UNIT – II

Chapter – 4

PROBABILITY

1. **Experiment :** A process that results in two or more outcomes is called as an experiment e.g. Tossing of a fair coin.
2. **Random experiment :** Any act that may be repeated under similar conditions resulting in a trial which yields an outcome.
3. **Outcome :** The result of a random experiment.
4. **Event :** One or more possible outcomes of an experiment.
5. **Compound event :** If 2 or more events occur in relation with on other, then their simultaneous occurrences is called a compound event.
6. **Mutually exclusive events :** If happening of an event precludes the happening of all other events.
7. **Sample space :** The set of all possible outcomes of a random process or experiment. Note that event is a subset of sample space.
8. **Independent events :** Two events are said to be independent if occurrence or non-occurrence of one does not affect the probability of the other.
9. **Dependent events :** Two events are said to be dependent if occurrence of one or non-occurrence of one affects the probability of occurrence of the other.
10. **Exhaustive events :** The total number of possible events.
11. **Equally likely events :** When an event does not occur more often than others it is said to be equally likely.
12. **Complementary event :** If A is an event defined over sample space 's', then event that A does not occur is called complementary event.

Equally Likely Probability Formula

If S is a finite sample space in which all outcomes are equally likely and E is a event in S ($E \subseteq S$), then the probability of E is denoted by P(E) and is given by,

$P(E) = \frac{m}{n}$, m = no. of outcomes favourable to E

n = total no. of possible outcomes = $n(S)$

Now : $0 \le P(E) \le 1$, $P(S) = 1$.

NUMERICALS

1. **A fair die is thrown. Find the probability that :**
   a) number 5 appears.
   b) odd number appears.
   c) multiple of 3 appears.

Solution :

$S = \{1, 2, 3, 4, 5, 6\}$

Let A, B, C be the events that number 5 appears, odd number appears and multiple of 3 appears respectively.

a) $A = \{5\}$ $\therefore$ $n(A) = 1 = m$

$n(S) = 6 = n$

$\therefore \quad P(A) = \frac{m}{n} = \frac{1}{6}$

b) $B = \{1, 3, 5\}$ $\therefore$ $n(B) = 3 = m$

$\therefore \quad P(B) = \frac{m}{n} = \frac{3}{6} = \frac{1}{2}$

c) $C = \{3, 6\}$ $\therefore$ $n(C) = 2 = m$

$\therefore \quad P(C) = \frac{m}{n} = \frac{2}{6} = \frac{1}{2}$

3. Write the sample space in case of following :

a) Two unbiased coins are tossed simultaneously.

b) Three unbiased coins are tossed simultaneously.

c) A die and a coin are tossed simultaneously.

d) One unbiased coin is tossed three times.

Solution :

a) S = {HH, HT, TH, TT}

b) S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

c) S = {(1, H), (2,H), (3, H), (4, H), (5, H), (6, H), (1, T), (2, T), (3, T), (4, T), (5, T), (6, T)}

d) S = {(H, H, H), (H, H, T), (H, T, H), (T, H, H), (H, T, T), (T, H, T), (T, T, H), (T, T, T)}

Note in Problem (d) (H, H, H) $\Rightarrow$ (Head in 1^{st} throw, Head in 2^{nd} throw, Head in 3^{rd} throw)

Note : atleast means minimum, "at the most" $\rightarrow$ maximum.

4. An unbaised coin in tossed 3 times.

Find the probability (a) of the event that exactly one toss results in head, (b) of te event that atleast two tosses result in head, (c) of the event that no head is obtained.

Solution :

Sample space S = {(H, H, H), (H, H, T), (H, T, H), (T, H, H), (H, T, T), (T, H, T), (T, T, H), (T, T, T)}

$\therefore$ $n(S) = 8$

a) Let A be event that exactly one toss results in a head.

A = {(H, T, T), (T, H, T), (T, T, H)}

$\therefore$ $n(A) = 3$

$$\therefore \; P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

b) Let B be the even that at least two tosses result in head.

B = {H, H, H), (H, H, T), (H, T, H), (T, H, H)}

$\therefore$ $n(B) = 4$ $\qquad$ $\therefore$ $P(B) = \frac{4}{8} = \frac{1}{2}$

c) Let C be the event that no head is obtained.

$\therefore$ C = {(T, T, T)} $\qquad$ $\therefore$ $n(C) = 1$ $\qquad$ $\therefore$ $P(C) = \frac{1}{8}$