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ORIGINAL ARTICLE

Designing specific tools to enhance the numeracy of adults with intellectual disabilities

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Abstract

Numeracy is important for everyday life. Being numerate has a positive impact on the quality of life of individuals, with positive economic, health, and social outcomes. Despite this, little is known about the role of numeracy in the lives of adults with intellectual disability (ID). Design research has been used to develop ways to support mathematical learning for typically developing students. This study investigates the use of design research to develop context-specific, physical tools to support adults with intellectual disability to improve their numeracy capabilities and engagement in daily tasks. Using observation and interview data, findings demonstrate increased engagement and participation in the numeracy demands of these tasks. Participants reported positive perceptions of improved competence and independence. This study demonstrates the application of design research to the field of numeracy for adults with intellectual disability, and the usefulness of designing context-specific tools to support their numeracy development and independence.

Keywords Numeracy · Adults · Intellectual disability · Tools · Design research · Mathematics

Introduction

Numeracy, literacy, and the ability to solve problems in a modern, technology-rich environment are essential for all adults to survive in today's world (OECD, 2012). Numeracy has come into focus in recent years with several reports such as The Centre for Education Statistics & Evaluation [CESE] (2016) and Tout (2020) emphasising the importance of numeracy for all individuals. Evidence from the 2012 Programme for International Assessment of Adult Competencies (PIAAC)

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demonstrated that individuals with low levels of numeracy experience a higher incidence of poor economic outcomes and poorer health and are less likely to be socially engaged. Conversely, higher levels of numeracy are associated with higher levels of health, employment, and wages (Jonas, 2018; OECD, 2016; Tout & Gal, 2015). Improving numeracy levels for adults with intellectual disability (ID) may have a positive impact on their health, economic status, and social engagement.

The term numeracy has evolved since first being coined in 1959 in England by Crowther (Ministry of Education, 1959). While early definitions predominantly focused on basic skills (Kirsch et al., 1993; Ministry of Education, 1963), current conceptualisations of numeracy (Geiger et al., 2015) value more than just basic mathematical knowledge. In today's world, an ability to apply mathematical knowledge (including numbers, algebra, and geometry) in different contexts, a positive disposition towards using and applying mathematical knowledge, and a willingness to engage with and solve problems involving mathematics, are considered vital qualities of a numerate individual. Additionally, being numerate includes a critical component, the ability and willingness to make judgements about mathematics in different situations (Goos et al., 2012; Steen, 2003). This understanding of numeracy is embedded in a socio-cultural theory of learning.

Conceptualisations of disability have also evolved over the last century. The medical model of disability considers disability as a deficit, intrinsic to the individual (Falvo, 2014; Longmore, 1995), and something to treat or cure. However, the social model of disability (Thurman & Fiorelli, 1979) maintains that disability is caused by societal and environmental barriers to the individual. The World Health Organisation [WHO] (2022) defines disability as resulting from "the interaction between individuals with a health condition … with personal and environmental factors" (para 2). The WHO's perspective of disability is in line with the Biopsychosocial Model of Disability (Engel, 1977). This multidimensional view of disability considers the biological impacts of disability on an individual as well as the role and context of social, psychological, and environmental conditions, which influence the individual impact of a disability. When considering adults with ID, it is important to understand the biological, psychological, and social components of disability and how they impact the development of a numerate individual.

In this context, the biological consideration of disability includes intellectual disability, defined by the Diagnostic and Statistical Manual (Version 5) [DSM5] (American Psychiatric Association, 2013), as difficulties with academic learning, including difficulties with "reasoning, problem-solving, planning, abstract thinking" (p. 66). It is also important to understand the psychological components of disability, such as how learning is influenced by past experiences (Piaget & Duckworth, 1970), in supporting the development of numeracy in this population. Finally, social components such as the social context in which learning is embedded (Lave, 1988) form a crucial part of the learning environment. The social context of understanding disability links to the sociocultural perspective of numeracy previously discussed. From this perspective, the disability-specific demands of activities, the numeracy demands of activities, individuals' past experiences, and the social context in which activities take place should all be considered within the learning environment being researched.

Students and adults with ID lack the opportunity and expectations to engage in numeracy learning (Tan et al., 2019). From school opportunities restricted by

exposure to a narrow curriculum (Faragher, 2019; Lemons et al., 2015), to post-school opportunities providing limited experiences in continued academic learning (Grigal et al., 2012; McManus et al., 2022), learners with ID have traditionally not been seen as learners of mathematics or as numerate individuals. In light of the reported life benefits of numeracy competence (Tout & Gal, 2015), learners with ID should have the opportunity to engage with numeracy learning at school and post-school.

While research on inclusive school mathematics education for students with ID is ongoing (c.f.Wehmeyer & Shogren, 2016), research into numeracy learning opportunities for adults with ID is sparse (Prendergast et al., 2017). This study aims to demonstrate one way of continuing to support numeracy learning for adults in work and social settings by investigating how specifically designed physical tools can support numeracy learning and task engagement.

Background

What counts as numeracy has changed in an emerging technological environment (Bennison et al., 2020). Gaining a mastery of computations and fluency with numbers, previously seen as the foundation of school mathematics, has evolved into an understanding that being numerate requires the ability and the disposition to use mathematics when solving problems in the context of home, community, and work (Geiger et al., 2015). Further, Faragher (2019) argues that mathematics for students with ID should include the consideration that students now need to master the use of appropriate tools, such as calculators or smartphone apps, to support them in completing basic mathematical tasks to develop more complex mathematical understandings. For example, students with ID may be able to efficiently learn to complete perimeter and area problems with their same-age peers if they have access to calculators for the computation steps of a problem.

One model of numeracy encompassing these conceptualisations is the 21st Century Model of Numeracy developed by Goos and colleagues (Geiger et al., 2015). This model consists of five elements: mathematical knowledge, tools, dispositions, context, and critical orientation. Numerate individuals can use mathematical knowledge and select useful tools to solve problems, making sense of mathematical situations. Thus, they demonstrate positive dispositions and a willingness to tackle problems involving mathematics. Additionally, the context of the problem can dictate the mathematical knowledge required and the tools available to support problem-solving (Geiger et al., 2015). Finally, an element considered essential by Goos et al. (2012) is a critical orientation to numeracy; the ability to challenge and critically evaluate a situation involving mathematics (Steen, 1990). The 21st Century Model of Numeracy has been used to frame this study of developing numeracy for adults with ID.

Opportunity to participate in mathematical learning in different contexts is essential to developing numerate individuals (Bennison et al., 2020; Schreiber-Barsch et al., 2020); however, for learners with ID, that opportunity is limited (Tan et al., 2019). More than 30 years ago, Mastropieri et al. (1991) identified differences in the research on mathematics and numeracy education for learners with ID and typically

developing learners. Learners with ID were mainly exposed to a narrow range of mathematics curricula, and teaching approaches constrained by behaviourist theories of learning. Mastropieri et al. (1991) noted the focus on constructivist approaches to mathematics education research for typically developing learners at that time and identified the need for mathematics research for students with ID to widen the range of mathematics topics and a variety of approaches. Despite this call to action, nearly 30 years later, Tan et al. (2019) found similar findings in mathematics research for students with ID still focused on narrow approaches and limited curriculum. The authors called for significant changes in mathematics education research "to work with students with intellectual disabilities as knowing participants with a genuine sense of belonging in mathematics" (p.10). More broadly, Lambert and Tan (2019, p. 28) called for research with students with various disabilities paying attention to "participation in general education mathematics" and research that documents development by students with disabilities in the "dominant pedagogical orientations in mathematics education". The current narrow focus of skills in the research leads to the segregation of students with ID into "low rigour" (Tan et al., 2022, p. 871) mathematics instruction and lower achievement expectations. Research in mathematics education for learners with ID that employs a deficit perspective is far more common than research with the view that individuals with ID are learners of mathematics (Tan et al., 2019, 2022).

Post-school, this lack of opportunity and expectations continues with Schreiber-Barsch et al. (2020) suggesting there are limited opportunities for adults with ID to continue learning when they leave school. Often, this lack of opportunity is because adults with ID are viewed as having limited capabilities in numeracy (Tan et al., 2022) and, consequently, have fewer opportunities to participate in learning (Prendergast et al., 2017). This lack of opportunity to learn contributes to a lack of opportunities in employment and results in an "ordinary life" (Lysaght & Cobigo, 2014). Faragher (2011) suggests that opportunities to learn mathematics can lead to further opportunities to engage in a variety of activities and can support further learning. Cuskelly et al., (2021, p. 325) note that for adults with ID, "formal education has benefits beyond learning", including higher levels of employment (Butler et al., 2016), greater friendship networks, and improved health (Butler et al., 2016; Cuskelly et al. 2021). Children with Disability Australia (CDA) commissioned a report on the post-school transition of children with disabilities in 2015. They found that people with disability in Australia "are only half as likely to be employed as people without disability" (Children with Disability Australia [CDA]., 2015, p. 19). Therefore, investigating ways to support adults with ID to continue to learn post-school, and specifically, to continue numeracy learning, needs to be investigated.

Prendergast et al. (2017) suggest that adults with ID want to learn mathematics that is meaningful and useful to them. Schreiber-Barsch et al. (2020) examined the types of numeracy practices undertaken by adults with ID in daily activities in Germany and the ways in which they engaged with the numeracy demands of those activities. Their methodology, focused on the social practice of acting out numeracy, highlighted the extant numeracy practices of a small number of adult learners, learners who too often are viewed from a deficit perspective. For example, in an adult housekeeping lesson on folding napkins, the teacher dismissed the capabilities of the adults stating the task was "surprisingly difficult" (p. 587) for the students and

not really involving any "everyday mathematics" (p. 587). One participant, however, identified the numeracy required by stating "If you need to fold at right angles to each other, you need to know what this is" (p. 587). While the teacher saw the students in the housekeeping lesson from a deficit perspective with very limited everyday mathematical skills, this participant demonstrated their spatial conceptual understanding, used the correct terminology (right angles), and provided a clear explanation about why the activity of folding napkins required numeracy.

In light of the evidence of the importance and benefits of developing numeracy in adults, this study investigates ways of further developing adults' numeracy that are meaningful and useful to adults with ID by working with participants with ID in their work or social contexts. The following research questions were explored:

1. In what ways can design research support the development of specifically designed tools to support numeracy learning and task engagement for adults with intellectual disability in social and work contexts?
2. How does greater engagement with the numeracy demands of tasks impact participants' views of themselves, and how others view them as users of numeracy?

Method

Qualitative research enables the collection of rich data (Merriam & Tisdell, 2016) and is best suited to situations where a deep understanding of social contexts and phenomena is required. In this study, qualitative approaches were chosen because a rich analysis of the context of numeracy in the actual experiences of adults with ID was required. Additionally, design research is used in mathematics education to study mathematical learning and the development of tools to support learning (Cobb et al., 2003). Because the current study required the designing of physical tools specifically targeting the participants' learning in the context of their daily tasks, design research was adapted to the context of adult learning.

Design research

Design research has been used in research into mathematics education to guide intervention and learning in mathematics (Cobb et al., 2003, 2008) and to investigate the development of mathematical learning through the participation of students in activities. Cobb et al. (2003) assert that design experiments are useful in researching mathematical learning because they study the process of learning, including the means of supporting learning, in interventions that specify the concept, and consider background conditions that may impact learning. When working with adults with ID, it is imperative to consider factors which impact the specific demands of different tasks. Design research methodology is a good fit for this study because it enables the development of an understanding of the process of learning, the means to support it, and the sociocultural factors shaping it.

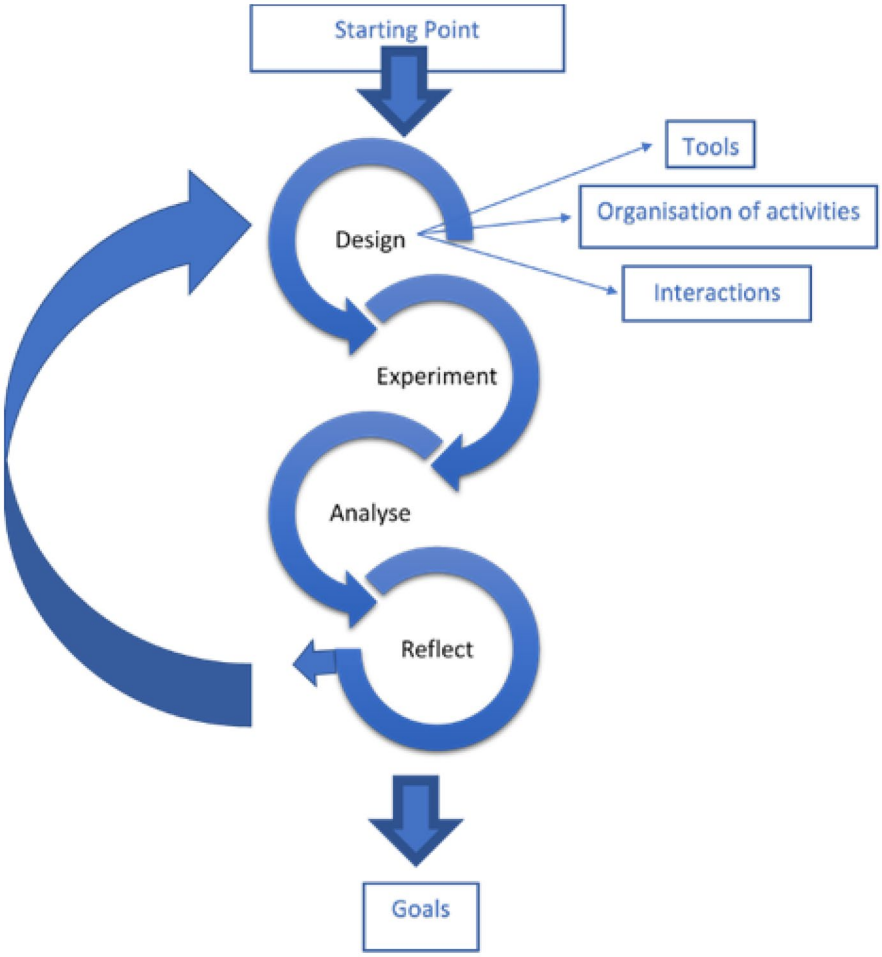


Fig. 1 The design research cycle

In conducting a design research experiment, the researcher first conjectures a learning trajectory (Simon, 1995) and possible tools and tasks to support the envisioned learning and then tests and modifies these conjectures in an iterative process of design, experiment, analysis, and reflection as the envisioned learning takes place (Cobb et al., 2003; Gravemeijer & Cobb, 2006). The learning process, the means by which it was supported, and the sociocultural context in which it took place are then analysed, and resources for supporting the learning are developed (Cortina et al., 2019). This iterative design process provides the scope to gauge and meet the needs of adult learners with ID in the context of their daily activities. Figure 1 shows this design research cycle.

In this study, design research was used to develop individualised learning pathways for the participants. First, individual learning goals and starting points were determined for each participant in each setting, with the researcher collaborating with each participant and significant others such as employers or support workers (Gaunt et al., 2019). These initial designs were discussed with the research team and refined. Second, the learning pathway and means of support needed to be conjectured, tested, and refined. This pathway was designed in collaboration with participants to consider ways to support the envisioned changes in adults' participation from their starting points to the achievement of goals, including tools designed to support participation (Prendergast et al., 2017). Cortina et al. (2019) caution that research design should not be reduced simply to designing tools and tasks to support learning. The organisation of activities, participation in discourse, and other sociocultural practices are equally important in developing positive dispositions and individuals' conceptual understandings. Hence, the design of tools to support participation in activities, ways of organising activities to support participants in using the tool, and the microculture of the setting in which learning took place were the main foci of the iterative design research cycle (Fig. 1). Each iteration of the cycle was reflected on by the lead researcher (first author) then discussed with the participants, their support team, and research team members. This reflection and collaboration led to incremental refinements in the development of the tool until the tool was fit for purpose.

Participants, recruitment of participants, and informed consent

Three males and one female, aged from 19 to 41 years, all with intellectual disability, participated in the study across four different contexts. Participants were recruited through convenience sampling with the first author approaching a local disability network she belongs to, and a local disability service organisation. Information sheets were provided, and the prospective participants were encouraged to take them home to discuss with their families and return the consent form if they were interested in participating. Table 1 provides details of the four participants across four contexts. This paper reports on three of the four participants, Ben, David, and Ellen and two of the four contexts, the bowling alley and the office. The reasons for choosing these contexts are discussed later in the paper. One participant, David*, was observed in two contexts.

Table 1 Overview of participants

Context	Participant	Age	Recruitment type
Ten-pin bowling	Ben	19	Disability organisation
	David*	41	Disability organisation
Restaurant	David*	41	Disability organisation
Office	Ellen	24	Disability network
Coffee run	Max	22	Disability network

Participants with ID are considered a vulnerable population by the (National Health and Medical Research Council (NHMRC), 2018). The NHMRC requires informed consent from all research participants, including an adult with ID if they have the capacity to do so. They also suggest another adult, not associated with the research, may be present during the consent process to ensure the adult with ID understands what is required of them. Research highlights the difficulties of gaining informed consent from participants with ID (Horner-Johnson & Bailey, 2013). Working with the participant's support personnel to determine if the individual wants to consent to the research, and questioning participants to determine their capacity to provide informed consent may help to remove this barrier to participation. For this study, participants were asked to explain the study in their own words and why they wanted to participate. Based on their responses, the three participants discussed in this article, David, Ben, and Ellen, were considered capable of providing informed consent, but they were also given the opportunity to have a support person attend the consent meeting. Ellen chose to bring her mother, Ben both his parents, and David chose to attend the meeting by himself. After having the research explained to them, and their freedom to withdraw at any time, David, Ben, and Ellen provided their informed consent and remained in the study for the duration of the research.

The researchers' stance in this study was to research *with* participants and not *on* participants. Walmsley (2001, p. 188) stated that research involving people with learning disabilities typically involves them being "tested, counted, observed, analysed, described and frequently pathologized, but never asked for their views". She suggests that inclusive research needs to involve people with different learning disabilities in the research process as much as possible. In this study, the context of the research, the numeracy demands being studied, and the tools developed were collaboratively discussed with the participants and their significance including employers, support workers, and parents, as well as other members of the research team.

Data collection

Using a qualitative research design, observations and interviews were conducted with the three participants in their work or social settings to determine their numeracy needs and design individual interventions. Based on these goals, design research was then used to develop, trial, and refine task-specific physical tools to support their numeracy learning. The study comprised two phases. Phase 1 documented the numeracy demands of the chosen tasks of the participants in their respective contexts. This involved audio recording six to eight observations each of 1 to 2 h duration across 3 to 12 weeks. Interviews with participants and significant others, such as support workers, were conducted at the end of Phase 1 to clarify researcher interpretations of observation data and provide participants with a voice. Observation and interview transcripts were combined as the data accumulated. Using the constant comparative method (Glaser & Strauss, 1967), chronological coding and comparisons with previous interpretations and conjectures searched for common and repeated themes. These data were used to identify learning goals (see Gaunt et al., 2019). In the second phase of the research study,

Table 2 Overview of data collection

Context	Phase 1		Phase 2	
	Interviews	Observations	Interviews	Observations
David is a male with ID of unknown aetiology, living in supported accommodation				
Ten-pin bowling alley	David	6 weeks	David	11 weeks
	Paul	9 weeks	Paul	16 weeks
	Support worker		Support worker	
Ben is a male with ID and ADHD of unknown aetiology, living at home with his parents				
Ten-pin bowling alley	Ben	8 weeks	Ben	11 weeks
	Paul	12 weeks	Paul	16 weeks
	Support worker		Support worker	
	Mother and father		Mother and father	
Ellen is a female with Down syndrome living at home. Her family employs independent support workers				
The office	Ellen	6 weeks	Ellen	18 weeks
	Mother	3 weeks	Mother	9 weeks

using design research, tools were designed to support participants' numeracy development. Phase 2 involved a further eight to 18 audio-recorded observations of 1- to 2-h duration across 9 to 16 weeks. Table 2 provides an overview of the data collection phase. Ongoing analysis throughout Phase 2 supported the design and modification of the tools. Phase 2 observation data were analysed with further interview data collected at the end of Phase 2 to determine the effectiveness of the tools designed to support participation in the numeracy demands of the selected tasks.

The development of viable tools using the design research cycle will be demonstrated using data from Phase 2 in the context of the bowling alley and the office. The bowling context was chosen as a number of iterations to the design of the tool were required before the resulting tool supported the participants' progress towards their goal. The office context was chosen to demonstrate the transformative nature possible using designed tools to support adults with ID.

Results

This section discusses the process of designing individualised tools to support learning. The bowling context is presented first to demonstrate the iterative nature of design research. Following the design research cycle illustrated in Fig. 1, the original design of goals and tools to support learning was based on observation and interview data from Phase 1 of the study. While this paper focuses on the implementation and refinement of these tools, through iterations of design research in Phase 2, each section begins with a description of the context and starting point.

Table 3 Scoreboard: Frames with a Discrepancy in Displayed Player's Score and Actual Running Totals Highlighted in Yellow

	Frame 1	Frame 2	Frame 3	Frame 4	Frame 5	Frame 6	Frame 7	Frame 8	Frame 9	Frame 10
Scores	7 /	8 -	9 -	X	X	X	X	7 1	9 -	9 -
Displayed total	0	26	35	35	35	65	95	158	167	176
Running Total	10	26	35	45	65	95	125	158	167	176

Designing a tool at the bowling alley: the scoresheet

Ten-pin bowling scoring is complex with scores awarded for the number of pins knocked down in each frame (two bowls) and extra points awarded if you score a spare (all ten pins knocked down in two bowls) or a strike (all pins knocked down in one bowl). Hence, spares and strikes are awarded 10 points, plus extra points, depending on future turns. At this bowling alley, the scoreboard shows complete and sometimes incomplete scores depending on how the game evolves. Complete scores are shown if no spare or strike is scored. For example, in Frame 3 in Table 3 the bowler scored nine points with two bowls and this score was added to his previous total of 26 to show his current score of 35. In this case, the score was accurate. Incomplete scores are shown if a spare or strike was scored because the scoreboard updated at the end of the scoring cycle, taking up to two extra turns to calculate the extra points required. Table 3 shows the scoreboard for one game where a number of strikes were scored in a row. The difference between the actual running total at the end of each frame (bottom row), and what the scoreboard displayed at the time (middle row), can be seen. The highlighted sections show that for 50% of this game, the scoreboard showed incomplete information for this bowler.

The starting point for this design research experiment was the identification of goals for Ben and David. The identification of goals was based on the analysis of Phase 1 data. Ben and David both indicated they wanted to be able to calculate their running total when a spare or strike was scored, so this was chosen as the goal for these two participants as it was important to them in the context of their bowling (see Gaunt et al., 2019 for detailed discussion of this phase). The next step in identifying the starting point was to determine the mathematical knowledge Ben and David required to calculate the running total after scoring a spare or a strike (add 10 to the current score) and the current capabilities of the two participants. From Phase 1 data, Ben and David demonstrated strength in reading, comparing, and understanding numbers (observed 85 times during Phase 1 with 100% accuracy from both bowlers). Both bowlers showed difficulties in determining their current score using mental calculations (out of 25 attempts during Phase 1 observations, 11 were correct and 14 were incorrect). Additionally, remembering information, such as how many points they scored for a strike, was difficult (out of 16 attempts, only David answered correctly, once). To support the participants' numeracy development, the first author designed a visual scoresheet to be used with the support of a calculator.

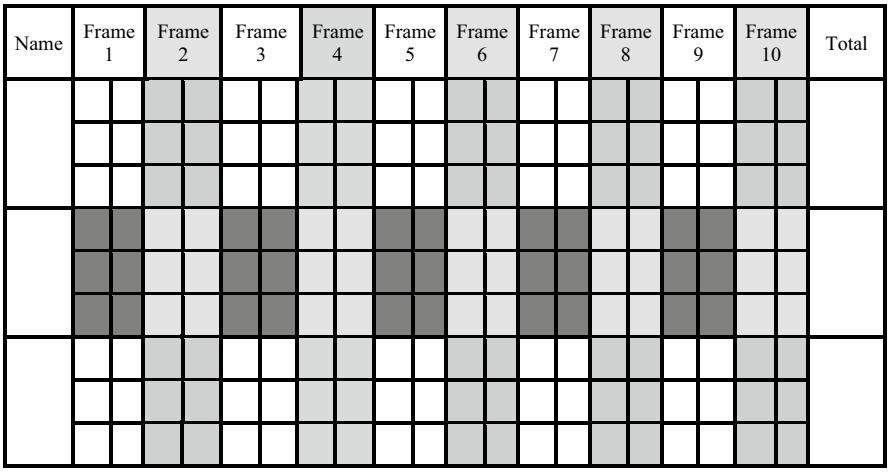


Fig. 2 Design 3 of the scoresheet

The decision to design a visual scoresheet similar in layout to the scoreboard was made as it was considered this would facilitate the bowlers' understanding and use of the tool (Prendergast et al., 2017). Design 1 of the scoresheet, developed and trialled during observations, included space for three bowlers[1] to record scores exactly as seen on the scoreboard. Following the design research cycle (Fig. 1), the way the design supported the participants and the potential difficulties it could present to them was analysed and considered. While Design 1 would support bowlers' understanding of the use of the tool due to its similarity to the scoreboard, it would potentially be difficult for the bowlers to track across the page, so alternate shading was added to the scoresheet. Design 2 included alternate shading to support ease of tracking across the page but did not provide any space to record the interim total if a spare or strike was scored. These two iterations of the design research cycle of analysis, reflection, redesign, and re-experiment resulted in Design 3, shown in Fig. 2, the first design trialled with the participants.

The Design 3 scoresheet was modelled on the scoreboard. It included alternate shading for ease of tracking across the page and provided space to record the scores for three bowlers. During Observation 1 of Phase 2, the researcher demonstrated the scoresheet to David and Ben. Both bowlers checked in after each turn and were shown how to record their scores and how to use the calculator to add 10. During this trial, it was conjectured that simplifying the amount of recording required for each person would further support the bowlers in completing their scoresheet independently, allowing them to concentrate on just their score for recording purposes. The subsequent Design 4 scoresheet, shown in Fig. 3, is intended for just one bowler to record the usual two games played in each competition.

[1] There were three bowlers on the team observed for this research; however, only two of the bowlers, Ben and David, were participants in this study. The third bowler took part in the scoring activity but was not recorded as part of the research.

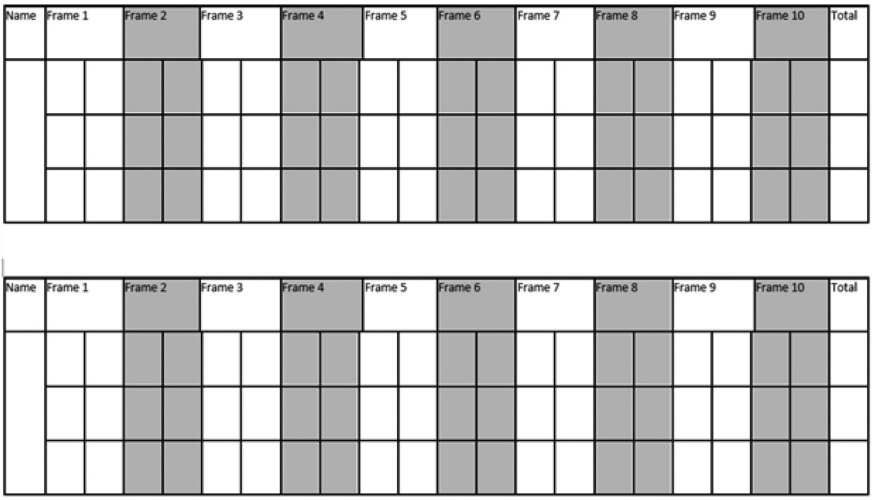


Fig. 3 Design 4 of the scoresheet

Design 4 was the first design where Ben and David completed recording their own scores. However, when the bowlers began using their scoresheets, it became apparent that Design 4 did not support the activity well. Figures 4 and 5 show copies of a section of the recorded scoresheet for Ben and David, respectively, compared with the same section of the researcher's master scoresheet.

As Fig. 4 shows, Ben had difficulty keeping the numerals within each box, and distinguishing the different frames on the scoresheet, even though they had been shaded alternately. His written numerals were large and even though the scoresheet covered the top half of the landscape, A4 page and each box was 12 × 14 mm, Ben struggled to fit the numerals within the boxes. Ben often rushed ahead to write his scores in, without assistance. In doing so, he had difficulty following the table setup, and as a result, Ben's final scoresheet contained a number of errors compared to the master scoresheet for his game.

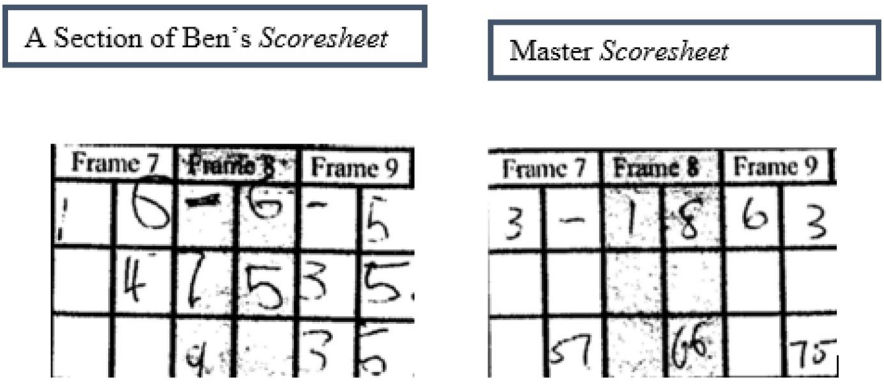


Fig. 4 First trial of Ben's design 4 scoresheet compared with the master scoresheet

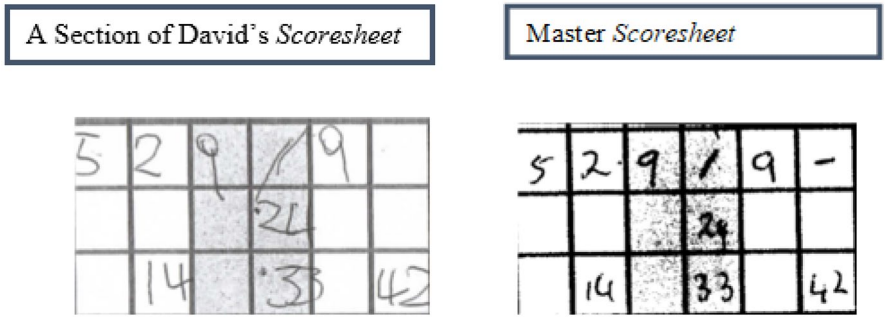


Fig. 5 First trial of David's design 4 scoresheet compared with the master scoresheet

David is hesitant to write numerals. He would look at the scoreboard (or calculator), back at the scoresheet, and then back to the scoreboard repeatedly before writing the number down. However, when asked what his score was, he could answer immediately. David had difficulty finding where to write each score. It was not sufficient to point to the sheet and say, "write your score in here" and then move away. The researcher had to hover the pen over the correct box and wait for David to check the scoreboard a few times before he wrote the number in the box. If the pen was moved away, David would not know where to write his score. As Fig. 5 shows, a dot was put in the empty box to add further visual support and indicate the correct place for David to write his score. However, this strategy was unsuccessful as it was even more difficult for David to fit his numbers in the square as he would not write the number over the dot. Despite these difficulties, Fig. 5 shows David's scoresheet accurately matched the master scoresheet for this section of the game.

Additionally, Ben and David had difficulty remembering the number of points to add (10) in order to calculate their running total when they scored a strike or a spare. During this observation, when asked, David responded with "seven points" three times, 10 once, and "I don't know" once. When he scored a spare, Ben responded with the first bowl of his spare (observed twice out of two times a spare was scored). On the one occasion Ben scored a strike, he responded with "I don't know".

Ongoing analysis and reflection of participants' interactions with the designed tool facilitated subsequent modifications within the design research cycle. At the end of this bowling session, the first author asked the participants what they thought of the scoresheet and both Ben and David indicated it was difficult to use. Ben stated, "It's a bit hard 'cos there's lots of boxes" and David said, "It's taking me a while to fill it in, but I think I will get better at it". The difficulties presented by participants in using the scoresheet informed specific adjustments that were subsequently trialled to support these adult learners. The next re-design of the scoresheet (Design 5) included the enlargement and separation of each frame shown in Fig. 6. Additionally, the scoresheet was made double sided so only one game was recorded on each side, allowing for larger boxes to record scores (18×14 mm). It was conjectured this would both support participants in finding the appropriate box more easily and allow for writing larger numerals.

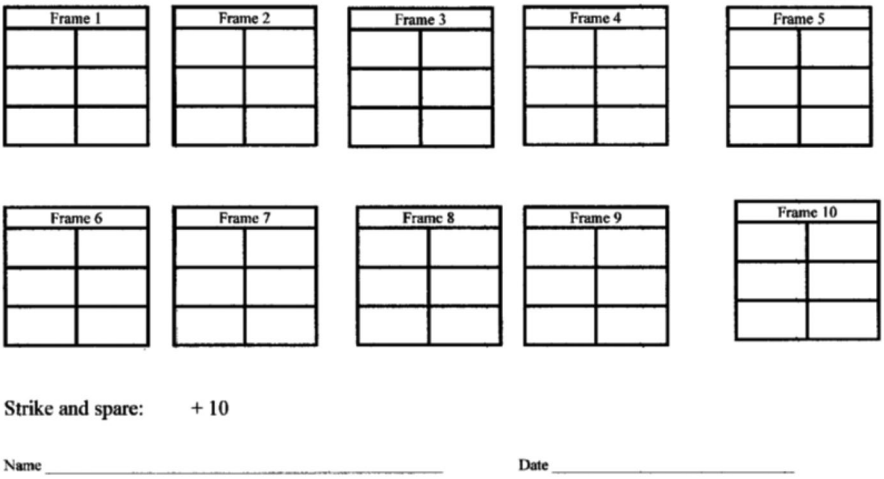


Fig. 6 Design 5 scoresheet

Additionally, further scaffolding was added by including written instructions to add 10 for spares and strikes.

Design 5 of the scoresheet assisted both Ben and David with a more accurate recording of their scores. The larger boxes made it easier to keep large numerals within the boxes. Separating each frame made it easier to track frames in the game. Ben required some assistance when recording scores for spares and strikes, but he independently recorded other scores. The separation of each frame made it easier for Ben to independently follow the scoresheet. As Ben could track across the scoresheet independently, he was keen to ask for assistance initially when he scored a spare or strike. While David still took time to write his score in, checking with the scoreboard or the computer a few times before writing on the scoresheet, the separate boxes made it easier for him to find where to write the score. Both participants said that Design 5 was much easier to follow with Ben stating, "This is easier to see than the last one" and David saying, "I was quicker today. I like this one better". This was the final design of the scoresheet used during Phase 2 of the research.

The design of the scoresheet supported both participants in reaching their goal of being able to independently calculate their running total when they scored a spare or a strike. While both participants achieved their numeracy goal, the impact of that achievement went beyond the simple ability of knowing their current score in the game. An increase in participation and engagement was observed as the participants progressed in their skills.

During Phase 1, particularly if several spares or strikes had been scored in a row, estimating who was currently in the lead was difficult. In such situations, both bowlers were frustrated by not knowing the current score. For example, in Transcript 1, Phase 1, Bowling, Observation 1[2] they said:

[2] Transcript excerpt lines are numbered sequentially throughout the paper for ease of referring to specific line items of data.

1	Ben	[Bowls 8 and checks his score] *64, yes!*
2	Bowlers	[All congratulate Ben. One person has bowled two strikes, so the scoreboard is inaccurate.]
3	Ben	*Thanks, but I don't know what your score is!*
4	Other Bowler	[Has turn and bowls a third strike.]
5	Ben	*Well done buddy! A turkey. Turkey dinner tonight*[3]*. We still don't know what your score is, but I think you are winning!*

Line 3 shows the evident frustration, particularly when several spares or strikes are scored in a row (line 5). Frustration with not knowing the current score was observed 39 times during Phase 1. In those situations, the bowlers could not calculate, or estimate accurately, the current score. Given the significant delay in the scoreboard displaying the score, and the complexity of updating the scores mentally, David and Ben were often unaware of their score and who was winning. Hence, while the bowlers used the scoreboard, it was not sufficient for their purposes. The scoresheet was a tool designed to support them in calculating and recording their scores, but knowing their scores also influenced their interactions with each other and their engagement and participation in the game.

As the participants became independent in calculating their scores, the focus of their conversations changed from discussing the score and guessing who was winning, to knowing their place in the game and discussing what was needed to maintain the lead or catch up. This conversation is from Transcript 2, Phase 2, Bowling, Observation 10:

6	David	[scored a strike and the onlookers cheered.]
7	Researcher	*Well done!* [To the third bowler] *I wonder if David has caught up to you?*
8	David	*Yep, I reckon I have, I'll work it out* [wrote X on the scoresheet, got calculator] …
9	David	[puts "+10=" in calculator] *65! I've nearly caught up to* [bowler] *He is on 71*
10	Ben	[Came to write in his score] *I have 49, I am not too far behind*
11	David	[Bowled 8 and wrote score in] *73, Now I am in front!*
12	Ben	*Yep, you are but I am not far behind*
13	David	*49, you have some catching up to do*
14	Ben	*I might need a strike then!*

Transcript 2 demonstrates the focus of conversations on scores with bowlers now discussing who they know is winning (lines 9–13) and what they need to do in their own game to change that (line 14). This focus is different from the earlier observations (Transcript 1) where conversations often focused on who the players thought might be winning (line 5). For Ben and David, achieving their numeracy goal impacted positively their engagement with and enjoyment of this social activity.

[3] In ten pin bolwing, a "Turkey" is three strikes in a row.

Expanding the bowlers' engagement in the game

The change in focus of conversations and engagement in the game was noticed by the bowlers. In the final interview, both bowlers were asked to comment on what they thought about using the scoresheet during their game. David said, "I enjoyed this, I liked working out the scores and knowing who was winning. I want to keep doing this". Ben said he thought keeping score was good because "we could see who was winning. It made me try harder to keep up because I knew the score".

Changes in participation were also noticed by the support worker and Ben's parents. Ben's father said, "Since he has been working with you (the first author), he's been keen to go. He would choose to miss some days at the Centre (disability support organisation), but he never misses bowling now". The support worker also noted the change in the bowlers indicating "Their conversations around bowling seem to be more frequent, even when they're not at the bowling alley, and they're always focused on the score. They were always pretty independent with their bowling, but they really seem much more interested in their scores now and they tell everyone who won". Knowing the current scores throughout the game appears to have transformed Ben and David's engagement with bowling and was noticed by both bowlers and other significant people in their worlds.

In the next section, the case of Ellen at the office is discussed to further demonstrate the impact of specifically designed tools. Ellen's case was chosen for this paper as using the tool specifically designed for her activity changed the activity for her and positively impacted how others saw her and how she viewed herself.

Designing a tool at the office

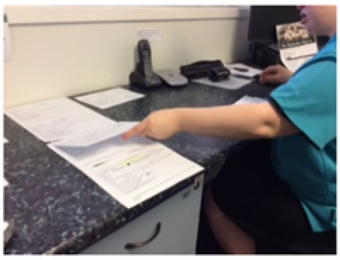
Ellen undertakes several tasks at the office, one of which is to file paperwork for the transport booking department. This department organises transport bookings for clients for social activities and medical appointments. The department uses four different folders for filing, each with different filing requirements. The transcript excerpts and data analysis presented here focus on medical transport bookings. Clients register their transport requirements, and transport runs are then organised for drivers to take clients to and from various appointments. Clients can book either a single or a series of appointments. These appointments are handwritten on booking sheets including the date(s) which are then filed in date order by the *latest date* (the date furthest into the future). Two transcript excerpts of observations from Ellen's context will be discussed. In the first observation, Transcript 3, Phase 2, Office, Observation 1, the researcher initially observed Ellen at work to identify the numeracy demands of her tasks.

15	Researcher	[After Ellen had discussed with a supervisor what her tasks were for the day] *What do you have to do today?*
16	Ellen	*I have filing to do, all of these* [carrying a few folders]
17	Ellen	*This folder* [Ellen picked up a manilla folder with previously filed documents and unfiled documents placed in various positions within the folder. It was the medical transport bookings folder]. *My boss has put these where they need to be filed. I have to open it open to the correct place* [opened the folder and unclipped the filing mechanism to allow a further sheet to be added to the file], *hole punch the sheet, like this* [used a hole punch and placed two holes in the sheet], *you need to be careful so the holes aren't too close to the edge because otherwise it will tear, and then I put it in the file close it up and do the next one.* [Ellen inserted the previously placed sheet and correctly filed it in the folder]
18	Researcher	*So, your boss sorts these out in the folder for you? In what order are they filed?*
19	Ellen	*Yes, my boss helps me. I used to have a trainer helping me, but she has gone, and I mess it up if I do it on my own. You have to look at the dates*
20	Researcher	[looking at the file, the documents were filed in date order] *Would you like it if I asked your boss if it was OK if I helped you to learn how to file these documents on your own?*
21	Ellen	*Oh yes, that would be awesome!*

After the discussion with both Ellen and her boss (lines 20 and 21), supporting Ellen to learn how to file documents in date order in the medical transport booking folder was chosen as the goal. As can be seen in Transcript 3, Ellen's task of filing the documents in the transport booking folder was a mechanical task of simply hole-punching the document already placed in the folder by another worker (Gaunt, 2022). To determine the starting point, Ellen's current mathematical knowledge was identified from Phase 1 data. Ellen demonstrated she was capable of ordering numbers correctly and reading dates in different formats (i.e., 19/8, August 19, 19/8/18), both demonstrated with 100% accuracy in all instances during Phase 1. Ellen also demonstrated she knew the order of the months and could order familiar events in date order. For example, in the initial interview, Ellen indicated the order of family and friends' birthdays and where they fit in with other important events, such as a concert and a party she was attending "I know all the important dates". While Ellen demonstrated strengths in numbers and dates, the complexity of sorting dated material and finding the correct place within the folder meant Ellen required support to complete this task. In collaboration with Ellen, the design research process was used to develop tools and structures to support Ellen, and a visual instruction sheet was developed, as shown in Fig. 7.

The visual instruction sheet contains three steps with pictures to support Ellen's understanding of the following steps (see Fig. 7); Step 1, sort booking sheets into date piles on the desk; Step 2, instructions if booking sheets contain more than one date; and Step 3, instructions on how to file sheets in the correct order. To further support Ellen, month tabs were added to the medical transport folder to enable Ellen to locate the correct month easily. Ellen developed increasing competence in the task of filling in date orders and was beginning to demonstrate independence in this task towards the end of Phase 2. Demonstrating this developing independence, the previous transcript is compared to an excerpt from Transcript 4, Phase 2, Office, Observation 15.

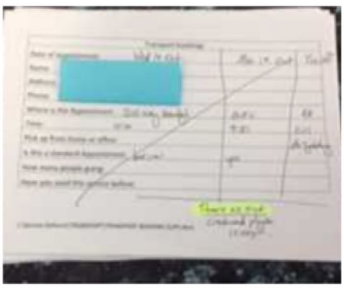
TRANSPORT BOOKINGS



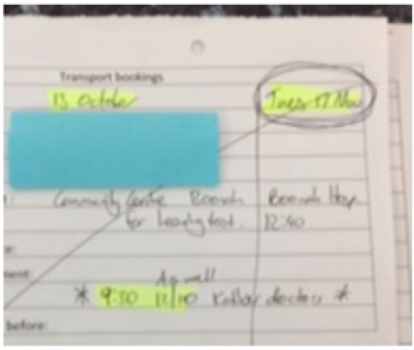
Step 1. Sort sheets into date piles.

Step 2. Sheets with more than one date:

If dates are in the same month
Highlight largest number



If there is more than one month
highlight largest number of latest month



Step 3: starting at the latest date, put sheets into folder in correct spot.

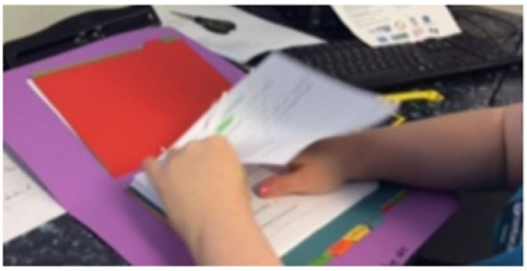


Fig. 7 Ellen's visual instruction sheet used to file medical transport booking sheets in date order

22	Ellen read the instruction sheet and began independently sorting booking sheets into piles in increasing date order in columns across her desk (Step 1). She encountered a booking sheet with three different dates on it and was unsure how to proceed	
23	Ellen	*What do I do here?* [pointing to the different dates]
24	Researcher	*Get out your instruction sheet*
25	Ellen	Opened the folder and looked at the tool
26	Researcher	*Look*, [pointing to the instruction sheet] *it is asking if the dates are the same months or different months*
27	Ellen	*Different months*
28	Researcher	*So, what does it say to do?* [pointing to the section of the instruction sheet for different months]
29	Ellen	*It says, "Highlight largest number of latest month"*
30	Researcher	*What months do you have?*
31	Ellen	*November, that is now, December, that is next and January, that is the latest month*
32	Researcher	*So, which one do you need to circle?*
33	Ellen	*That one* [pointing to the date in January]
34	Researcher	*That's it* [Ellen put the sheet on its own as there weren't any other bookings dated for January yet]
35	Ellen continued to sort booking sheets in increasing date order in columns across her desk as per Step 1 on her instruction sheet. Twice, Ellen came across sheets with multiple dates. She identified the latest date and checked with me first, both times she was correct. The third time, Ellen identified the latest date correctly and placed it correctly on the pile without checking with me	
36	On completion of Step 1, Ellen checked the ordered sheets with me	
37	Ellen	*That's all done, that's all correct* [proudly]
38	Researcher	*They are all correct. You can put them all in the folder now. Where do you start from?*
39	Ellen	*I will start in December and do these from December and November and then I will file the January ones last because they will be on the top again*
40	Researcher	*That's it, off you go then*
41	Ellen	*I am getting good at this! I won't need you soon. My boss is really happy I can do this now!* [Filed all the sheets correctly in the folder.]

Transcript 4 is representative of Ellen's ability at this stage of the study, towards the end of Phase 2, and demonstrates how the designed tools played a key role in orienting her work. The month tabs added to the filing folder provided her with an easier way of finding months, while the visual instruction sheet provided a step-by-step procedure to guide Ellen in completing this multistep task.

Ellen's use of the designed tool supported her independence in this task. In fact, Ellen completed Step 1 of the visual instruction sheet (line 22) independently since Observation 12. The second step provided support for Ellen when more than one

date was recorded on the booking sheet. Ellen requested help to determine the latest date (line 23). With verbal prompting (lines 24, 26, 28, 30, and 32), Ellen correctly identified the latest date by first attending to the latest month and then attending to the latest date in the month. The Visual Instruction Sheet supported Ellen by breaking this process into two steps. On two further occasions, Ellen checked that she had completed the task correctly, and another time, completed the task independently (line 35). Over the next three observations, across 2 weeks, Ellen completed this task independently five out of six times. On one occasion when Ellen made a mistake with this filing task, she did not notice a second date had been written on the booking sheet at the bottom of the page, instead of in the usual place, at the top of the page. Step 3 on the Visual Instruction Sheet was to start at the latest date and place the booking sheets in order in the file. Ellen completed this task independently (line 41). Ellen had been independently completing this task since Observation 11.

It is significant to note that during this observation Ellen recognised that the January documents needed to be filed last, not first. The booking folder was organised with the month of January on top and December at the bottom. Step 3 instructions stated to begin filing documents from the *latest date*. In this excerpt, there were some bookings for January the following year (the *latest date*), and hence, following the instructions, these should have been filed first. However, due to the structure of the file, the January bookings needed to go on the top of the folder and thus filed last. Ellen recognised that she needed to begin filing documents dated the *latest date* in the *year*, and documents from January, although later chronologically, needed to be filed last (line 39). This demonstrates a sophisticated understanding of the task and the tools she was using. Ellen correctly filed documents dated in the following year in January and February on three occasions in the final three observations over the next 2 weeks.

The support provided by the tools allowed Ellen to achieve her goal of filing the booking sheets independently. Ellen's statements "I mess it up if I do it on my own" (line 19) and "Oh yes, that would be awesome" (line 21), demonstrate her awareness of her difficulties in the task but also her willingness to overcome these difficulties. While this shows a positive disposition towards wanting to complete the filing task, it also demonstrates Ellen's lack of confidence in her ability. The visual instruction sheet and month tabs supported Ellen's independence in this filing task. She was proud of her achievement (line 41) and was now confident in her ability to complete this task independently.

Expanding Ellen's work opportunities

Ellen's competence in completing the filing task independently was noted by her boss and other employees, providing further evidence of Ellen's developing numeracy competence in her filing task. In the interview at the end of Phase 2, Ellen's mother stated Ellen's work colleagues had reported to her their admiration of the progress Ellen had made at work, stating "The positive feedback has been really beneficial for Ellen". In Ellen's interview, she stated "She [Ellen's boss] is really happy with me". Ellen elaborated that now she could do the filing herself, it was

one less thing the boss had to do. From a practical perspective, this led Ellen's colleagues to seek further tasks Ellen could complete at work, and thus, provided opportunities for her further numeracy growth. Ellen's colleagues viewed her as a mathematics learner.

One of these further tasks was to determine how much money was collected in the charity donation tin kept at reception. Ellen's boss contacted the first author and asked for some guidance on developing a tool to support Ellen in counting donations. This provides evidence that the boss recognised the role played by the designed tools in supporting Ellen's competence in the numeracy demands of her work tasks. Ellen's success in improving her numeracy skills in filing impacted Ellen's view of herself "I am getting good at this" and impacted how others viewed Ellen "She [Ellen's boss] is really happy with me". Ellen's improved numeracy skills stimulated a further opportunity for Ellen to continue her numeracy development in her workplace.

Discussion

Two findings are demonstrated by this research. First, (Research Question 1) design research is helpful to frame the development of tools that support numeracy learning for adult learners with ID. Second, the documented changes in participation and disposition towards the numeracy demands of these tasks evidence the development of numeracy for these participants, but more importantly, (Research Question 2) the positive impact that has on how adult learners view themselves, and how others view the adult learner.

In response to the first research question, this study demonstrates the usefulness of design research to frame the design of tools to support the achievement of numeracy learning goals for adults with intellectual disability. In design research in mathematics education, the design research cycle focuses on the tools, activities, and other means to support students' progress from their current understanding towards a goal, usually predetermined by the curriculum (Cobb et al., 2003). In contrast, when using design research to support the individual numeracy learning of adults with ID, goals are designed for the context of the adult's activity (Gaunt et al., 2019), and the design tools that supports adults to achieve numeracy goals can lead to adults with ID demonstrating greater participation and engagement in the activity or task. This was seen in the changes in the conversations about scoring demonstrated in Transcript 2 (lines 10–14) and the transformation of Ellen's task from one of hole-punching to one of sorting and filing documents in date order, as seen in Transcripts 3 and 4.

In designing tools to support the adult learner with ID, it is important to design and trial these tools within the context in which the learner requires them (Faragher, 2019; Prendergast et al., 2017). Developed from a socio-cultural perspective, the 21st-century numeracy model highlights the importance of context to numeracy (Geiger et al., 2015), and the value of selecting and using tools to support numerate activities. This study has not only highlighted how tools can support adults with ID to tackle the numeracy demands of daily tasks and activities, but it has also demonstrated that individual tools designed and implemented within the context of those

tasks and activities have great potential to support engagement and participation in the numeracy demands of tasks. The design process in Fig. 1 of trialling tools, with ongoing analysis and reflection on the impact and ease of use, is an important factor in the resulting successful development of the tool. While the mathematical aspects of the task are the focus of the initial design of the tool, collaboration with and consideration of the adult learner is a vital aspect in the design process for a successful outcome. This was seen in the enlargement and separation of the boxes for recording scores on the scoresheets for Ben and David, and the inclusion of photos on the visual instruction sheet to support Ellen. The design research cycle has much to offer learners with ID and has been shown to be valuable in supporting adults with ID in this study.

With reference to the second research question, it is clear that for adults with ID, continued learning post-school which supports their increased participation and engagement in a task or activity can lead to greater independence in or enjoyment of that task. Cuskelly et al. (2021) identified how the benefits of continued learning for adults with ID go beyond simply achieving a learning goal and include better outcomes in employment, health, and friendships. The increases in engagement and participation indicated by the transcript excerpts in this paper provide initial evidence of the development of personal numeracy and its impact on individual outcomes. The 21st Century Numeracy Model identifies the uses of numeracy in three different contexts, personal and social, work, and citizenship. Transcript 2 demonstrates both the bowlers increasing personal and social numeracy capabilities, in line with aspects of the 21st Century Numeracy Model (Goos et al., 2012), such as increased mathematical knowledge and use of tools as well as changes to the social interactions between the bowlers, indicating their mathematical dispositions. Their camaraderie, competition, and friendship are clear from lines 11 to 15 of the transcript. The feedback provided by the bowlers and others (parents and support workers) demonstrates changes in the way both bowlers engaged with the game, which was noticed by bowlers, parents, and support workers. This demonstrates the power that improving numeracy can have on an individual's engagement with the world around them.

In transcript 4, the designed tool supported Ellen's competence in the numeracy demands of the work task in such a way that it transformed the task from one of simply hole-punching and placing a document in a predetermined place in a folder (see line 21), to independently sorting and determining the correct place to file the document in date order. The transformation in this task may have contributed to changes in Ellen's self-concept and also to how others at work saw her, as a "mathematic doers and thinker" (Tan et al., 2019, p. 1). Both Ellen and her mother commented on feedback received from Ellen's boss and co-workers about how impressed they were with the improvements in Ellen's numeracy capabilities which supported her in being able to complete the filing task at work independently. This, in turn, increased Ellen's opportunities to participate in other work tasks as demonstrated by her boss investigating ways of supporting Ellen to count cash donations. This documents how changes in participation in numeracy activities positively impact how adult learners view themselves and how others view them.

Limitations and further research

While the current study does not provide sufficient evidence to make definitive claims, the increase in participation and engagement demonstrated in this study is an area that warrants further research. This study was limited in sample size as it investigated only four cases (see Table 1). While the qualitative approach in the methodology might be seen as a limitation, the depth of understanding gained from the intensive investigations of these limited cases and contexts shows the possibilities design research provides to enable the creation of individualised tools. These tools may support the development of personal numeracy. The possible transformative nature of numeracy competence on the participation and engagement of adults with ID in work and social contexts warrants further investigation in the use of design research using a wider sample.

Conclusion

Findings from this study demonstrate how designing specific tools to support numeracy learning in specific contexts may have benefits beyond the achievement of a learning goal. While access to continued learning post-school is still limited for adults with ID, the benefits shown by this research and others (c.f. Schreiber-Barsch et al., 2020) demonstrate the value of continuing to advocate for and undertake research that will provide more opportunities for adults with ID to have access to post-school learning. This research offers a new and compelling way to understand and develop numeracy for adults with ID by using design research (Cobb et al., 2003) to support the design of tools specific to the context and tasks of adults' everyday activities. The documented changes in dispositions are evidence of numeracy development for these participants. The design of tools to support participants and participants' take-up of those tools also evidences the changes in participants' personal numeracy skills, mathematical knowledge, and their capacity to independently complete their tasks. Ben, David, and Ellen all indicated their desire to continue to use the tools after the research study ended. These participants saw the value of tools designed to support their continued engagement in their daily activities.

These findings present compelling evidence that demonstrates the value of supporting numeracy development for adults with intellectual disability. This value is seen not just in improving an individual's numeracy capabilities in a specific work or social task, but in the impact that improved capability has on an adult with ID. Improving numeracy for adults with ID can impact the way in which they engage with the world around them and change the way others view them, supporting the call from Tan et al., (2019, p. 1) that individuals with intellectual disabilities be "honoured as mathematics doers and thinkers". In Ellen's workplace, other employees and her boss began to see Ellen as someone who could learn a new work task, and her boss actively sought to support Ellen in expanding her work opportunities. Everyone, including adults with intellectual disability, has the right to continue learning post-school. With the potential outcomes of improved numeracy leading to potentially greater life outcomes, further research into continued numeracy learning for this population should be a priority for researchers.

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Data availability Due to the vulnerable population involved in this research, for those wishing to access data, please contact the lead author.

Declarations

Ethical approval Ethical approval for the research study that underpins this publication was provided by the University of Queensland, registration number 2013000802.

Informed consent The NHMRC states that consent should be sought from an adult with learning disabilities if they have the capacity to do so. They also suggest that another adult, not associated with the research, may be present during the consent process to support the adult with learning disabilities in understanding what is required of them. To determine if the participating adults were able to give informed consent, participants were questioned by the researcher regarding their understanding of the research and the consent process. To be able to provide informed consent gave agency to participants over their own choices. For this study, participants were asked to explain the study in their own words and why they wanted to participate. Based on their responses, Ben, David, and Ellen demonstrated capability in providing informed consent; however, they were given the choice of bringing a support person to the initial meeting if they wished. Ellen and Ben chose to bring a parent as a support person. Parents observed the consent discussion but did not intervene. David chose to attend the meeting independently. Max's mother provided consent for Max's participation as he did not have the capacity to provide informed consent. After having the research explained to them, and their freedom to withdraw at any time, participants completed the consent to participant.

Conflict of interest The authors declare no competing interests.

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