BINOMIAL DISTRIBUTION

The binomial distribution is one of the widely used probability distributions of discrete random variable. It describes discrete, non-continuous data resulting from an experiment that is known as Bernoulli Process (named after Jacob Bernoulli, a Swiss Mathematician of the 17th Century).

The problems relating to tossing of a coin or throwing of dice or drawing cards from a pack of cards with replacement lead to binomial probability distribution.

Characteristics of Bernoulli Process

1. Each trial will have only two possible outcomes : (Success or Failure).
2. The probability of the outcome of any trial remains constant over time.
3. The outcome of one trial cannot influence the outcome of any other trial and each trial is statistically independent.

Example of Bernoulli process : tossing of a fair coin a fixed number of times.

The probability of "r" success in "n" trails is given by :

$P(X = r) = \frac{n!}{r!\,(n-r)!} p^r q^{n-r}$ where "p" is the probability of success in each trial and "q" is the probability of failure in each trial, with $p + q = 1$

Mean $E(X) = np$ and $V(X) = npq$

Problem : 1

A fair coin is tossed ten times. If getting head is defined as success, find out the probability of getting 4 success in the ten trials.

Solution :

Here p = probability of getting head in one trial = 0.5

$q = 1 - p = 0.5$

$n = 10$

$r = 4$

Required Probability : $= P(X = r) = \frac{n!}{r!\,(n-r)!} p^r q^{n-r}$

$$P(X = 4) = \frac{10!}{4!\,(10-4)!} (0.5)^4 (0.5)^{10-4} = 0.2051$$

Problem : 2

In a pharmacy firm, there are 5 workers who often come late. The owner has studied the situation over a period of time and determined that there is 0.4 chance of any employee being late and that they arrive independently of one and another. Find the probability that (i) 2 workers come late and (ii) 4 workers come late.

Solution :

Here $n = 5$

$p = 0.4$ and $q = 1 - p = 0.6$

i) $r = 2$

$$P(X = 2) = \frac{5!}{2!\,(5-2)!}(0.4)^2\,(0.6)^{5-2} = 0.3456$$

ii) $$P(X = 4) = \frac{5!}{4!\,(5-4)!}(0.4)^4\,(0.6)^{5-4} = 0.0768$$

Problem : 4

Find the binomial distribution if its mean is 48 and standard deviation is 4.

Solution :

Given Mean = 48

$\therefore np = 48$ ...

SD = 4

$\therefore \sqrt{npq} = 4$

$npq = 16$ ... (

Dividing (1) by (2) we get :

$$\frac{np}{npq} = \frac{48}{4}$$

$$\therefore \quad q = \frac{1}{3} \text{ and hence } p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$$

Now $np = 48 \therefore n = 72$

Problem : 5

A fair die is tossed 7 times. We say that a toss is a success if a 5 or 6 appears; otherwise it's a failure. What is the distribution of the random variable X representing the number of successes out of the 7 tosses? What is the probability that there are exactly 3 successes? What is the probability that there are no successes?

Solution :

Here $p = \frac{1}{3}$ {when a die is thrown the possibilities are {1, 2, 3, 4, 5, 6}, since success is defined as occurring of 5 or 6, there are two chances favourable to it, therefore $p = 2/6 = 1/3$}

$$q = \frac{2}{3} \quad n = 7$$

$$P(X = 3) = \frac{7!}{3!(7-3)!}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^{7-3} = \frac{560}{2187}$$

$$P(X = 0) = \frac{7!}{0!\,(7-0)!}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^{7-0} = \frac{128}{2187}$$

Problem 6 :

The probability of a defective bolt is 0.2. Find the mean and SD for the distribution of defective bolt in a total of 1000.

Solution :

Given : $p = 0.2$, $q = 1 - p = 0.8$, $n = 1000$

Mean $= np = 1000 \times 0.2 = 200$

$$SD = \sqrt{npq} = \sqrt{1000 \times 0.2 \times 0.8} = 12.6$$