Trigonometry 10

10.1 Introduction

EMCCE

In this chapter we will build on the trigonometry from previous years by looking at the result applying trigonometric functions to sums and differences of angles. We will also apply trigonometry to solving geometric problems in two and three dimensions.

See introductory video: VMhtu at www.everythingmaths.co.za

10.2 Compound Angle Identities

EMCCF

Derivation of $\sin(\alpha + \beta)$

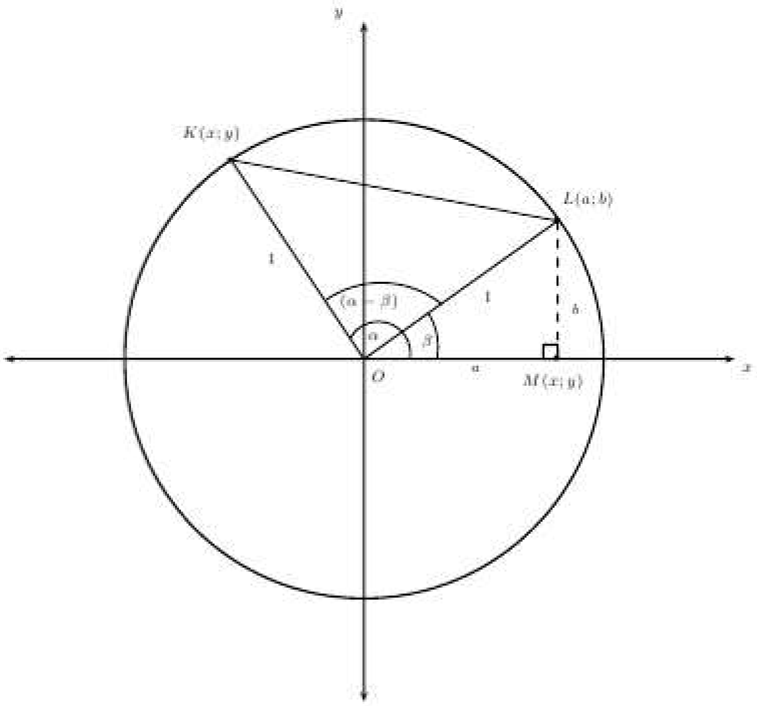
EMCCG

We have, for any angles α and β, that

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

How do we derive this identity? It is tricky, so follow closely.

Suppose we have the unit circle shown below. The two points $L(a;b)$ and $K(x;y)$ are on the circle.



We can get the coordinates of L and K in terms of the angles α and β. For the triangle LOM, we have that

$$\begin{aligned} \sin\beta = \frac{b}{1} \quad &\Longrightarrow \quad b = \sin\beta \\ \cos\beta = \frac{a}{1} \quad &\Longrightarrow \quad a = \cos\beta \end{aligned}$$

Thus the coordinates of L are $(\cos\beta; \sin\beta)$. In the same way as above, we can see that the coordinates of K are $(\cos\alpha; \sin\alpha)$. The identity for $\cos(\alpha - \beta)$ is now determined by calculating KL^2 in two ways. Using the distance formula (i.e. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ or $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$), we can find KL^2:

$$\begin{aligned} KL^2 &= (\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2 \\ &= \cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta + \sin^2\alpha - 2\sin\alpha\sin\beta + \sin^2\beta \\ &= (\cos^2\alpha + \sin^2\alpha) + (\cos^2\beta + \sin^2\beta) - 2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta \\ &= 1 + 1 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) \\ &= 2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) \end{aligned}$$

The second way we can determine KL^2 is by using the cosine rule for $\triangle KOL$:

$$\begin{aligned} KL^2 &= KO^2 + LO^2 - 2 \,.\, KO \,.\, LO \,.\, \cos(\alpha - \beta) \\ &= 1^2 + 1^2 - 2(1)(1)\cos(\alpha - \beta) \\ &= 2 - 2 \,.\, \cos(\alpha - \beta) \end{aligned}$$

Equating our two values for KL^2, we have

$$\begin{aligned} 2 - 2 \,.\, \cos(\alpha - \beta) &= 2 - 2(\cos\alpha\cos\beta + \sin\alpha \,.\, \sin\beta) \\ \Longrightarrow \quad \cos(\alpha - \beta) &= \cos\alpha \,.\, \cos\beta + \sin\alpha \,.\, \sin\beta \end{aligned}$$

Now let $\alpha \to 90^\circ - \alpha$. Then

$$\begin{aligned} \cos(90^\circ - \alpha - \beta) &= \cos(90^\circ - \alpha)\cos\beta + \sin(90^\circ - \alpha)\sin\beta \\ &= \sin\alpha \,.\, \cos\beta + \cos\alpha \,.\, \sin\beta \end{aligned}$$

But $\cos(90^\circ - (\alpha + \beta)) = \sin(\alpha + \beta)$. Thus

$$\sin(\alpha + \beta) = \sin\alpha \,.\, \cos\beta + \cos\alpha \,.\, \sin\beta$$

Derivation of $\sin(\alpha - \beta)$

EMCCH

We can use

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

to show that

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

We know that

$$\sin(-\theta) = -\sin(\theta)$$

and

$$\cos(-\theta) = \cos\theta$$

Therefore,

$$\begin{aligned}\sin(\alpha - \beta) &= \sin(\alpha + (-\beta)) \\ &= \sin\alpha\cos(-\beta) + \cos\alpha\sin(-\beta) \\ &= \sin\alpha\cos\beta - \cos\alpha\sin\beta\end{aligned}$$

Derivation of $\cos(\alpha + \beta)$

EMCCI

We can use

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$$

to show that

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

We know that

$$\sin(\theta) = \cos(90 - \theta).$$

Therefore,

$$\begin{aligned}\cos(\alpha + \beta) &= \sin(90 - (\alpha + \beta)) \\ &= \sin((90 - \alpha) - \beta)) \\ &= \sin(90 - \alpha)\cos\beta - \sin\beta\cos(90 - \alpha) \\ &= \cos\alpha\cos\beta - \sin\beta\sin\alpha\end{aligned}$$

Derivation of $\cos(\alpha - \beta)$

EMCCJ

We found this identity in our derivation of the $\sin(\alpha + \beta)$ identity. We can also use the fact that

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

to derive that

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

As

$$\cos(\theta) = \sin(90 - \theta),$$

we have that

$$\begin{aligned}\cos(\alpha-\beta) &= \sin(90-(\alpha-\beta))\\ &= \sin((90-\alpha)+\beta))\\ &= \sin(90-\alpha)\cos\beta+\cos(90-\alpha)\sin\beta\\ &= \cos\alpha\cos\beta+\sin\alpha\sin\beta\end{aligned}$$

Derivation of $\sin 2\alpha$

EMCCK

We know that

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

When $\alpha=\beta$, we have that

$$\begin{aligned}\sin(2\alpha)=\sin(\alpha+\alpha) &= \sin\alpha\cos\alpha+\cos\alpha\sin\alpha\\ &= 2\sin\alpha\cos\alpha\end{aligned}$$

Derivation of $\cos 2\alpha$

EMCCL

We know that

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

When $\alpha=\beta$, we have that

$$\begin{aligned}\cos(2\alpha)=\cos(\alpha+\alpha) &= \cos\alpha\cos\alpha-\sin\alpha\sin\alpha\\ &= \cos^2\alpha-\sin^2\alpha\end{aligned}$$

However, we can also write

$$\cos 2\alpha = 2\cos^2\alpha-1$$

and

$$\cos 2\alpha = 1-2\sin^2\alpha$$

by using

$$\sin^2\alpha+\cos^2\alpha=1.$$

Activity:	*The $\cos 2\alpha$ Identity*

Use

$$\sin^2\alpha+\cos^2\alpha=1$$

to show that:

$$\cos 2\alpha = 2\cos^2\alpha-1=1-2\sin^2\alpha$$

Problem-solving Strategy for Identities

EMCCM

Tip

When proving trigonometric identities, never assume that the left hand side is equal to the right hand side. You need to **show** that both sides are equal.

The most important thing to remember when asked to prove identities is:

A suggestion for proving identities: It is usually much easier simplifying the more complex side of an identity to get the simpler side than the other way round.

Example 1: *Trigonometric Identities 1*

QUESTION

Prove that $\sin 75^\circ = \frac{\sqrt{2}(\sqrt{3}+1)}{4}$ *without using a calculator.*

SOLUTION

Step 1 : ***Identify a strategy***

We only know the exact values of the trig functions for a few special angles (30°; 45°; 60°; etc.). We can see that $75^\circ = 30^\circ + 45^\circ$. Thus we can use our double-angle identity for $\sin(\alpha + \beta)$ to express $\sin 75^\circ$ in terms of known trig function values.

Step 2 : ***Execute strategy***

$$\begin{aligned}
\sin 75^\circ &= \sin(45^\circ + 30^\circ) \\
&= \sin(45^\circ)\cos(30^\circ) + \cos(45^\circ)\sin(30^\circ) \\
&= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \\
&= \frac{\sqrt{3}+1}{2\sqrt{2}} \\
&= \frac{\sqrt{3}+1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\
&= \frac{\sqrt{2}(\sqrt{3}+1)}{4}
\end{aligned}$$

Example 2: Trigonometric Identities 2

QUESTION

Deduce a formula for $\tan(\alpha+\beta)$ in terms of $\tan\alpha$ and $\tan\beta$.
Hint: Use the formulae for $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$

SOLUTION

Step 1 : ***Identify a strategy***

We can express $\tan(\alpha+\beta)$ in terms of cosines and sines, and then use the double-angle formulae for these. We then manipulate the resulting expression in order to get it in terms of $\tan\alpha$ and $\tan\beta$.

Step 2 : ***Execute strategy***

$$\begin{aligned}
\tan(\alpha+\beta) &= \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} \\
&= \frac{\sin\alpha\,.\,\cos\beta+\cos\alpha\,.\,\sin\beta}{\cos\alpha\,.\,\cos\beta-\sin\alpha\,.\,\sin\beta} \\
&= \frac{\frac{\sin\alpha\,.\,\cos\beta}{\cos\alpha\,.\,\cos\beta}+\frac{\cos\alpha\,.\,\sin\beta}{\cos\alpha\,.\,\cos\beta}}{\frac{\cos\alpha\,.\,\cos\beta}{\cos\alpha\,.\,\cos\beta}-\frac{\sin\alpha\,.\,\sin\beta}{\cos\alpha\,.\,\cos\beta}} \\
&= \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\,.\,\tan\beta}
\end{aligned}$$

Example 3: Trigonometric Identities 3

QUESTION

Prove that

$$\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}=\tan\theta$$

In fact, this identity is not valid for all values of θ. Which values are those?

SOLUTION

Step 1 : ***Identify a strategy***

The right-hand side (RHS) of the identity cannot be simplified. Thus we should try simplify the left-hand side (LHS). We can also notice that the trig function on the RHS does not have a 2θ dependence. Thus we will need to use the double-angle formulae to simplify the $\sin 2\theta$ and $\cos 2\theta$ on the LHS. We know that $\tan\theta$

is undefined for some angles θ. Thus the identity is also undefined for these θ, and hence is not valid for these angles. Also, for some θ, we might have division by zero in the LHS, which is not allowed. Thus the identity won't hold for these angles also.

Step 2 : ***Execute the strategy***

$$\begin{aligned}
\text{LHS} &= \frac{\sin\theta + 2\sin\theta\cos\theta}{1+\cos\theta+(2\cos^2\theta-1)} \\
&= \frac{\sin\theta(1+2\cos\theta)}{\cos\theta(1+2\cos\theta)} \\
&= \frac{\sin\theta}{\cos\theta} \\
&= \tan\theta \\
&= \text{RHS}
\end{aligned}$$

We know that $\tan\theta$ is undefined when $\theta = 90^\circ + 180^\circ n$, where n is an integer. The LHS is undefined when $1+\cos\theta+\cos 2\theta = 0$. Thus we need to solve this equation.

$$\begin{aligned}
1+\cos\theta+\cos 2\theta &= 0 \\
\Longrightarrow \quad \cos\theta(1+2\cos\theta) &= 0
\end{aligned}$$

The above has solutions when $\cos\theta = 0$, which occurs when $\theta = 90^\circ + 180^\circ n$, where n is an integer. These are the same values when $\tan\theta$ is undefined. It also has solutions when $1+2\cos\theta = 0$. This is true when $\cos\theta = -\frac{1}{2}$, and thus $\theta = \ldots - 240^\circ, -120^\circ, 120^\circ, 240^\circ, \ldots$. To summarise, the identity is not valid when $\theta = \ldots - 270^\circ; -240^\circ; -120^\circ; -90^\circ; 90^\circ; 120^\circ; 240^\circ; 270^\circ; \ldots$

Example 4: *Trigonometric Equations*

QUESTION

Solve the following equation for y without using a calculator:

$$\frac{1-\sin y-\cos 2y}{\sin 2y-\cos y} = -1$$

SOLUTION

Step 1 : ***Identify a strategy***

Before we are able to solve the equation, we first need to simplify the left-hand side. We do this by using the double-angle formulae.

Step 2 : ***Execute the strategy***

$$\begin{aligned} \frac{1-\sin y-(1-2\sin^2 y)}{2\sin y\cos y-\cos y} &= -1 \\ \Longrightarrow \quad \frac{2\sin^2 y-\sin y}{\cos y(2\sin y-1)} &= -1 \\ \Longrightarrow \quad \frac{\sin y(2\sin y-1)}{\cos y(2\sin y-1)} &= -1 \\ \Longrightarrow \quad \tan y &= -1 \\ \Longrightarrow \quad y = 135^\circ + 180^\circ n;\ n \in \mathbb{Z} & \end{aligned}$$

10.3 Applications of Trigonometric Functions

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Problems in Two Dimensions

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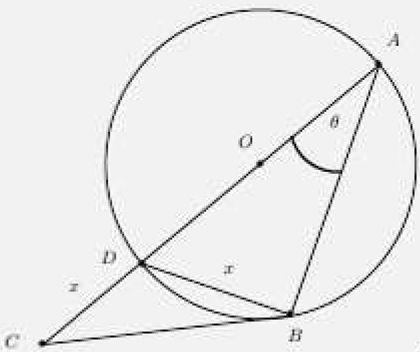
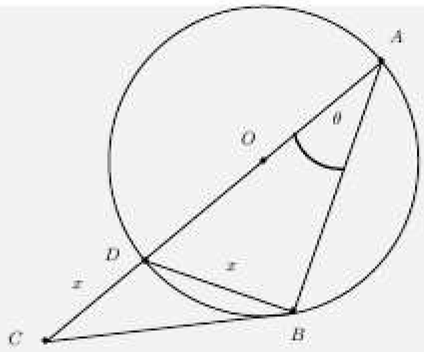
Example 5: Problem in Two Dimensions

QUESTION



For the figure below, we are given that $CD = BD = x$.

Show that $BC^2 = 2x^2(1 + \sin\theta)$.



SOLUTION

Step 1 : ***Identify a strategy***

We want CB, and we have CD and BD. If we could get the angle $B\hat{D}C$, then we could use the cosine rule to determine BC. This is possible, as $\triangle ABD$ is a right-angled triangle. We know this from circle geometry, that any triangle circumscribed by a circle with one side going through the origin, is right-angled. As we have two angles of $\triangle ABD$, we know $A\hat{D}B$ and hence $B\hat{D}C$. Using the cosine rule, we can get BC^2.

Step 2 : ***Execute the strategy***

$$A\hat{D}B = 180^\circ - \theta - 90^\circ = 90^\circ - \theta$$

Thus

$$\begin{aligned} B\hat{D}C &= 180^\circ - A\hat{D}B \\ &= 180^\circ - (90^\circ - \theta) \\ &= 90^\circ + \theta \end{aligned}$$

Now the cosine rule gives

$$\begin{aligned} BC^2 &= CD^2 + BD^2 - 2\,.\,CD\,.\,BD\,.\,\cos(B\hat{D}C) \\ &= x^2 + x^2 - 2\,.\,x^2\,.\,\cos(90^\circ + \theta) \\ &= 2x^2 + 2x^2\left[\sin(90^\circ)\cos(\theta) + \sin(\theta)\cos(90^\circ)\right] \\ &= 2x^2 + 2x^2\left[1\,.\,\cos(\theta) + \sin(\theta)\,.\,0\right] \\ &= 2x^2(1 - \sin\theta) \end{aligned}$$

Exercise 10 - 1

1. For the diagram on the right,

(a) Find $A\hat{O}C$ in terms of θ.

(b) Find an expression for:

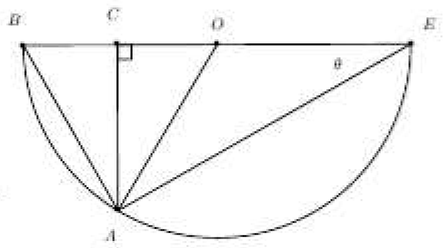
i. $\cos\theta$

ii. $\sin\theta$

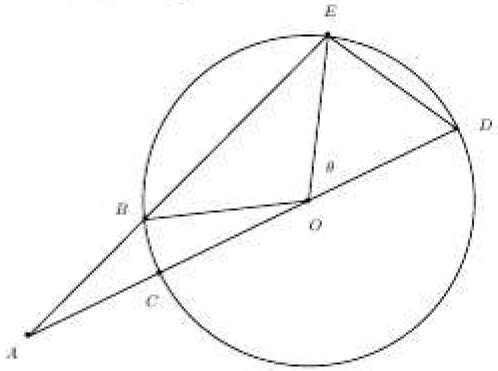
iii. $\sin 2\theta$

(c) Using the above, show that $\sin 2\theta = 2\sin\theta\cos\theta$.

(d) Now do the same for $\cos 2\theta$ and $\tan\theta$.



2. DC is a diameter of circle O with radius r. $CA = r$, $AB = DE$ and $D\hat{O}E = \theta$. Show that $\cos\theta = \frac{1}{4}$.



3. The figure below shows a cyclic quadrilateral with $\frac{BC}{CD} = \frac{AD}{AB}$.

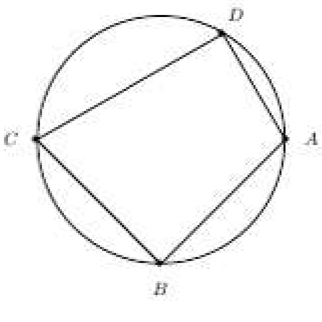
(a) Show that the area of the cyclic quadrilateral is $DC \,.\, DA \,.\, \sin\hat{D}$.

(b) Find expressions for $\cos\hat{D}$ and $\cos\hat{B}$ in terms of the quadrilateral sides.

(c) Show that $2CA^2 = CD^2 + DA^2 + AB^2 + BC^2$.

(d) Suppose that $BC = 10$, $CD = 15$, $AD = 4$ and $AB = 6$. Find CA^2.

(e) Find the angle $\hat{D}$ using your expression for $\cos\hat{D}$. Hence find the area of $ABCD$.



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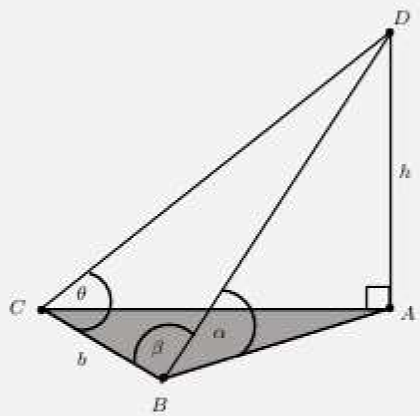
Problems in 3 Dimensions

EMCCP

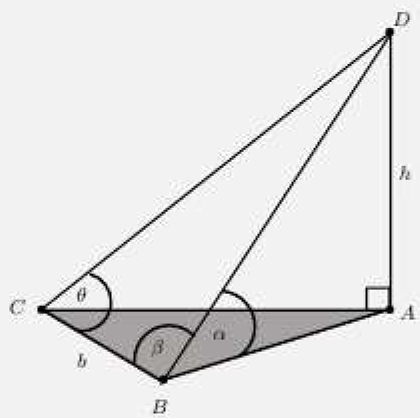
Example 6: Height of tower

QUESTION



D is the top of a tower of height h. Its base is at A. The triangle ABC lies on the ground (a horizontal plane). If we have that $BC = b$, $D\hat{B}A = \alpha$, $D\hat{B}C = \beta$ and $D\hat{C}B = \theta$, show that

$$h = \frac{b \sin\alpha \sin\theta}{\sin(\beta + \theta)}$$



SOLUTION

Step 1 : ***Identify a strategy***

We have that the triangle ABD is right-angled. Thus we can relate the height h with the angle α and either the length BA or BD (using sines or cosines). But we have two angles and a length for $\triangle BCD$, and thus can work out all the remaining lengths and angles of this triangle. We can thus work out BD.

Step 2 : ***Execute the strategy***

We have that

$$\begin{aligned} \frac{h}{BD} &= \sin\alpha \\ \Longrightarrow \quad h &= BD\sin\alpha \end{aligned}$$

Now we need BD in terms of the given angles and length b. Considering the triangle BCD, we see that we can use the sine rule.

$$\begin{aligned} \frac{\sin\theta}{BD} &= \frac{\sin(B\hat{D}C)}{b} \\ \Longrightarrow \quad BD &= \frac{b\sin\theta}{\sin(B\hat{D}C)} \end{aligned}$$

But $B\hat{D}C = 180^\circ - \beta - \theta$, and

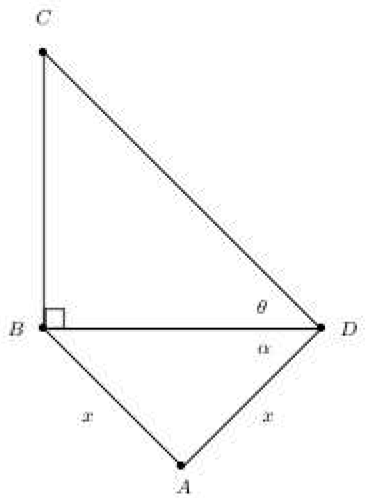
$$\begin{aligned} \sin(180^\circ - \beta - \theta) &= -\sin(-\beta-\theta) \\ &= \sin(\beta+\theta) \end{aligned}$$

So

$$\begin{aligned} BD &= \frac{b\sin\theta}{\sin(B\hat{D}C)} \\ &= \frac{b\sin\theta}{\sin(\beta+\theta)} \\ \therefore h &= BD\sin\alpha \\ &= \frac{b\sin\alpha\sin\theta}{\sin(\beta+\theta)} \end{aligned}$$

Exercise 10 - 2

1. The line BC represents a tall tower, with B at its foot. Its angle of elevation from D is θ. We are also given that $BA = AD = x$.



2. Find the height of the tower BC in terms of x, $\tan\theta$ and $\cos 2\alpha$.

3. Find BC if we are given that $x = 140\ \text{m}$, $\alpha = 21^\circ$ and $\theta = 9^\circ$.

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10.4 Other Geometries EMCCQ

Taxicab Geometry EMCCR

Taxicab geometry, considered by Hermann Minkowski in the 19th century, is a form of geometry in which the usual metric of Euclidean geometry is replaced by a new metric in which the distance between two points is the sum of the (absolute) differences of their coordinates.

Manhattan Distance EMCCS

The metric in taxi-cab geometry, is known as the *Manhattan distance*, between two points in an Euclidean space with fixed Cartesian coordinate system as the sum of the lengths of the projections of the line segment between the points onto the coordinate axes.

For example, the Manhattan distance between the point P_1 with coordinates $(x_1; y_1)$ and the point P_2 at $(x_2; y_2)$ is

$$|x_1 - x_2| + |y_1 - y_2| \tag{10.1}$$

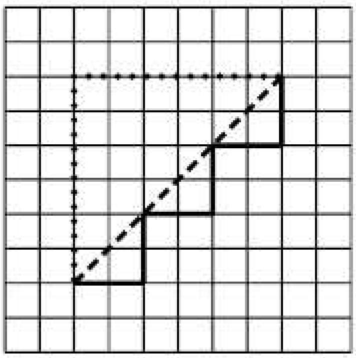

Figure 10.1: Manhattan distance (dotted and solid) compared to Euclidean distance (dashed). In each case the Manhattan distance is 12 units, while the Euclidean distance is $\sqrt{36}$

The Manhattan distance changes if the coordinate system is rotated, but does not depend on the translation of the coordinate system or its reflection with respect to a coordinate axis.

Manhattan distance is also known as city block distance or taxi-cab distance. It is given these names because it is the shortest distance a car would drive in a city laid out in square blocks.

Taxicab geometry satisfies all of Euclid's axioms except for the side-angle-side axiom, as one can generate two triangles with two sides and the angle between them the same and have them not be congruent. In particular, the parallel postulate holds.

A circle in taxicab geometry consists of those points that are a fixed Manhattan distance from the centre. These circles are squares whose sides make a $45°$ angle with the coordinate axes.

Summary of the Trigonometric Rules and Identities

EMCCT

Pythagorean Identity

$\cos^2\theta + \sin^2\theta = 1$

Cofunction Identities

$\sin(90^\circ - \theta) = \cos\theta$
$\cos(90^\circ - \theta) = \sin\theta$

Ratio Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Odd/Even Identities

$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$
$\tan(-\theta) = -\tan\theta$

Periodicity Identities

$\sin(\theta \pm 360^\circ) = \sin\theta$
$\cos(\theta \pm 360^\circ) = \cos\theta$
$\tan(\theta \pm 180^\circ) = \tan\theta$

Double Angle Identities

$\sin(2\theta) = 2\sin\theta\cos\theta$
$\cos(2\theta) = \cos^2\theta - \sin^2\theta$
$\cos(2\theta) = 1 - 2\sin^2\theta$
$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

Addition/Subtraction Identities

$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$
$\sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$
$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$
$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$
$\tan(\theta + \phi) = \frac{\tan\phi + \tan\theta}{1-\tan\theta\tan\phi}$
$\tan(\theta - \phi) = \frac{\tan\phi - \tan\theta}{1+\tan\theta\tan\phi}$

Area Rule

$\text{Area} = \frac{1}{2}bc\sin A$
$\text{Area} = \frac{1}{2}ab\sin C$
$Area = \frac{1}{2}ac\sin B$

Cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$
$b^2 = a^2 + c^2 - 2ac\cos B$
$c^2 = a^2 + b^2 - 2ab\cos C$

Sine Rule

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Chapter 10 End of Chapter Exercises

Do the following without using a calculator.

1. Suppose $\cos\theta = 0{,}7$. Find $\cos 2\theta$ and $\cos 4\theta$.
2. If $\sin\theta = \frac{4}{7}$, again find $\cos 2\theta$ and $\cos 4\theta$.
3. Work out the following:
   (a) $\cos 15^\circ$
   (b) $\cos 75^\circ$
   (c) $\tan 105^\circ$
   (d) $\cos 15^\circ$
   (e) $\cos 3^\circ\cos 42^\circ - \sin 3^\circ\sin 42^\circ$
   (f) $1 - 2\sin^2(22{,}5^\circ)$
4. Solve the following equations:
   (a) $\cos 3\theta \,.\, \cos\theta - \sin 3\theta \,.\, \sin\theta = -\frac{1}{2}$

   (b) $3\sin\theta = 2\cos^2\theta$

5. Prove the following identities
   (a) $\sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$
   (b) $\cos^2\alpha(1 - \tan^2\alpha) = \cos 2\alpha$
   (c) $4\sin\theta \,.\, \cos\theta \,.\, \cos 2\theta = \sin 4\theta$
   (d) $4\cos^3 x - 3\cos x = \cos 3x$
   (e) $\tan y = \frac{\sin 2y}{\cos 2y + 1}$
6. (Challenge question!) If $a + b + c = 180^\circ$, prove that

   $$\sin^3 a + \sin^3 b + \sin^3 c = 3\cos(a/2)\cos(b/2)\cos(c/2) + \cos(3a/2)\cos(3b/2)\cos(3c/2)$$

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