Assignment: Binomial Distribution

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1. Find the probability of exactly 15 successes in 121 trials with $p = 0.1$

   The probability that a random variable X with binomial distribution $B(n, p)$ is equal to the value k, where $k = 0, 1, ...n$, is given by

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

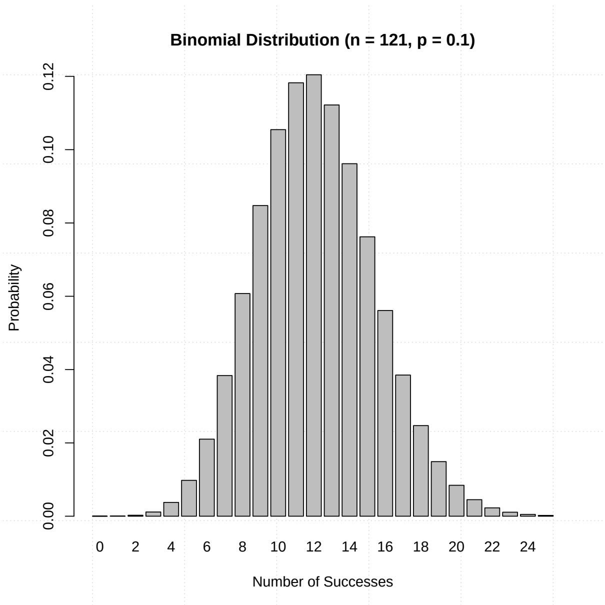
$$\begin{aligned} P(X = 15) &= \binom{121}{15} (0.1)^{15} (1-0.1)^{106} \\ &= \frac{121!}{15!(121-15)!} (0.1)^{15} (0.9)^{106} \\ &= 0.07622452 \end{aligned}$$

   Script:

```
#!/usr/bin/env Rscript

# binomial distribution
binomial <- function(k, n, p) {
    return(factorial(n)/(factorial(k) * factorial(n - k)) * p**(k) * (1 - p)**(n - k))
}

binomial(15, 121, 0.1)

# compute probability mass function (PMF)
# P(X = 15)
dbinom(x = 15, size = 121, prob = 0.1)
```

   Output:

```
[1] 0.07622452
[1] 0.07622452
```



2. Find the probability that in 30 tosses of a fair coin the head comes up with less than 24 times.

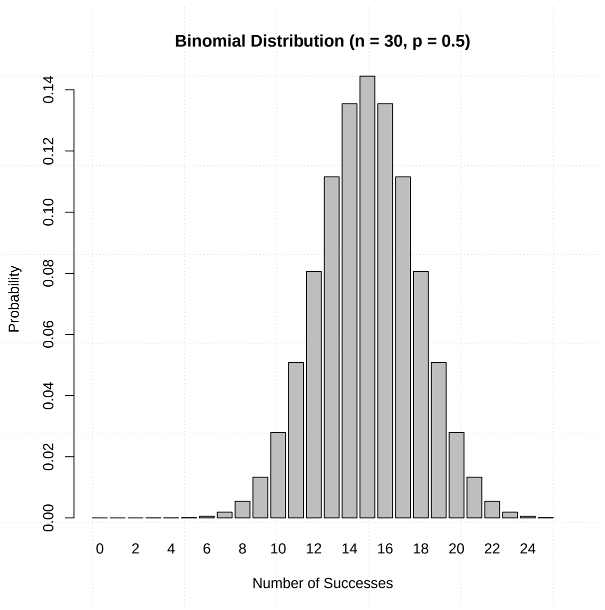
$$
\begin{aligned}
P(X < 24) &= P(X \leq 23) \\
&= 1 - (1 - P(X \leq 23)) \\
&= 1 - P(X \geq 24) \\
&= 1 - \Big(P(X = 24) + P(X = 25) + P(X = 26) \\
&\quad + P(X = 27) + P(X = 28) + P(X = 29) + P(X = 30)\Big) \\
&= 1 - \Bigg(\binom{30}{24}(0.5)^{24}(1 - 0.5)^{6} + \binom{30}{25}(0.5)^{25}(1 - 0.5)^{5} + \binom{30}{26}(0.5)^{26}(1 - 0.5)^{4} \\
&\quad + \binom{30}{27}(0.5)^{27}(1 - 0.5)^{3} + \binom{30}{28}(0.5)^{28}(1 - 0.5)^{2} + \binom{30}{29}(0.5)^{29}(1 - 0.5) + \binom{30}{30}(0.5)^{30}\Bigg) \\
&= 1 - \Bigg(\frac{30!}{24!(30-24)!}(0.5)^{24}(0.5)^{6} + \frac{30!}{25!(30-25)!}(0.5)^{25}(0.5)^{5} + \frac{30!}{26!(30-26)!}(0.5)^{26}(0.9)^{4} \\
&\quad + \frac{30!}{27!(30-27)!}(0.5)^{27}(0.5)^{3} + \frac{30!}{28!(30-28)!}(0.5)^{28}(0.5)^{2} + \frac{30!}{29!(30-29)!}(0.5)^{29}(0.5) \\
&\quad + \frac{30!}{30!(30-30)!}(0.5)^{30}\Bigg) \\
&= 1 - \Big(0.0005529961 + 0.0001327191 + 0.0000255229 \\
&\quad + 0.0000037812 + 0.0000004051 + 0.0000000279 + 0.0000000009\Big) \\
&= 0.9992845
\end{aligned}
$$

Script:

```
#!/usr/bin/env Rscript

# binomial distribution
binomial <- function(k, n, p) {
    return(factorial(n)/(factorial(k) * factorial(n - k)) * p**(k) * (1 - p)**(n - k))
}

sum(binomial(0:23, 30, 0.5))
1 - sum(binomial(24:30, 30, 0.5))

# compute cumulative distribution function (CDF)
# P(X <= 23)
sum(dbinom(x = 0:23, size = 30, prob = 0.5))
1 - sum(dbinom(x = 24:30, size = 30, prob = 0.5))
pbinom(23, size = 30, prob = 0.5)
```

Output:

```
[1] 0.9992845
[1] 0.9992845
[1] 0.9992845
[1] 0.9992845
[1] 0.9992845
```



3. Find the probability that in 75 tosses of a fair coin the tail comes up between 28 and 32 times.

$$
\begin{aligned}
P(28 \le X \le 32) &= P(X=28) + P(X=29) + P(X=30) + P(X=31) + P(X=32) \\
&= \binom{75}{28}(0.5)^{28}(1-0.5)^{47} + \binom{75}{29}(0.5)^{29}(1-0.5)^{46} + \binom{75}{30}(0.5)^{30}(1-0.5)^{45} \\
&\quad + \binom{75}{31}(0.5)^{31}(1-0.5)^{44} + \binom{75}{32}(0.5)^{32}(1-0.5)^{43} \\
&= \frac{75!}{28!(75-28)!}(0.5)^{28}(0.5)^{47} + \frac{75!}{29!(75-29)!}(0.5)^{29}(0.5)^{46} + \frac{75!}{30!(75-30)!}(0.5)^{30}(0.5)^{45} \\
&\quad + \frac{75!}{31!(75-31)!}(0.5)^{31}(0.5)^{44} + \frac{75!}{32!(75-32)!}(0.5)^{32}(0.5)^{43} \\
&= 0.00832825 + 0.01349751 + 0.02069618 + 0.03004284 + 0.0413089 \\
&= 0.1138737
\end{aligned}
$$

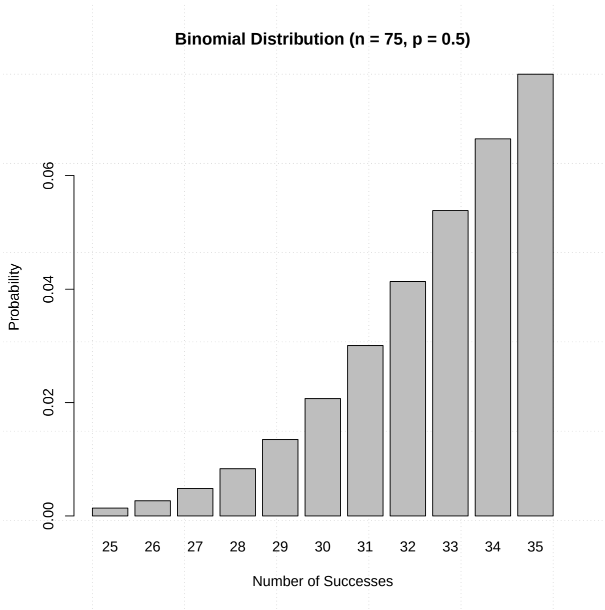
Script:

```
#!/usr/bin/env Rscript

# binomial distribution
binomial <- function(k, n, p) {
    return(factorial(n)/(factorial(k) * factorial(n - k)) * p**(k) * (1 - p)**(n - k))
}

sum(binomial(28:32, 75, 0.5))
1 - (sum(binomial(0:27, 75, 0.5)) + sum(binomial(33:75, 75, 0.5)))

# compute cumulative distribution function (CDF)
# P(28 <= X <= 32)
sum(dbinom(x = 28:32, size = 75, prob = 0.5))
1 - (sum(dbinom(0:27, 75, 0.5)) + sum(dbinom(33:75, 75, 0.5)))
pbinom(32, size = 75, prob = 0.5) - pbinom(27, size = 75, prob = 0.5)
```

Output:

```
[1] 0.1138737
[1] 0.1138737
[1] 0.1138737
```

```
[1] 0.1138737
[1] 0.1138737
```



4. How many heads will have a probability of 0.80 will come out when a coin is tossed 25 times.

   Script:

```
#!/usr/bin/env Rscript

# compute inverse cumulative distribution function (quantiles)
qbinom(p = 0.80, size = 25, prob = 0.5)
```

   Output:

```
[1] 15
```