Assignment t-test

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t-test test the null hypothesis H_0 against the alternative hypothesis H_1.

For univariate samples, t-test performs a Student t test. The test statistic is assumed to follow a Student t-Distribution $[df]$.

For multivariate samples, t-test performs Hotelling's t^2 test. The test statistic is assumed to follow a Hotelling t-square distribution $[p, df]$ where p is the dimension of data.

The degrees of freedom df, used to specify the distribution of the test statistic, depend on the sample size, number of samples, and in the case of two univariate samples, the results of a test for equal variances.

For the t-test, a cutoff α is chosen such that H_0 is rejected only if $p < \alpha$. The value of α used for the "Test Conclusion" and "Short Test Conclusion" properties is controlled by the Significance Level option. This value α is also used in diagnostic tests of assumptions, including tests for normality, equal variance, and symmetry. By default, α is set to 0.05.

1. Certain refined edible oil is packed in tins holding 16 kg each. The filling machine can maintain this but with a standard deviation of 0.5 kg. Samples of 25 are taken from the production line. If a sample means are (i) 16.35 kg, (ii) 15.8 kg. Can we be 95 per cent sure that the sample has come from a population of 16 kg tins?

   Solution:

   Null hypothesis: $H_0 : \mu = 16$
   Alternative hypothesis: $H_1 : \mu \neq 16$ (two tailed test)

   (i) given, $\bar{x} = 16.35$, $s = 0.5$ and $n = 25$

   t-statistic:

   $$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{16.35 - 16}{0.5/\sqrt{25}} = 3.5$$

   degrees of freedom:

   $$d.o.f = n - 1 = 25 - 1 = 24$$

   The critical value for t (from t-distribution table) with degrees of freedom $= 24$ and $\alpha = 0.05$ is 2.064

   Standard Error:

   $$S.E = \frac{s}{\sqrt{n}} = \frac{0.5}{\sqrt{25}} = 0.1$$

   Conclusion: $|t| > t_{0.05/2}$, we reject the Null hypothesis. That is the sample mean is differ from the intended weight and hence does not come from a population with mean 16 kg tins.

   (ii) given, $\bar{x} = 15.8$, $s = 0.5$ and $n = 25$

   t-statistic:

   $$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{15.8 - 16}{0.5/\sqrt{25}} = -2.0$$

   degrees of freedom:

   $$d.o.f = n - 1 = 25 - 1 = 24$$

   The critical value for t (from t-distribution table) with degrees of freedom $= 24$ and $\alpha = 0.05$ is 2.064

Standard Error:

$$S.E = \frac{s}{\sqrt{n}} = \frac{0.5}{\sqrt{25}} = 0.1$$

Conclusion: $|t| < t_{0.05}$, we fail to reject the Null hypothesis. That is the sample mean is not differ from the intended weight and hence come from a population of 16 kg tins.

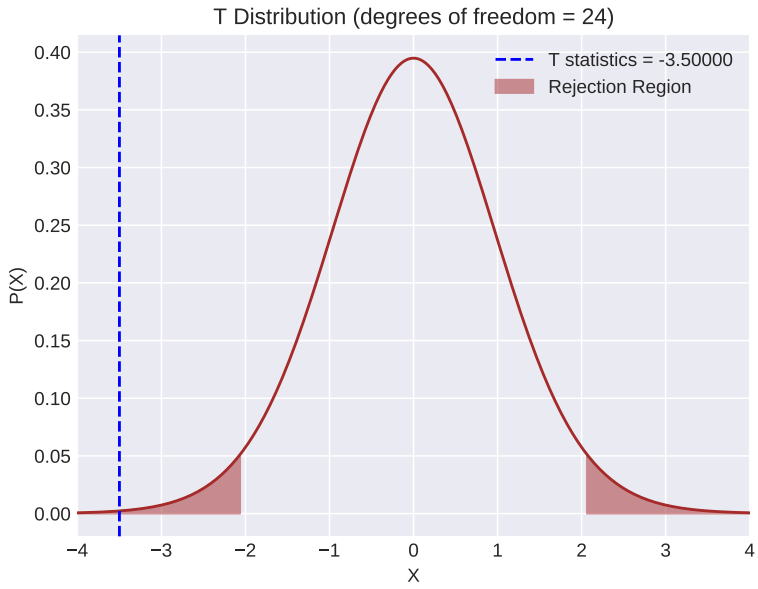
Program:

```
#!/usr/bin/env python3
import numpy as np
from scipy.stats import t
import matplotlib.pyplot as plt
plt.style.use('seaborn-darkgrid')

print("__________t-test__________")

p_mean = float(input("\nPopulation mean: "))
s_mean = float(input("Sample mean: "))
s_stdv = float(input("Sample stand_dev: "))
n = float(input("Sample length: "))

# degrees of freedom, level of significance, confidence level
dof = n - 1; alpha = 0.05; clevel = 1 - alpha

# calculation of t-statistic
tstatistic = (s_mean - p_mean)/(s_stdv/np.sqrt(n))
tstatistic = -tstatistic if tstatistic > 0 else tstatistic
print("\nt-statistic: {:.5f}".format(tstatistic))

# calculation of critical values
tcritical_l = t.ppf(q = alpha/2, df = dof)
tcritical_u = -tcritical_l
print("\nCritical values are {:.5f}, {:.5f}".format(tcritical_l, tcritical_u))

# decision making - using T-statistic and Critical values
if tstatistic < tcritical_l or tstatistic > tcritical_u:
    print("Reject the Null hypothesis.")
else: print("Fail to reject the Null hypothesis.")

# calculation of p-value
pvalue = 2*t.cdf(tstatistic, df = dof)
print("\np-value: {:.5f}".format(pvalue))

# decision making - using p-value
if pvalue < alpha: print("Reject the Null hypothesis.")
else: print("Fail to reject the Null hypothesis.")

# standard error
std_err = s_stdv/np.sqrt(n)
print("\nStandard Error: {:.5f}".format(std_err))

# confidence interval
cnf_int = s_mean + std_err * np.array([tcritical_l, tcritical_u])
print("Confidence Interval: {}".format(cnf_int))

# plot script
x1=np.linspace(-10,tcritical_l,1000); y1=t.pdf(x1,df=dof)
x2=np.linspace(tcritical_u,10,1000);  y2=t.pdf(x2,df=dof)
x3=np.linspace(-4,4,1000); y3=t.pdf(x3,df=dof)
ax=plt.figure().add_subplot(111); ax.plot(x3,y3,color='brown')
ax.fill_between(x1,y1,0,alpha=0.5,color='brown',label='Rejection Region')
ax.fill_between(x2,y2,0,alpha=0.5,color='brown'); ax.set(xlabel='X',ylabel='P(X)')
ax.axvline(x=tstatistic,ls='--',c='b',label='t-statistic = {:.5f}'.format(tstatistic))
```

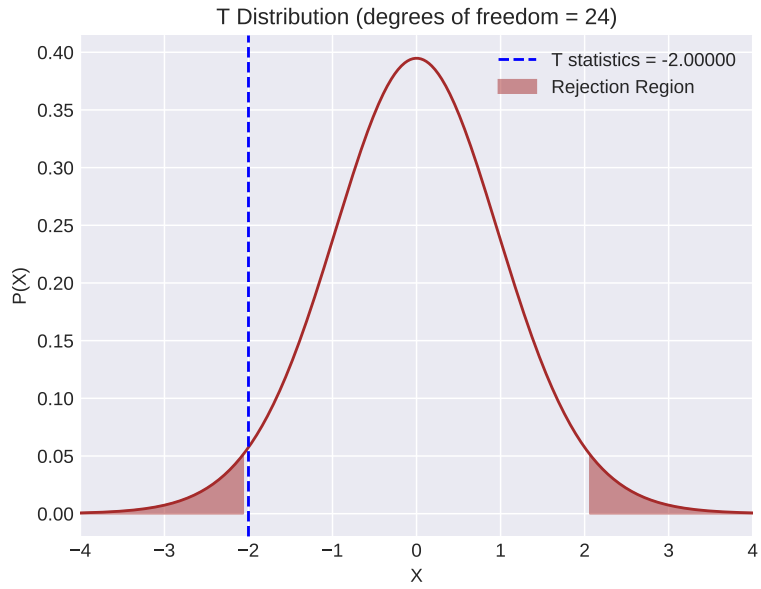
```
ax.set(xlim=[-4,4],title="t-distribution (degrees of freedom = {:.0f})".format(dof))
plt.legend(); plt.show()
```

Output:

```
__________T-test__________

Population mean: 16
Sample mean: 16.35
Sample stand_dev: 0.5
Sample length: 25

T-statistics: -3.50000

Critical values are -2.06390, 2.06390
Reject the Null hypothesis.

p-value: 0.00184
Reject the Null hypothesis.

Standard Error: 0.10000
Confidence Interval: [16.14361014 16.55638986]
```




```
__________T-test__________

Population mean: 16
Sample mean: 15.8
Sample stand_dev: 0.5
Sample length: 25

T-statistics: -2.00000

Critical values are -2.06390, 2.06390
Fail to reject the Null hypothesis.

p-value: 0.05694
Fail to reject the Null hypothesis.

Standard Error: 0.10000
Confidence Interval: [15.59361014 16.00638986]
```



2. A filling machine is expected to fill 5 Kg of powder into bags. A sample of 10 bags gave the weighs 4.7, 4.9, 5.0, 5.1, 5.4, 5.2, 4.6, 5.1, 4.6 and 4.7 Test whether the machine is working properly.

Solution:

Null hypothesis: $H_0 : \mu = 5$
Alternative hypothesis: $H_1 : \mu \neq 5$ (two tailed test)

Sample mean:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{4.7 + 4.9 + 5.0 + 5.1 + 5.4 + 5.2 + 4.6 + 5.1 + 4.6 + 4.7}{10} = 4.93$$

Sample variance:

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} = \frac{\sum_{i=1}^{10} (x_i - 4.93)^2}{10-1} = 0.07567$$

Sample standard deviation:

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{10} (x_i - 4.93)^2}{10-1}} = \sqrt{0.07567} = 0.275076$$

t-statistic:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{4.93 - 5}{0.275076/\sqrt{10}} = -0.80472$$

degrees of freedom:

$$d.o.f = n - 1 = 10 - 1 = 9$$

The critical value for t for a two-tailed test at 5% level of significane with $10 - 1 = 9$ degrees of freedom is 2.262

Standard Error:

$$S.E = \frac{s}{\sqrt{n}} = \frac{0.275076}{\sqrt{10}} = 0.08699$$

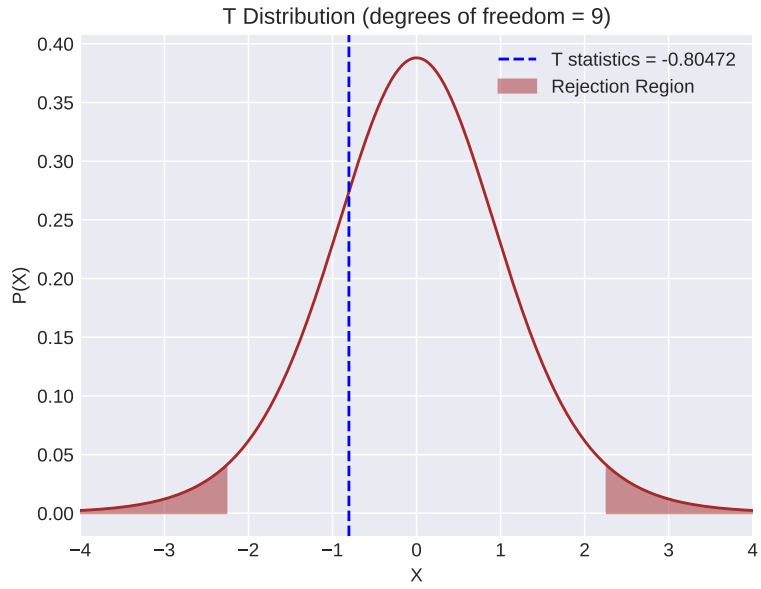
Conclusion: $|t| < t_{0.05}$, we fail to reject the Null hypothesis. That is the sample data are consistent with the assumption of mean 5 kg in the population.

Program:

```
#!/usr/bin/env python3
import numpy as np
from scipy.stats import t, ttest_1samp
import matplotlib.pyplot as plt
plt.style.use('seaborn-darkgrid')

print("__________T-test (One Sample)__________")

p_mean = float(input("\nPopulation mean: "))
n = int(input("Sample length: "))
sample = [float(value) for value in input("Sample values: ").split()]
s_mean = np.mean(sample); s_stdv = np.std(sample, ddof=1)

# degrees of freedom, level of significance, confidence level
dof = n - 1; los = 0.05; cnl = 1 - los

# calculation of T-statistics and p-value
tstatistics, pvalue = ttest_1samp(sample, p_mean)
print("\nT statistics: {:.5f}".format(tstatistics))

# calculation of Critical values
tcritical_l = t.ppf(q = los/2, df = dof)
tcritical_u = -tcritical_l
print("\nCritical values are {:.5f}, {:.5f}".format(tcritical_l, tcritical_u))

# decision making - using T-statistics and Critical values
if tstatistics < tcritical_l or tstatistics > tcritical_u:
    print("Reject the Null hypothesis.")
else: print("Fail to reject the Null hypothesis.")

print("\np-value: {:.5f}".format(pvalue))

# decision making - using p-value
if pvalue < los: print("Reject the Null hypothesis.")
else: print("Fail to reject the Null hypothesis.")

# standard error
std_err = s_stdv/np.sqrt(n)
print("\nStandard Error: {:.5f}".format(std_err))

# confidence interval
cnf_int = s_mean + std_err * np.array([tcritical_l, tcritical_u])
print("Confidence Interval: {}".format(cnf_int))

# plot script
x1=np.linspace(-10,tcritical_l,1000); y1=t.pdf(x1,df=dof)
x2=np.linspace(tcritical_u,10,1000);  y2=t.pdf(x2,df=dof)
x3=np.linspace(-4,4,1000); y3=t.pdf(x3,df=dof)
ax=plt.figure().add_subplot(111); ax.plot(x3,y3,color='brown')
ax.fill_between(x1,y1,0,alpha=0.5,color='brown',label='Rejection Region')
ax.fill_between(x2,y2,0,alpha=0.5,color='brown'); ax.set(xlabel='X',ylabel='P(X)')
ax.axvline(x=tstatistics,ls='--',c='b',label='T statistics = {:.5f}'.format(tstatistics))
ax.set(xlim=[-4,4],title="T Distribution (degrees of freedom = {:.0f})".format(dof))
plt.legend(); plt.savefig('tscript2.pdf',dpi=72,bbox_inches='tight'); plt.show()
```

Output:

```
__________T-test (One Sample)__________

Population mean: 5
Sample length: 10
Sample values: 4.7 4.9 5.0 5.1 5.4 5.2 4.6 5.1 4.6 4.7

T statistics: -0.80472

Critical values are -2.26216, 2.26216
Fail to reject the Null hypothesis.

p-value: 0.44172
Fail to reject the Null hypothesis.

Standard Error: 0.08699
Confidence Interval: [4.73322266 5.12677734]
```



3. Two sets of ten students selected at random from a college were taken. One set was given memory test as they were and the other was given the memory test after two weeks of training and the scores are given below.

   Set A: 10 8 7 9 8 10 9 6 7 8

   Set B: 12 8 8 10 8 11 9 8 9 9

   Do you think there is a significant effect due to training?

Solution:

Null hypothesis: $H_0 : \mu_1 = \mu_2$, No significant effect due to training
Alternative hypothesis: $H_1 : \mu_1 \neq \mu_2$, Significant effect due to training

Set A:

Mean:

$$\bar{x}_1 = \frac{\sum_{i=1}^{n1} x_{1i}}{n1} = \frac{10+8+7+9+8+10+9+6+7+8}{10} = 8.2$$

Variance:

$$s_1^2 = \frac{\sum_{i=1}^{n1}(x_{1i} - \bar{x_1})^2}{n1-1} = \frac{\sum_{i=1}^{10}(x_{1i} - 8.2)^2}{10-1} = 1.73333$$

Standard Deviation:

$$s_1 = \sqrt{\frac{\sum_{i=1}^{n1}(x_{1i} - \bar{x_1})^2}{n1 - 1}} = \sqrt{\frac{\sum_{i=1}^{10}(x_{1i} - 8.2)^2}{10 - 1}} = \sqrt{1.73333} = 1.31656$$

Set B:

Mean:

$$\bar{x}_2 = \frac{\sum_{i=1}^{n2} x_{2i}}{n2} = \frac{12 + 8 + 8 + 10 + 8 + 11 + 9 + 8 + 9 + 9}{10} = 9.2$$

Variance:

$$s_2^2 = \frac{\sum_{i=1}^{n2}(x_{2i} - \bar{x_2})^2}{n2 - 1} = \frac{\sum_{i=1}^{10}(x_{2i} - 9.2)^2}{10 - 1} = 1.95556$$

Standard Deviation:

$$s_2 = \sqrt{\frac{\sum_{i=1}^{n2}(x_{2i} - \bar{x}_2)^2}{n2 - 1}} = \sqrt{\frac{\sum_{i=1}^{10}(x_{2i} - 9.2)^2}{10 - 1}} = \sqrt{1.95556} = 1.39841$$

Calculation of $s1/s2$:

$$\frac{s_1}{s_2} = \frac{1.31656}{1.39841} = 0.94147$$

Test statistic:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n1 + s_2^2/n2}}$$

where $n1$ and $n2$ are the sample sizes, $\bar{x}_1$ and $\bar{x}_2$ are the sample means, and s_1^2 and s_2^2 are the sample variances. If equal variances are assumed $(0.5 < s1/s2 < 2)$, then the formula reduces to:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{1/n1 + 1/n2}}$$

where

$$s_p^2 = \frac{(n1 - 1)s_1^2 + (n2 - 1)s_2^2}{n1 + n2 - 2}$$

Calculation of s_p:

$$s_p = \sqrt{\frac{(10 - 1) \times 1.31656^2 + (10 - 1) \times 1.39841^2}{10 + 10 - 2}} = 1.35810$$

t-statistic:

$$t = \frac{8.2 - 9.2}{1.35810 \times \sqrt{1/10 + 1/10}} = -1.64646$$

degrees of freedom:

$$d.o.f = n_1 + n_2 - 2 = 10 + 10 - 2 = 18$$

The critical value for t (from t-distribution table) with degrees of freedom $= 18$ and $\alpha = 0.05$ is 2.101

Standard Error:

$$S.E = sp\sqrt{1/n1 + 1/n2} = 1.35810 \times \sqrt{1/10 + 1/10} = 0.60736$$

Conclusion: $|t| < t_{0.05}$, we fail to reject the Null hypothesis. That is there is no significant effect in the scores of the memeory test due to training.

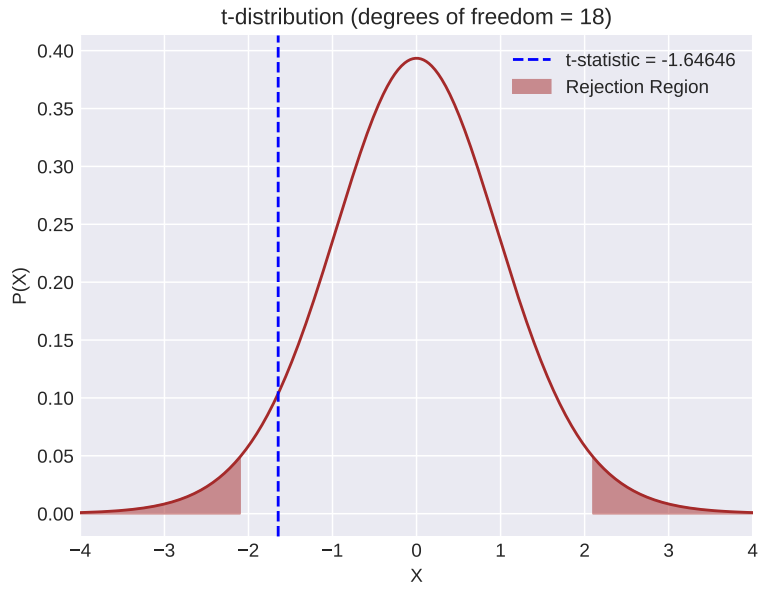
Program:

```
#!/usr/bin/env python3
import numpy as np
from scipy.stats import t
import matplotlib.pyplot as plt
plt.style.use('seaborn-darkgrid')

def similar_variance(n1,s1_mean,n2,s2_mean):
    dof = n1+n2-2
    sp = np.sqrt(((n1-1)*s1_stdv**2+(n2-1)*s2_stdv**2)/(dof))
    return (s1_mean-s2_mean)/(sp * np.sqrt(1/n1 + 1/n2)), dof, sp

def non_similar_variance(n1,s1_mean,n2,s2_mean):
    sd = np.sqrt(s1_stdv**2/n1 + s2_stdv**2/n2)
    dof = (sd**4)/(((s1_std**2/n1)**2/(n1-1)) + ((s2_std**2/n2)**2/(n2-1)))
    return (s1_mean-s2_mean)/sd, dof, sd

def ttest_and_variance(s1_stdv, s2_stdv):
    if 0.5 < s1_stdv/s2_stdv < 2:
        return similar_variance(n1,s1_mean,n2,s2_mean)
    else:
        return non_similar_variance(n1,s1_mean,n2,s2_mean)

print("__________Two-sample t-test for unpaired data__________\n")

sample1 = [float(value) for value in input("Sample_1 values: ").split()]
sample2 = [float(value) for value in input("Sample_2 values: ").split()]

# Number of Observations
n1 = len(sample1); n2 = len(sample2)

# Mean and Standard Deviation
s1_mean = np.mean(sample1); s1_stdv = np.std(sample1, ddof=1)
s2_mean = np.mean(sample2); s2_stdv = np.std(sample2, ddof=1)

# level of significance, confidence level
los = 0.05; cnl = 1 - los

# standard error (SE) of mean of sample1
std_err1 = s1_stdv/np.sqrt(n1)

print("\nSample 1: \n\t Number of Observations = {} \n\t Mean = {:.5f}".format(n1,s1_mean))
print("\t Standard Deviation = {:.5f}".format(s1_stdv))
print("\t Standard Error of the Mean = {:.5f}".format(std_err1))

# standard error (SE) of mean of sample2
std_err2 = s2_stdv/np.sqrt(n2)

print("\nSample 2: \n\t Number of Observations = {} \n\t Mean = {:.5f}".format(n2,s2_mean))
print("\t Standard Deviation = {:.5f}".format(s2_stdv))
print("\t Standard Error of the Mean = {:.5f}".format(std_err2))

# calculation of t-statistic and degrees of freedom
tstatistic, dof, sp = ttest_and_variance(s1_stdv, s2_stdv)
print("\nt-statistic: {:.5f}".format(tstatistic))

# calculation of Critical values
tcritical_l = t.ppf(q = los/2, df = dof)
tcritical_u = -tcritical_l
print("\nCritical values are {:.5f}, {:.5f}".format(tcritical_l, tcritical_u))
```

```
# decision making: t-statistic and Critical values
if tstatistic < tcritical_l or tstatistic > tcritical_u:
    print("Reject the Null hypothesis.")
else: print("Fail to reject the Null hypothesis.")

# calculation of p-value
pvalue = 2*t.cdf(tstatistic, df = dof)
print("\np-value: {:.5f}".format(pvalue))

# decision making: p-value and level of significance
if pvalue < los: print("Reject the Null hypothesis.")
else: print("Fail to reject the Null hypothesis.")

# standard error
std_error = sp * np.sqrt(1/n1 + 1/n2)
print("\nStandard Error: {:.5f}".format(std_error))

# confidence interval
cnf_int = (s1_mean - s2_mean) + std_error * np.array([tcritical_l, tcritical_u])
print("Confidence Interval: {}".format(cnf_int))

x1=np.linspace(-10,tcritical_l,1000); y1=t.pdf(x1,df=dof)
x2=np.linspace(tcritical_u,10,1000);  y2=t.pdf(x2,df=dof)
x3=np.linspace(-4,4,1000); y3=t.pdf(x3,df=dof)
ax=plt.figure().add_subplot(111); ax.plot(x3,y3,color='brown')
ax.fill_between(x1,y1,0,alpha=0.5,color='brown',label='Rejection Region')
ax.fill_between(x2,y2,0,alpha=0.5,color='brown'); ax.set(xlabel='X',ylabel='P(X)')
ax.axvline(x=tstatistic,ls='--',c='b',label='t-statistic = {:.5f}'.format(tstatistic))
ax.set(xlim=[-4,4],title="t-distribution (degrees of freedom = {:.0f})".format(dof))
plt.legend(); plt.savefig('tscript3.pdf',dpi=72,bbox_inches='tight'); plt.show()
```

Output:

```
__________Two-sample t-test for unpaired data__________

Sample_1 values: 10 8 7 9 8 10 9 6 7 8
Sample_2 values: 12 8 8 10 8 11 9 8 9 9

Sample 1:
     Number of Observations = 10
     Mean = 8.20000
     Standard Deviation = 1.31656
     Standard Error of the Mean = 0.41633

Sample 2:
     Number of Observations = 10
     Mean = 9.20000
     Standard Deviation = 1.39841
     Standard Error of the Mean = 0.44222

t-statistic: -1.64646

Critical values are -2.10092, 2.10092
Fail to reject the Null hypothesis.

p-value: 0.11702
Fail to reject the Null hypothesis.

Standard Error: 0.60736
Confidence Interval: [-2.27602071  0.27602071]
```



Program: t-test with in-built scipy.stats.ttest

```
#!/usr/bin/env python3
import numpy as np
from scipy.stats import t, ttest_ind
import matplotlib.pyplot as plt
plt.style.use('seaborn-darkgrid')

print("__________Two-sample t-test for unpaired data__________\n")

sample1 = [float(value) for value in input("Sample_1 values: ").split()]
sample2 = [float(value) for value in input("Sample_2 values: ").split()]

# Number of Observations
n1 = len(sample1); n2 = len(sample2)

# Mean and Standard Deviation
s1_mean = np.mean(sample1); s1_stdv = np.std(sample1, ddof=1)
s2_mean = np.mean(sample2); s2_stdv = np.std(sample2, ddof=1)

# calculation of degrees of freedom and pooled variance
if 0.5 < s1_stdv/s2_stdv < 2:
    dof = n1+n2-2
    sp = np.sqrt(((n1-1)*s1_stdv**2+(n2-1)*s2_stdv**2)/(dof))
else:
    sp = np.sqrt(s1_stdv**2/n1 + s2_stdv**2/n2)
    dof = (sp**4)/(((s1_std**2/n1)**2/(n1-1)) + ((s2_std**2/n2)**2/(n2-1)))

# level of significance, confidence level
los = 0.05; cnl = 1 - los

# standard error (SE) of mean of sample1
std_err1 = s1_stdv/np.sqrt(n1)

print("\nSample 1: \n\t Number of Observations = {} \n\t Mean = {:.5f}".format(n1,s1_mean))
print("\t Standard Deviation = {:.5f}".format(s1_stdv))
print("\t Standard Error of the Mean = {:.5f}".format(std_err1))

# standard error (SE) of mean of sample2
std_err2 = s2_stdv/np.sqrt(n2)

print("\nSample 2: \n\t Number of Observations = {} \n\t Mean = {:.5f}".format(n2,s2_mean))
print("\t Standard Deviation = {:.5f}".format(s2_stdv))
```

```
print("\t Standard Error of the Mean = {:.5f}".format(std_err2))

# calculation of t-statistic and p-value
tstatistic, pvalue = ttest_ind(sample1, sample2)
print("\nt-statistic: {:.5f}".format(tstatistic))

# calculation of Critical values
tcritical_l = t.ppf(q = los/2, df = dof)
tcritical_u = -tcritical_l
print("\nCritical values are {:.5f}, {:.5f}".format(tcritical_l, tcritical_u))

# decision making: t-statistic and Critical values
if tstatistic < tcritical_l or tstatistic > tcritical_u:
    print("Reject the Null hypothesis.")
else: print("Fail to reject the Null hypothesis.")

print("\np-value: {:.5f}".format(pvalue))

# decision making: p-value and level of significance
if pvalue < los: print("Reject the Null hypothesis.")
else: print("Fail to reject the Null hypothesis.")

# standard error
std_error = sp * np.sqrt(1/n1 + 1/n2)
print("\nStandard Error: {:.5f}".format(std_error))

# confidence interval
cnf_int = (s1_mean - s2_mean) + std_error * np.array([tcritical_l, tcritical_u])
print("Confidence Interval: {}".format(cnf_int))

x1=np.linspace(-10,tcritical_l,1000); y1=t.pdf(x1,df=dof)
x2=np.linspace(tcritical_u,10,1000);  y2=t.pdf(x2,df=dof)
x3=np.linspace(-4,4,1000); y3=t.pdf(x3,df=dof)
ax=plt.figure().add_subplot(111); ax.plot(x3,y3,color='brown')
ax.fill_between(x1,y1,0,alpha=0.5,color='brown',label='Rejection Region')
ax.fill_between(x2,y2,0,alpha=0.5,color='brown'); ax.set(xlabel='X',ylabel='P(X)')
ax.axvline(x=tstatistic,ls='--',c='b',label='t-statistic = {:.5f}'.format(tstatistic))
ax.set(xlim=[-4,4],title="t-distribution (degrees of freedom = {:.0f})".format(dof))
plt.legend(); plt.savefig('ttest2.pdf',dpi=72,bbox_inches='tight'); plt.show()
```

Output:

```
__________Two-sample t-test for unpaired data__________

Sample_1 values: 10 8 7 9 8 10 9 6 7 8
Sample_2 values: 12 8 8 10 8 11 9 8 9 9

Sample 1:
     Number of Observations = 10
     Mean = 8.20000
     Standard Deviation = 1.31656
     Standard Error of the Mean = 0.41633

Sample 2:
     Number of Observations = 10
     Mean = 9.20000
     Standard Deviation = 1.39841
     Standard Error of the Mean = 0.44222

t-statistic: -1.64646

Critical values are -2.10092, 2.10092
Fail to reject the Null hypothesis.
```

```
p-value: 0.11702
Fail to reject the Null hypothesis.

Standard Error: 0.60736
Confidence Interval: [-2.27602071  0.27602071]
```