Punto 5.

Si definimos

$\bar{r} = r/d \rightarrow r = \bar{r} d$

$\bar{p}_r = p_r/md \rightarrow p_r = \bar{p}_r md$

$\bar{p}_\phi = p_\phi/md^2 \rightarrow p_\phi = \bar{p}_\phi md^2$

$\Delta = GM_T/d^3$

$\mu = m_L/m_T$

$\bar{r}' = \sqrt{1+\bar{r}^2 - 2\bar{r}\cos(\phi - \omega t)}$

Tenemos que

$\dot{r} = \dfrac{p_r}{m}$ $\quad\quad \dot{r} = \dot{\bar{r}} d, \; p_r = \bar{p}_r md$

$\therefore \quad \dot{\bar{r}} \cancel{d} = \dfrac{\bar{p}_r \cancel{md}}{\cancel{m}} \rightarrow \dot{\bar{r}} = \bar{p}_r$

También

$\dot{\phi} = \dfrac{p_\phi}{mr^2}$

$\therefore \quad \dot{\phi} = \dfrac{\bar{p}_\phi \cancel{m}\cancel{d^2}}{\cancel{m}\bar{r}^2\cancel{d^2}} \rightarrow \dot{\phi} = \dfrac{\bar{p}_\phi}{\bar{r}^2}$

Para la otra

$$\therefore \quad \dot\phi = \frac{P_\phi \cancel{m} d^2}{\cancel{m} \bar r^2 \cancel{d^2}} \longrightarrow \dot\phi = \frac{\bar P_\phi}{\bar r^2}$$

Para la otra

$$\dot p_r = \frac{p_\phi^2}{m r^3} - \frac{G m m_T}{r^2} - \frac{G m m_L}{r_L(r,\phi,t)^3}\left[r - d\cos(\phi - \omega t) \right]$$

$$P_\phi = \bar P_\phi \, m d^2$$

$$\mu \equiv m_L / m_T$$

$$\therefore \quad \dot{\bar p}_r \cancel{m} d = \frac{\bar P_\phi^2 \cancel{m^2} \cancel{d^4}}{\cancel{m} \bar r^3 \cancel{d^3}} - \frac{G m_T \cancel{m}}{\bar r^2 d^2} - \frac{G \cancel{m} \mu m_L}{\bar r'^3 d^3}\left[\bar r d - d\cos(\phi - \omega t) \right]$$

$$\dot{\bar P}_r d = \frac{\bar P_\phi}{m \bar r} d - \frac{G m_T}{d^2}\left[\frac{1}{\bar r^2} + \frac{\mu}{\bar r'^3}\left(\bar r - \cos(\phi - \omega t) \right) \right]$$

$$\dot{\bar P}_r = \frac{\bar P_\phi}{m \bar r} - \underbrace{\frac{G m_T}{d^3}}_{\Delta}\left[\frac{1}{\bar r^2} + \frac{\mu}{\bar r'^3}\left(\bar r - \cos(\phi - \omega t) \right) \right]$$

$$\therefore \quad \dot{\bar P}_r = \frac{\bar P_\phi}{m \bar r} - \Delta \left[\frac{1}{\bar r^2} + \frac{\mu}{\bar r'^3}\left(\bar r - \cos(\phi - \omega t) \right) \right]$$

$$\dot{p}_\phi = -\frac{G\, m\, m_L}{r_L(r,\phi,t)^3}\, r\, d \sin(\phi - \omega t)$$

$$\dot{\bar{p}}_\phi\, \cancel{\mu}\, \cancel{d^2} = -\frac{G\, \cancel{\mu}\, m_T\, \mu\, \bar{r}\, \cancel{d^2}}{d^3\, \bar{r}'^3} \sin(\phi - \omega t)$$

$$\dot{\bar{p}}_\phi = -\frac{G\, m_T}{d^3}\, \frac{\mu\, \bar{r}}{\bar{r}'^3} \sin(\phi - \omega t)$$

$$\dot{\bar{p}}_\phi = -\Delta\, \frac{\mu\, \bar{r}}{\bar{r}'^3} \sin(\phi - \omega t)$$

Punto c.

Para la nave, la parametrización de las coordenadas

$x = r(t)\,\text{Cos}(\theta(t))$ y $y = r(t)\,\text{Sen}(\theta(t))$

Mientras que para la luna

$x_L = d\cos(\omega t)$ y $y_L = d\,\text{Sen}(\omega t)$

Luego la magnitud de la distancia entre la nave y la luna

$r_L = |r - r_L|$ y $r - r_L = (r\,\text{Cos}\,\theta - d\,\text{Cos}\,\omega t,\ r\,\text{Sen}\,\theta - d\,\text{Sen}\,\omega t)$

$\therefore$

$$r_L = \sqrt{(r\,\text{Cos}\,\theta - d\,\text{Cos}\,\omega t)^2 + (r\,\text{Sen}\,\theta - d\,\text{Sen}\,\omega t)^2}$$

$$= \sqrt{r^2\,\text{Cos}^2\theta + d^2\,\text{Cos}^2\,\omega t - rd\,\text{Cos}\,\theta\,\text{Cos}\,\omega t + r^2\,\text{sen}^2\,\theta + d^2\,\text{sen}^2\,\omega t - rd\,\text{Sen}\,\theta\,\text{Sen}\,\omega t}$$

$$= \sqrt{r^2 + d^2 - 2rd\,(\underbrace{\text{Cos}\,\theta\,\text{Cos}\,\omega t + \text{Sen}\,\theta\,\text{Sen}\,\omega t}_{\Downarrow\ \cos(\theta - \omega t)})}$$

$$= \sqrt{r(t)^2 + d^2 - 2r(t)\,d\,\text{Cos}(\theta(t) - \omega t)}$$