Image Processing
Homework 1

Problem 1 (20points):

Originally designed by Jane Doe. Given a set of N data, each with D entries, organized in the form of an $N \times D$ matrix,

$$X = \begin{bmatrix} x_{0,0} & x_{0,1} & \cdots & x_{0,D-1} \\ x_{1,0} & x_{1,1} & \cdots & x_{1,D-1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-1,0} & x_{N-1,1} & \cdots & x_{N-1,D-1} \end{bmatrix}$$

Each row of X is a horizontal vector $\overrightarrow{x_i}^T$, in which $\overrightarrow{x_i}$ is $D \times 1$.

The goal is to compute an $N \times N$ matrix, whose entries are the pairwise Euclidean distance between any two data.

$$Z = \begin{pmatrix} \|\overrightarrow{x_0} - \overrightarrow{x_0}\| & \|\overrightarrow{x_0} - \overrightarrow{x_1}\| & \cdots & \|\overrightarrow{x_0} - \overrightarrow{x_{N-1}}\| \\ \|\overrightarrow{x_1} - \overrightarrow{x_0}\| & \|\overrightarrow{x_1} - \overrightarrow{x_1}\| & \cdots & \|\overrightarrow{x_1} - \overrightarrow{x_{N-1}}\| \\ \vdots & \vdots & \ddots & \vdots \\ \|\overrightarrow{x_{N-1}} - \overrightarrow{x_0}\| & \|\overrightarrow{x_{N-1}} - \overrightarrow{x_1}\| & \cdots & \|\overrightarrow{x_{N-1}} - \overrightarrow{x_{N-1}}\| \end{pmatrix}$$

You need to implement two Matlab functions:

1. The first one, uses a two level nested loop, iterating through all pairs (i, j) and compute the corresponding entry $Z_{i,j}$.
2. The second one, compute Z without any loop, but use **basic matrix operations**, including element-wise operations. This is vectorization. You are not allowed to use any build-in functions, such as plist() and squareform(), for this job.

Generate random matrices of various sizes and profile the performance as plots of data size v.s. execution time (see CompareEntropyFig.pdf for an example). The x-axis shows the control variable in the experiment, and the y-axis shows the results of the experiments.

Hint:

$$\|\overrightarrow{x_i} - \overrightarrow{x_j}\| = \sqrt{(\overrightarrow{x_i} - \overrightarrow{x_j})^T(\overrightarrow{x_i} - \overrightarrow{x_j})} = \sqrt{\|\overrightarrow{x_i}\|^2 - 2\overrightarrow{x_i}^T\overrightarrow{x_j} + \|\overrightarrow{x_j}\|^2}$$

Think about how to compute each of the three entries (for all (i, j) pairs) from the data matrix X.

In the report, include the following:

- Your algorithms. The description should be independent of the programming language (so this is not an explanation of your code). Clearly indicate the dimensions of matrices in your work. **The dimensions should match!**
- Results. The timing plot and the discussion of the plot. What is the control variable in this experiment? What did you observe? What do you learn from this observation?

Problem 2 (10 points):

The effect of double exposure can be created by adding two images together. Write a Matlab function to add two images and produce an output image with the following constaints:

- The output image has the maximum width and height of input images.
- Resizing of the input images is not allowed.
- Image saturation has to be avoided. Image saturation happens when intensity values exceeding 255 are truncated.

Test your function with Everest_expedition.jpg and Everest_kalapatthar.jpg, but your program should work for images of any sizes, not just the two given test images.

In the report, include the following:

- Your algorithm. The description should be independent of the programming language (so this is not an explanation of your code).
- Result. The input images, the output image, and the discussion. Does the output meet your expectation? How did you check the correctness of your work?

To Submit:

- You are highly encouraged to pair up with another student to complete this homework.
- At the end of your report, please include a section titled: "Resources that Helped Me."
- Submit one single **.zip** file (NOT .rar or .tar.gz), containing well-documented Matlab files (.m) and the report. It is preferred that the file is named "LastNames-HW.zip".
- Only one submission per group, and **full names** of the members should be included in the Comments box of the submission page on Blackboard.