DD | MM | AA

Ecuación diferencial no lineal.

1. $\frac{dv}{dt} = v^{q}$

Solución

Si $q = 1$

$\frac{dv}{dt} = v \rightarrow \int \frac{dv}{v} = \int dt \rightarrow \log(v) + C_1 = t + C_2$

$\therefore \ln(v) = t + c \longrightarrow v(t) = A e^{t}$

Si $q < 1$

$\frac{dv}{dt} = v^{q} \longrightarrow \int \frac{dv}{v^{q}} = \int dt \longrightarrow \int \frac{dv}{v^{q}} = t + C_1$

$\int \frac{dv}{v^{q}} \leftarrow \int x^{n} dx = \frac{x^{n+1}}{n+1}$ Si $n = -q$

$\downarrow$

$\frac{v^{1-q}}{1-q} + C_2 = t + C_1 \longrightarrow v^{1-q} = (1-q)(t + c)$

$\therefore v = \left((1-q)(t+c)\right)^{1/1-q}$

Debido a que no tenemos condiciones iniciales, podemos elegir las constantes del problema

Luego, si

$q = 1 \longrightarrow v(t) = e^{t}$

$q < 1 \longrightarrow v(t) = \left((1-q)t + 1\right)^{1/1-q}$

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Polinomio interpolador de orden k:

$$P_k(x) = \sum_{i=0}^{k} f(x_i) \mathcal{L}_i(x),$$

$$\mathcal{L}_i(x) = \prod_{j=0,\ j\neq i}^{k} \frac{x - x_j}{x_i - x_j}$$

Cambio de variable

$[t_{n-2}, t_{n-1}, t_n, t_{n+1}, t_{n+2}]$

$[-2h, -h, 0, h, 2h]$

① método Adams - Bashforth de 3 pasos:

Puntos: $(f_n, t_n), (f_{n-1}, t_{n-1}) (f_{n-2}, t_{n-2})$

$n = 3$ (t_n, t_{n-1}, t_{n-2})

$i = 0, 1, 2$

$j = 0, 1, 2$

$$\mathcal{L}_0(t) = \frac{t - t_{n-1}}{t_n - t_{n-1}} \cdot \frac{t - t_{n-2}}{t_n - t_{n-2}} \rightarrow P_1 = f_n \cdot \mathcal{L}_0(t)$$

$$\mathcal{L}_1(t) = \frac{t - t_n}{t_{n-1} - t_n} \cdot \frac{t - t_{n-2}}{t_{n-1} - t_{n-2}} \rightarrow P_2 = f_{n-1}\, \mathcal{L}_1(t)$$

$$\mathcal{L}_2(t) = \frac{t - t_n}{t_{n-2} - t_n} \cdot \frac{t - t_{n-1}}{t_{n-2} - t_{n-1}} \rightarrow P_3 = f_{n-2}\, \mathcal{L}_2(t)$$

$$P_1 = \frac{t-(-h) \cdot t - (-2h)}{2h^2} f_n \ //$$

$$P_2 = -\frac{t \cdot (t - (-2h))}{h^2} f_{n-1} \ //$$

$$P_3 = \frac{t \cdot (t - (-h))}{2h^2} f_{n-2} \ //$$

② Método Adams Bashforth de 4 pasos:

Puntos $(t_n, t_{n-1}, t_{n-2}, t_{n-3})$

$n = 4$

$i = 0, 1, 2, 3$

$j = 0, 1, 2, 3$

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$$\mathcal{L}_0(t) = \frac{t - t_{n-1}}{\cancel{t_n} - t_{n-1}} \cdot \frac{t - t_{n-2}}{\cancel{t_n} - t_{n-2}} \cdot \frac{t - t_{n-3}}{\cancel{t_n} - t_{n-3}}$$

$t_n = 0$

$$\mathcal{L}_1(t) = \frac{t - \cancel{t_n}}{t_{n-1} - \cancel{t_n}} \cdot \frac{t - t_{n-2}}{t_{n-1} - t_{n-2}} \cdot \frac{t - t_{n-3}}{t_{n-1} - t_{n-3}}$$

$$\mathcal{L}_2(t) = \frac{t - \cancel{t_n}}{t_{n-2} - \cancel{t_n}} \cdot \frac{t - t_{n-1}}{t_{n-2} - t_{n-1}} \cdot \frac{t - t_{n-3}}{t_{n-2} - t_{n-3}}$$

$$\mathcal{L}_3(t) = \frac{t - \cancel{t_n}}{t_{n-3} - \cancel{t_n}} \cdot \frac{t - t_{n-1}}{t_{n-3} - t_{n-1}} \cdot \frac{t - t_{n-2}}{t_{n-3} - t_{n-2}}$$

$$P_1 = \frac{(t-(-h))\cdot(t-(-2h))\cdot(t-(-3h))}{(-(-h))\cdot(-(-2h))\cdot(-(-3h))} f_n$$

$$P_2 = \frac{t\cdot(t-(-2h))\cdot(t-(-3h))}{-h\cdot(-h-(-2h))\cdot(-h-(-3h))} f_{n-1}$$

$$P_3 = \frac{t\cdot(t-(-h))\cdot(t-(-3h))}{-2h\cdot(-2h-(-h))\cdot(-2h-(-3h))} f_{n-2}$$

$$P_4 = \frac{t\cdot(t-(-h))\cdot(t-(-2h))}{-3h\cdot(-3h-(-h))\cdot(-3h-(-2h))} f_{n-3}$$

3 Método de Adams-Moulton: tres pasos

Puntos $= (t_{n+1}, t_n, t_{n-1})$

$n = 3$
$i = 0, 1, 2$
$j = 0, 1, 2$

$$\mathcal{L}_0(t) = \frac{t - t_n}{t_{n+1} - t_n} \cdot \frac{t - t_{n-1}}{t_{n+1} - t_{n-1}} \quad ; \quad \mathcal{L}_1(t) = \frac{t - t_{n+1}}{t_n - t_{n+1}} \cdot \frac{t - t_{n-1}}{t_n - t_{n-1}}$$

$$\mathcal{L}_2(t) = \frac{t - t_{n+1}}{t_{n-1} - t_{n+1}} \cdot \frac{t - t_n}{t_{n-1} - t_n}$$

$$P_1 = \frac{t \cdot (t-(-h))}{h \cdot (h-(-h))} f_{n+1}$$

$$P_2 = \frac{(t-h) \cdot (t-(-h))}{-h \cdot (-(-h))} f_n$$

$$P_3 = \frac{t \cdot (t-h)}{(-h-(h)) \cdot (-h)} f_{n-1}$$

④ método Adams - Moulton 4 pasos

Puntos: $(t_{n+1}, t_n, t_{n-1}, t_{n-2})$

$n = 4$
$i = 0, 1, 2, 3$
$j = 0, 1, 2, 3$

$$L_0(t) = \frac{t - t_n}{t_{n+1} - t_n} \cdot \frac{t - t_{n-1}}{t_{n+1} - t_{n-1}} \cdot \frac{t - t_{n-2}}{t_{n+1} - t_{n-2}}$$

$$L_1(t) = \frac{t - t_{n+1}}{t_n - t_{n+1}} \cdot \frac{t - t_{n-1}}{t_n - t_{n-1}} \cdot \frac{t - t_{n-2}}{t_n - t_{n-2}}$$

$$L_2(t) = \frac{t - t_{n+1}}{t_{n-1} - t_{n+1}} \cdot \frac{t - t_n}{t_{n-1} - t_n} \cdot \frac{t - t_{n-2}}{t_{n-1} - t_{n-2}}$$

$$L_3(t) = \frac{t - t_{n+1}}{t_{n-2} - t_{n+1}} \cdot \frac{t - t_n}{t_{n-2} - t_n} \cdot \frac{t - t_{n-1}}{t_{n-2} - t_{n-1}}$$

$$P_1 : \frac{t \cdot (t-(-h)) \cdot (t-(-2h))}{h \cdot (h-(-h)) \cdot (h-(-2h))} f_{n+1}$$

$$P_2 : \frac{(t-h) \cdot (t-(-h)) \cdot (t-(-2h))}{-h \cdot (-(-h)) \cdot (-(-2h))} f_n$$

$$P_3 : \frac{(t-h) \cdot (t) \cdot (t-(-2h))}{(-h-h) \cdot (-h) \cdot (-h-(-2h))} f_{n-1}$$

$$P_4 : \frac{(t-h) \cdot t \cdot (t-(-h))}{(-2h-h) \cdot (-2h) \cdot (-2h-(-h))} f_{n-2}$$