Assignment Poisson Distribution
Date: 04/05/2020   Name: D.Saravanan

1. Find the probability that at most 5 defective fuses will be found in a box of 200 fuses if experience shows that 2 percent of such fuses are defective.

   Poisson Distribution Formula:
$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

   where,
   $k = 0, 1, 2, 3, ..., \infty$
   λ = mean number of occurrences in the interval
   e = Euler's constant ≈ 2.71828

   $N = 200$ and $p = 1/50$
   Expectation value of k, $E[k] = \lambda = 200 \times 1/50 = 4$

$$P(X \leq 5) = \sum_{k=0}^{5} \frac{\lambda^k e^{-\lambda}}{k!} = \sum_{k=0}^{5} \frac{4^k e^{-4}}{k!} = 0.7851$$

   Program:

```
#!/usr/bin/env python3
from scipy.stats import poisson

# compute cumulative distribution function (CDF)
# P(X <= 5)
pms = poisson.cdf(k=5, mu=4, loc=0)
print("The probability that at most 5 defective fuses: {:.4f}".format(pms))
```

   Output:

```
The probability that at most 5 defective fuses: 0.7851
```

2. The number of accidents in a year attributed to taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers, find approximetly the number of drivers with

   a) No accidents in a year
$$P(X = 0) = \frac{3^0 e^{-3}}{0!} = 0.0498$$

   b) More than 3 accidents in a year
$$\begin{aligned}
P(X > 3) &= 1 - P(X \leq 3) \\
&= 1 - (P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)) \\
&= 1 - \left(\frac{3^0 e^{-3}}{0!} + \frac{3^1 e^{-3}}{1!} + \frac{3^2 e^{-3}}{2!} + \frac{3^3 e^{-3}}{3!}\right) \\
&= 1 - (0.0498 + 0.1494 + 0.2240 + 0.2240) \\
&= 0.3528
\end{aligned}$$

Program:

```
#!/usr/bin/env python3
import math
import numpy as np

print("\n__________Poisson Distribution__________\n")
mu = float(input("Number lambda: "))
option = input("Type L for at least, M for at most, E for exact,\
 LT for less than, MT for more than: ")

if option == 'L':
    k = int(input("Number of at least: "))
    sum = 0
    for n in range(k):
        sum += (mu)**n*np.exp(-mu)/math.factorial(n)

    sum = 1-sum
    print("\nProbability: {:.4f}\n".format(sum))

if option == 'M':
    k = int(input("Number of at most: "))
    sum = 0
    while(k >= 0):
        sum += (mu)**k*np.exp(-mu)/math.factorial(k)
        k -= 1

    print("\nProbability: {:.4f}\n".format(sum))

if option == 'E':
    k = int(input("Number of exact: "))
    sum = (mu)**k*np.exp(-mu)/math.factorial(k)
    print("\nProbability: {:.4f}\n".format(sum))

if option == 'LT':
    k = int(input("Number of less than: "))
    sum = 0
    for n in range(k):
        sum += (mu)**n*np.exp(-mu)/math.factorial(n)

    sum = 1-sum
    sum = 1-sum
    print("\nProbability: {:.4f}\n".format(sum))

if option == 'MT':
    k = int(input("Number of more than: "))
    sum = 0
    while(k >= 0):
        sum += (mu)**k*np.exp(-mu)/math.factorial(k)
        k -= 1

    sum = 1-sum
    print("\nProbability: {:.4f}\n".format(sum))
```

Output:

```
__________Poisson Distribution__________

Number lambda: 3
Type L for at least, M for at most, E for exact, LT for less than, MT for more than: E
Number of exact: 0

Probability: 0.0498

__________Poisson Distribution__________

Number lambda: 3
Type L for at least, M for at most, E for exact, LT for less than, MT for more than: MT
Number of more than: 3

Probability: 0.3528
```

Program: Scipy package

```
#!/usr/bin/env python3
from scipy.stats import poisson

# compute probability mass function (PMF)
# P(X = 0)
pes = poisson.pmf(k=0, mu=3, loc=0)
print("The probability that no accidents in a year: {:.4f}".format(pes))

# compute survival function (SF)
# P(X > 3)
pgs = poisson.sf(k=3, mu=3, loc=0)
print("The probability that more than 3 accidents in a year: {:.4f}".format(pgs))
```

Output:

```
The probability that no accidents in a year: 0.0498
The probability that more than 3 accidents in a year: 0.3528
```

3. From the records of 10 Indian Army corps kept over 20 years the following data were obtained showing the number of deaths caused by the horse. Calculate the theoretical Poisson frequencies.

No. of Deaths:	0	1	2	3	4
Frequency :	109	65	22	3	1

$N = 109 + 65 + 22 + 3 + 1 = 200$

$\sum k \cdot F = 0 \cdot 109 + 1 \cdot 65 + 2 \cdot 22 + 3 \cdot 3 + 4 \cdot 1 = 122$

Mean, $\lambda = (\sum k \cdot F)/N = 122/200 = 0.61$

Calculation of Probability:

$$P(X = 0) = \frac{(0.61)^0 e^{-0.61}}{0!} = 0.5433$$
$$P(X = 1) = \frac{(0.61)^1 e^{-0.61}}{1!} = 0.3314$$
$$P(X = 2) = \frac{(0.61)^2 e^{-0.61}}{2!} = 0.1011$$
$$P(X = 3) = \frac{(0.61)^3 e^{-0.61}}{3!} = 0.0206$$
$$P(X = 4) = \frac{(0.61)^4 e^{-0.61}}{4!} = 0.0031$$

No. of Deaths :	0	1	2	3	4
Frequency :	109	65	22	3	1
Calculated Frequency, $N \cdot P(k)$:	108.7	66.3	20.2	4.1	0.6

Program:

```
#!/usr/bin/env python3
import math
import numpy as np

k_value = [0, 1, 2, 3, 4]
Frequency = [109, 65, 22, 3, 1]

N = np.sum(Frequency)
mu = np.sum(np.multiply(k_value, Frequency))/N

Calculated = []
for n in k_value:
    Calculated.append("%.1f" %round(N * ((mu)**(n) * np.exp(-mu)/math.factorial(n)), 1))

print("The Calculated theoretical poisson frequencies: ", Calculated)
```

Output:

```
The Calculated theoretical poisson frequencies:  ['108.7', '66.3', '20.2', '4.1', '0.6']
```

Program: Scipy package

```
#!/usr/bin/env python3
import numpy as np
from scipy.stats import poisson
import matplotlib.pyplot as plt
plt.style.use('seaborn')

k_value = [0, 1, 2, 3, 4]
Frequency = [109, 65, 22, 3, 1]

N = np.sum(Frequency)
mu = np.sum(np.multiply(k_value, Frequency))/N

Calculated = []
for n in k_value:
    Calculated.append(N * (poisson.pmf(k=n, mu=mu, loc=0)))

ax = plt.figure().add_subplot(111)
ax.plot(k_value, Frequency, '--bo', label="Observed")
ax.plot(k_value, Calculated, '--ro', label="Expected")
ax.set(xlabel='k', ylabel='Frequency', title='Calculation of Poisson frequencies')
plt.legend(); plt.savefig('frequency.pdf')
```