Term 1

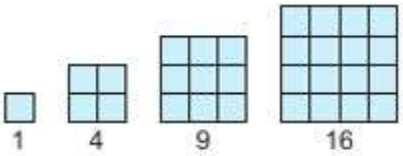
Unit 4 Comparing and representing numbers in exponential form

In this unit you will:

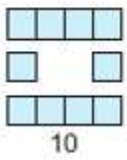
compare and represent whole numbers in exponential form.

Getting started Squares and cubes

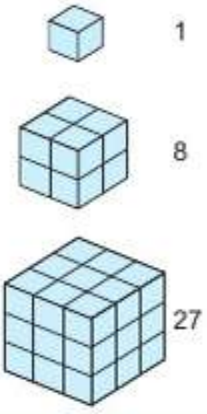
Look at these *square* numbers:



1. Why do we call the numbers 1; 4; 9 and 16 square numbers?
2. What are the next three square numbers?
3. The whole number 10 is not a square number, although the picture drawn here looks 'square'. Explain what is wrong with this square picture of ten.



4. Look at these *cubic* numbers:
   a) Why do we call the numbers 1, 8 and 27 cubes?
   b) What are the next two cubic numbers?
5. How can you find the squares and cubes without using a drawing?
6. We carry on the pattern of squares and cubes shown above and alongside. What is the tenth square number and the tenth cubic number?



Key ideas

- Multiplying each whole number by itself makes a square number. For example, $1 \times 1 = 1$; $2 \times 2 = 4$; $3 \times 3 = 9$, etc. The seventh square number is $7 \times 7 = 49$. We say that the number 49 is the square of 7.
- For the cubic numbers the whole number is multiplied by itself twice. For example, $1 \times 1 \times 1 = 1$; $2 \times 2 \times 2 = 8$; $3 \times 3 \times 3 = 27$, etc. The cube of 5 is $5 \times 5 \times 5$ or 125.

Exercise 4.1 Square numbers and cubic numbers

1. Copy and complete the table below.

Number	Squared	Number	Squared	Number	Cubed
1	$1 \times 1 = 1$	7		1	
2		8		2	$2 \times 2 \times 2 = 8$
3		9		3	
4		10		4	
5		11		5	
6		12		6	

2. a) What is a square number?
   b) What is the square of 12?
   c) 169 is the square of 13. Find the number that 625 is the square of. Use a calculator.
   d) Fill in the missing numbers: $\square \times \square = 289$. The same number must go in both blocks.
3. a) Show how you work out the cube of 9.
   b) 27 is the cube of 3.What is 216 the cube of?
   c) The same number goes in each block in $\square \times \square \times \square = 64$. What is the number?
4. A palindrome is a number or word that reads the same forwards or backwards, e.g. aha, 101, racecar.
   Using the digits 1, 2, 3 and 4, find:
   a) a three-digit square number that is a palindrome
   b) a three-digit cubic number that is a palindrome.

Activity 4.1 Making sense of exponential notation

There is a shorter way of writing $2 \times 2 \times 2 \times 2$ and of writing 9×9. We call this the exponential form:

$2 \times 2 \times 2 \times 2 = 2^4$ We call 2 the base and 4 the exponent
and $9 \times 9 = 9^2$ We call 9 the base and 2 the exponent

1. What does the exponent '4' stand for in 2^4?
2. What does the exponent '2' stand for in 9^2?
3. Write the following in a shorter way. Use the exponential form.
   a) $5 \times 5 \times 5$ b) $12 \times 12 \times 12$ c) 24×24 d) 53×53

Key ideas

- $2 \times 2 \times 2 = 8$. We get 8 by multiplying 2 by 2 by 2.
- We can write this in a short way: $2^3 = 8$. We say: '2 to the *power* of 3 equals 8' or '2 *cubed* equals 8'. We do not say 'two to the three equals 8'.
- We can write 9×9 as $9^2 = 81$. We say: 9 *squared* equals 81. We do not say 'nine to the two equals 81'.
- We say that 9^2 is written in *exponential form*. We call the number '9' the *base* and the number '2' the *exponent*. The exponent tells us how many times the base is multiplied by itself.
- Any number to the power of one stays that number. $4^1 = 4$.

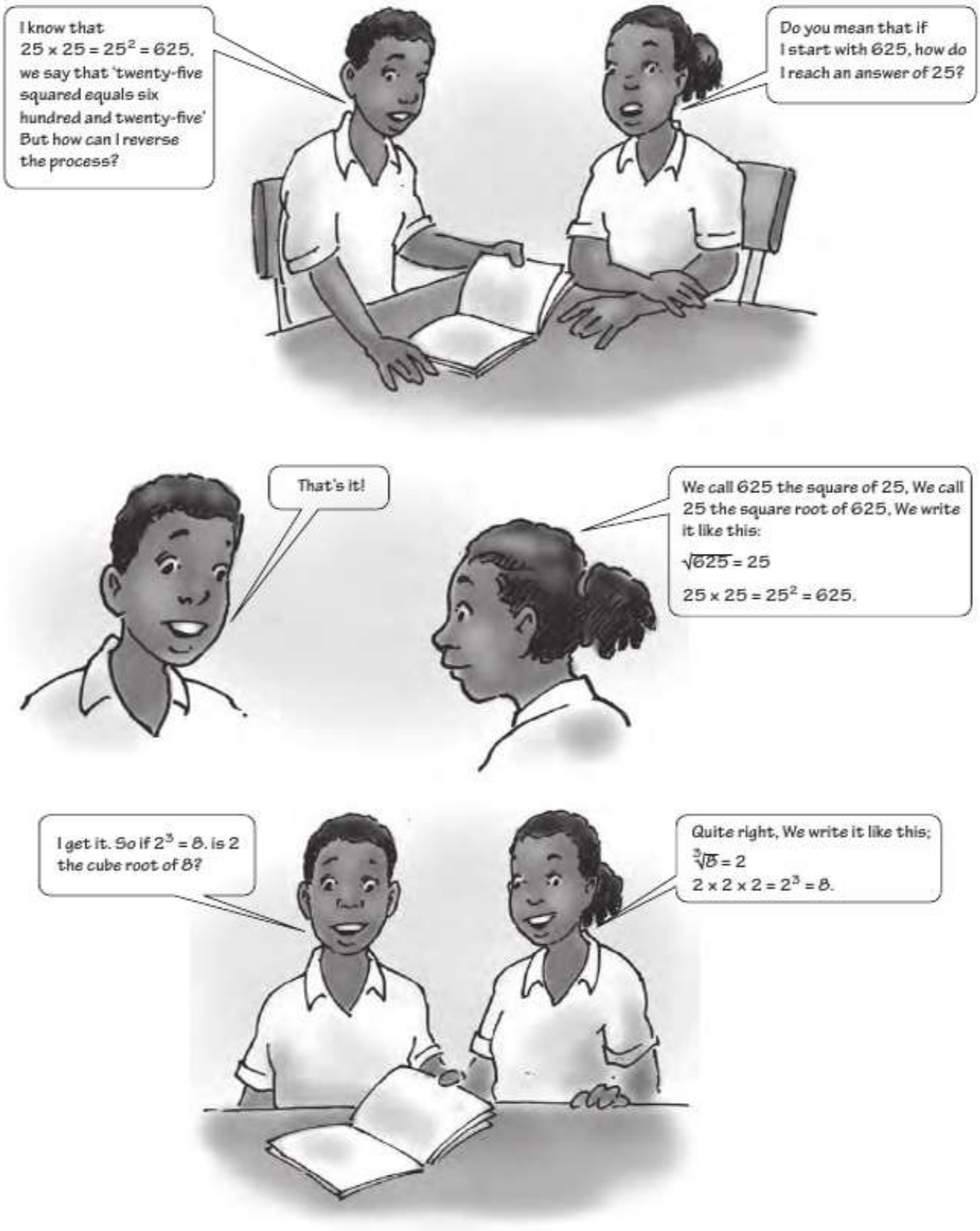
Exercise 4.2 Writing in exponential form

1. Copy and complete the table below. The first row has been done for you.

Repeated multiplication	Exponential form	We say	Value
6×6	6^2	Six squared	36
		Ten squared	100
	12^2		
		Five cubed	
$1 \times 1 \times 1 \times 1$	1^4	One to the power of four	1
	5^4		625
$10 \times 10 \times 10 \times 10$			

2. What does six to the power of one equal?
3. Is $3^2 + 4^2$ equal to 7^2? Show your calculations.

Activity 4.2 Squares and square roots, cubes and cube roots



1. Explain why 25 is the square root of 625.
2. Find the square root of 121. Use the same reasoning.
3. Find the cube root of 125 in the same way.

Key ideas

- The sign $\sqrt{\ }$ stands for the square root of a number. We read $\sqrt{64} = 8$ as 'the square root of 64 is equal to 8'. The square root of 64 is 8 because $8 \times 8 = 64$.
- The sign $\sqrt[3]{\ }$ stands for the cube root of a number. We read $\sqrt[3]{64} = 4$ as 'the cube root of 64 is equal to 4'. The cube root of 64 is 4 because $4 \times 4 \times 4 = 64$.
- With the square root sign, $\sqrt{\ }$, you expect a little 2, like the little 3 in the cube root sign. The little 2 is usually left out, but you must imagine that it is there.
- If you have a calculator, check to see if it has $\sqrt{\ }$ and $\sqrt[3]{\ }$ keys. Use these to work out square and cube roots. On some calculators you need to first put in the number and then press the $\sqrt{\ }$ or $\sqrt[3]{\ }$ key. On other calculators you need to first press the $\sqrt{\ }$ or $\sqrt[3]{\ }$ key and then put in the number.

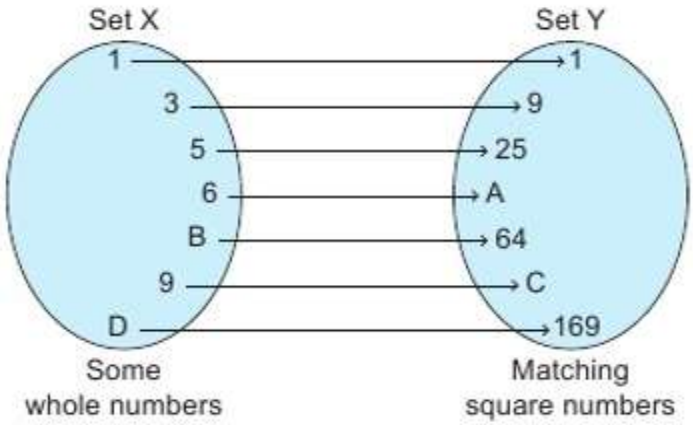
Exercise 4.3 Square and cube roots

Copy and complete the table below.

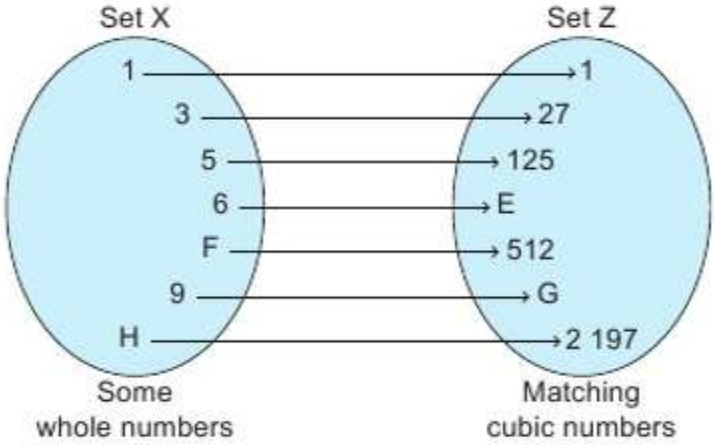
We say	We write	Repeated multiplication	Value
The square root of 36	$\sqrt{36}$	$= \sqrt{6 \times 6}$	$= 6$
		$= \sqrt{7 \times 7}$	
The square root of nine			
	$\sqrt{144}$		
		$= \sqrt{4 \times 4}$	
The cube root of 27	$\sqrt[3]{27}$	$= \sqrt[3]{3 \times 3 \times 3}$	$= 3$
		$= \sqrt[3]{6 \times 6 \times 6}$	
The cube root of eight			
	$\sqrt[3]{125}$		
		$= \sqrt[3]{1 \times 1 \times 1}$	$= 1$
	$\sqrt[3]{64}$		

Activity 4.3 Inverse operations of squaring and cubing

Work with a friend. Discuss and answer the questions.



1. Some whole numbers are shown in Set X. Their square numbers are shown in Set Y.
   a) Find the values of A, B, C and D.
   b) Explain how you worked out the missing square numbers in Set Y.
   c) Explain how you worked out the missing whole numbers in Set X.
2. Some whole numbers are shown in Set X and their cubic numbers are shown in Set Z.



   a) Find the values of E, F, G and H.
   b) Explain how you worked out the missing cubic numbers in Set Z.
   c) Explain how you worked out the missing whole numbers in Set X.
3. Compare and discuss your answers in class.

Key ideas

- You get the square number in Set Y by multiplying the whole number by itself. So, the square of 6 is 36, because $6 \times 6 = 36$. We also write this as $6^2 = 36$.
- You get the cubic number in Set Z by multiplying the whole number by itself twice. So, the cube of 6 is $6 \times 6 \times 6 = 216$. We also write this as $6^3 = 216$.
- To work out the whole number in Set X, you have to answer the question, 'What number multiplied by itself, once or twice, will give me the number in Set Y or Set Z?' Then you find, for example, that $8^2 = 64$ and $8^3 = 512$.
- Square roots and cube roots are the inverse operations of squaring and cubing numbers.
- Any number to the power of one stays that number. Example: $4^1 = 4$

Exercise 4.4 Mixed calculations

1. Find the values of each of the following sets of calculations. Do not use a calculator.

Set 1	Set 2	Set 3	Set 4
a) 8^2	a) 4^2	a) $\sqrt{1}$	a) $\sqrt{144}$
b) 3^2	b) 3^3	b) $\sqrt{9}$	b) $\sqrt[3]{8}$
c) 5^2	c) $\sqrt{36}$	c) $\sqrt{64}$	c) 10^2
d) $\sqrt{4}$	d) $\sqrt[3]{8}$	d) $\sqrt[3]{125}$	d) 6^3
e) $\sqrt{100}$	e) 10^2	e) $\sqrt[3]{8}$	e) $\sqrt{81}$

2. Say which of the following are true or false. If the statement is false, make it true.

a) $12^2 = 144$ b) $2^2 = 12 \times 2$ c) $50^1 = 50 \times 50$

d) $12 \times 2 = 24$ e) $12 \times 12 = 12 \times 2$ f) $1 \times 1 \times 1 \times 1 = 4$

g) $1^4 = 1 \times 1 \times 1 \times 1$ h) $\sqrt{9} = 81$ i) $9^2 = 3$

Summary

- Multiplying each whole number by itself makes a square number.
- A cubic number is formed when a whole number is multiplied by itself twice.
- We can write 9×9 as $9^2 = 81$.We say: 9 *squared* equals 81.
- We say that 9^2 is written in *exponential form*. We call the number '9' the base and the number '2' the *exponent*. The exponent tells us how many times the base is multiplied by itself.

- Any number to the power of one stays that number. Example: $4^1 = 4$.
- The sign $\sqrt{\ }$ stands for the square root of a number.
- The sign $\sqrt[3]{\ }$ stands for the cube root of a number. Example: $\sqrt[3]{64} = 4$
- Finding the square roots and cube roots of numbers are the inverse operations of squaring and cubing numbers.

Check what you know

1. Write each of the following in exponential form.
   a) $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$
   b) $11 \times 11 \times 11 \times 11$
   c) $145 \times 145 \times 145$
   d) $17 \times 17 \times 17 \times 17 \times 17 \times 17$
   e) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$
2. We can write the number 84 as the product of prime numbers: $84 = 2 \times 2 \times 3 \times 7$. We can write this in exponential form: $84 = 2^2 \times 3 \times 7$. Write the following numbers as products of their prime numbers. Use the exponential form.
   a) 36 b) 40 c) 54 d) 198 e) 525 f) 48
3. Which is larger: 3^4 or 4^3? Explain your answer.
4. Evaluate the following:
   a) 3^2 b) 5^1 c) 7^2 d) 4^3 e) 10^1
5. Complete the table below. Fill in:
   - each number squared
   - the number cubed
   - your answer square rooted
   - your answer cube rooted.

x	Square (x^2)	Square root of the square $\sqrt{x^2}$	Cube (x^3)	Cube root of the cube $\sqrt[3]{x^3}$
3	$3^2 = 3 \times 3 = 9$	$\sqrt{9} = \sqrt{3^2} = 3$	$3^3 = 3 \times 3 \times 3 = 27$	$\sqrt[3]{27} = \sqrt[3]{3^3} = 3$
4				
5				
11				
12				

6. Solve the following. Do not use your calculator. You may have to use prime factorisation in some cases.
   a) $\sqrt{196}$ b) $\sqrt{10\,000}$ c) $\sqrt{225}$
7. The number 100 is a perfect square. Its square root is a whole number.
   a) Is 14 a perfect square? Explain why or why not.
   b) List all the perfect squares from 0 to 100.