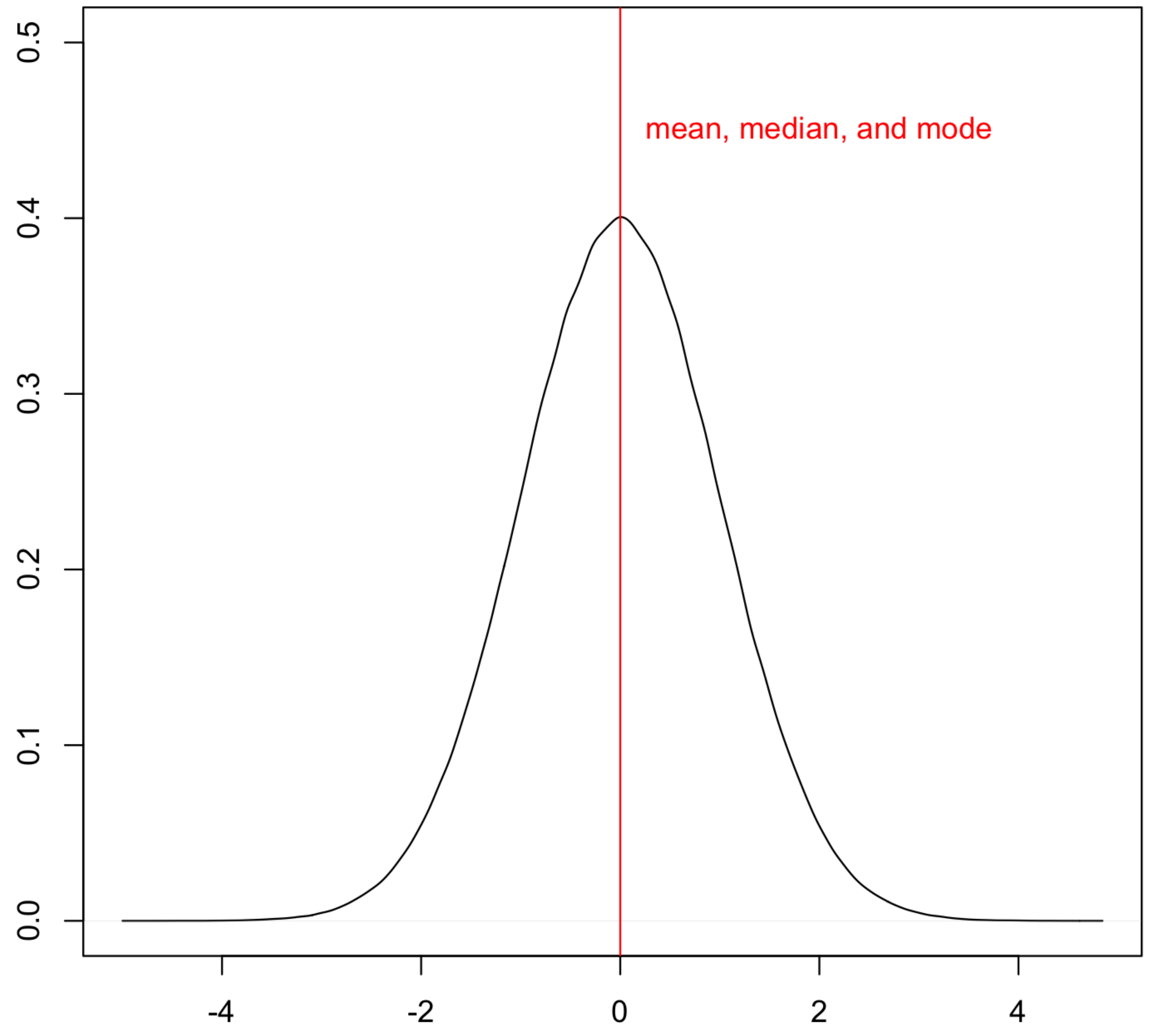

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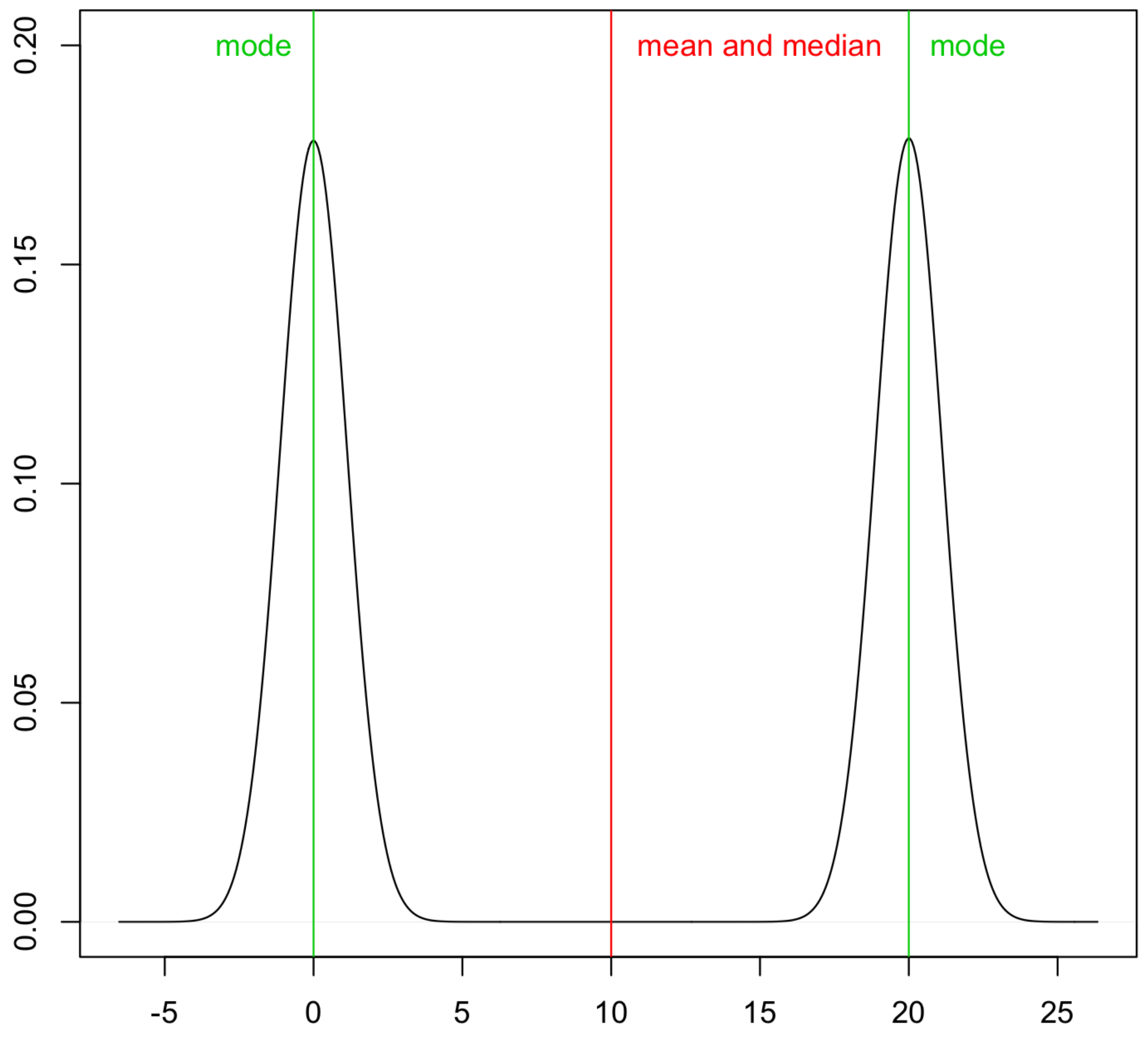

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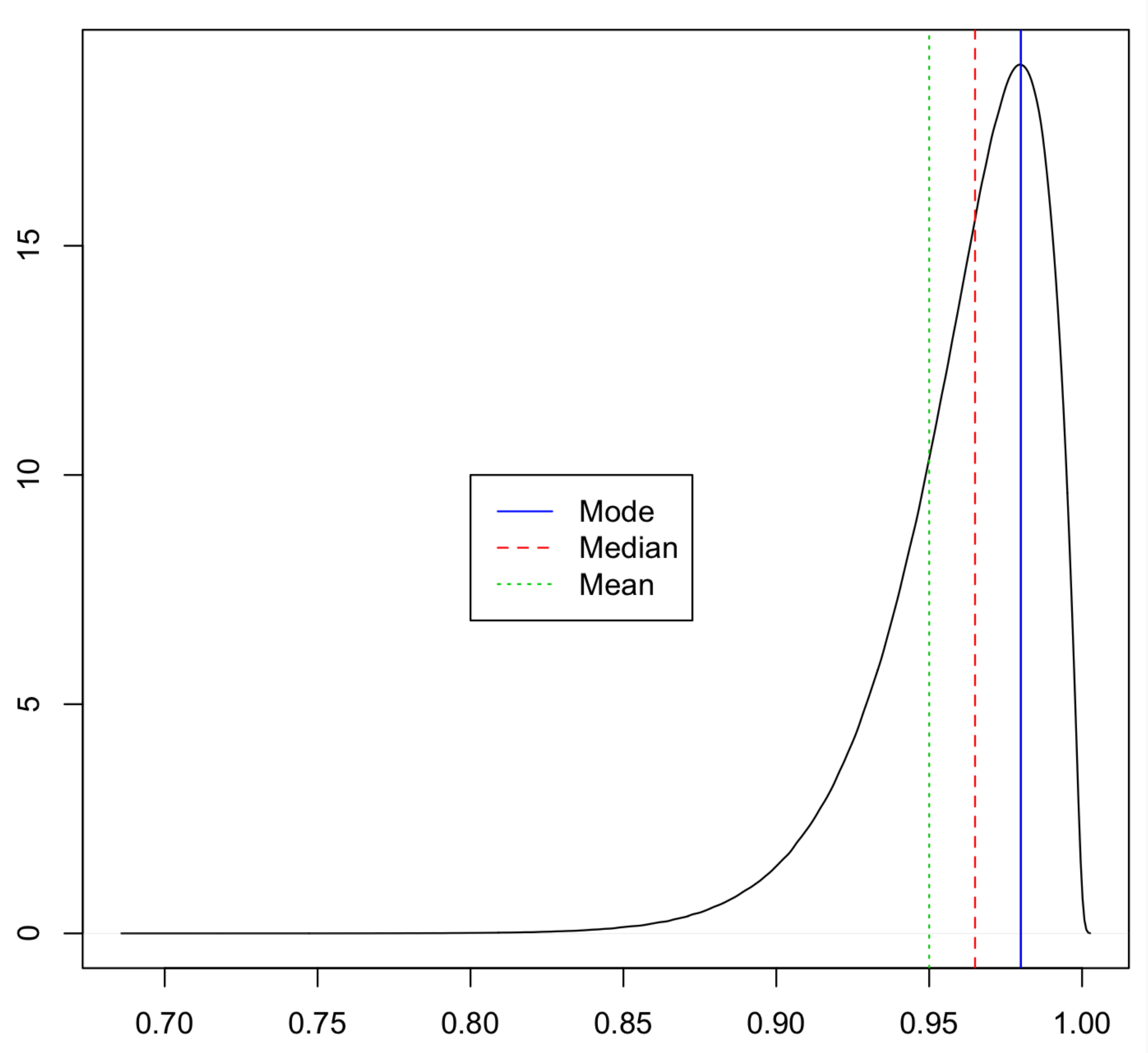

Left-skewed
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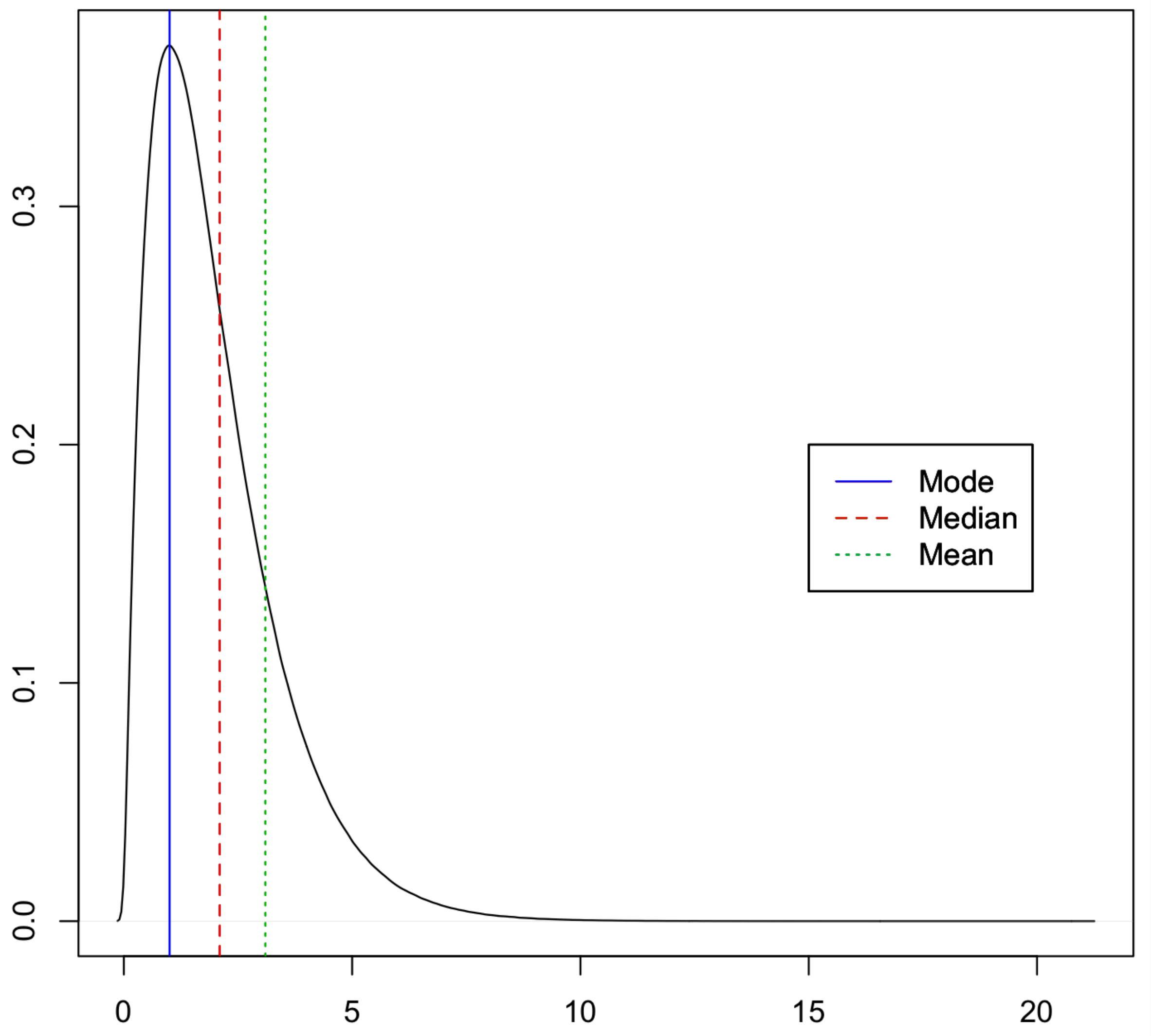

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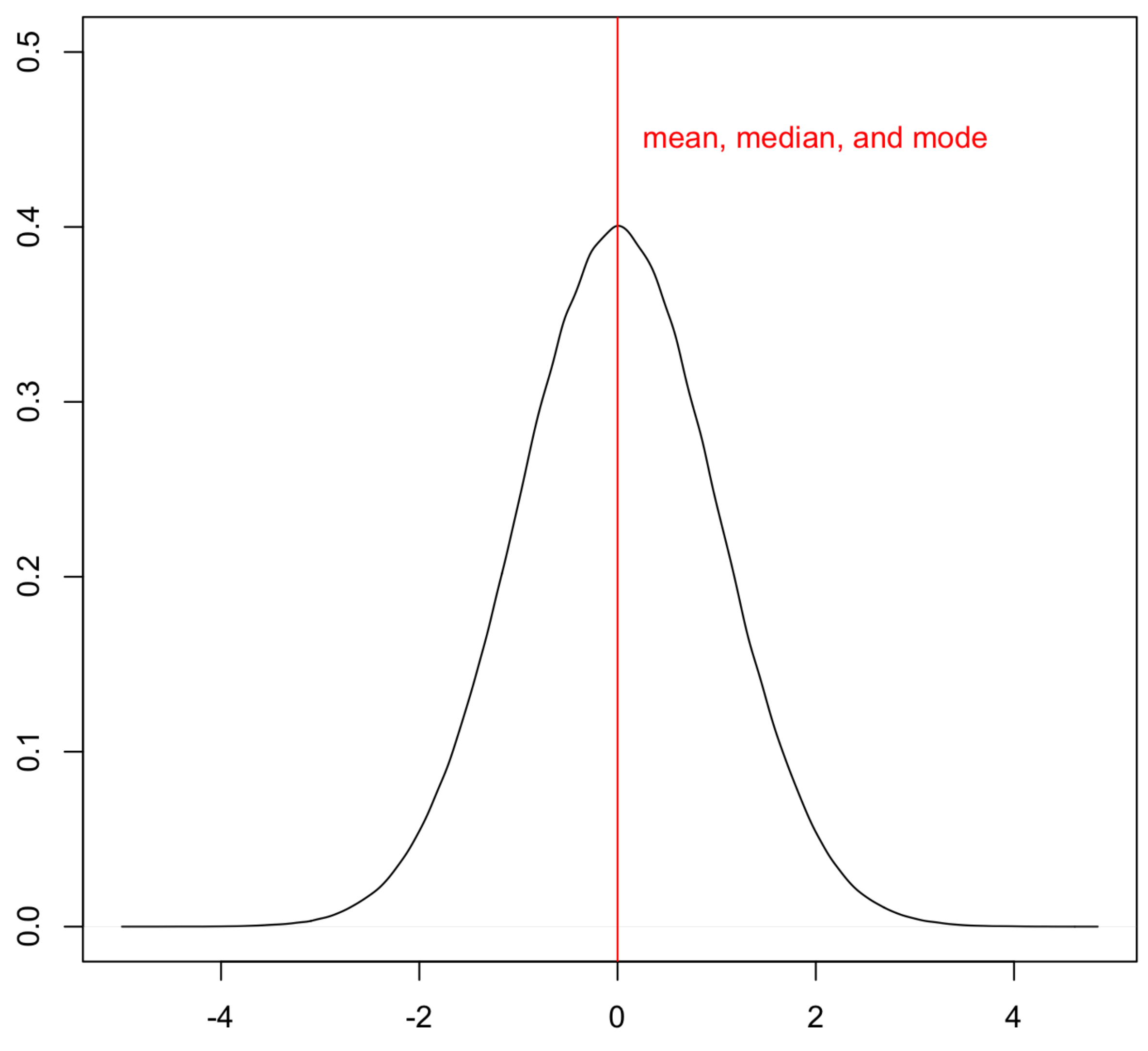

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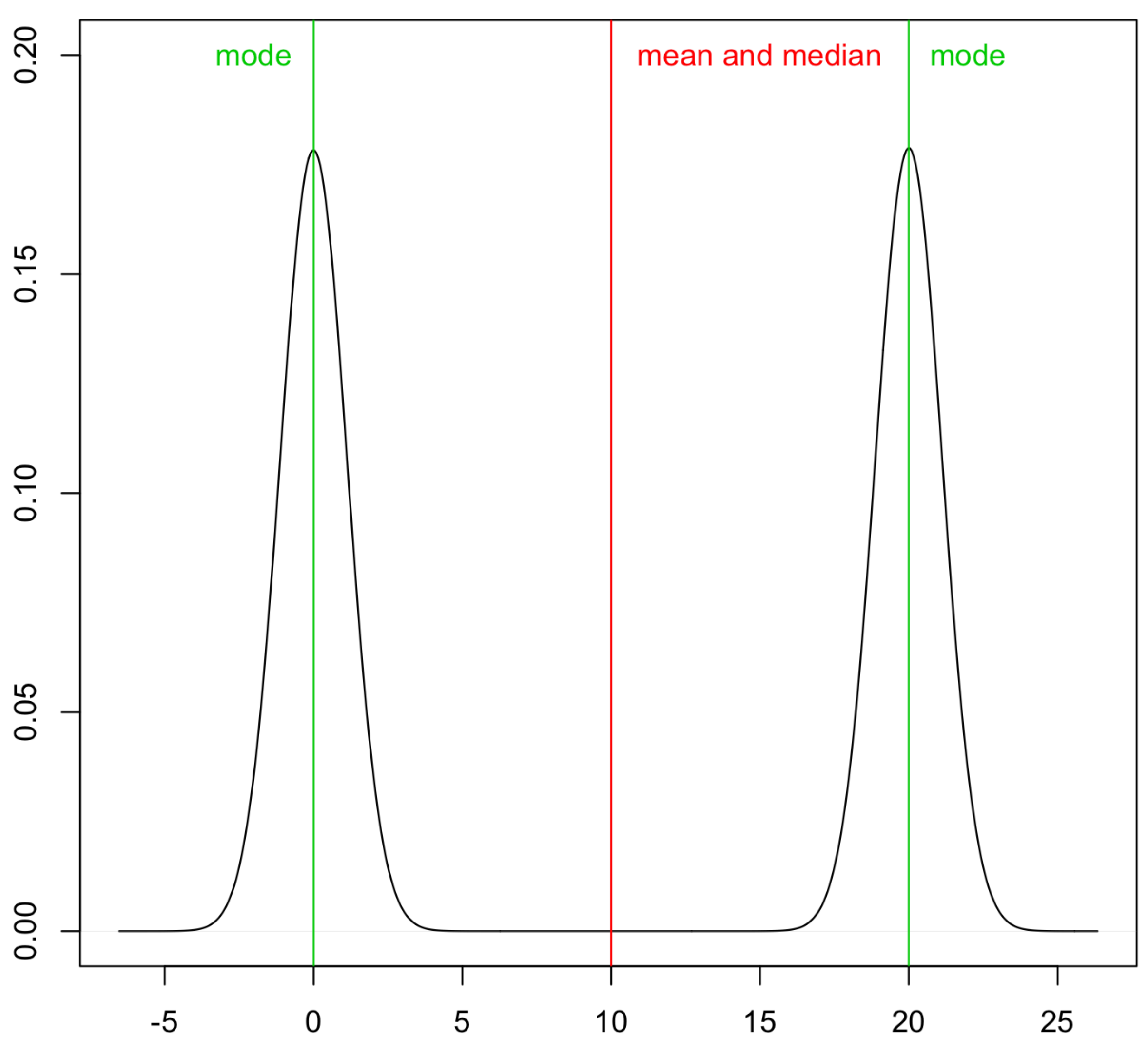

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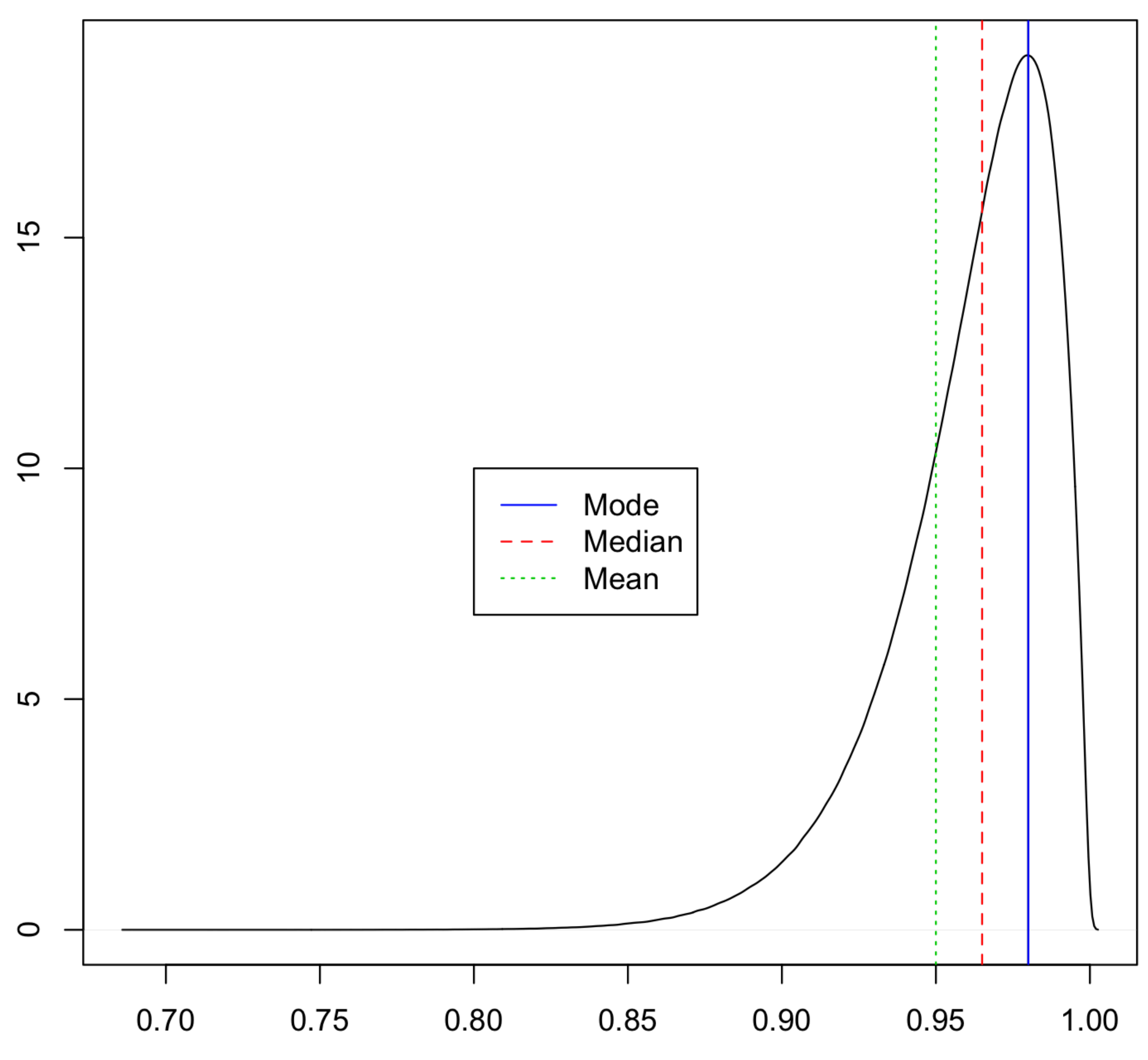

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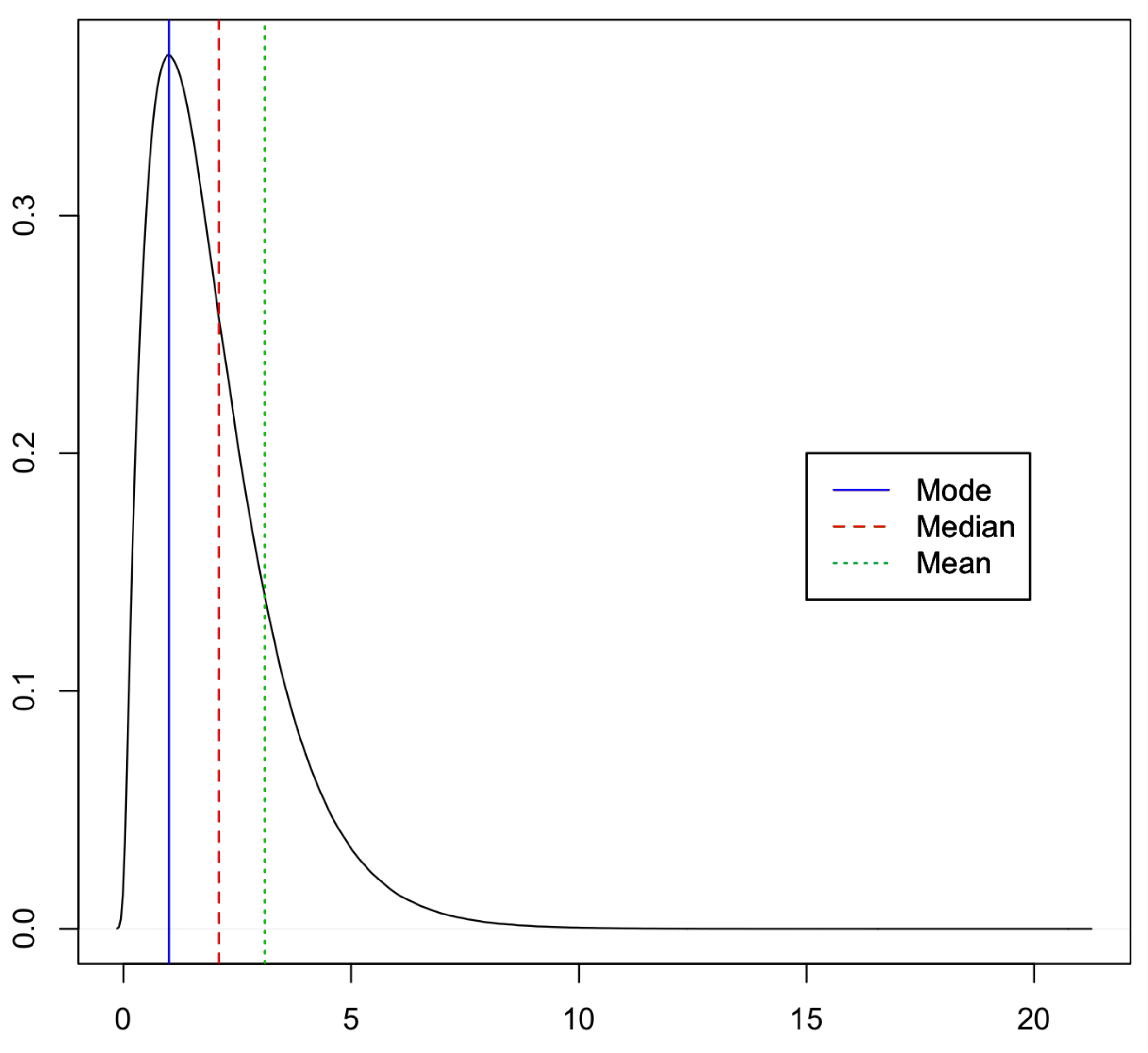

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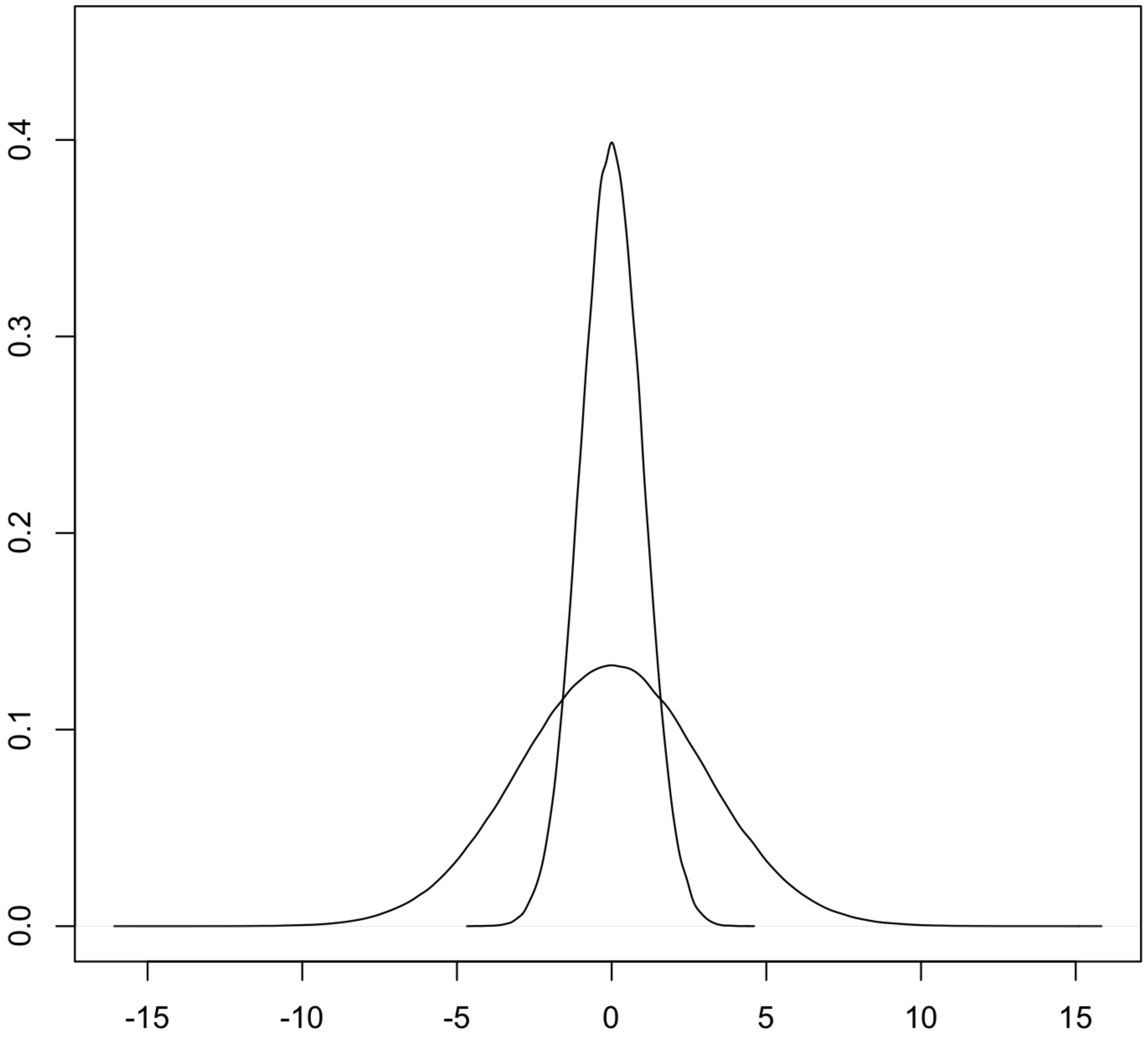
Measures of Center: Not the Whole Picture



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- While knowing where the center of a distribution is may be important, it is also essential to know how disperse the data is around that center to better understand how the variable acts

Quantiles (and Percentiles and Quartiles)

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    - Default: `type=7`

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  OR (Option 2) `quantile(data,0.75)-quantile(data,0.25)`

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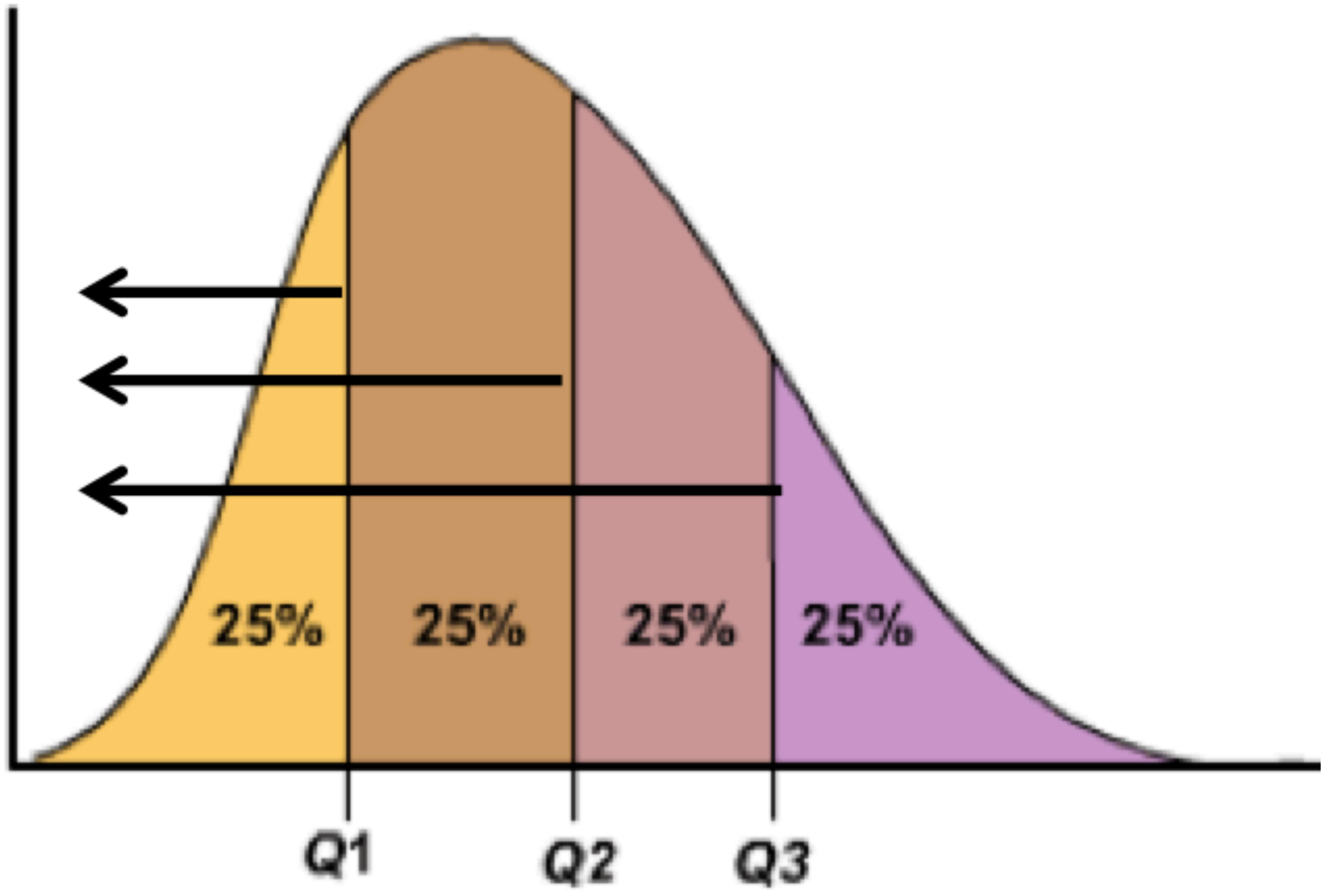

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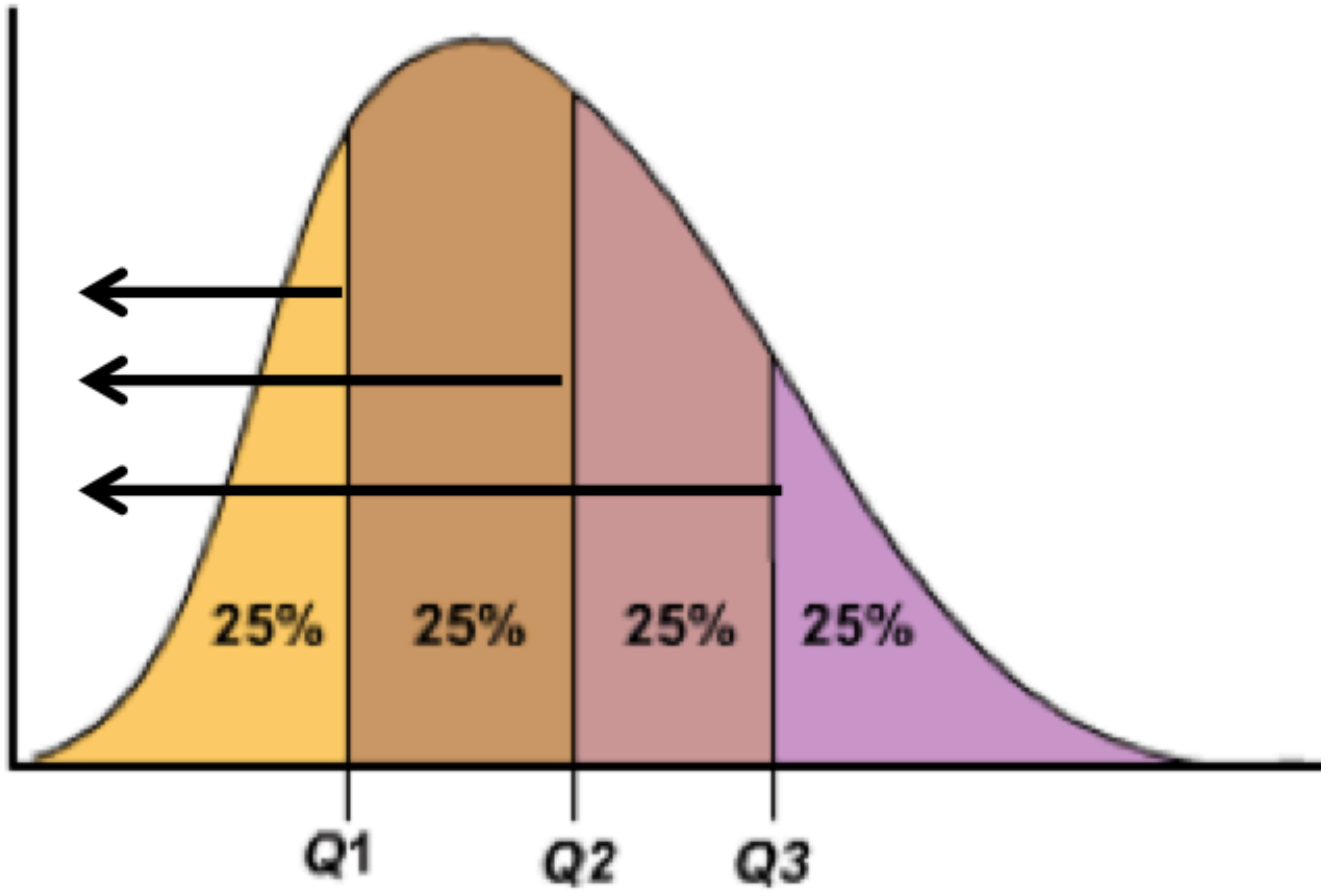

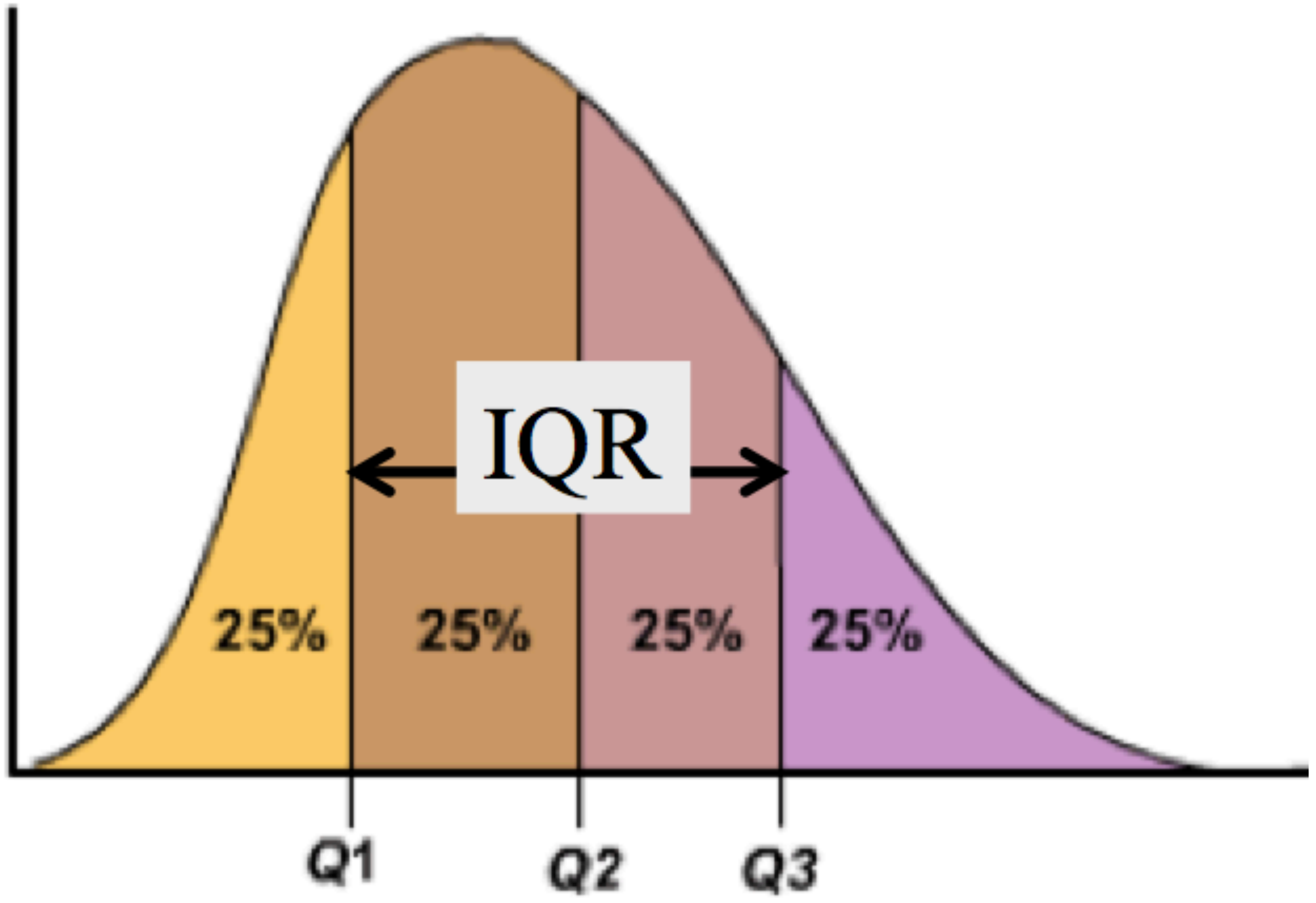

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- IQR: Q3 - Q1

Interquartile Range (IQR): Example, $n = 14$

11, 14, 15, 20, 41, 45, 61, 71, 74, 80, 93, 95, 97, 100

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  - 2^{nd} quartile (Q2): 50^{th} percentile (median)
  - 3^{rd} quartile (Q3): 75^{th} percentile

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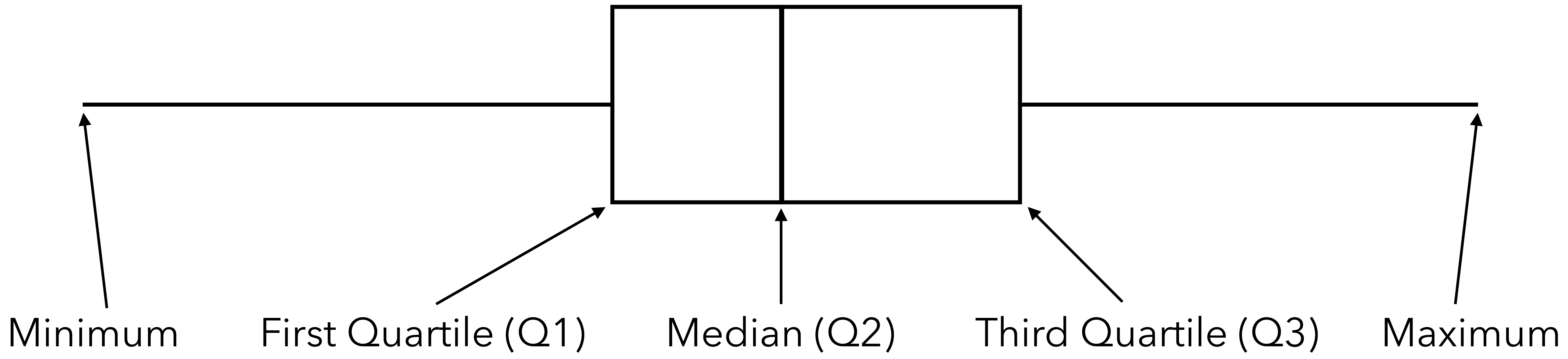
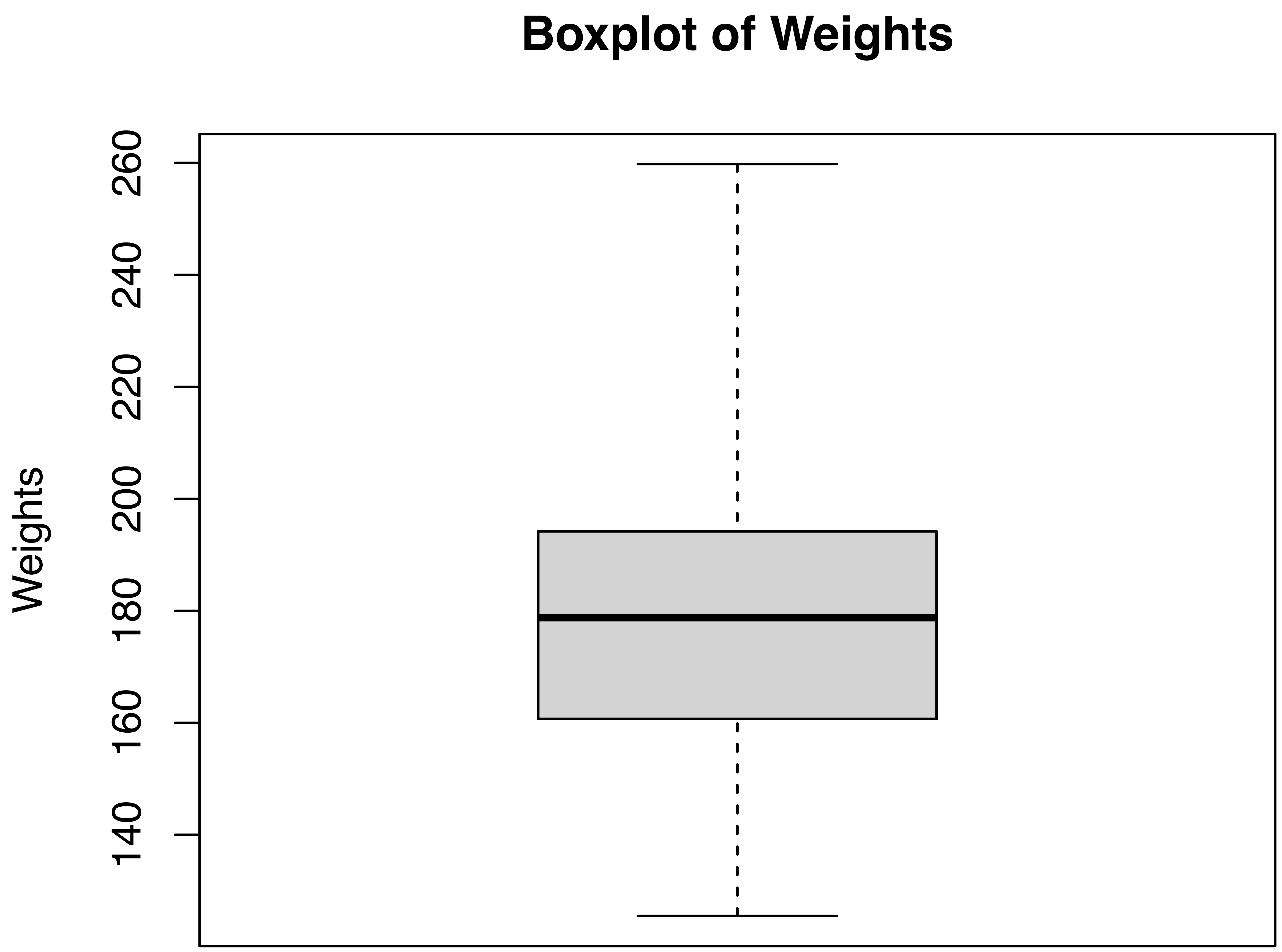


Boxplot: Example




```
boxplot(weights,range=0,main="Boxplot of Weights",ylab="Weights")
```

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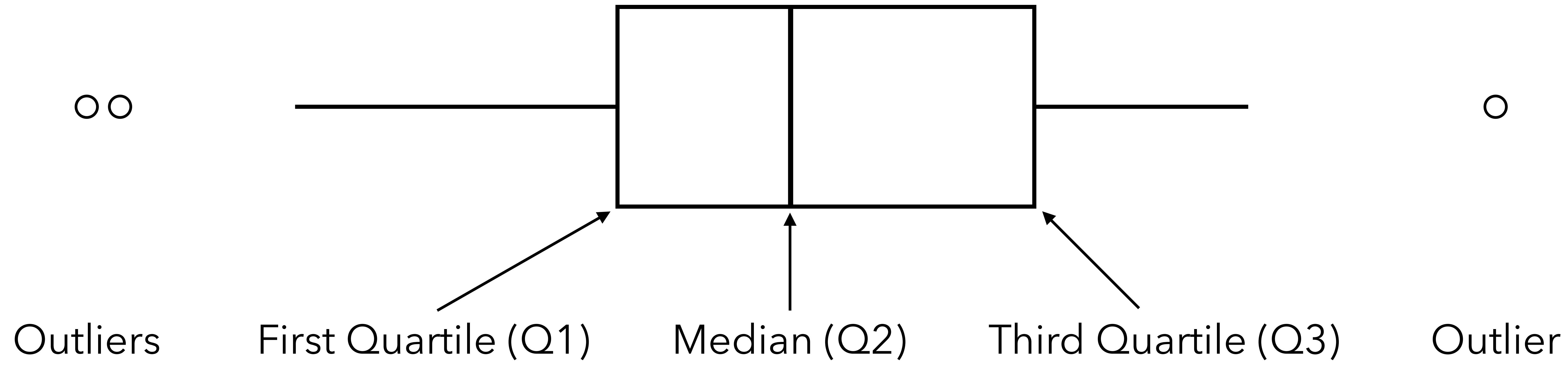
- Now, whiskers extend only to the highest / lowest non-outlier points, and the outliers are distinguished as separate points

Modified Boxplot



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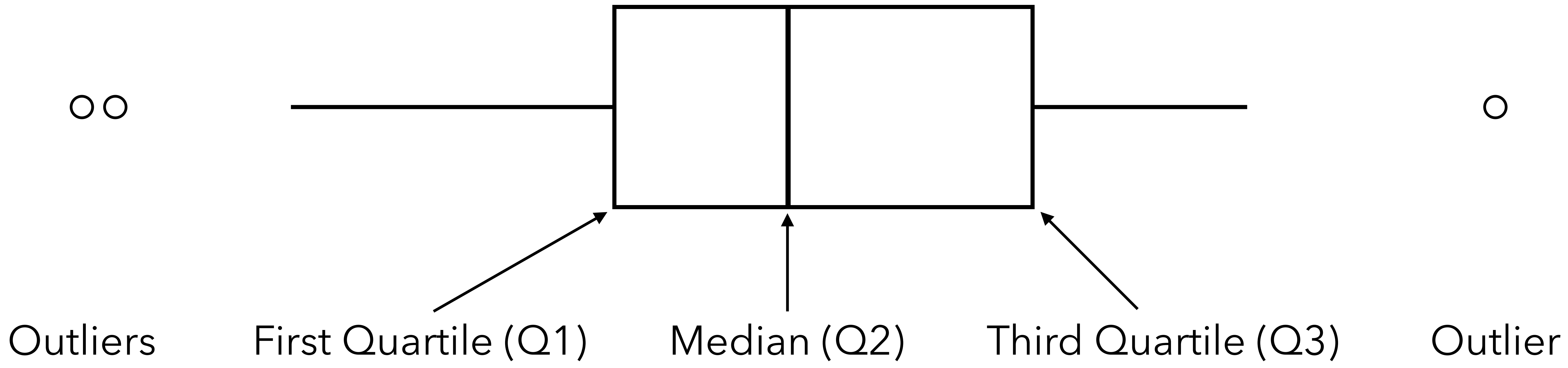
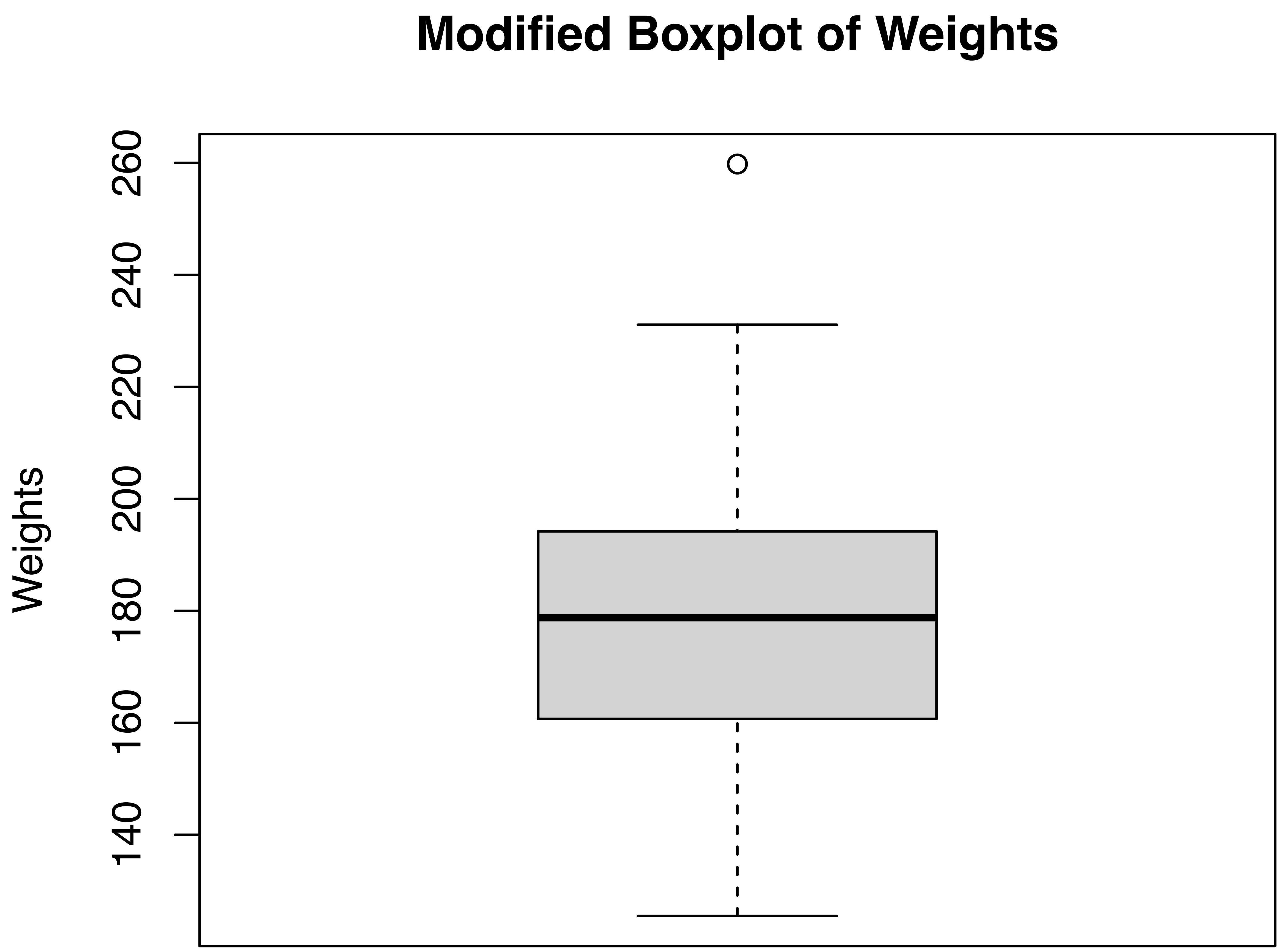


- Note that whiskers extend to the most extreme observation that isn't an outlier, and there can be multiple outliers beyond the whiskers

Modified Boxplot: Example

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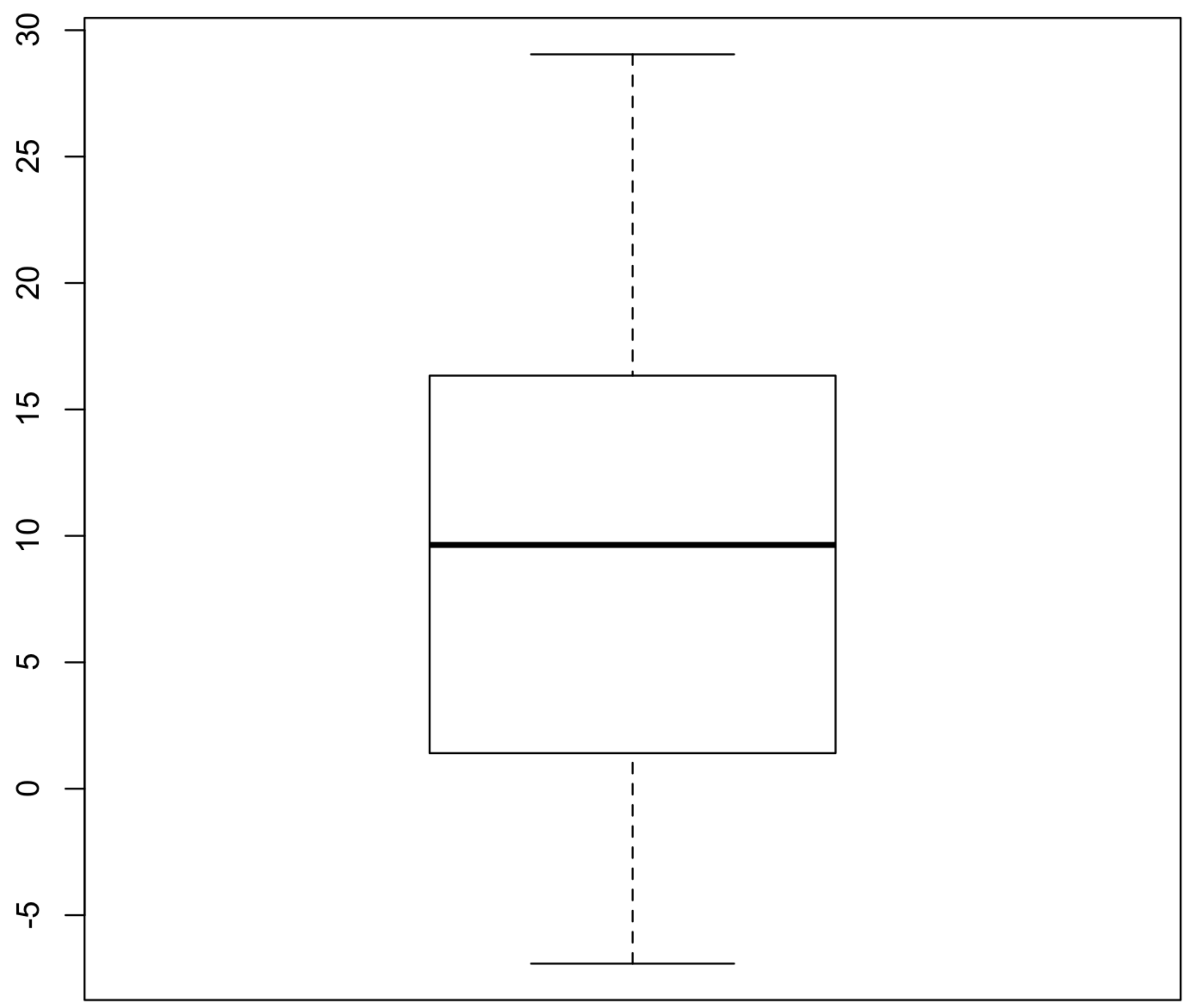
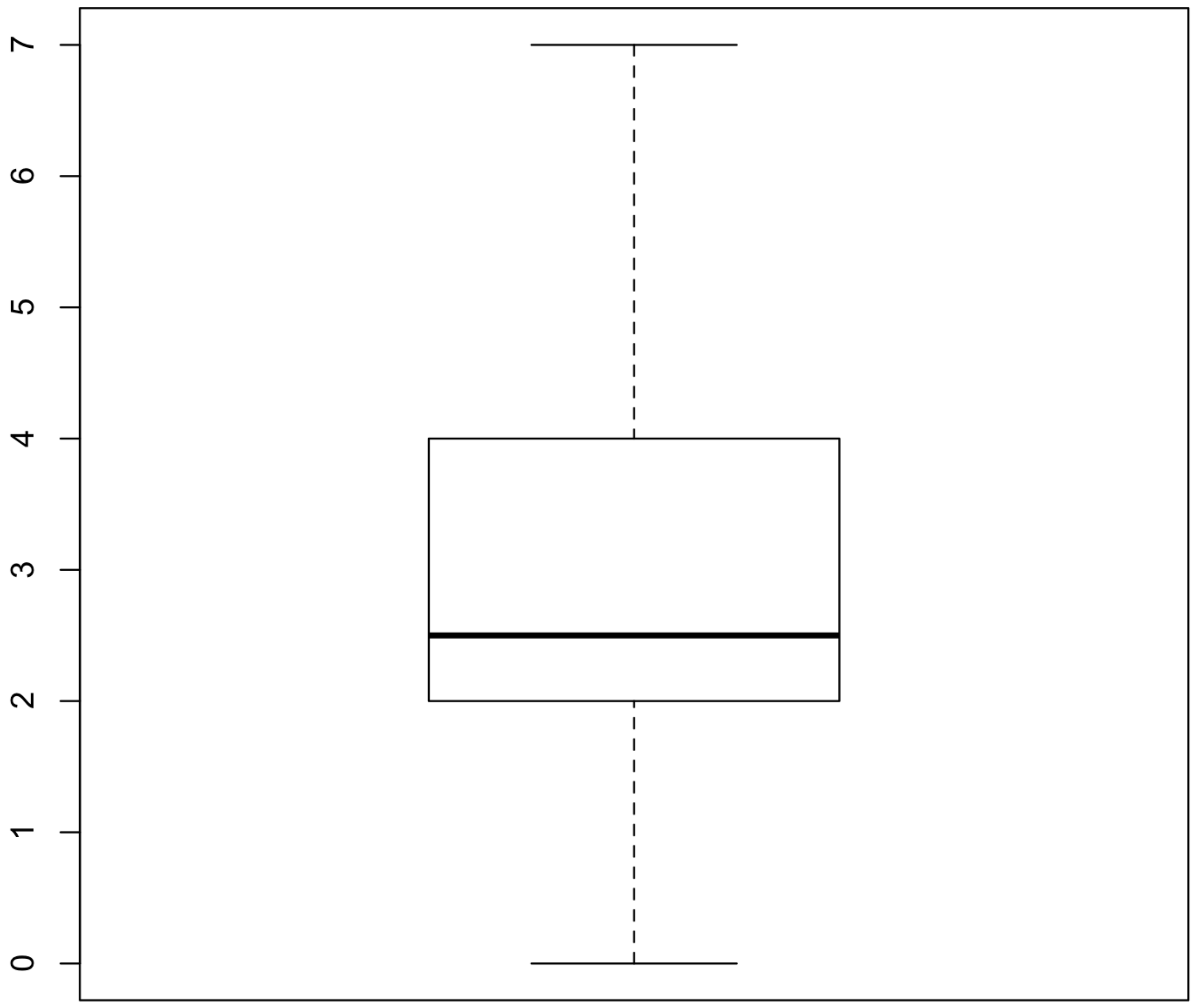
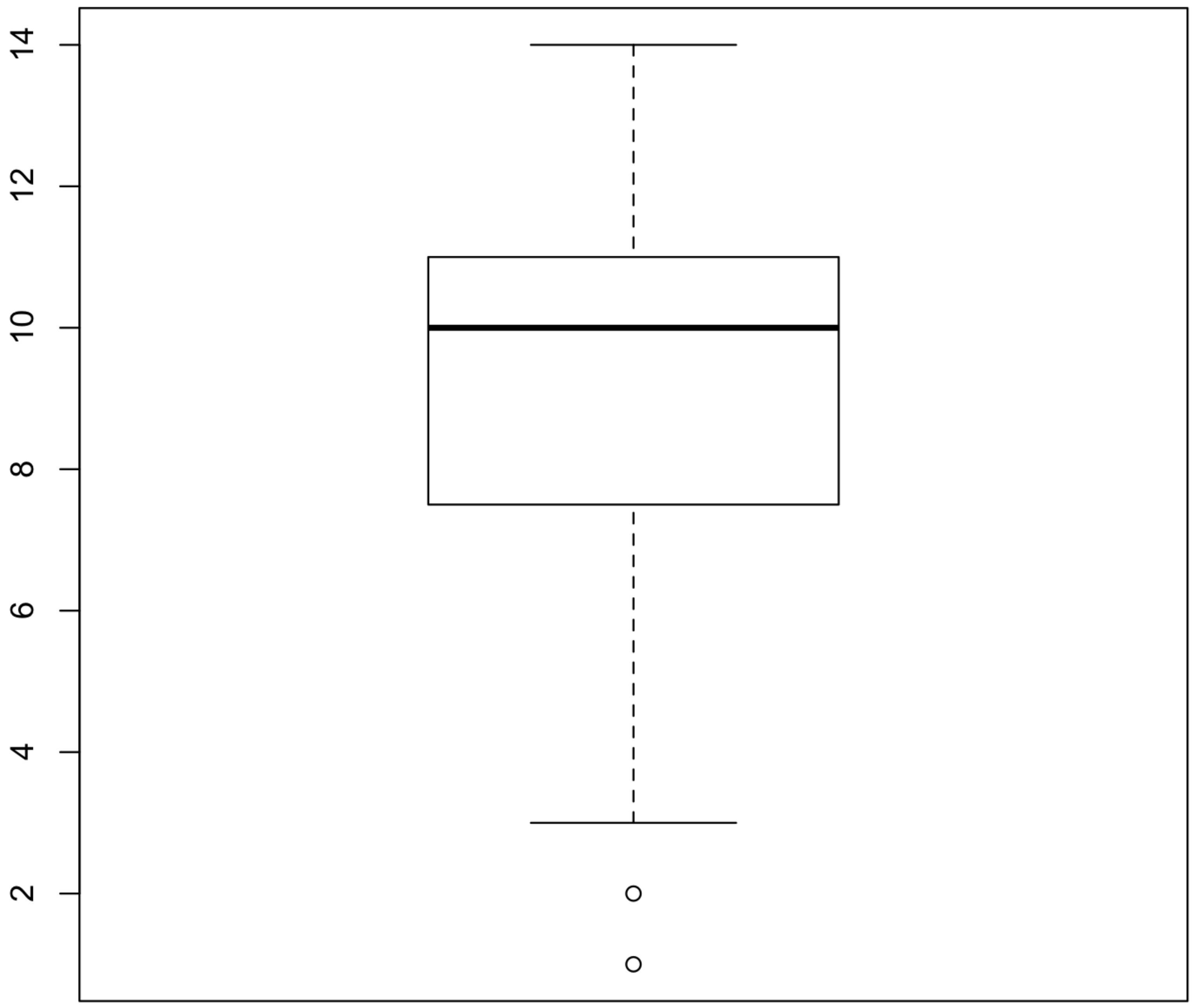
- Indicate the upper whisker, given by the largest observation that is less than the upper fence
- Indicate the lower whisker, given by the smallest observation that is greater than the lower fence
- Mark outliers as individual points beyond the fences

(Modified) Boxplot Examples







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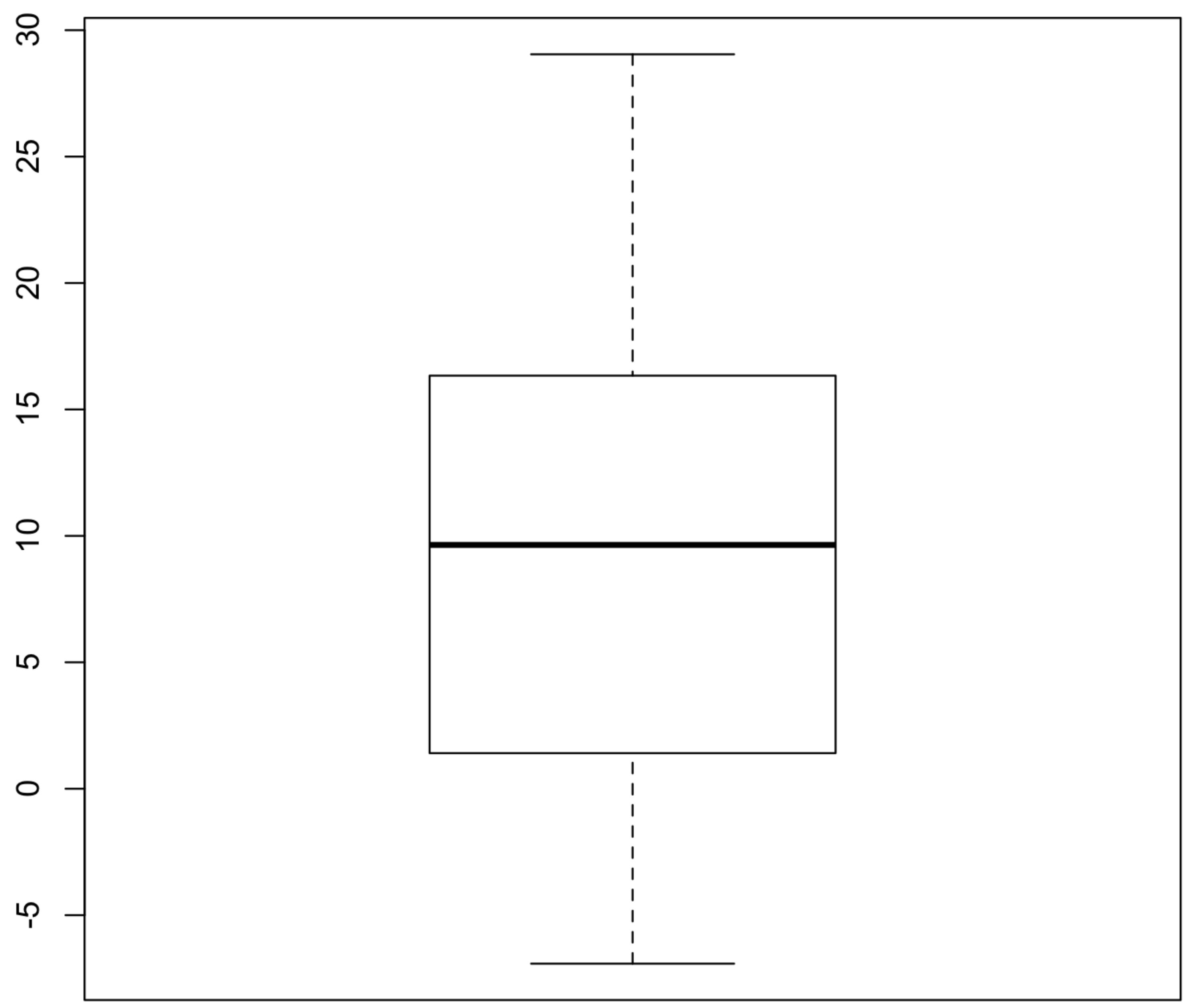

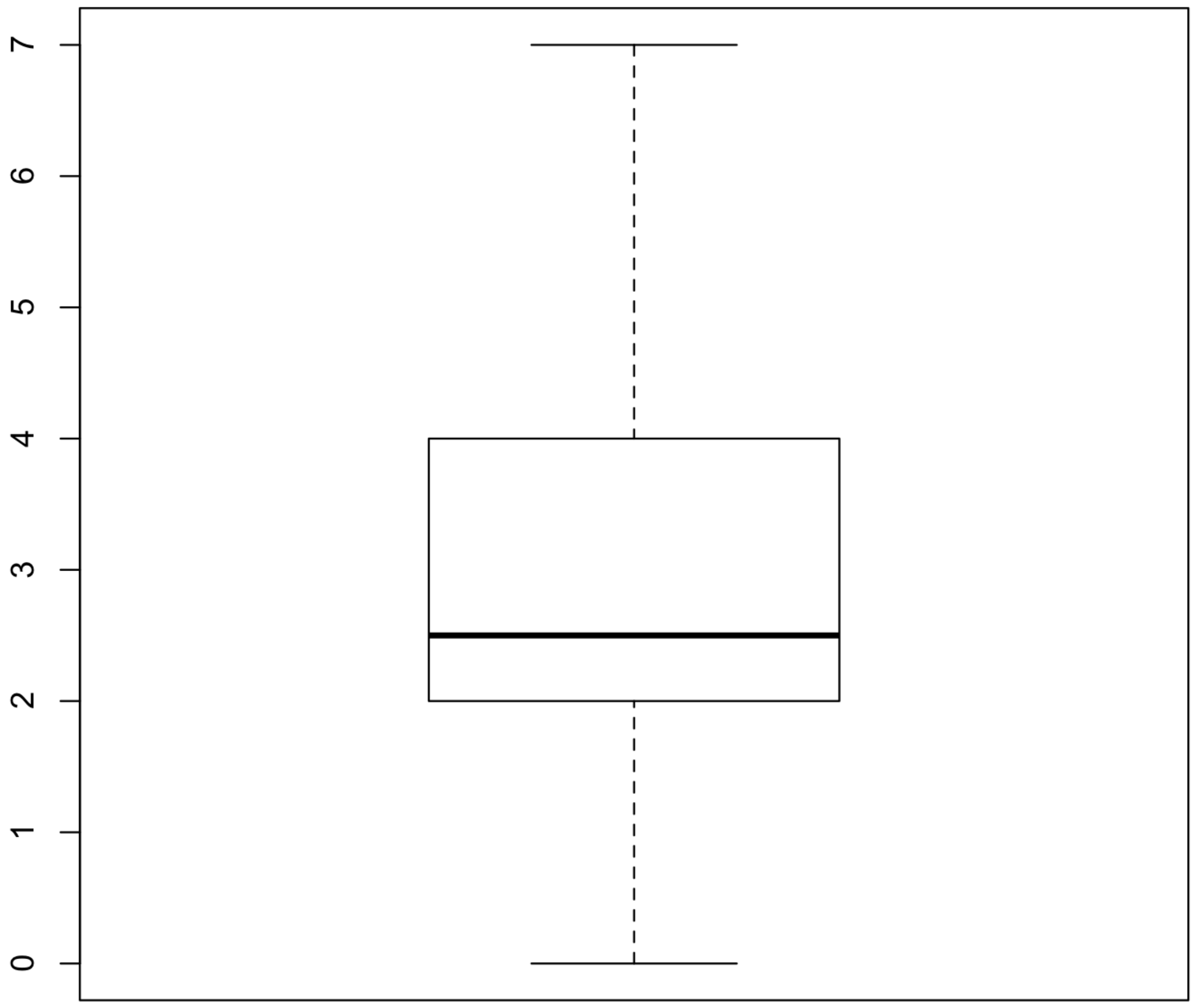

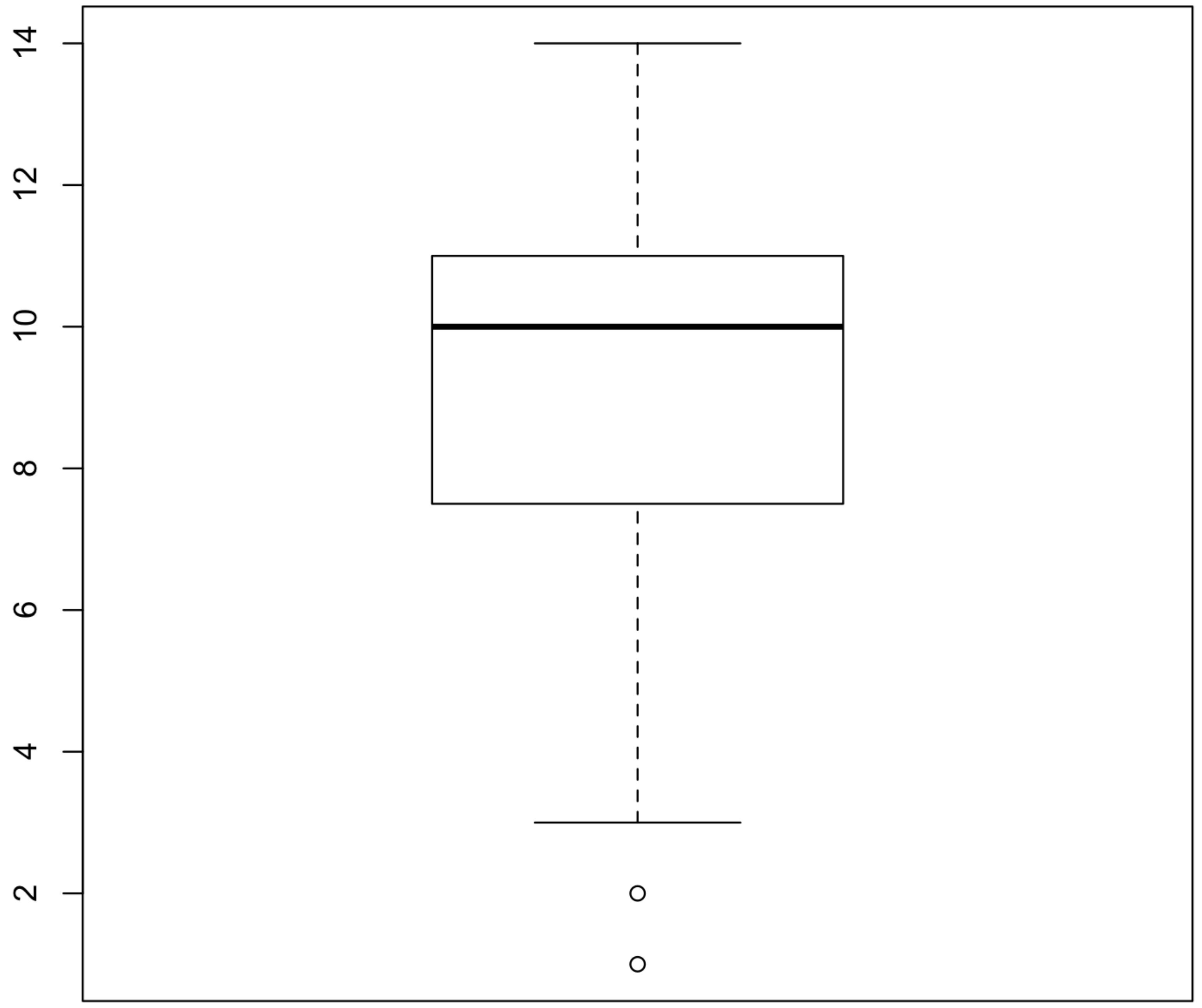

Symmetric

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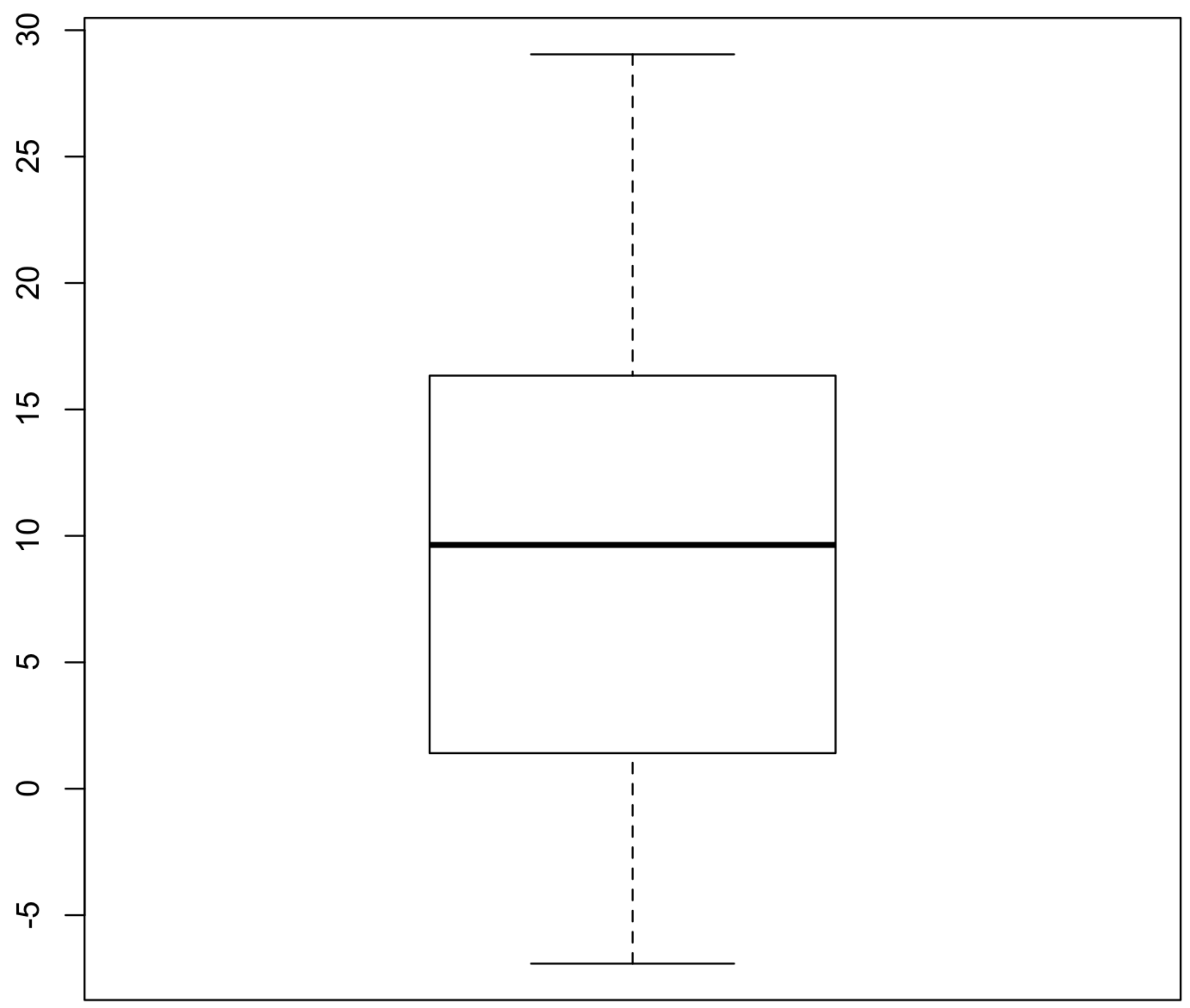

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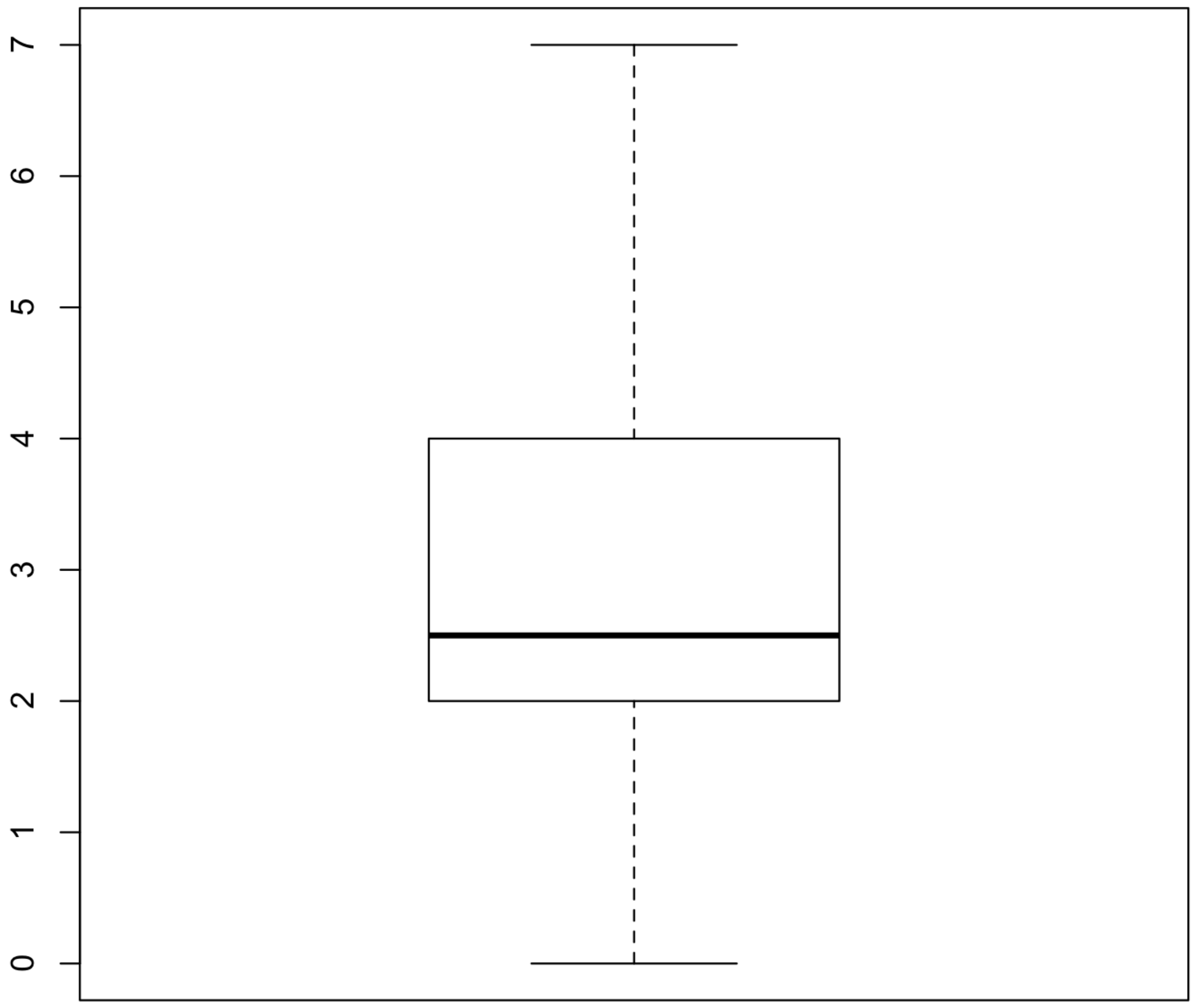

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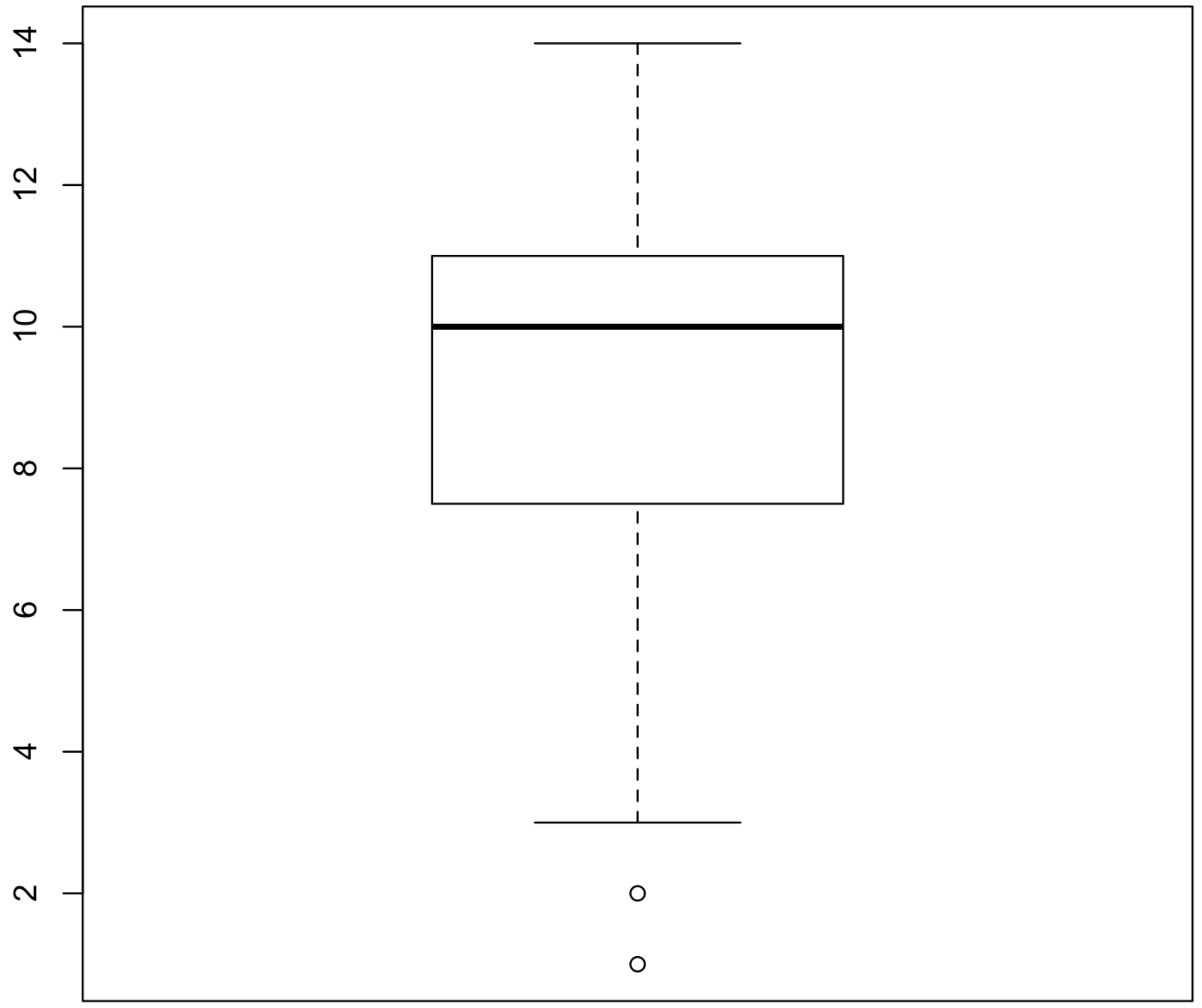

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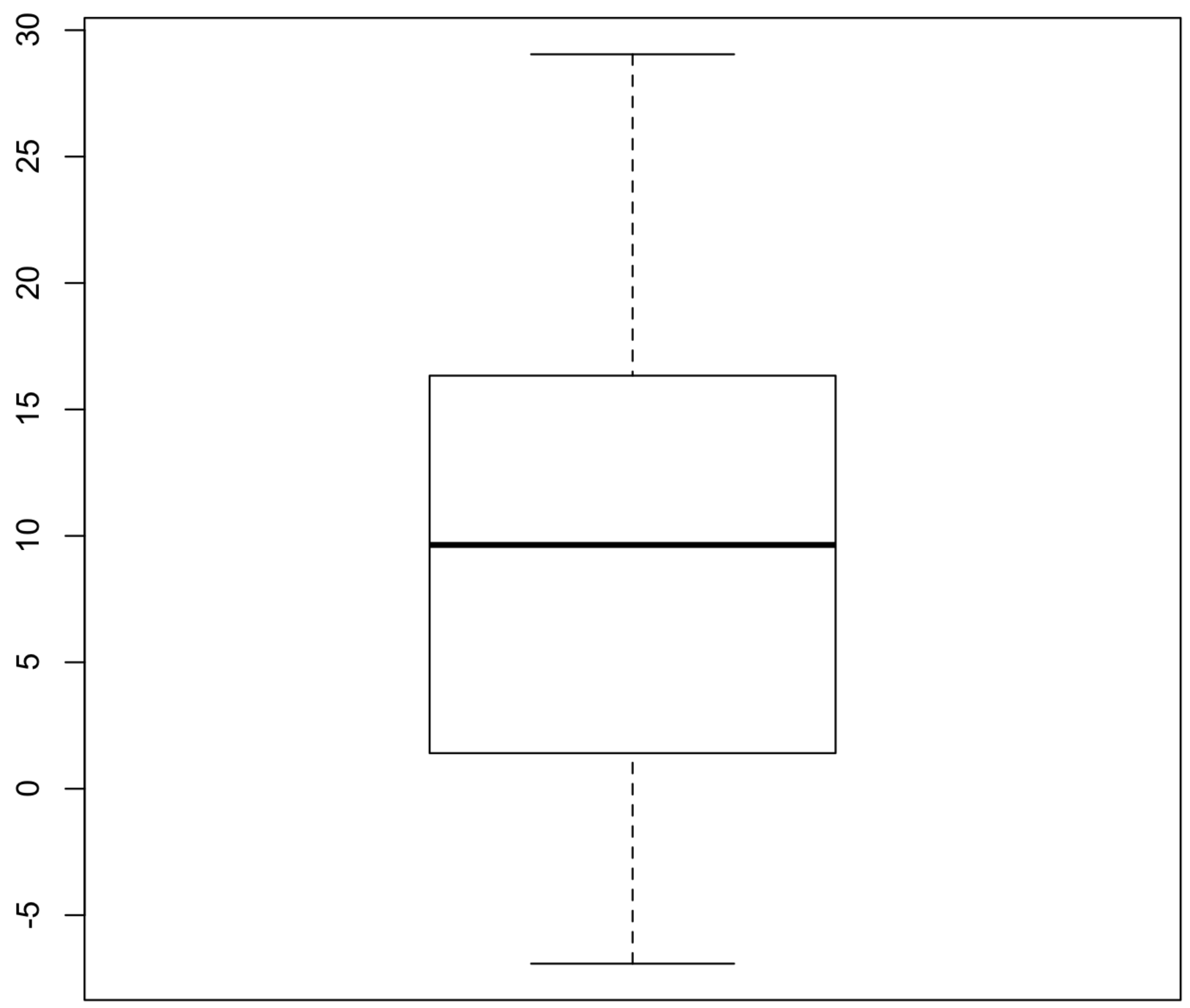

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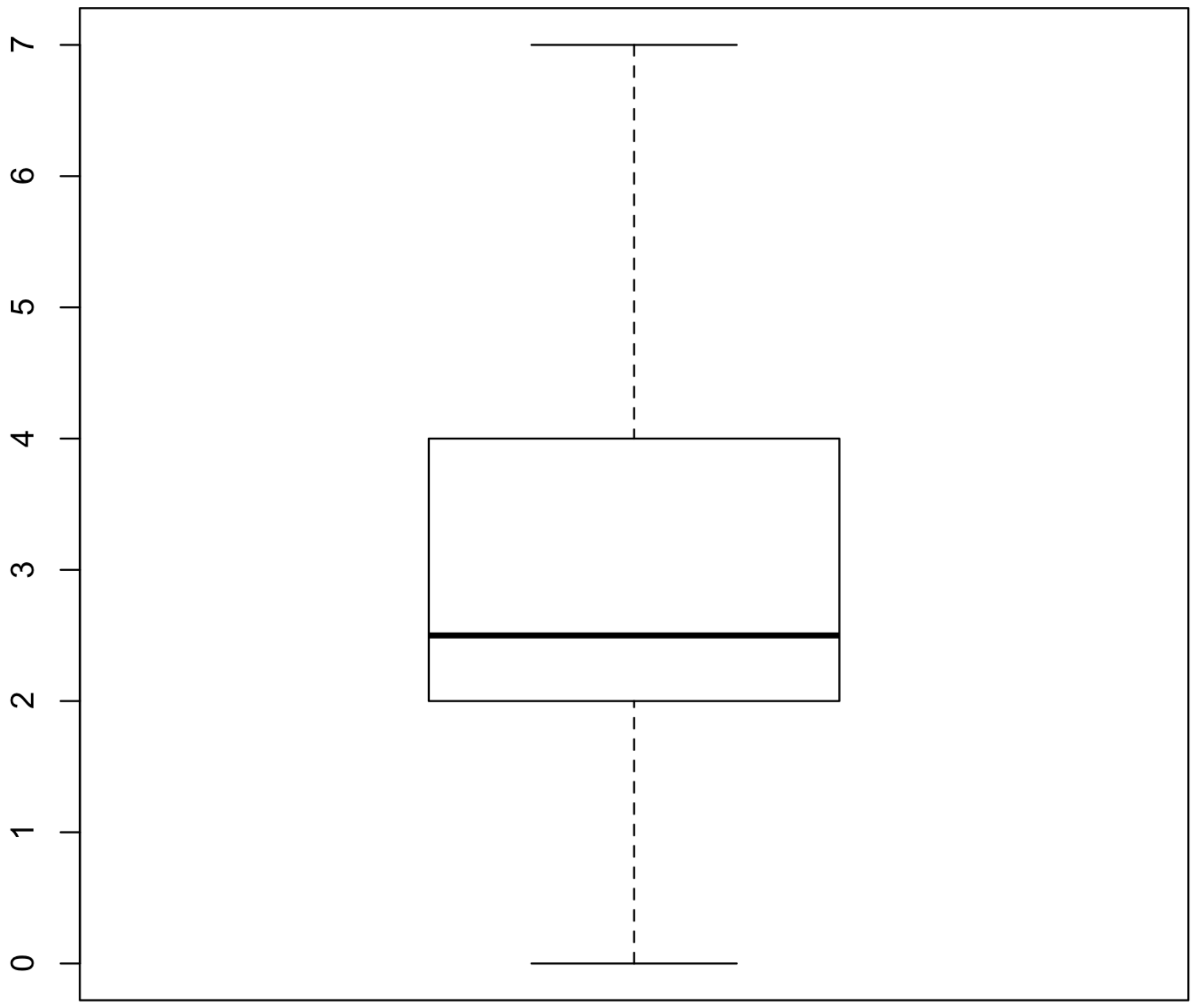

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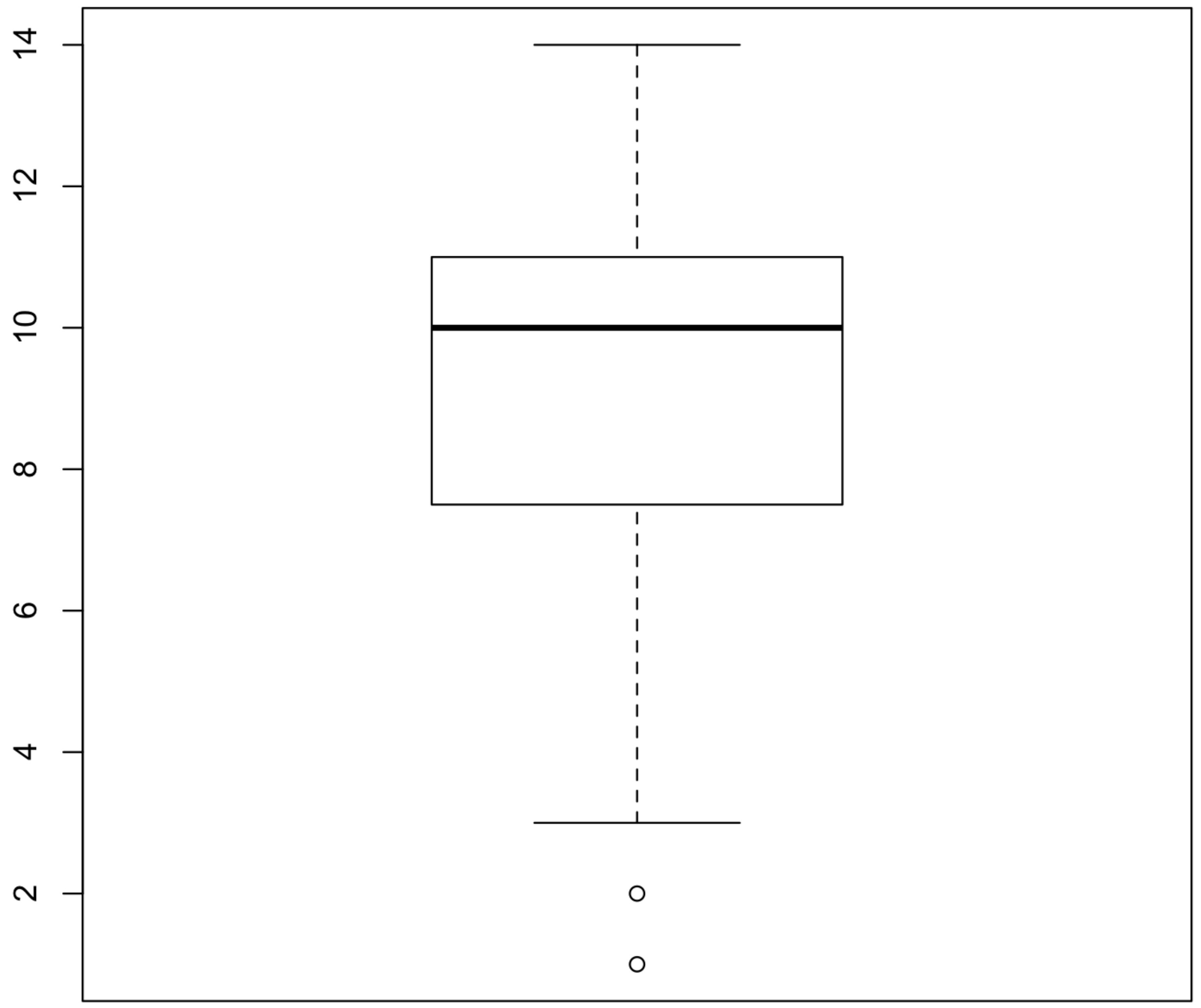

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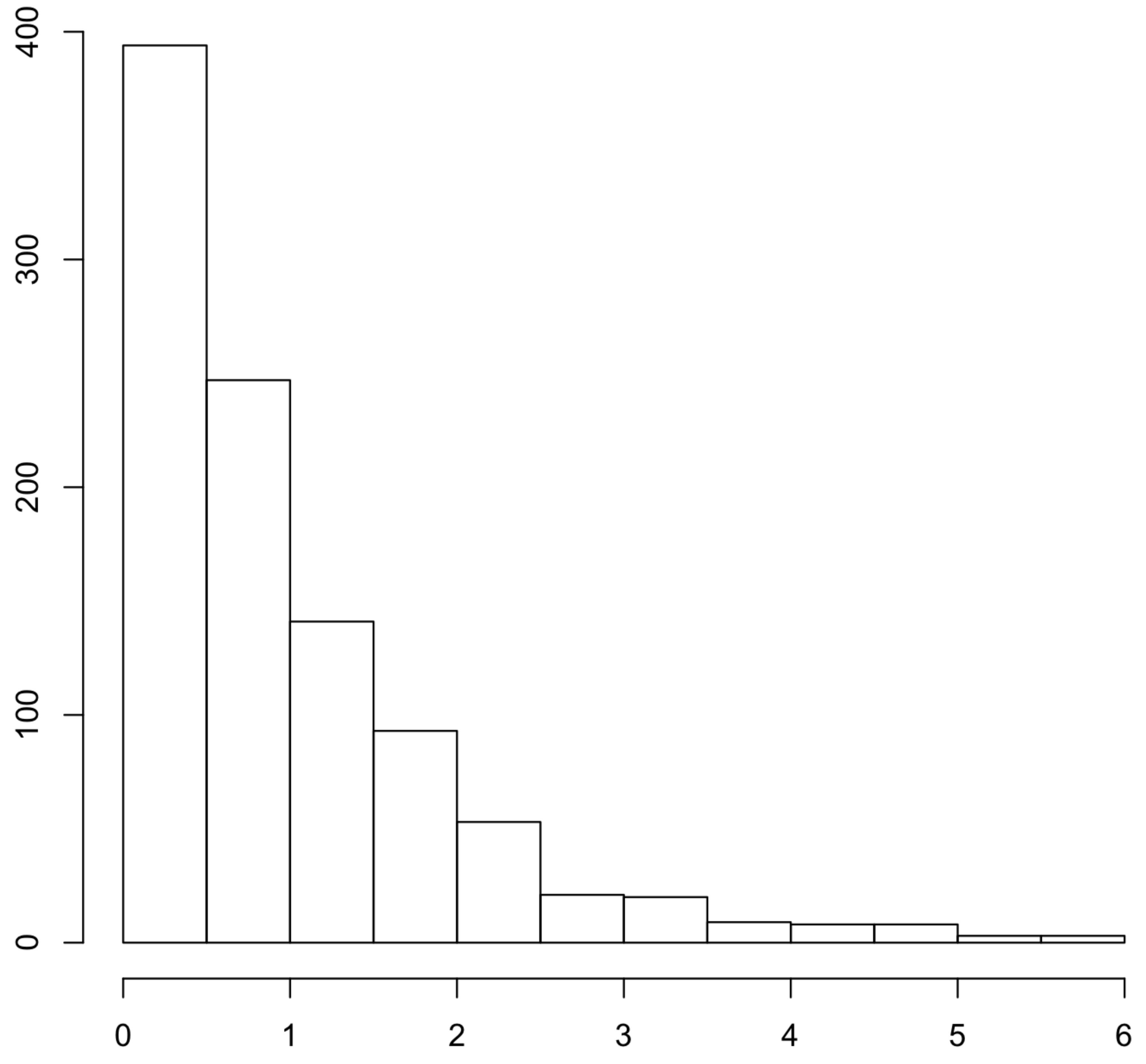
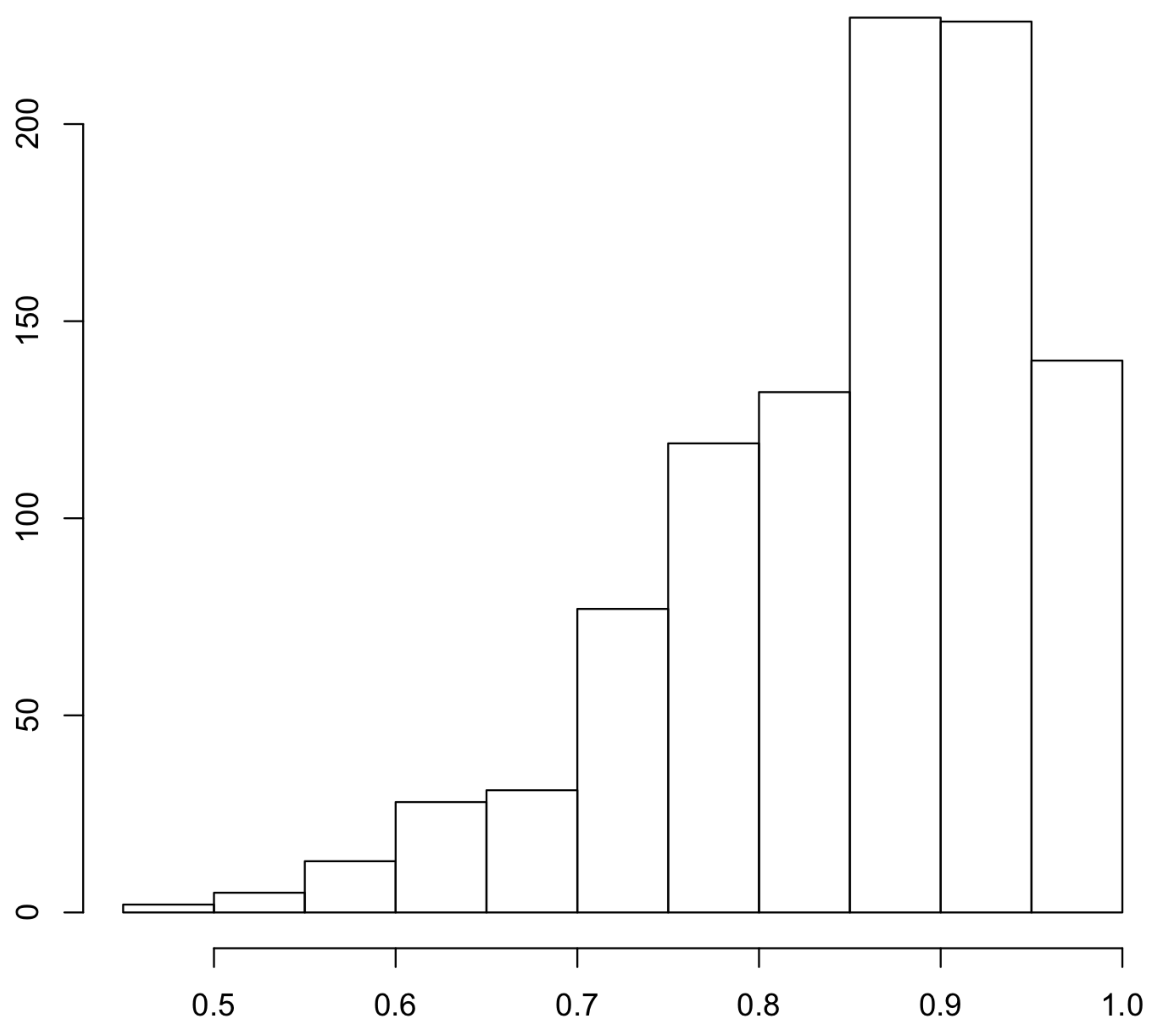
- R code: `library(moments); skewness(data)`

Skewness: Examples

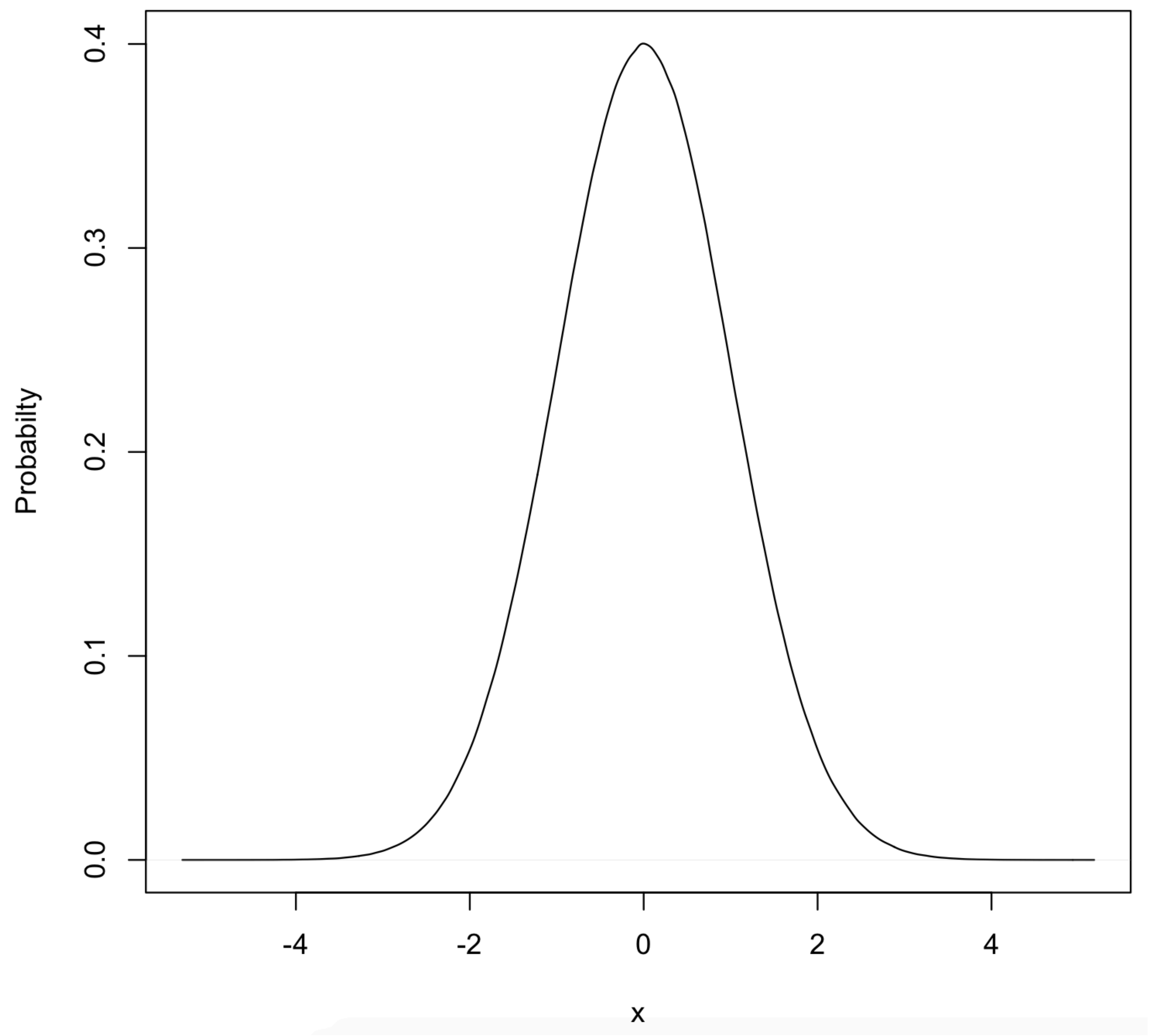


Right Skew (1.890)



Left Skew (-0.963)

Describing Distributions: Normality



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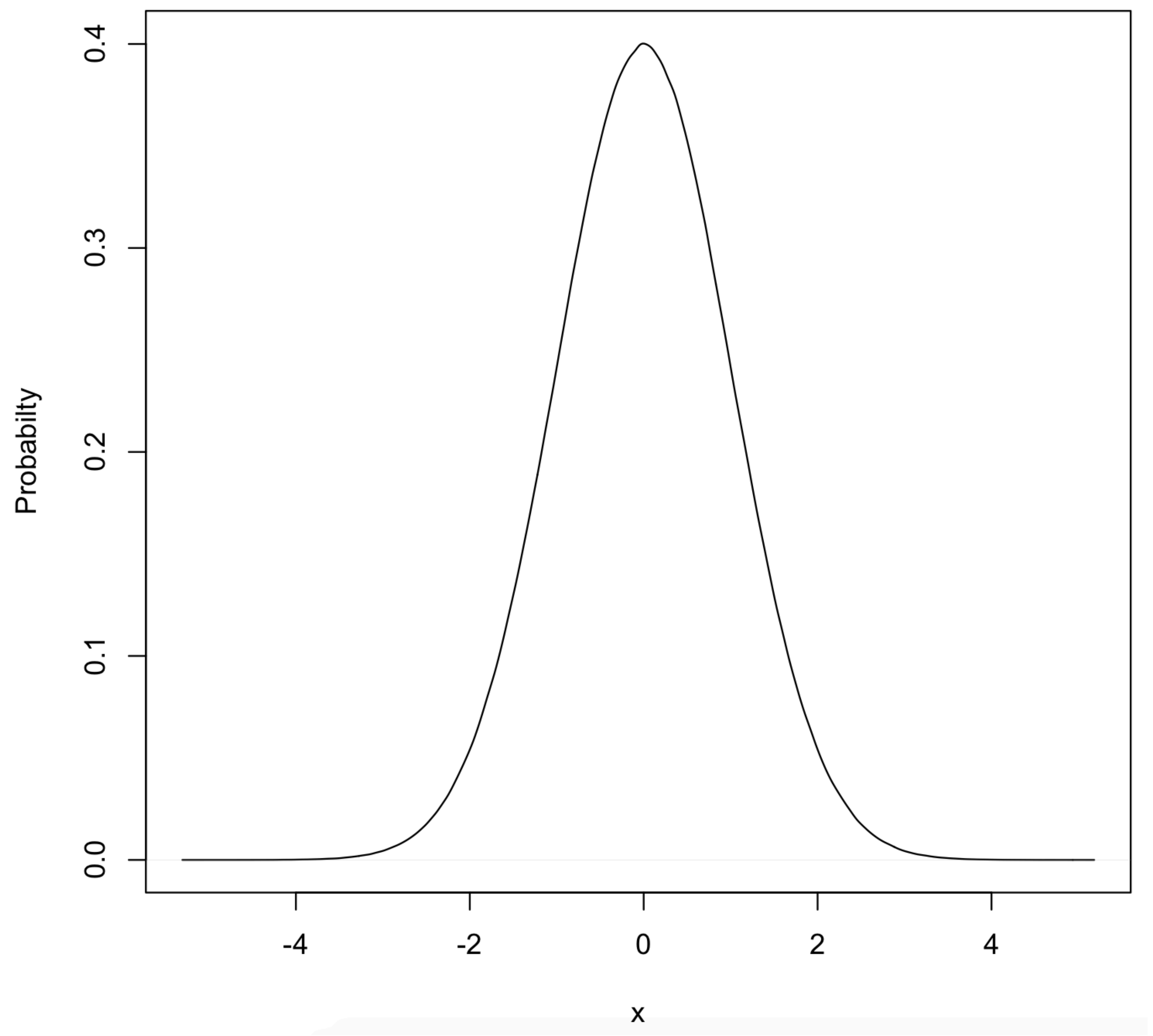

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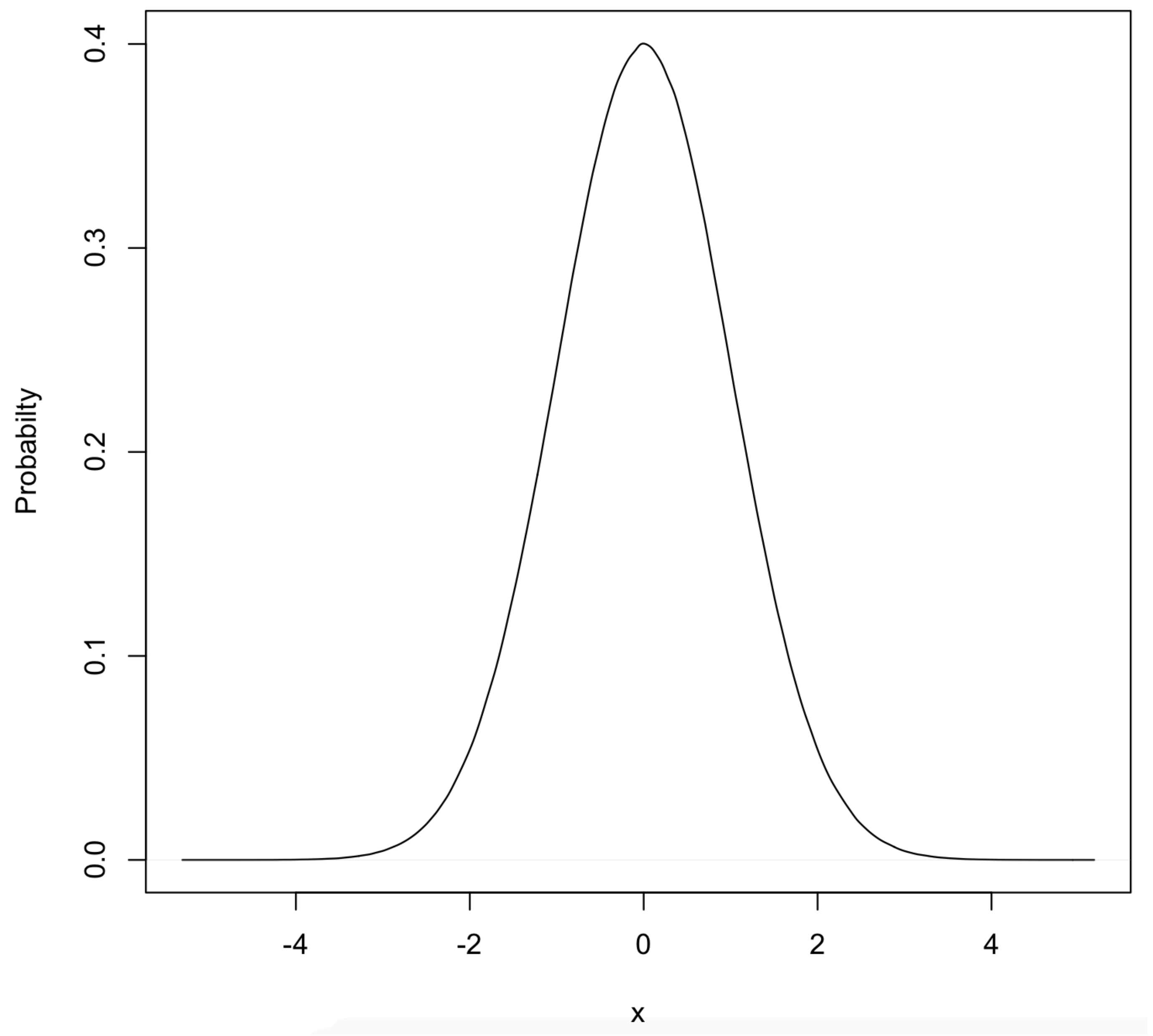

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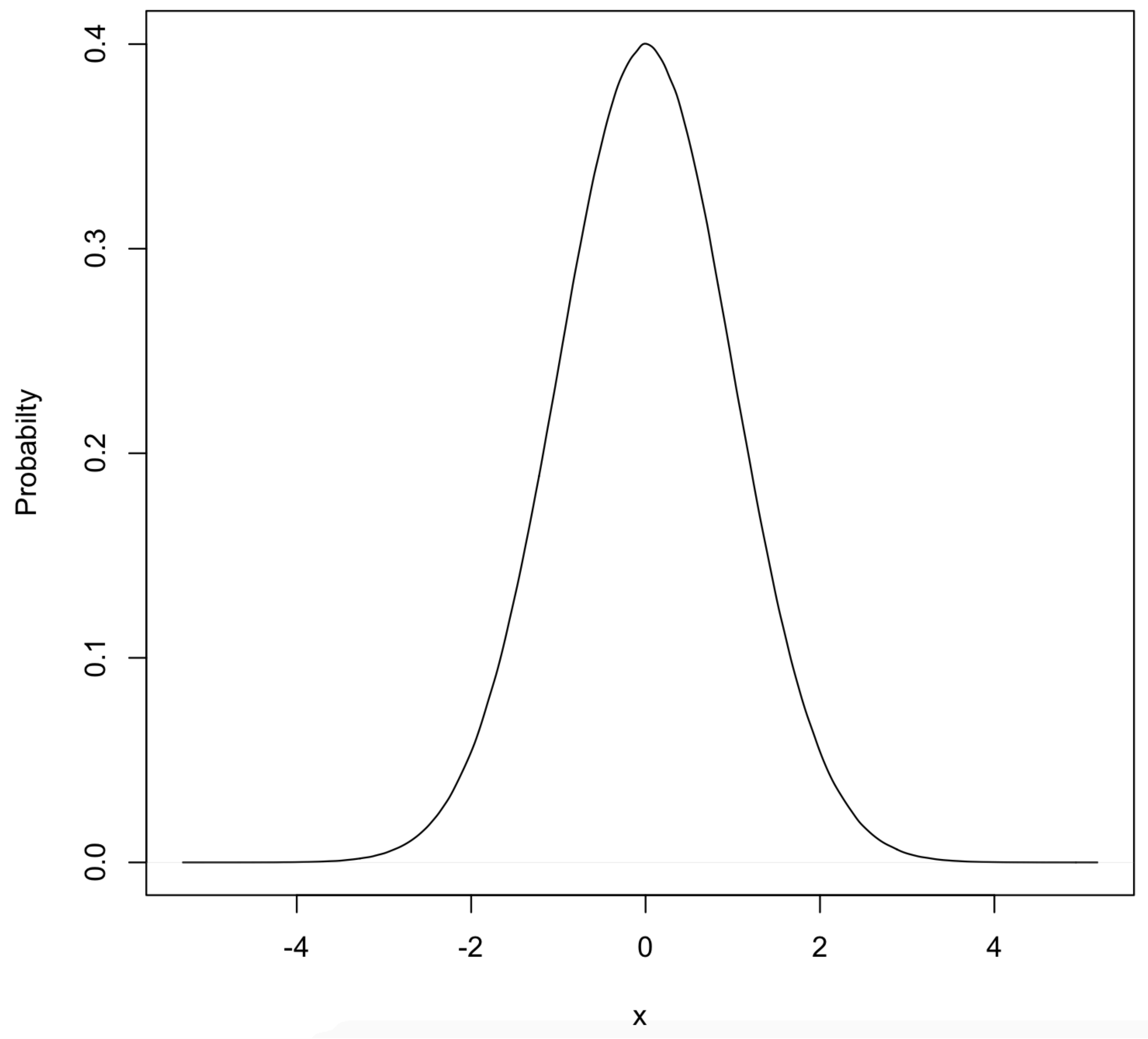
- In particular, in order to apply many common statistical procedures, data should follow a *normal (Gaussian) distribution*
- To see if the data is approximately normal, we can use the *Empirical Rule*



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- A student takes two tests. The mean on the first test is 86 with a standard deviation of 4. The mean on the second test is 400 with a standard deviation of 15. The student scored 91 on the first test and 425 on the second. Which test did the student score better on relative to the other test takers?

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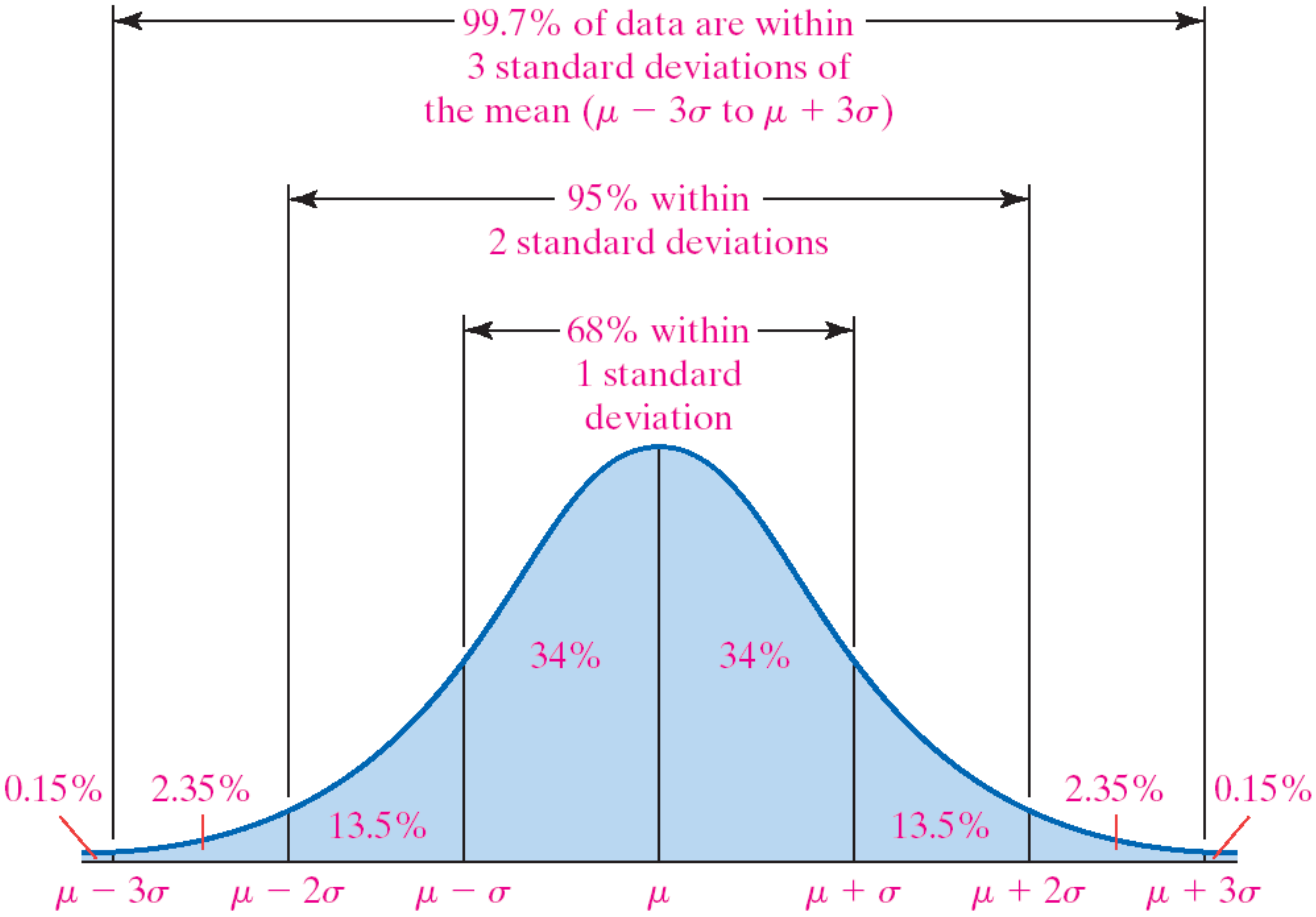

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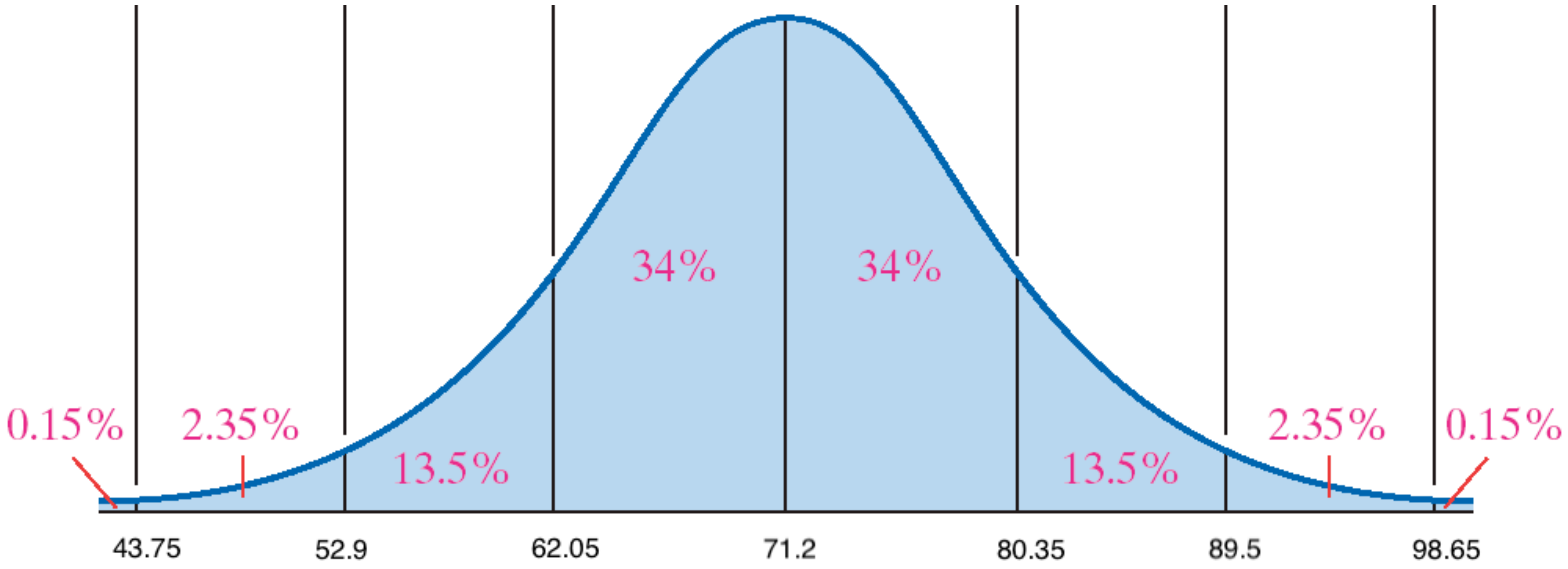

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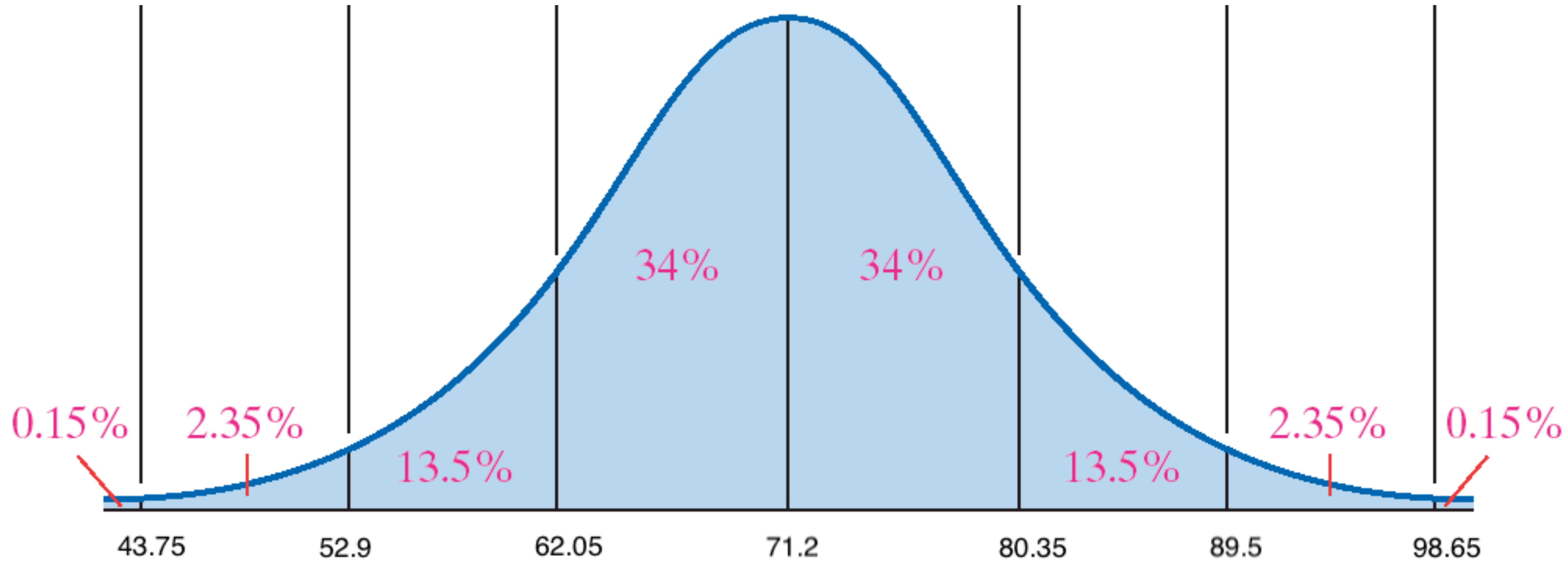

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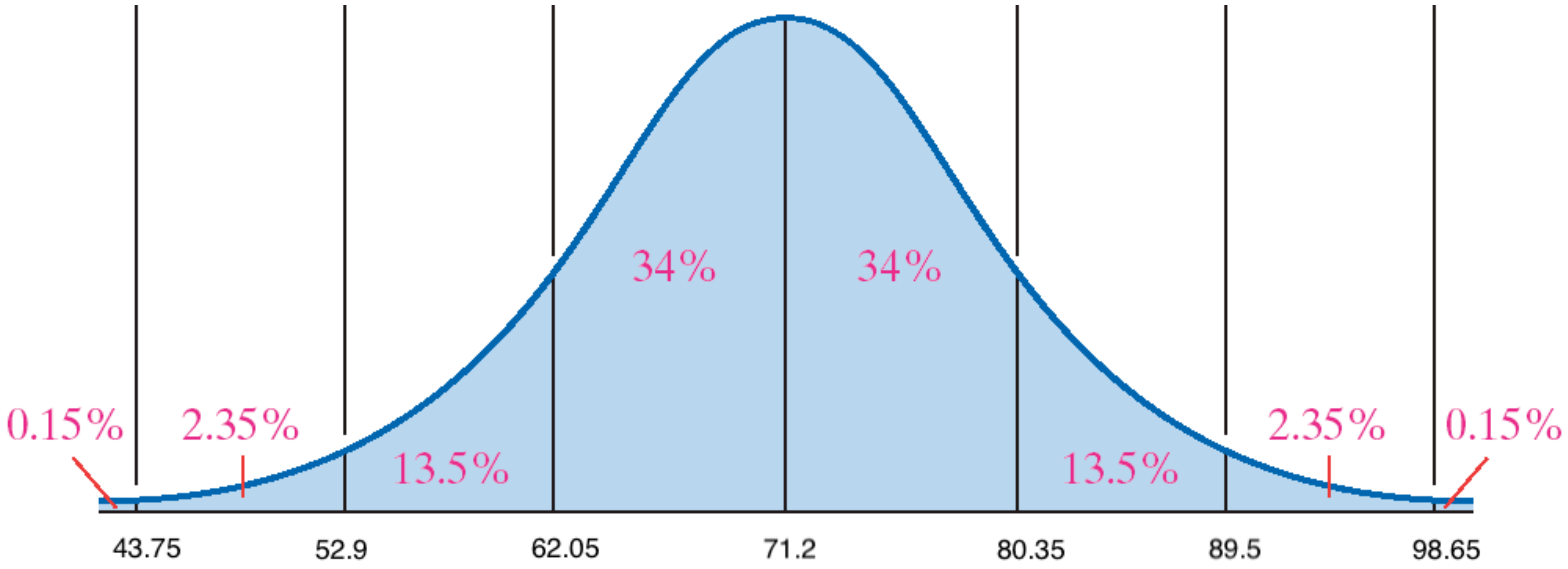

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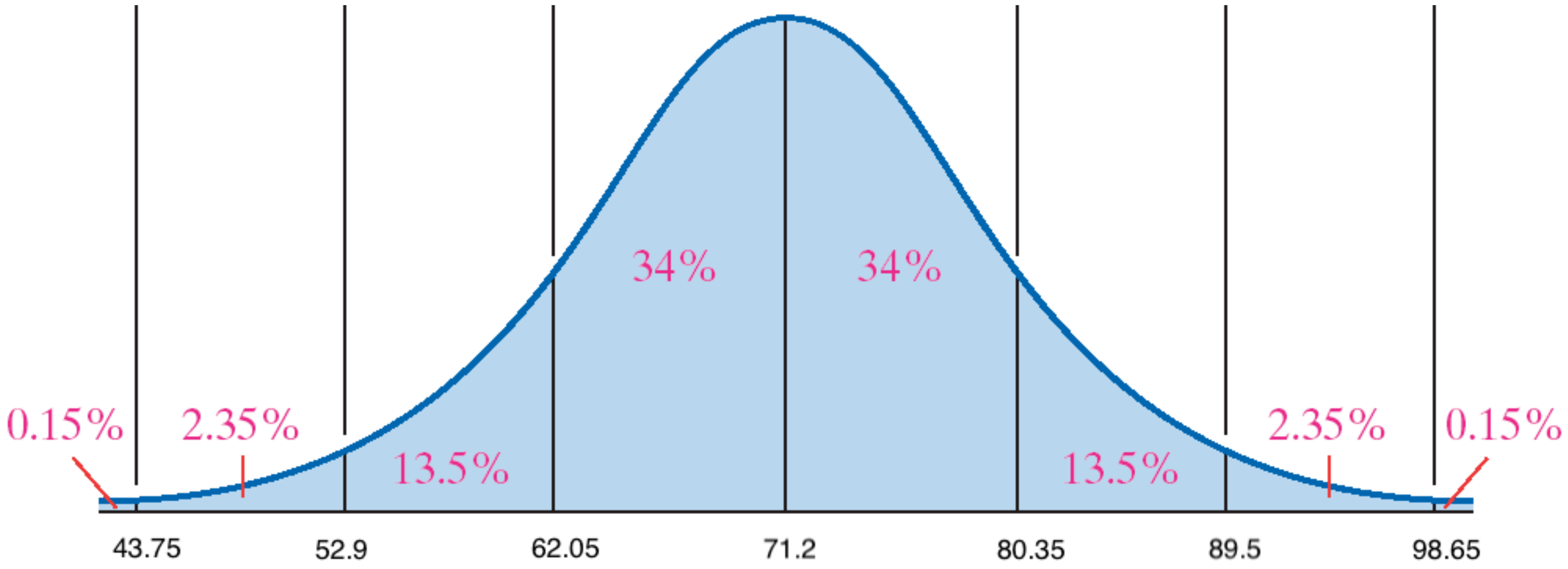

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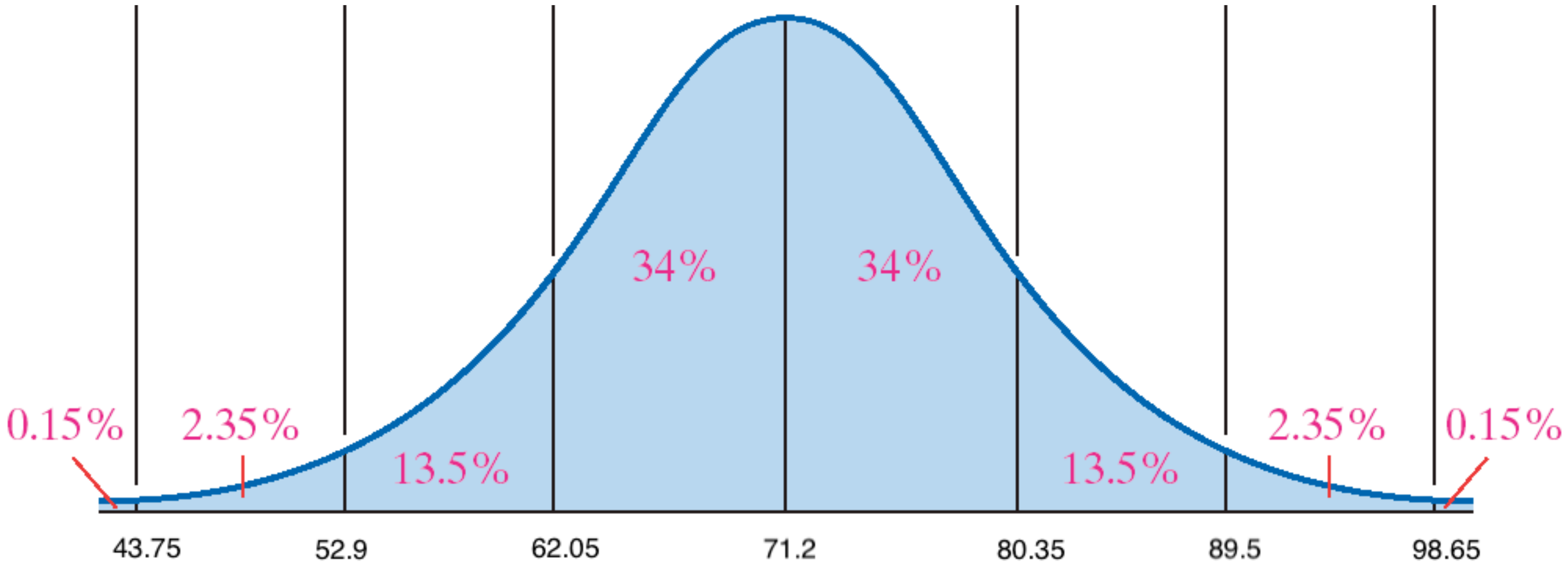

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- Weaker than the Empirical Rule because it makes fewer assumptions

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- R code: `qqnorm(data); qqline(data)`

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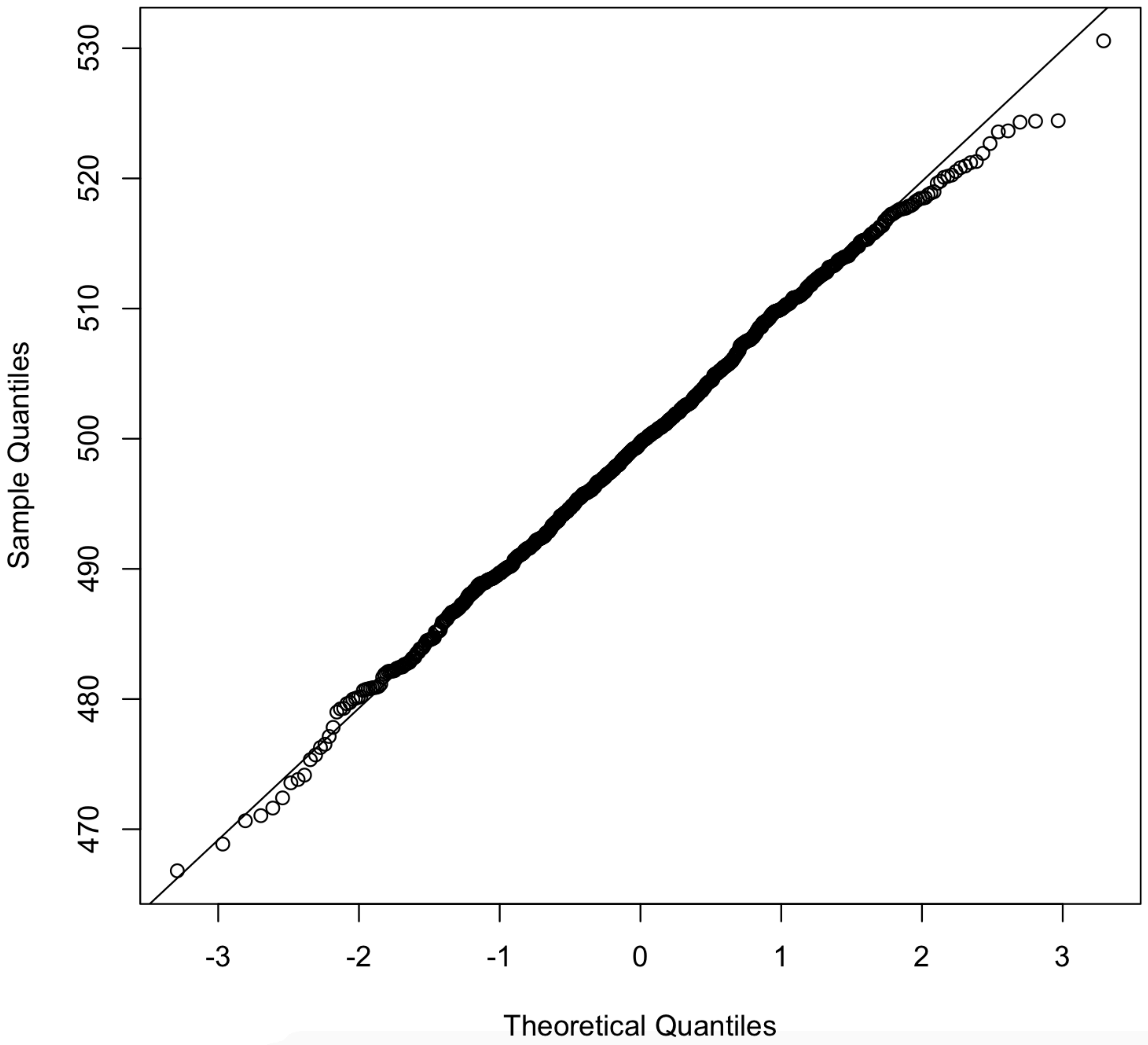

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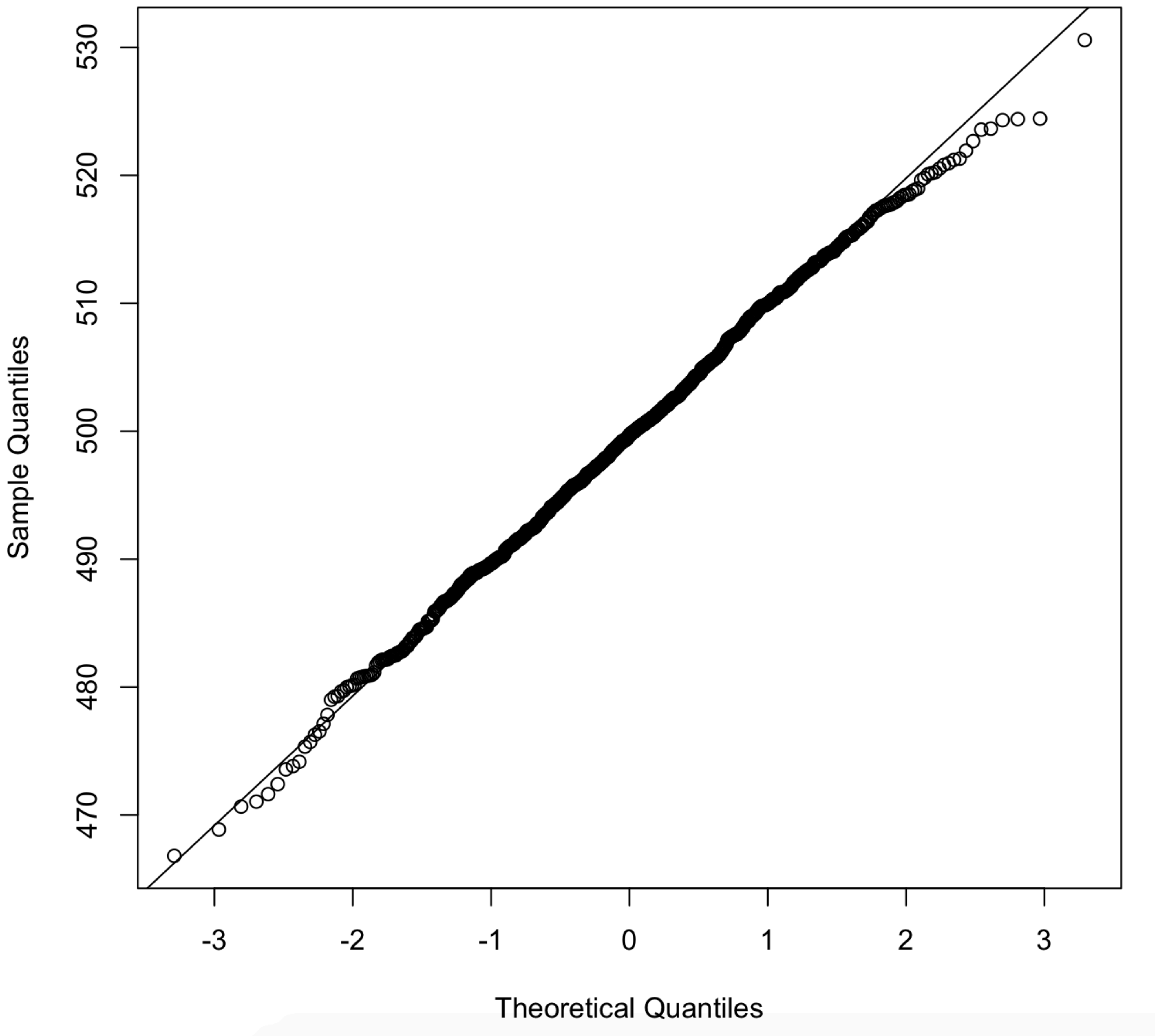

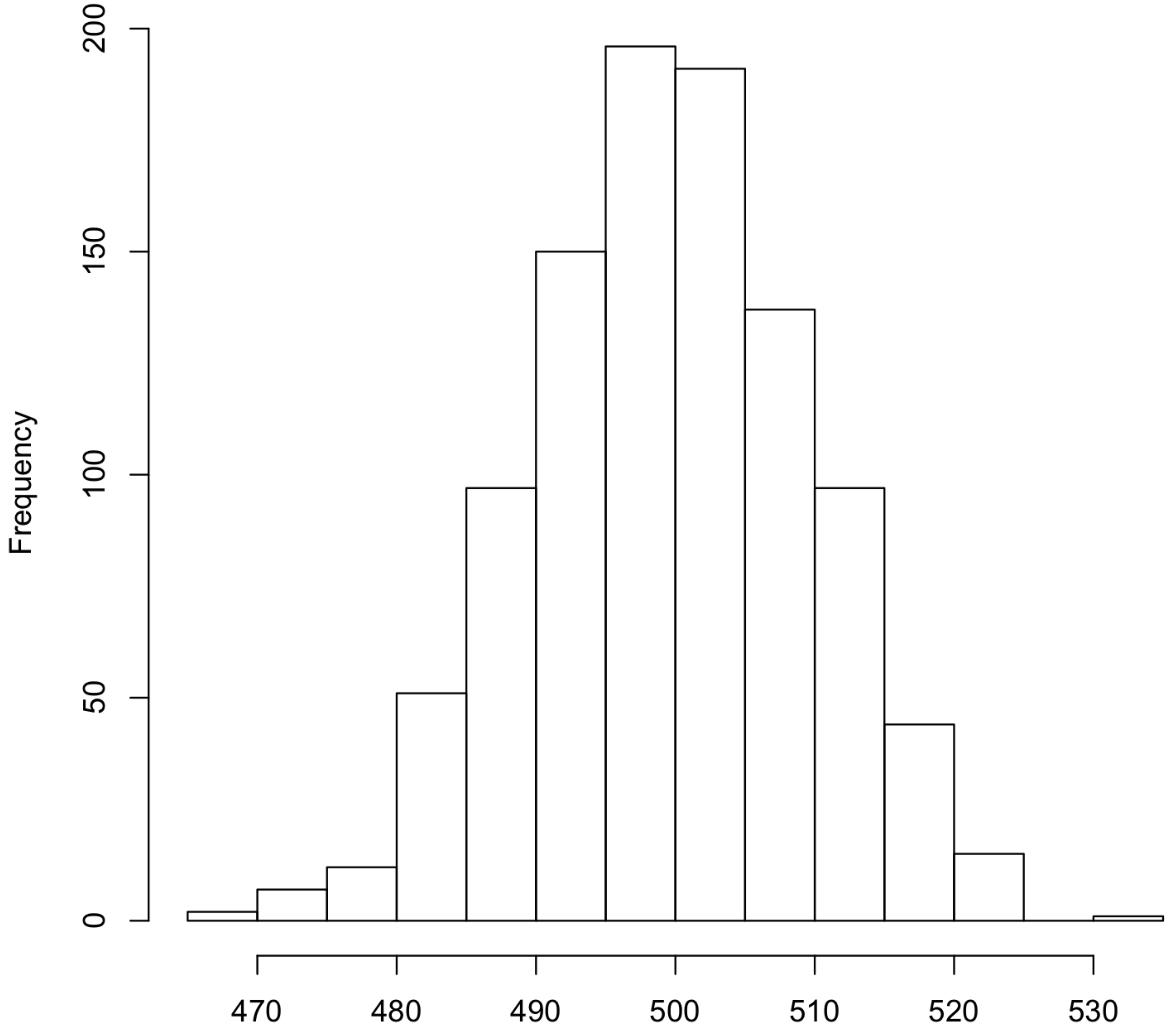

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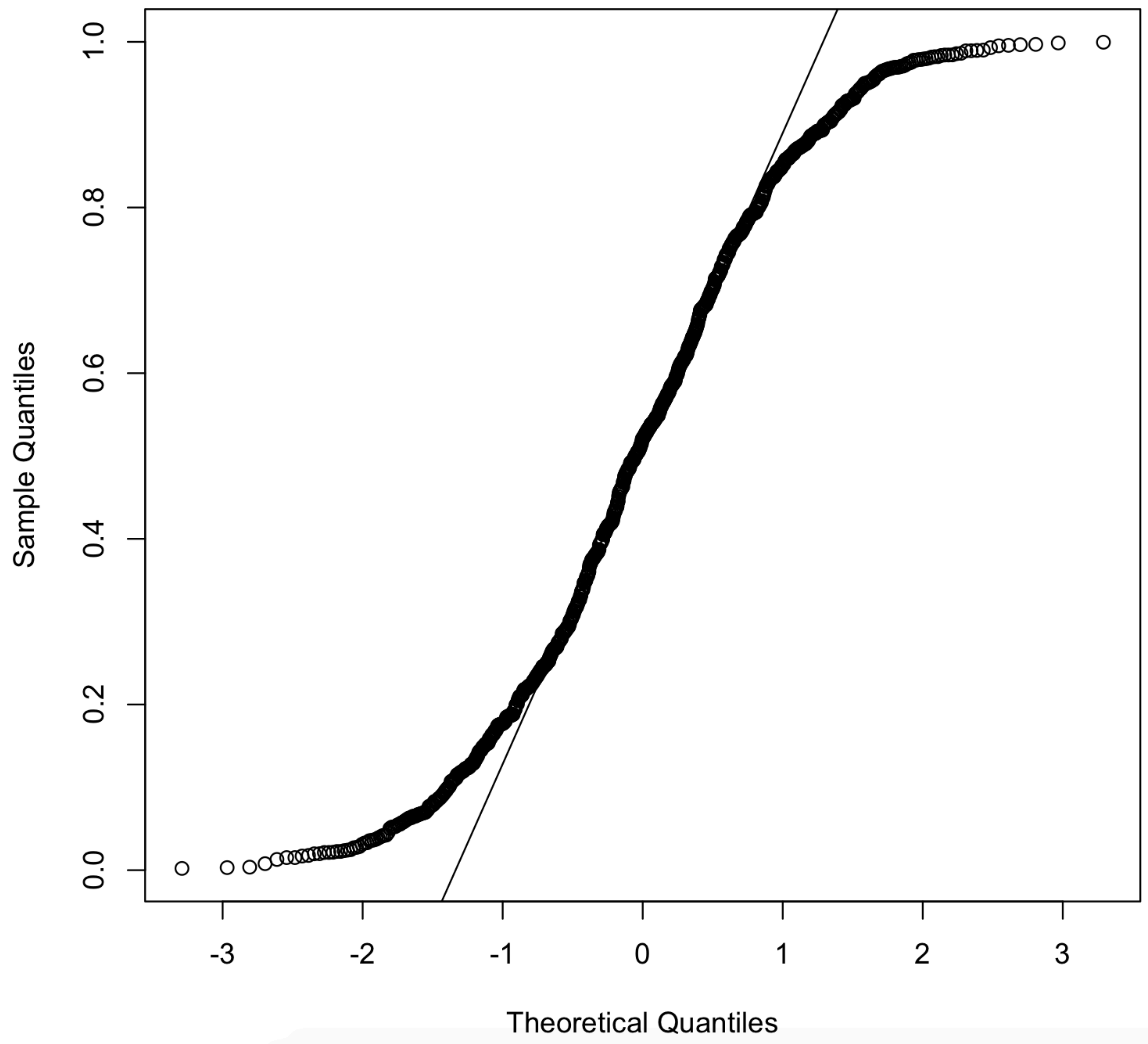

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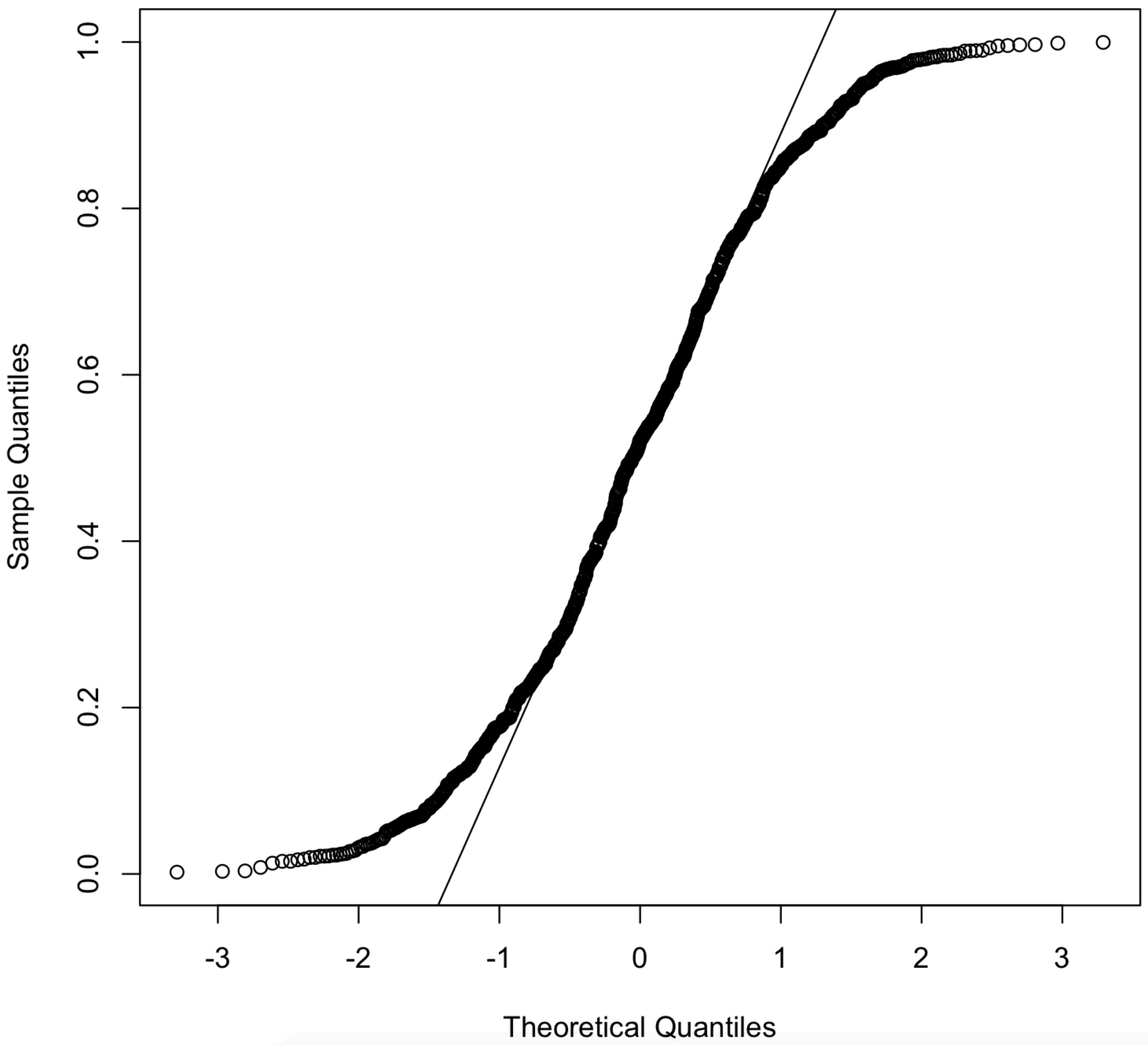

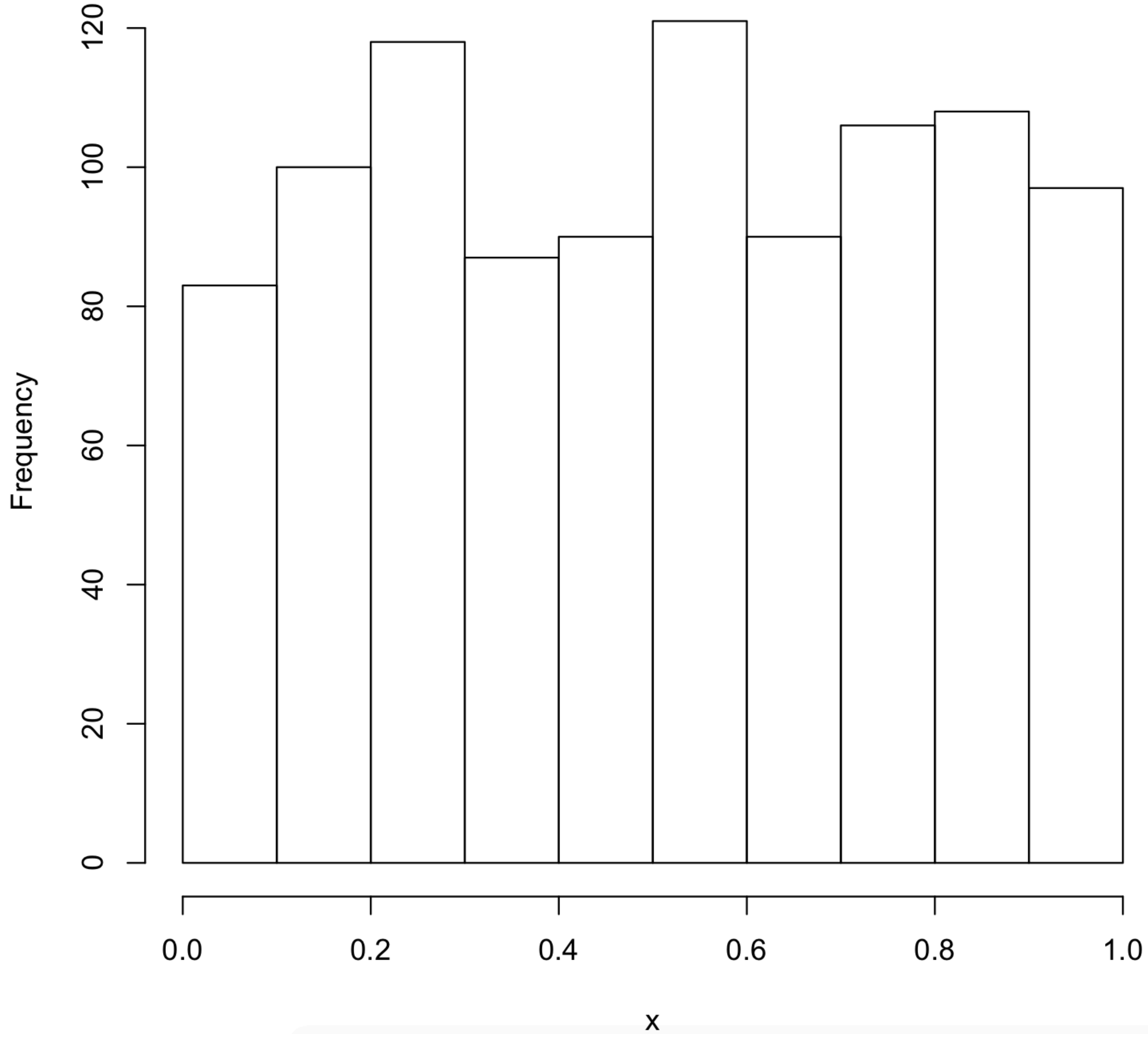

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- Often, when doing a transformation, we attempt to take skewed data and transform it onto a scale in which the data is then approximately normally distributed

Linear Transformations

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- Example: When converting Celsius to Fahrenheit, we have $F = 1.8C + 32$ with $a = 1.8$ and $b = 32$

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Statistic	Original	Transformed
Mean	$\bar{x}$	$a\bar{x} + b$
Median	$\tilde{x}$	$a\tilde{x} + b$
Trimmed Mean	$\bar{x}_{K\%}$	$a\bar{x}_{K\%} + b$
Variance	s^2	$a^2 s^2$
Standard Deviation	s	$\|a\|s$
Interquartile Range	IQR	$\|a\|$IQR
Lower Quartile	Q_1	$aQ_1 + b$ if $a > 0$
		$aQ_3 + b$ if $a < 0$
Upper Quartile	Q_2	$aQ_3 + b$ if $a > 0$
		$aQ_1 + b$ if $a < 0$
Quantile	$x_{K\%}$	$ax_{K\%} + b$ if $a > 0$
		$ax_{(100-K)\%} + b$ if $a < 0$

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- Transform data to meet assumptions needed for analyses

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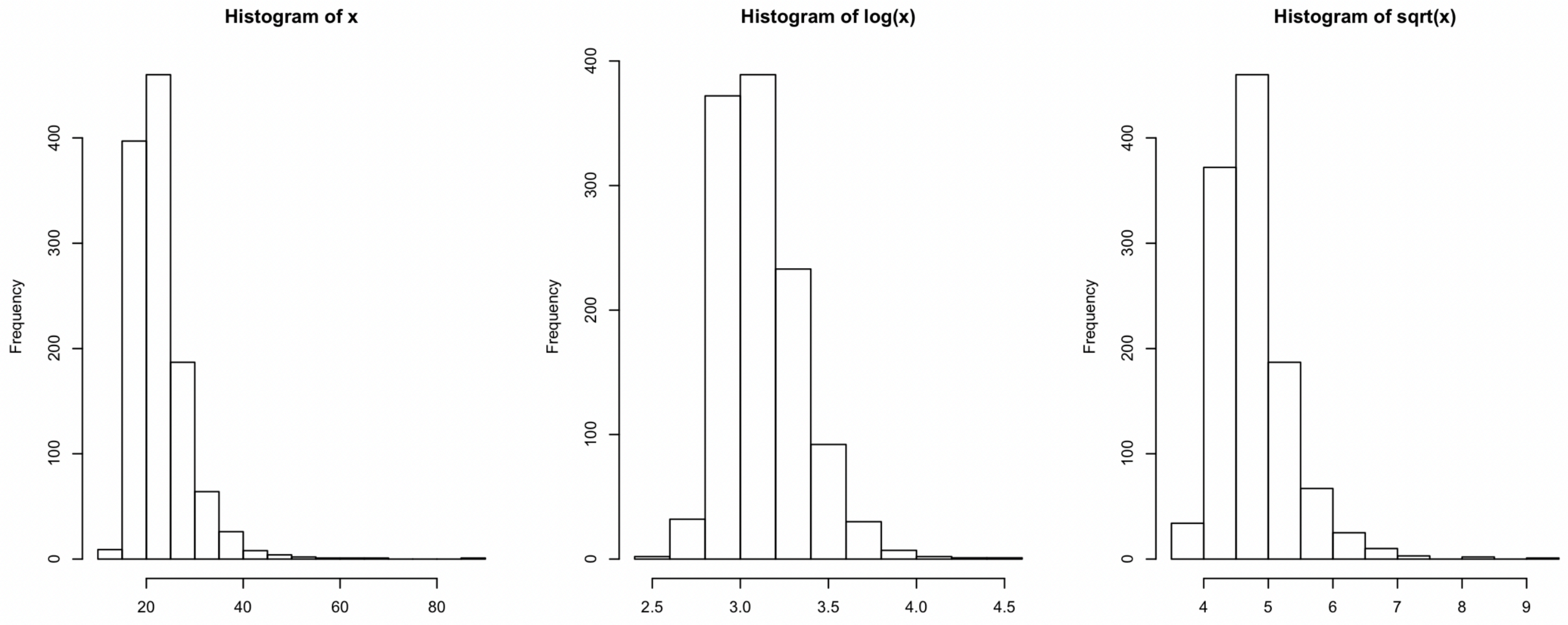
Transformations



Transformations

	k	$\bar{x} - ks$	$\bar{x} + ks$	Theoretical % in Range	Actual % in Range
x	1	17.20	29.45	68	81.6
	2	11.08	35.58	95	96.2
	3	4.95	41.71	99.7	98.4
$\log(x)$	1	2.90	3.34	68	69.6
	2	2.68	3.57	95	95.8
	3	2.46	3.79	99.7	99.1
$\sqrt{x}$	1	4.22	5.37	68	79.7
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$$y_\lambda = \begin{cases} \frac{x^\lambda - 1}{\lambda}, & \lambda \neq 0 \\ \log(x), & \lambda = 0 \end{cases}$$

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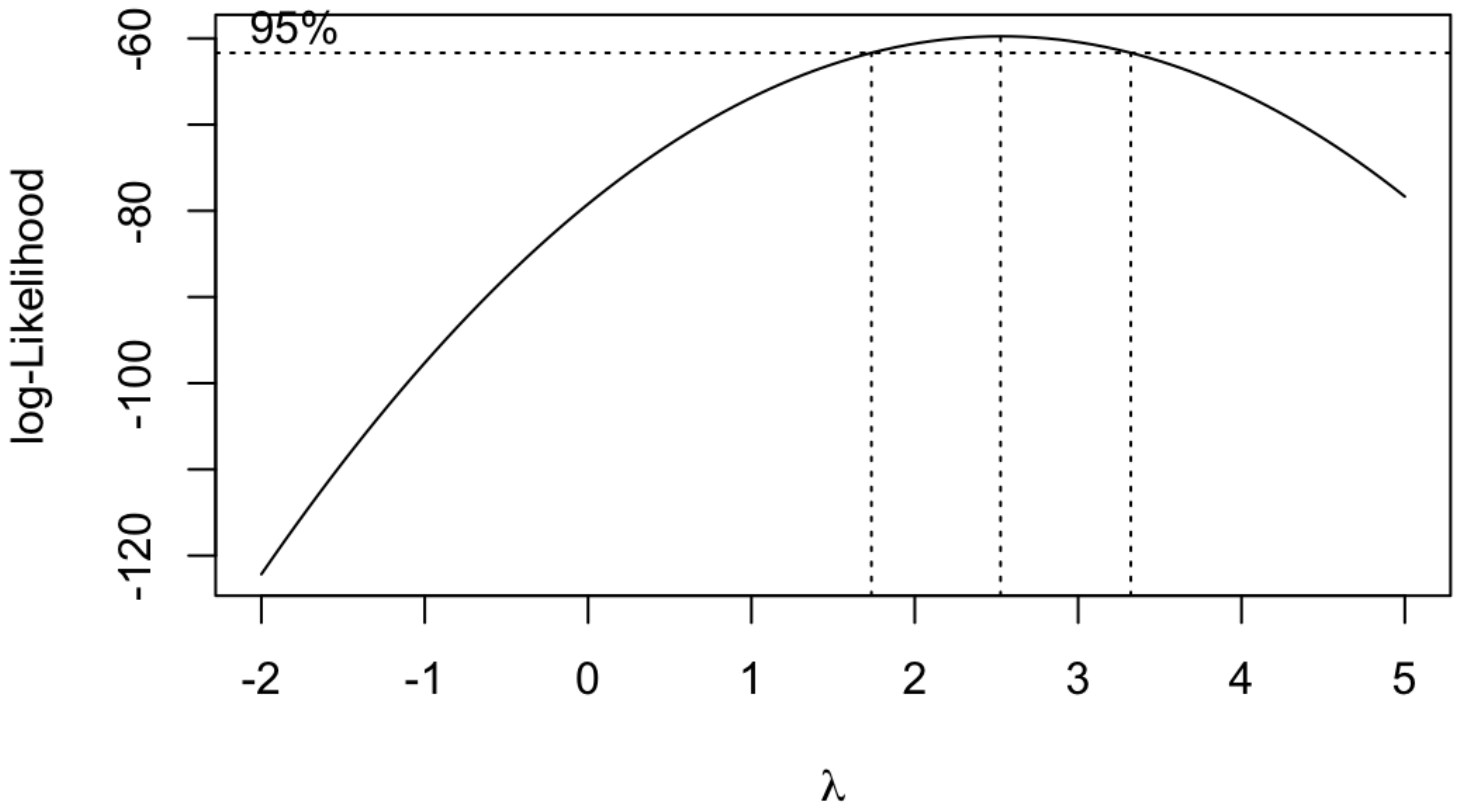

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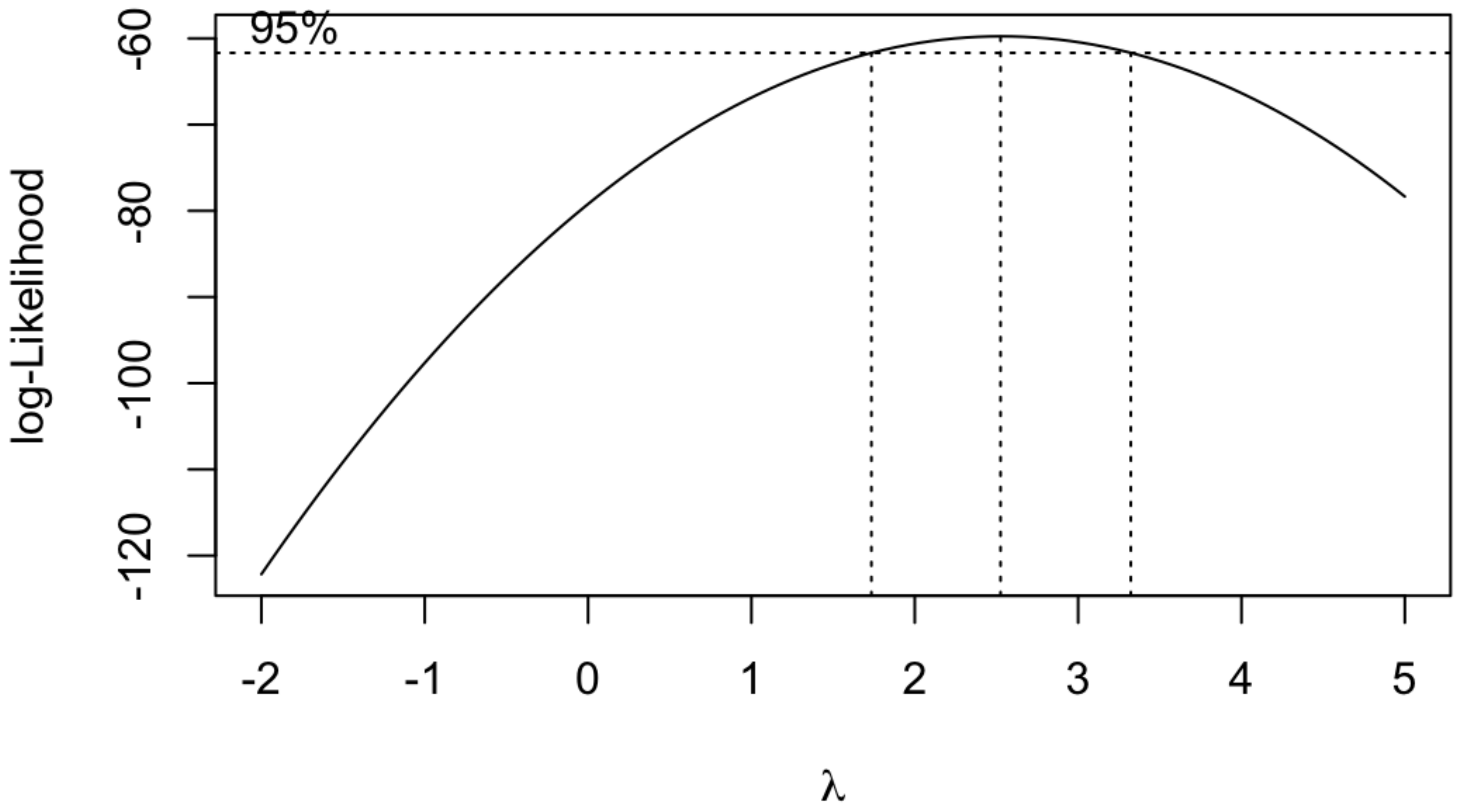

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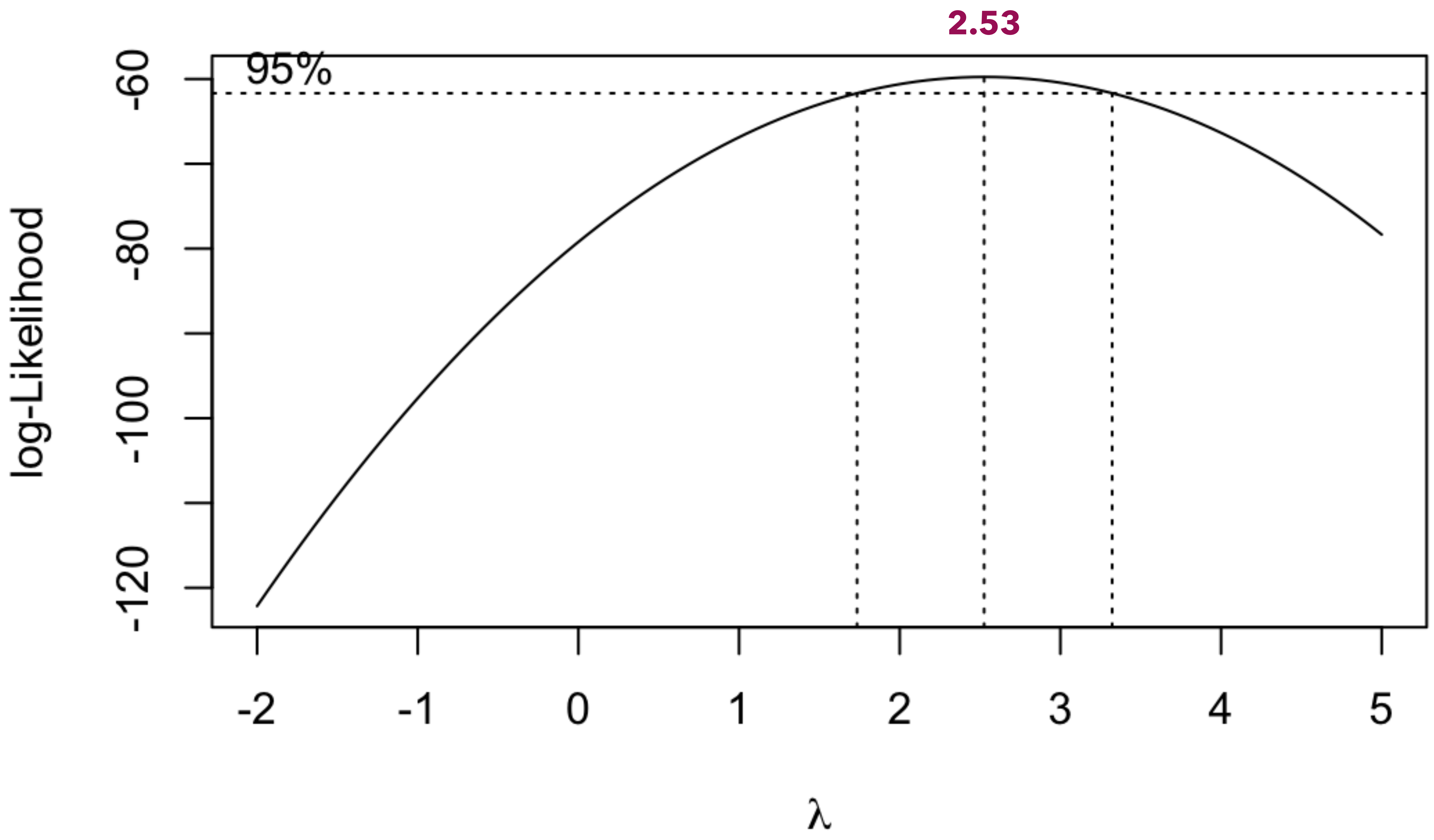

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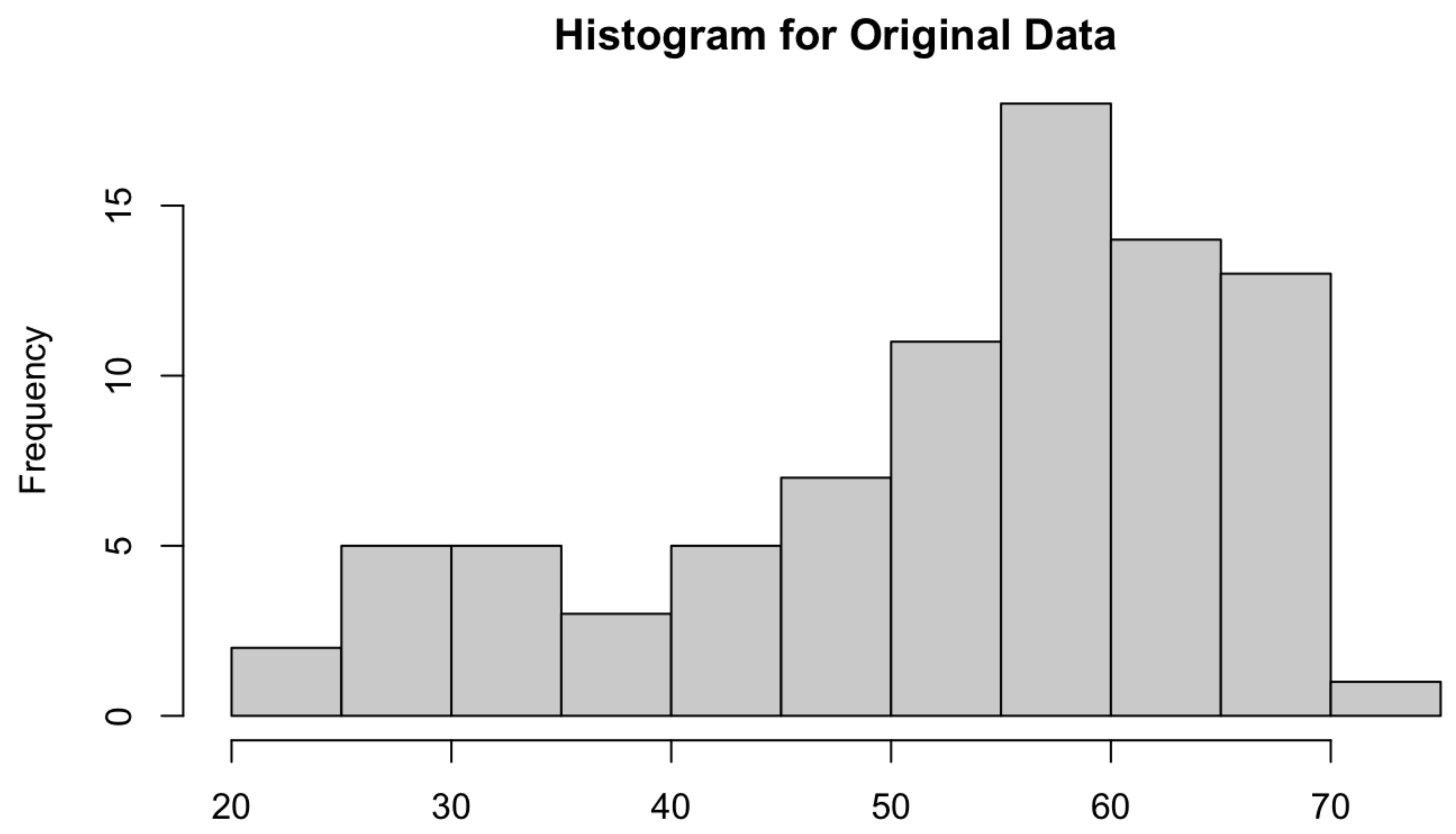

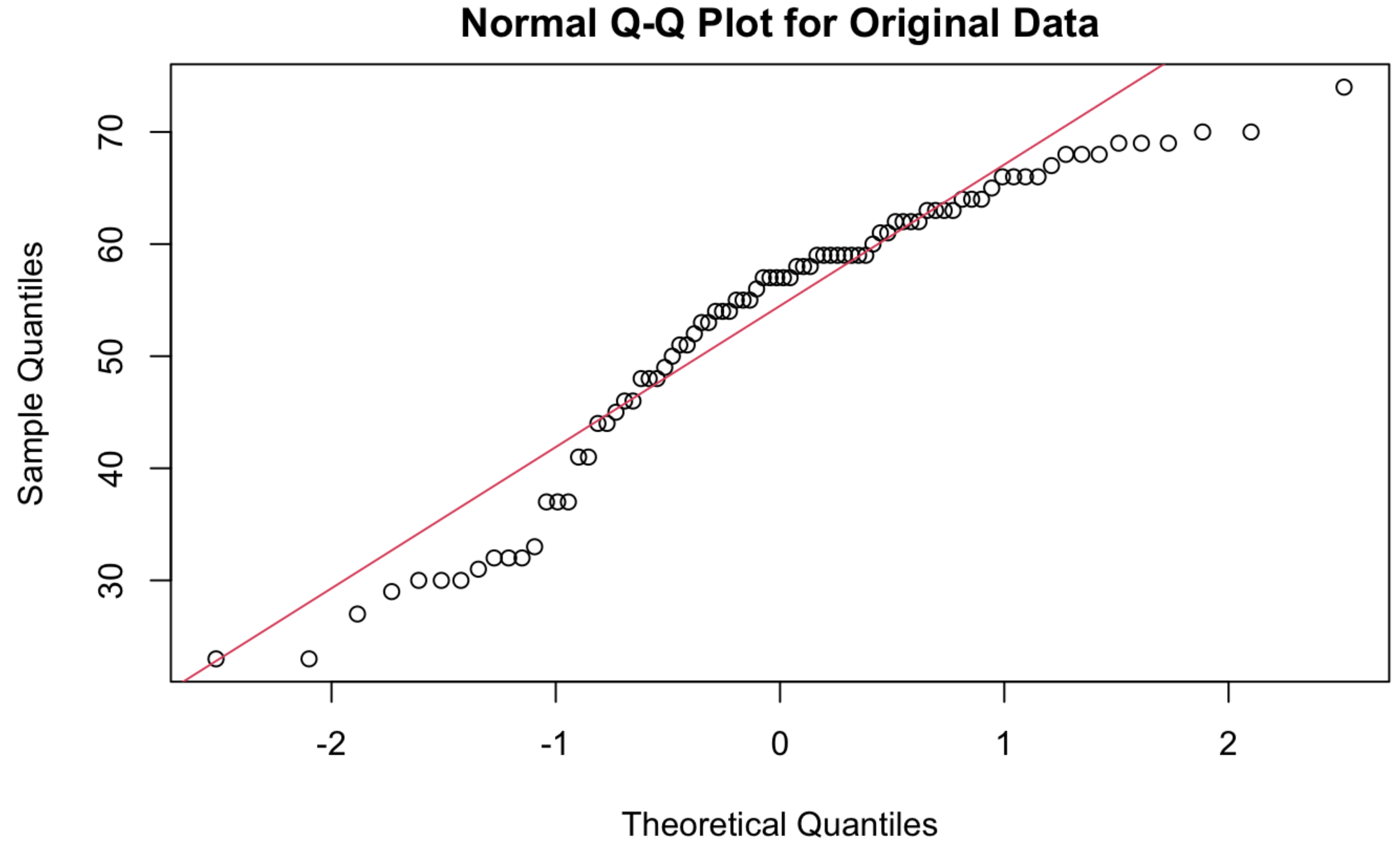

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