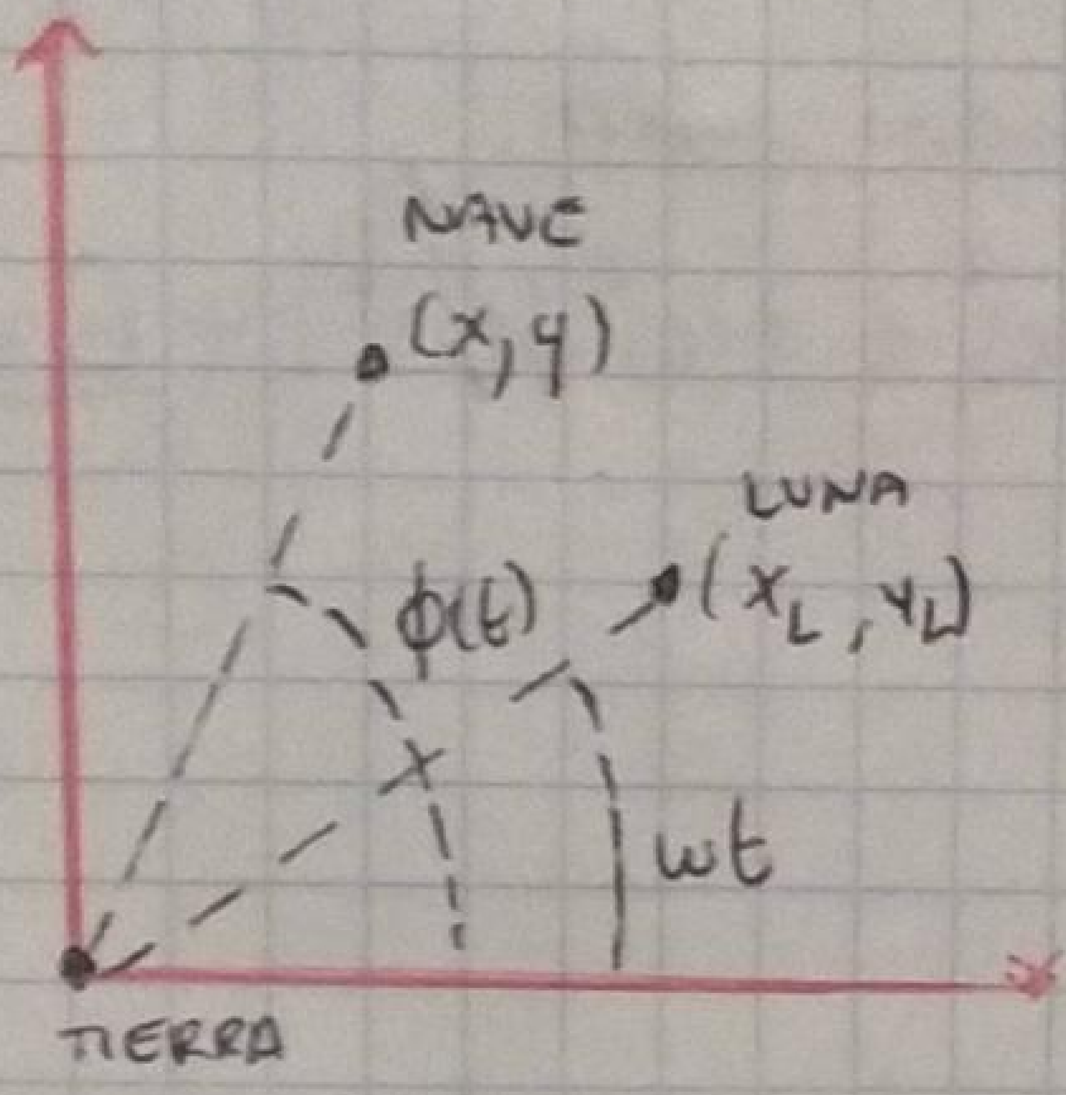

d) Usando esta distancia, muestre que el Hamiltoniano de la nave está dado por:

$$H = p_r \dot{r} + p_\phi \dot{\phi} - L = \frac{p_r^2}{2m} + \frac{p_\phi^2}{2mr^2} - G\frac{mm_T}{r} - G\frac{mm_L}{r_L(r,\phi,t)}$$

donde L es la energía cinética menos la potencial de la nave en coordenadas polares

distancia nave-luna

$$r_L(r,\phi,t) = \sqrt{r(t)^2 + d^2 - 2r(t)d\cos(\phi - \omega t)}$$

$x = r(t)\cos\phi(t)$ | $\dot{x} = \dot{r}(t)\cos\phi(t) - r(t)\operatorname{sen}\phi(t)\dot{\phi}(t)$

$y = r(t)\operatorname{sen}\phi(t)$ | $\dot{y} = r(t)\cos\phi(t)\dot{\phi}(t) + \dot{r}(t)\operatorname{sen}\phi(t)$

Ahora, definimos T y U:

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) = \frac{1}{2}m\Big(\dot{r}(t)^2\cos^2\phi(t) - \cancel{2\dot{r}(t)\cos\phi(t)r(t)}\operatorname{sen}\phi(t)\dot{\phi}(t) + r(t)^2\operatorname{sen}^2\phi(t)\dot{\phi}(t) + r(t)^2\cos^2\phi(t)\dot{\phi}(t)^2 + \cancel{2r(t)\dot{r}(t)}\cos(t)\dot{\phi}(t)\operatorname{sen}\phi(t) + \dot{r}(t)^2\operatorname{sen}^2\phi(t)\Big)$$

$$T = \frac{1}{2}m\Big(\dot{r}(t)^2\cos^2\phi(t) + r(t)^2\operatorname{sen}^2\phi(t)\dot{\phi}(t) + r(t)^2\cos^2\phi(t)\dot{\phi}(t)^2 + \dot{r}(t)^2\operatorname{sen}^2\phi(t)\Big)$$

$$= \frac{1}{2}m\Big(\dot{r}(t)^2\cancelto{1}{(\cos^2\phi(t) + \operatorname{sen}^2\phi(t))} + r(t)^2\dot{\phi}(t)^2\cancelto{1}{(\cos^2\phi(t) + \operatorname{sen}^2\phi(t))}\Big)$$

$$T = \frac{1}{2}m(\dot{r}(t)^2 + r(t)^2\dot{\phi}(t)^2) \;//$$

$$U = \left(-G\frac{Mm}{r}\right) = -\frac{Gmm_T}{r} - \frac{Gmm_L}{r_L(r,\phi,t)} \;//$$

entonces nuestro lagrangiano sería

$$L = T - U = \frac{1}{2} m\left(\dot{r}(t)^2 + r(t)^2 \dot{\phi}(t)^2\right) + \frac{6\, m m_T}{r} + \frac{6\, m m_L}{r_L(r,\phi,t)}$$

Hallamos P_r y P_ϕ:

$$P_r = \frac{\partial L}{\partial \dot{r}} = m\, \dot{r}(t)$$

$$P_\phi = \frac{\partial L}{\partial \dot{\phi}} = m\, r(t)^2 \dot{\phi}(t)$$

Nuestro hamiltoniano quedaría,

$$H = P_r \dot{r} + P_\phi \dot{\phi} - L$$

$$= m\dot{r}^2 + m r^2 \dot{\phi}^2 - \frac{1}{2} m(\dot{r}^2 + r^2\dot{\phi}^2) - \frac{6\, m m_T}{r} - \frac{6\, m m_L}{r_L(r,\phi,t)}$$

$$= \frac{m\dot{r}^2}{2} + \frac{m r^2 \dot{\phi}^2}{2} - \frac{6\, m m_T}{r} - \frac{6\, m m_L}{r_L(r,\phi,t)}$$

$$H = \frac{P_r^2}{2m} + \frac{P_\phi^2}{2mr^2} - \frac{6\, m m_T}{r} - \frac{6\, m m_L}{r_L(r,\phi,t)} \quad //$$

e) Muestre que las ecuaciones de Hamilton, que son las ecuaciones de movimiento estás dadas por:

$$\dot{r} = \frac{\partial H}{\partial P_r} = \frac{P_r}{m}$$

$$\dot{\phi} = \frac{\partial H}{\partial P_\phi} = \frac{P_\phi}{mr^2}$$

$$\dot{P}_r = -\frac{\partial H}{\partial r} = \frac{P_\phi^2}{mr^3} - \frac{6\, m m_T}{r^2} - \frac{6\, m m_L}{r_L(r,\phi,t)^3}\left[r - d\cos(\phi - \omega t)\right]$$

$$\dot{P}_\phi = -\frac{\partial H}{\partial \phi} = -\frac{6\, m m_L}{r_L(r,\phi,t)^3}\, r d \sin(\phi - \omega t)$$

$$\dot{r} = \frac{\partial}{\partial p_r}\left(\frac{p_r^2}{2m} + \frac{p_\phi^2}{2mr^2} - \frac{Gmm_T}{r} - \frac{Gmm_L}{r_L(r,\phi,t)}\right)$$

$$= \frac{\cancel{2}p_r}{\cancel{2}m} = \frac{p_r}{m} \; //$$

$$\dot{\phi} = \frac{\partial}{\partial p_\phi}\left(\frac{p_r^2}{2m} + \frac{p_\phi^2}{2mr^2} - \frac{Gmm_T}{r} - \frac{Gmm_L}{r_L(r,\phi,t)}\right)$$

$$= \frac{\cancel{2}p_\phi}{\cancel{2}mr^2} = \frac{p_\phi}{mr^2} \; //$$

$$\dot{p}_r = -\frac{\partial}{\partial r}\left(\frac{p_r^2}{2m} + \frac{p_\phi^2}{2mr^2} - \frac{Gmm_T}{r} - \frac{Gmm_L}{r_L(r,\phi,t)}\right)$$

$$= -\left[-\frac{2p_\phi^2 r^{-3}}{2m} + Gmm_T r^{-2} - Gmm_L\frac{\partial}{\partial r}\left(\frac{1}{\sqrt{r(t)^2 + d^2 - 2r(t)d\cos(\phi-\omega t)}}\right)\right]$$

$$= -\left[-\frac{p_\phi^2}{mr^3} + \frac{Gmm_T}{r^2} - Gmm_L\frac{\partial}{\partial r}\left((r(t)^2 + d^2 - 2r(t)d\cos(\phi-\omega t))^{-\frac{1}{2}}\right)\right]$$

Resolvemos la derivada:

$$\frac{\partial}{\partial r}\left(r(t)^2 + d^2 - 2r(t)d\cos(\phi-\omega t)\right)^{-1/2} = -\frac{1}{2}\left(r(t)^2 + d^2 - 2r(t)d\cos(\phi-\omega t)\right)^{-\frac{3}{2}}$$

$$\cdot\, 2r(t) - 2d\cos(\phi-\omega t))$$

$$= -\frac{1}{\cancel{2}}\frac{(\cancel{2}r(t) - \cancel{2}d\cos(\phi-\omega t))}{\left((r(t)^2 + d^2 - 2r(t)d\cos(\phi-\omega t))^{\frac{1}{2}}\right)^3}$$

$$= -\frac{(r(t) - d\cos(\phi-\omega t))}{r_L(r,\phi,t)^3}$$

$$\dot{p}_r = \frac{p_\phi^2}{mr^3} - \frac{Gmm_T}{r^2} - \frac{Gmm_L}{r_L(r,\phi,t)^3}\left[r - d\cos(\phi-\omega t)\right]$$

//

$$\dot{P}_\phi = -\frac{\partial H}{\partial \phi} = -\frac{\partial}{\partial \phi}\left(\frac{P_r^2}{2m} + \frac{P_\phi^2}{2mr^2} - \frac{GmmT}{r} - \frac{Gmm_L}{r_L(r,\phi,t)}\right)$$

$$= Gmm_L \cdot \frac{\partial}{\partial \phi} r_L(r,\phi,t)^{-1}$$

$$= Gmm_L \cdot \frac{\partial}{\partial \phi}\left(r(t)^2 + d^2 - 2r(t)d\cos(\phi - wt)\right)^{-1/2}$$

$$= Gmm_L \cdot \left(-\frac{1}{2}\right)\left(r(t)^2 + d^2 - 2r(t)d\cos(\phi - wt)\right)^{-\frac{3}{2}} \cdot$$

$$(+\ 2r(t)d\ \mathrm{sen}(\phi - wt))$$

$$= -Gmm_L \frac{\cancel{2}r(t)d\ \mathrm{sen}(\phi - wt)}{\cancel{2}\ r_L(r,\phi,t)^3}$$

$$\dot{P}_\phi = \frac{-Gmm_L\ rd\ \mathrm{sen}(\phi - wt)}{r_L(r,\phi,t)^3}$$

//