Assignment Normal Distribution

Date: 04/05/2020   Name: D.Saravanan

1. Find the area under the standard normal curve which lie

   A normal distribution in a variate X with mean μ and variance σ^2 is a statistic distribution with probability density function

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

   The standard normal distribution is given by taking $\mu = 0$ and $\sigma^2 = 1$ in a general normal distribution. An arbitrary normal distribution can be converted to a standard normal distribution by changing variables to $Z \equiv (X - \mu)/\sigma$, so $dz = dx/\sigma$, yielding

$$P(x)dx = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$$

   Normal distributions have many convenient properties, so random variates with unknown distributions are often assumed to be normal. It is often a good approximation due to the central limit theorem. This theorem states that the mean of any set of variates with any distribution having a finite mean and variance tends to the normal distribution. Many common attributes such as test scores, height, etc., follow roughly normal distributions, with few members at the high and low ends and many in the middle.

a) To the right of $Z = 2.70$

$$P(Z > 2.70) = \frac{1}{\sqrt{2\pi}} \int_{2.70}^{\infty} e^{-z^2/2} dz = 0.00347$$

Program:

```
#!/usr/bin/env python3
import numpy as np
from scipy.integrate import quad

def integrand(x):
    return 1/np.sqrt(2*np.pi) * np.exp(-x**2/2)

ans, err = quad(integrand, 2.70, np.inf)
print("The area under the standard normal curve P(Z > 2.70) is {:.5f}".format(ans))
```

Output:

```
The area under the standard normal curve P(Z > 2.70) is 0.00347
```

Program: Normal Distribution using scipy package

```
#!/usr/bin/env python3
import numpy as np
from scipy.stats import norm
import matplotlib.pyplot as plt
plt.style.use('seaborn')

# compute survival function (SF)
# P(Z > 2.70)
x = 2.70; loc = 0; scale = 1
pls = norm.sf(x, loc, scale)
print("The area under the standard normal curve P(Z > 2.70) is {:.5f}".format(pls))

x1=np.linspace(x,10,1000); y1=norm.pdf(x1,loc,scale)
x2=np.linspace(-4,4,1000); y2=norm.pdf(x2,loc,scale)
ax = plt.figure().add_subplot(111);  ax.minorticks_on()
ax.plot(x2,y2,label='P(Z > 2.70) = {:.5f}'.format(pls))
ax.fill_between(x1,y1,0,alpha=0.5,color='b')
ax.fill_between(x2,y2,0,alpha=0.1,color='b')
ax.set(xlim=[-4,4], yticklabels=[]);   ax.legend(handlelength=0)
plt.savefig('norm1a.pdf',dpi=72,bbox_inches='tight'); plt.show()
```

Output:

```
The area under the standard normal curve P(Z > 2.70) is 0.00347
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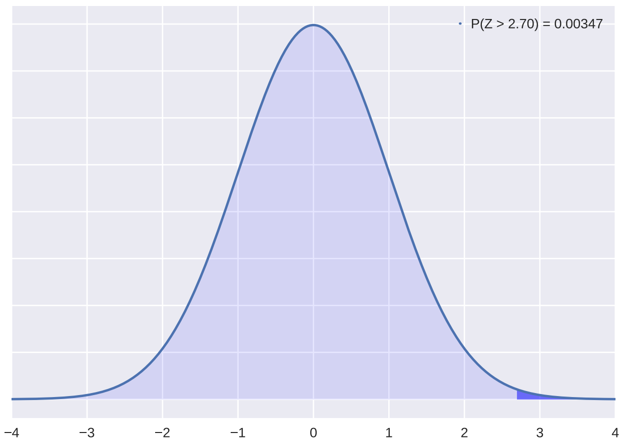

b) To the left of $Z = 1.73$

$$P(Z < 1.73) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{1.73} e^{-z^2/2} dz = 0.95818$$

Program:

```
#!/usr/bin/env python3
import numpy as np
from scipy.integrate import quad

def integrand(x):
    return 1/np.sqrt(2*np.pi) * np.exp(-x**2/2)

ans, err = quad(integrand, 2.70, np.inf)
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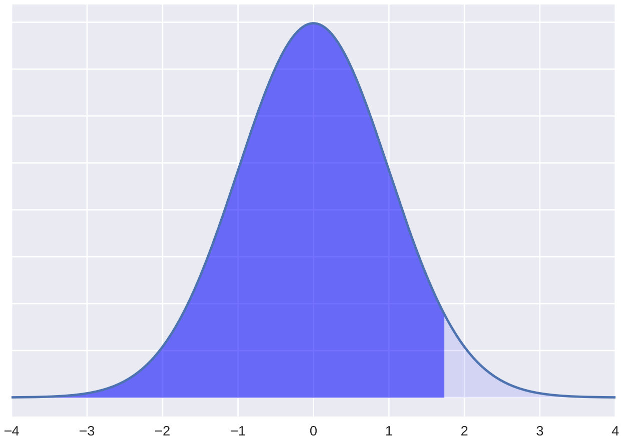

c) To the right of $Z = -0.66$

$$P(Z > -0.66) = \frac{1}{\sqrt{2\pi}} \int_{-0.66}^{\infty} e^{-z^2/2} dz = 0.74537$$

Program:

```
#!/usr/bin/env python3
import numpy as np
from scipy.integrate import quad

def integrand(x):
    return 1/np.sqrt(2*np.pi) * np.exp(-x**2/2)

ans, err = quad(integrand, 2.70, np.inf)
print("The area under the standard normal curve P(Z > 2.70) is {:.5f}".format(ans))
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Output:

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Program: Normal Distribution using scipy package

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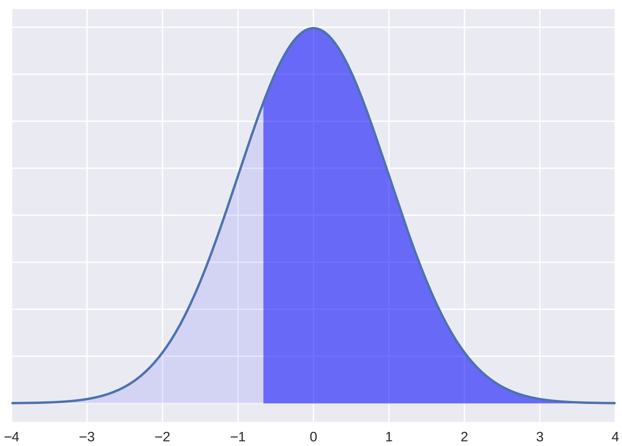

d) To the left of $Z = -1.88$

$$P(Z < -1.88) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-1.88} e^{-z^2/2} dz = 0.03005$$

Program:

```
#!/usr/bin/env python3
import numpy as np
from scipy.integrate import quad

def integrand(x):
    return 1/np.sqrt(2*np.pi) * np.exp(-x**2/2)

ans, err = quad(integrand, 2.70, np.inf)
print("The area under the standard normal curve P(Z > 2.70) is {:.5f}".format(ans))
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Output:

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The area under the standard normal curve P(Z > 2.70) is 0.00347
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Program: Normal Distribution using scipy package

```
#!/usr/bin/env python3
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import matplotlib.pyplot as plt
plt.style.use('seaborn')

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print("The area under the standard normal curve P(Z > 2.70) is {:.5f}".format(pls))

x1=np.linspace(x,10,1000); y1=norm.pdf(x1,loc,scale)
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ax = plt.figure().add_subplot(111);  ax.minorticks_on()
ax.plot(x2,y2,label='P(Z > 2.70) = {:.5f}'.format(pls))
ax.fill_between(x1,y1,0,alpha=0.5,color='b')
ax.fill_between(x2,y2,0,alpha=0.1,color='b')
ax.set(xlim=[-4,4], yticklabels=[]);   ax.legend(handlelength=0)
plt.savefig('norm1a.pdf',dpi=72,bbox_inches='tight'); plt.show()
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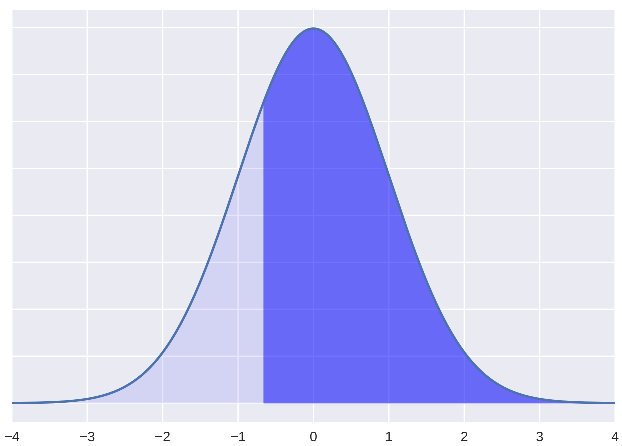

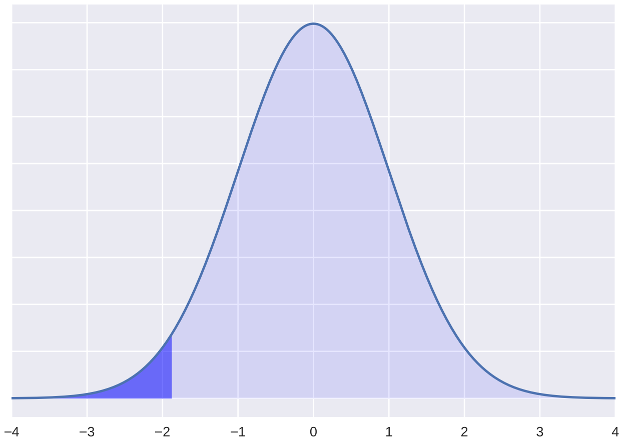

e) Between $Z = -0.90$ and $Z = -1.85$

$$P(-1.85 < Z < -0.90) = \frac{1}{\sqrt{2\pi}} \int_{-1.85}^{-0.90} e^{-z^2/2} dz$$

Program:

```
#!/usr/bin/env python3
import numpy as np
from scipy.integrate import quad

def integrand(x):
    return 1/np.sqrt(2*np.pi) * np.exp(-x**2/2)

ans, err = quad(integrand, 2.70, np.inf)
print("The area under the standard normal curve P(Z > 2.70) is {:.5f}".format(ans))
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Output:

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Program: Normal Distribution using scipy package

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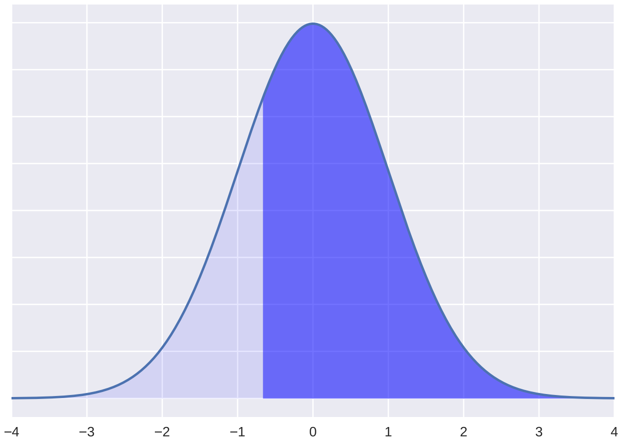

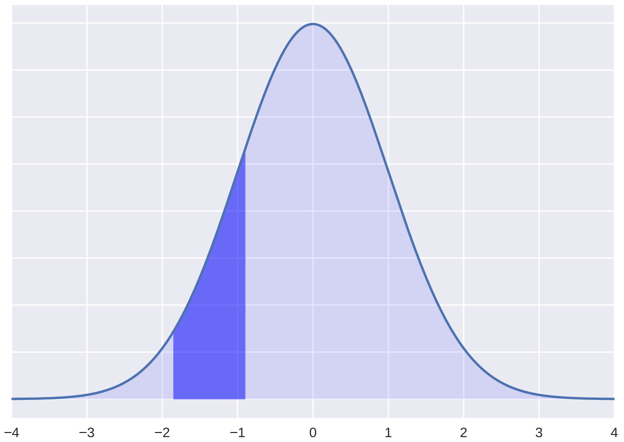

f) Between $Z = -1.45$ and $Z = 1.45$

$$P(-1.45 < Z < 1.45) = \frac{1}{\sqrt{2\pi}} \int_{-1.45}^{1.45} e^{-z^2/2} dz$$

Program:

```
#!/usr/bin/env python3
import numpy as np
from scipy.integrate import quad

def integrand(x):
    return 1/np.sqrt(2*np.pi) * np.exp(-x**2/2)

ans, err = quad(integrand, 2.70, np.inf)
print("The area under the standard normal curve P(Z > 2.70) is {:.5f}".format(ans))
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Output:

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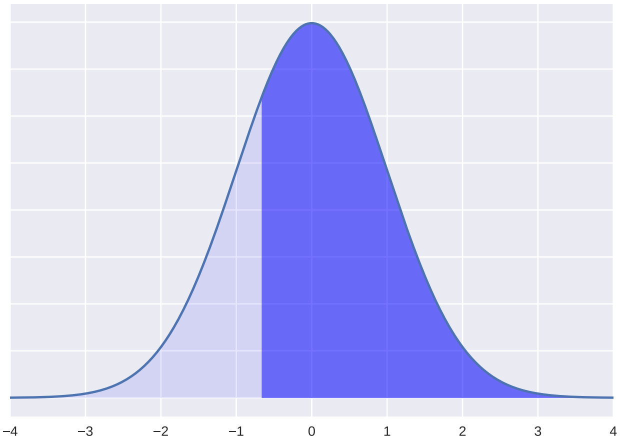

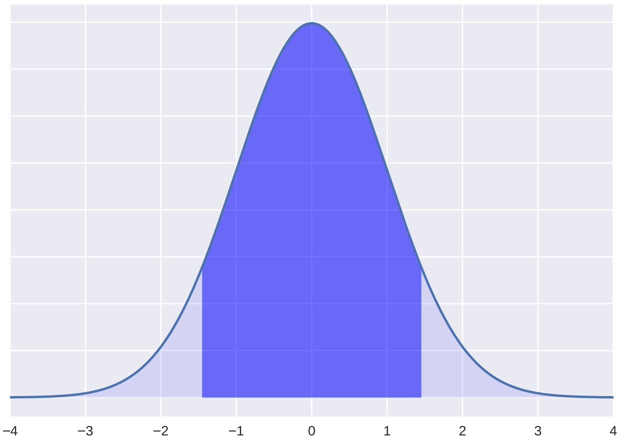

g) Between $Z = -0.90$ and $Z = 1.58$

$$P(-0.90 < Z < 1.58) = \frac{1}{\sqrt{2\pi}} \int_{-0.90}^{1.58} e^{-z^2/2} dz$$

Program:

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def integrand(x):
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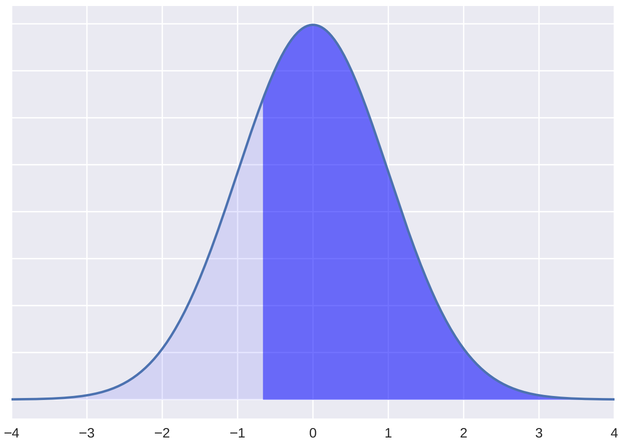
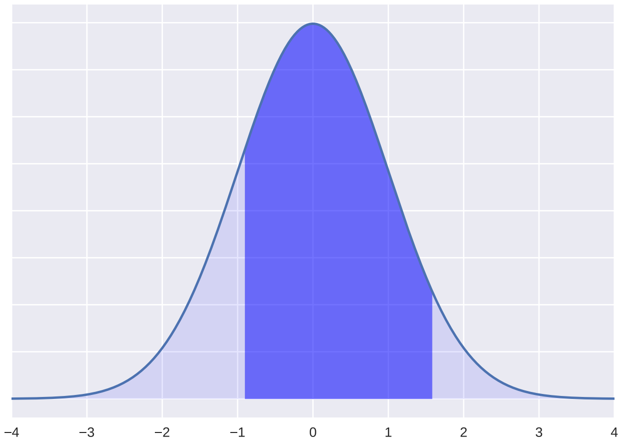


2. The life of a certain kind of electronic device has a mean of 300 hours and a standard deviation of 25 hours. Assuming that the distribution of life times which are measured to the nearest hours can be approximated closely with a normal curve.

   a) Find the probability that any one of these devices will have a lifetime of more than 350 hours.

   b) What percentage will have life time from 220 to 260 hours?



3. The customer accounts of a certain departmental store have an average balance of Rs.120 and standard deviation of Rs.40 Assuming that the account balances are normally distributed, find

    a) What proportion of accounts is over Rs.150?

    b) What proportion of accounts is between Rs.100 and Rs.150?

    c) What proportion of accounts is between Rs.60 and Rs.90?