3.4 Total Numerical Errors

- The total numerical error is the sum of the truncation and round-off errors. In general, the only way to reduce round off errors is to increase the number of significant figures of the computer.
- In addition, we noted that the round-off error would increase due to subtractive cancellation or due to an increase in the number of counts in an analysis.
- The truncation error is reduced by decreasing the step size. Since the decrease in step size can cancel the subtraction or increase the count, the truncation errors decrease when the round-off errors increase.
- Therefore, we are faced with the following dilemma. The strategy of reducing one component of the total error leads to or increases of the other component.

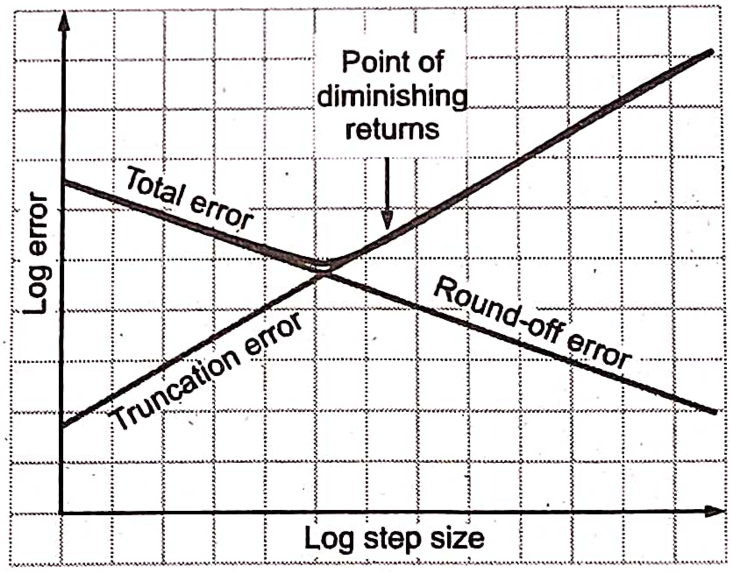

Fig. 3.4.1

- A graphical depiction of the trade-off between round-off and truncation error that sometimes comes into play in the course of a numerical method.
- The point of diminishing returns is shown, where the round-off error begins to negate the benefits of step- size reduction.
- One component of the total error occurs to an increase of the other component.

- In a computation, we can reduce the step size to reduce truncation errors only to discover that in doing so, the round-off error begins to dominate the solution and the total error increases.
- Thus, our solution becomes our problem (Fig. 3.4.1). One challenge we face is to determine an appropriate step size for a particular calculation.
- We would like to choose a larger step size in order to reduce the amount of computations and round-off errors without penalty of a large truncation error.
- If the total error is shown in Fig. 3.4.1, the challenge is to identify the point of diminishing returns, where round-off error begins to negate the benefits of step-size reduction.
- In real cases, however, such situations are relatively uncommon because most computers carry sufficiently significant figures that round-off errors do not predominate.
- Nevertheless, they sometimes do occur and suggest a type of "numerical uncertainty principle".
- There is a complete limit on the accuracy that can be achieved using some computerized numerical methods. We explore such a case in the following section.

Syllabus Topic : Formulation Errors and Data Uncertainty

3.5 Formulation Errors and Data Uncertainty

- Although the following sources of error are not directly connected with most of the numerical methods in this book, they can sometimes have great impact on the success of a modelling effort.
- Thus, they must always be kept in mind when applying numerical techniques in the context of real-world problems.

3.5.1 Blunders

- We are all familiar with gross errors, or blunder. In the early years of computers, erroneous numerical results could sometimes be attributed to malfunctions of the computer itself.
- Today, this source of error is highly unlikely, and most blunders must be attributed to human imperfections. Blunders can occur at any stage of the mathematical modelling process and can contribute to all the other components of error.
- They can be avoided only by sound knowledge of fundamental principles and by the care with which you approach and design your solution to a problem.
- Blunders are usually disregarded in discussions of numerical methods.
- This is no doubt due to the fact that, try as we may, mistakes are to a certain extent unavoidable. However, we believe that there are a number of ways in which their occurrence can be minimized.
- In addition, there are usually simple ways to check whether a particular numerical method is working properly.

3.5.2 Formulation Errors

- Formulation or model, errors relate to bias that can be ascribed to incomplete mathematical models. An example of a negligible formulation error is the fact that Newton's second law does not account for relativistic effects.
- This does not detract from the adequacy of the solution because these errors are minimal on the time and space seals associated with the falling parachutist problem.
- However, suppose that the air resistance is not linearly proportional to fall velocity but is a function of square velocity.

- It this were the case, booth analytical and numerical solution obtained would be erroneous because of the formulation error.
- Further considerations of formulations error is induced in some of the engineering applications.

3.5.3 Data Uncertainty

- Errors sometimes enter into an analysis because of uncertainty in the physical data upon which a model is based.
- For instance, suppose we wanted to test the falling parachutist model by having an individual make repeated jumps and then measuring his or her velocity after a specified time interval.
- Uncertainty would undoubtedly be associated with these measurements, since the parachutist would fall faster during some jumps than during others.
- These errors can exhibit both inaccuracy and imprecision. If our equipment consistently underestimates the velocity, we are dealing with an inaccurate or biased, instrument.
- On the other hand, if the measurements are randomly high and low, then we are dealing with the question of accuracy.
- Measurement errors can be determined by summing data with one or more streamlined statistics that give as much information as possible about the specific characteristics of the data.