Assignment Binomial Distribution

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1. For a Binomial Distribution parameter $n = 5$ and $p = 0.3$ Find the probabilities of getting

   a) At least 3 successes
   The probability that a random variable X with binomial distribution $B(n, p)$ is equal to the value k, where $k = 0, 1, ...n$, is given by

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

$$\begin{aligned}
P(X \geq 3) &= P(X = 3) + P(X = 4) + P(X = 5) \\
&= \binom{5}{3}(0.3)^3(1-0.3)^2 + \binom{5}{4}(0.3)^4(1-0.3) + \binom{5}{5}(0.3)^5 \\
&= \frac{5!}{3!(5-3)!}(0.3)^3(0.7)^2 + \frac{5!}{4!(5-4)!}(0.3)^4(0.7) + \frac{5!}{5!(5-5)!}(0.3)^5 \\
&= 0.1323 + 0.0283 + 0.0024 \\
&= 0.1630
\end{aligned}$$

   b) At most 3 successes

   Method 1:

$$\begin{aligned}
P(X \leq 3) &= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) \\
&= \binom{5}{0}(1-0.3)^5 + \binom{5}{1}(0.3)(1-0.3)^4 + \binom{5}{2}(0.3)^2(1-0.3)^3 + \binom{5}{3}(0.3)^3(1-0.3)^2 \\
&= \frac{5!}{0!(5-0)!}(0.7)^5 + \frac{5!}{1!(5-1)!}(0.3)(0.7)^4 + \frac{5!}{2!(5-2)!}(0.3)^2(0.7)^3 + \frac{5!}{3!(5-3)!}(0.3)^3(0.7)^2 \\
&= 0.1681 + 0.3601 + 0.3087 + 0.1323 \\
&= 0.9692
\end{aligned}$$

   Method 2:

$$\begin{aligned}
P(X \leq 3) &= 1 - P(X > 3) \\
&= 1 - (P(X = 4) + P(X = 5)) \\
&= 1 - \left(\binom{5}{4}(0.3)^4(1-0.3) + \binom{5}{5}(0.3)^5\right) \\
&= 1 - \left(\frac{5!}{4!(5-4)!}(0.3)^4(0.7) + \frac{5!}{5!(5-5)!}(0.3)^5\right) \\
&= 1 - (0.0283 + 0.0024) \\
&= 0.9692
\end{aligned}$$

c) Exactly 3 failures

$$
\begin{aligned}
P(X=2) &= \binom{5}{2}(0.3)^2(1-0.3)^3 \\
&= \frac{5!}{2!(5-2)!}(0.3)^2(0.7)^3 \\
&= 0.3087
\end{aligned}
$$

Program:

```
#!/usr/bin/env python3

def fact(n):
    fact = 1
    for value in range(1, n+1):
        fact *= value
    return(fact)

print("\n__________Binomial Distribution__________\n")

n = int(input("Number of trials: "))
p = float(input("Probability of getting a success in one trial: "))
option1 = input("Type L for at least, M for at most, E for exact: ")
option2 = input("Type S for success, F for failure: ")

if option2 == 'S':
    x = int(input("Number of success desired: "))
    chosen = 'success'
    k = x

else:
    x = int(input("Number of failure desired: "))
    chosen = 'failure'
    k = n-x

if option1 == 'L':
    sum = 0
    while(n >= k):
        sum += fact(n)/(fact(n-k)*fact(k)) * (p)**(k) * (1-p)**(n-k)
        k += 1

    print("\nThe probability of getting at least {} {}: {:.4f}\n".format(x, chosen, sum))

if option1 == 'M':
    sum = 0
    while(k >= 0):
        sum += fact(n)/(fact(n-k)*fact(k)) * (p)**(k) * (1-p)**(n-k)
        k -= 1

    print("\nThe probability of getting at most {} {}: {:.4f}\n".format(x, chosen, sum))

if option1 == 'E':
    sum = fact(n)/(fact(n-k)*fact(k)) * (p)**(k) * (1-p)**(n-k)
    print("\nThe probability of getting exactly {} {}: {:.4f}\n".format(x, chosen, sum))
```

Output:

```
__________Binomial Distribution__________

Number of trials: 5
Probability of getting a success in one trial: 0.3
Type L for at least, M for at most, E for exact: L
Type S for success, F for failure: S
Number of success desired: 3

The probability of getting at least 3 success: 0.1631



__________Binomial Distribution__________

Number of trials: 5
Probability of getting a success in one trial: 0.3
Type L for at least, M for at most, E for exact: M
Type S for success, F for failure: S
Number of success desired: 3

The probability of getting at most 3 success: 0.9692



__________Binomial Distribution__________

Number of trials: 5
Probability of getting a success in one trial: 0.3
Type L for at least, M for at most, E for exact: E
Type S for success, F for failure: F
Number of failure desired: 3

The probability of getting exactly 3 failure: 0.3087
```

Program: Binomial Distribution using Scipy package

```python
#!/usr/bin/env python3
from scipy.stats import binom

# compute survival function (SF)
# P(X >= 3)
pls = binom.sf(k=3, n=5, p=0.3, loc=1)
print("The probability of getting at least 3 success: {:.4f}".format(pls))

# compute cumulative density function (CDF)
# P(X <= 3)
pms = binom.cdf(k=3, n=5, p=0.3, loc=0)
print("The probability of getting at most 3 success: {:.4f}".format(pms))

# compute probability mass function (PMF)
# P(X = 2)
pef = binom.pmf(k=2, n=5, p=0.3, loc=0)
print("The probability of getting exactly 3 failure: {:.4f}".format(pef))
```

Output:

```
The probability of getting at least 3 success: 0.1631
The probability of getting at most 3 success: 0.9692
The probability of getting exactly 3 failure: 0.3087
```

2. If on an average one vessel in every ten is wrecked, find the probability that out of five vessels expected to arrive, four at least will arrive safely.

   The probability of success, $p = 0.9$ and the number of observations, $n = 5$.

$$
\begin{aligned}
P(X \geq 4) &= P(X = 4) + P(X = 5) \\
&= \binom{5}{4}(0.9)^4(1 - 0.9) + \binom{5}{5}(0.9)^5 \\
&= \frac{5!}{4!(5-4)!}(0.9)^4(0.1) + \frac{5!}{5!(5-5)!}(0.9)^5 \\
&= 0.3280 + 0.5905 \\
&= 0.9185
\end{aligned}
$$

   Program:

```
#!/usr/bin/env python3
from scipy.stats import binom

# probability of success, p = 0.9 and number of observations, n = 5
# P(X >= 4)
pls = binom.sf(k=4, n=5, p=0.9, loc=1)
print("The probability of getting at least 4 success is {:.4f}".format(pls))
```

   Output:

```
The probability of getting at least 4 success is 0.9185
```

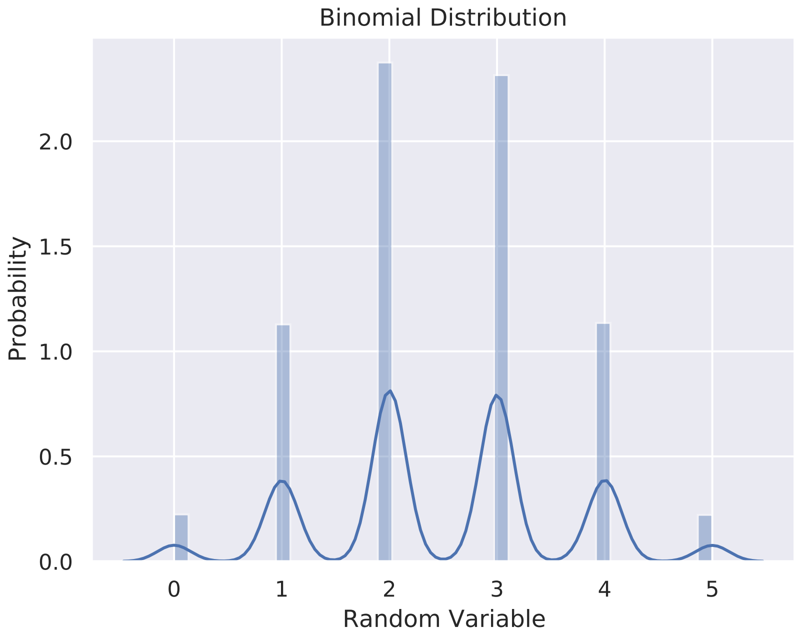
3. Five coins are tossed 3,200 times.

   Program:

```
#!/usr/bin/env python3

import numpy as np
import pandas as pd
from scipy.stats import binom
import matplotlib.pyplot as plt
import seaborn as sns
sns.set()

data = binom.rvs(n=5, p=0.5, size=3200)
heads, freq = np.unique(data, return_counts=True)

dframe = pd.DataFrame({'Heads':heads, 'Frequencies':freq})
print("Frequencies of distribution of heads: \n{}".format(dframe))

print("Mean: {:.3f}".format(data.mean()))
print("Standard Deviation: {:.3f}".format(data.std()))

plt.figure(dpi=120)
sns.distplot(data)
plt.xlabel("Random Variable")
plt.ylabel("Probability")
plt.title("Binomial Distribution")
plt.savefig("binormdistplot.pdf")
plt.show()
```

a) Find the Frequencies of the distribution of heads and tabulate the results.

```
Frequencies of distribution of heads:
   Heads  Frequencies
      0           97
      1          488
      2         1027
      3         1001
      4          491
      5           96
```



b) Calculate the mean number of sucess and standard deviation.

Mean, $\mu_X = np = 5 \times 0.5 = 2.5$
Variance, $\sigma_X^2 = np(1-p) = 5 \times 0.5 \times (1 - 0.5) = 1.25$
Standard Deviation, $\sigma_X = \sqrt{np(1-p)} = \sqrt{1.25} = 1.12$

Output:

```
Mean: 2.496
Standard Deviation: 1.106
```