Group 34

Optimization in Finance

-Assigned by Prof. Nabinkumar Sahu

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Problem Statement

- In financial literature, a portfolio is considered as an appropriate collection of investments held by an individual or a financial institution.
- Portfolio optimization is a decision-making problem in how we allocate our funding to different possible investment options so that we can get the maximum return.

Portfolio Rate of Return

$$Portfolio\ Return = \sum_i Weight_i \cdot Asset\ Return_i$$

where

$$\sum_i Weight_i = 1$$

Portfolio's mean rate of return is given by

$$E[Portfolio\ Return] = \sum_i Weight_i \cdot E[Asset\ Return_i]$$

Variance is given by

$$Var[Portfolio\ Return] = \sum_{i,j} Weight_i \cdot Weight_j \cdot Cov[Asset\ Return_i, Asset\ Return_j]$$

Markowitz Mathematical Model

Markowitz Model which maximizes the portfolio's mean return and minimizes the variance. Such portfolios are called mean-variance optimal.

Our aim is to

$$\text{Maximize: } E[Portfolio\ Rate\ of\ Return]$$

$$\text{Minimize: } Var[Portfolio\ Rate\ of\ Return]$$

$$\text{minimize } \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}$$

With respective to constraints

$$\sum_{i=1}^{N} w_i = 1.$$

$$\sum_{i=1}^{N} w_i \overline{R}_i = \mu_P$$

Variance of Portfolio is given by

$$\sigma^2_{portfolio} = E[(r_i - \mu_i)^2] = \sum_i \sum_j w_i \, w_j \, \sigma_{ij} = \underline{W}^T \underline{\underline{\Sigma}} \, \underline{W}$$

Kelly Mathematical Model

Kelly proposed a formula for bet sizing that leads almost surely to higher wealth compared to any other strategy in the long run (i.e. the limit as the number of bets goes to infinity). In this , the idea is to maximise mean logarithmic growth-rate which results into sure profits in the long run .

$$Kelly\ Criterion = E[\log(1 + Portfolio\ Rate\ of\ Return)]$$

$$= E\left[\log\left(1 + \sum_i Asset\ Weight_i \cdot Asset\ Rate\ of\ Return_i\right)\right]$$

In the long term, after n times where n is large, the investor is expected to have w * n wins and (1 – w)n losses. The portfolio P will be worth:

$$P_n = (1 + kB)^{wn} \cdot (1 - kA)^{(1-w)n}$$

We would like to solve for the optimal k:

$$\log(P_n) = \log((1 + kB)^{wn} \cdot (1 - kA)^{(1-w)n}) = \text{n} \cdot \log(1 + kB)^{w} + \text{n} \cdot \log(1 - kA)^{(1-w)}$$

$$\frac{\log(P_n)}{n} = \log(1 + kB)^{w} + \log(1 - kA)^{(1-w)} = \text{w} \cdot \log(1 + kB) + (1 - \text{w}) \cdot \log(1 - kA)$$

To maximize , we take its derivative with respect to k and set it to 0:

k=w-(1-w)/R

Markowitz vs Kelly

Here, return of asset A is negative

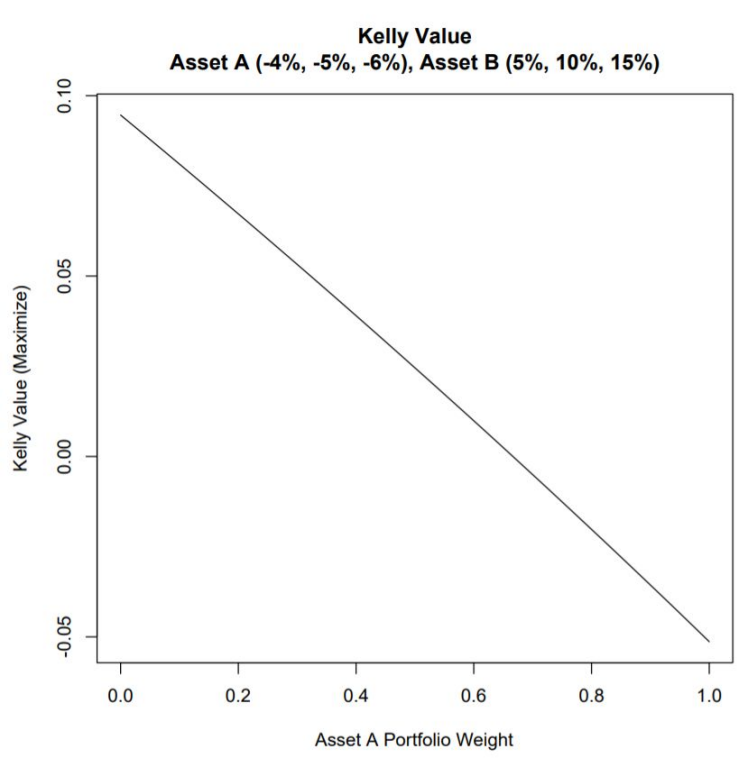
Asset	Possible Returns			Mean	Stdev
A	(4%)	(5%)	(6%)	(5%)	1%
B	5%	10%	15%	10%	5%

By Markowitz - minimum variance portfolio:

Asset A weight = ⅝;

Asset B weight = ⅛;

While by Kelly Criterion we found following solution:



Case Study - KOSPI200

As we are comparing the two Mathematical model , a great case study which was published in the Proceedings of the 2014 International Conference on Industrial Engineering and Operations Management Bali, Indonesia, January 7– 9, 2014 named as KOSPI200. The expected return with Kelly Mathematical Model can be seen as 0.8790% and with the Markowitz Mathematical Model can be seen as 0.8100% over the return of 200 stocks included in KOSPI 200.

Conclusion

Markowitz Model:

- Markowitz portfolios do not maximize return and minimize risk as we all believe, even when given the true probability distribution of returns are there.
- Markowitz portfolios are diversified which may give an illusion of safety.

Kelly Model:

- Kelly portfolio optimization does what it is supposed to: Favours assets with better return distributions.
- By limiting the weight assets in Kelly , Diversification can take place int this model.

References

Books

[1] Risk-Return Analysis: The Theory and Practice of Rational Investing- by Harry M. Markowitz.

[2] Mathematical Methods for Finance: Tools for Asset and Risk Management (Frank J. Fabozzi Series) by Sergio, Frank and Turan.

[3] Kelly Capital Growth Investment Criterion, The: Theory And Practice (World Scientific Handbook in Financial Economics Series) by - Leonard MacLaren ,Edward Thorp and William Ziemba.

[4] Modern Portfolio Theory and Investment Analysis - by

William N.Goetzmann ,Edwin J. Elton, Martin J. Gruber, Stephen J. Brown[wiley].

References

Websites:

[1] http://ieomsociety.org/ieom2014/pdfs/192.pdf (journal used for case study by Gyutai Kim and Suhee Jung.

[2] https://en.wikipedia.org/wiki/Portfolio_optimization (Modern Portfolio Theory)

[3] https://en.wikipedia.org/wiki/Kelly_criterion (Kelly Criterion)

[4]http://www.hvass-labs.org/people/magnus/publications/pedersen2014portfolio-optimization.pdf

[5] **http://www.iemsjl.org/journal/article.php?code=54793 (** Details for Journal For KOSPI 200).

Thank You!