Assignment z-test

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1. A sample of 400 male students is found to have a mean height of 171.38 cm. Can it be reasonably regarded as a sample from a large population with mean height 171.17 cm and standard deviation 3.30 cm.

   Solution:

   Given that $n = 400$, $\bar{x} = 171.38$ cm, $\mu = 171.17$ cm, $\sigma = 3.30$ cm

   Sample proportion:

   $$p = \frac{\bar{x}}{n} = \frac{171.38}{400} = 0.42845$$

   Null hypothesis: $H_0 : \mu = 171.17$
   Alternative hypothesis: $H_1 : \mu \neq 171.17$ (two tailed test)

   z-statistic:

   $$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{171.38 - 171.17}{3.30/\sqrt{400}} = 1.27273$$

   At 5% significance level the tabulated value for z_α is 1.96

   Conclusion: $|z| < z_\alpha$, we accept the Null hypothesis. That is there is no significant difference between the sample mean and the population mean.

   Standard Error:

   $$S.E = \frac{\sigma}{\sqrt{n}} = \frac{3.30}{\sqrt{400}} = 0.165$$

   Confidence Interval: 95% fiducial limits
   The confidence interval for the population mean is given by

   $$\bar{x} \pm z_\alpha \frac{\sigma}{\sqrt{n}}$$
   $$171.38 \pm 1.96 \frac{3.30}{\sqrt{400}} = 171.38 \pm 0.3234$$
   $$171.0566 < \mu < 171.7034$$

2. A sample of 900 items has mean 3.4 and standard deviation 2.61 Can the sample be regarded as drawn from a population with mean 3.25 at 1 percent level of significance.

   Solution:

   Given that $n = 900$, $\bar{x} = 3.4$, $s = 2.61$, $\mu = 3.25$

   Sample propotion:

   $$p = \frac{\bar{x}}{n} = \frac{3.4}{900} = 0.00378$$

   Null hypothesis: $H_0 : \mu = 3.25$
   Alternative hypothesis: $H_1 : \mu \neq 3.25$ (two tailed test)

   z-statistic:

   $$z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{3.4 - 3.25}{2.61/\sqrt{900}} = 1.72414$$

   At 1% significance level the tabulated value for z_α is 2.576

   Conclusion: $|z| < z_\alpha$, we accept the Null hypothesis. The sample can be regarded as drawn from a population with mean 3.25 at 1 percent level of significance.

Standard Error:

$$S.E = \frac{s}{\sqrt{n}} = \frac{2.61}{\sqrt{900}} = 0.087$$

Confidence Interval: 99% fiducial limits
The confidence interval for the population mean is given by

$$\bar{x} \pm z_\alpha \frac{s}{\sqrt{n}}$$
$$3.4 \pm 2.576 \frac{2.61}{\sqrt{900}} = 3.4 \pm 0.2241$$
$$3.1759 < \mu < 3.6241$$