Term 1

Unit 5 Calculations with numbers in exponential form

In this unit you will:

- perform calculations with numbers in exponential form
- explore the order of operations with numbers involving exponents, and square and cube roots
- solve problems involving numbers in exponential form.

Getting started Order of operations

Complete these calculations. Use the rules for the order of operations.

1. $11 \times 2 + 5 \times 8$
2. $11 \times (2 + 5) \times 8$
3. $25 \times 4 - 3$
4. $25 \times (4 - 3)$
5. $88 - 24 \div 8$
6. $(88 - 24) \div 8$
7. $12 \div 12 \times 4 + 218 \times 0$
8. $12 \div 12 \times (4 + 218) \times 0$

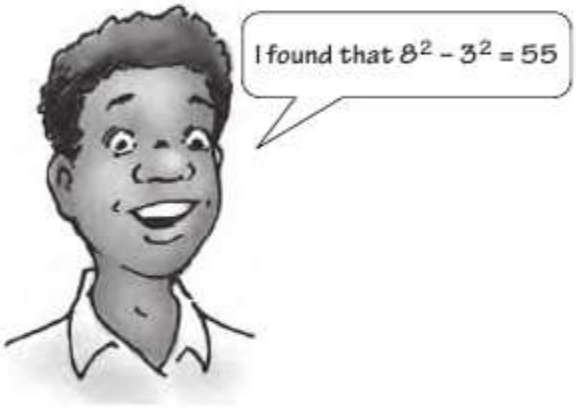
Key ideas

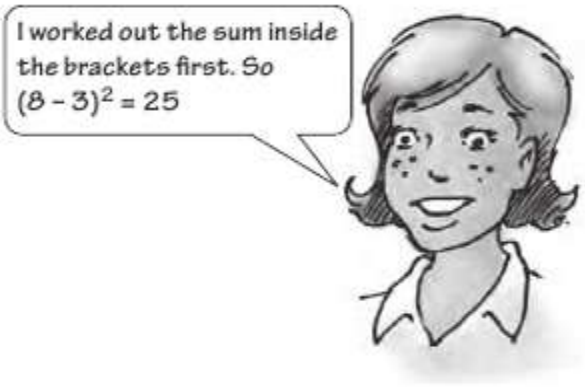
When expressions do not have exponents, the rules for the order of operations are:

1. Simplify the operations inside the brackets first.
2. Then do the multiplication and division operations. Work from left to right.
3. Finally do the addition and subtraction. Work from left to right.

Activity 5.1 Order of calculations involving exponents

Is $(8 - 3)^2$ the same as $8^2 - 3^2$?
Mthunzi and Gloria write down their solutions. Compare their solutions with your own.





Mthunzi

$$(8 - 3)^2 = (8 - 3) \times (8 - 3)$$
$$= 5 \times 5$$
$$= 25$$

Gloria

$$8^2 - 3^2 = (8 \times 8) - (3 \times 3)$$
$$= 64 - 9$$
$$= 55$$

We know that $25 \neq 55$ so $(8 - 3)^2 \neq 8^2 - 3^2$

Key ideas

We need to add a new rule:

1. Simplify the operations inside the brackets first.
2. *Simplify all exponents. Work from left to right.*
3. Then do the multiplication and division operations. Work from left to right.
4. Finally do the addition and subtraction. Work from left to right.

Worked example

Solve $(8 - 6)^3 \times 4^2 - 5$

SOLUTION

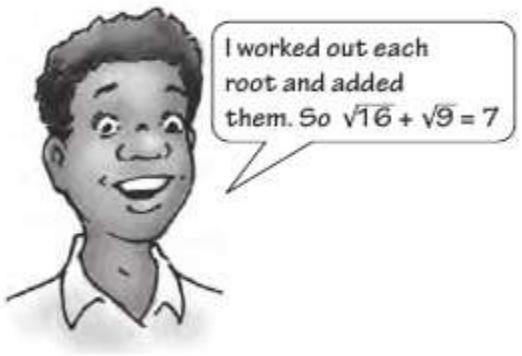
$$(8 - 6)^3 \times 4^2 - 5$$
$$= 2^3 \times 4^2 - 5$$
$$= 8 \times 16 - 5$$
$$= 128 - 5$$
$$= 123$$

Exercise 5.1 Calculations with exponents

1. Evaluate the following pairs of expressions:
   a) i) $(7 + 4)^2$ ii) $7^2 + 4^2$
   b) i) $(10 - 5)^2$ ii) $10^2 - 5^2$
   c) Are any of the pairs equal? Explain.
2. Now evaluate the following pairs of expressions:
   a) i) $(3 \times 2)^3$ ii) $3^3 \times 2^3$
   b) i) $(9 \div 3)^2$ ii) $9^2 \div 3^2$
   c) Are any of the pairs equal? Explain.
3. Evaluate the following expressions. Use your new rules.
   a) $(4 - 1)^2 \div 3$ b) $(4^2 - 1^2) \div 3$
   c) $8 + (2 \times 5) \times 3^4 \div 9$ d) $(8 + 2) \times 5 \times 3^4 \div 9$
   e) $(27 + 42) \div 3 - 5 \times 2^2$ f) $75 - (9 - 4)^2 \div 5$

Activity 5.2 Order of calculations involving roots

Is $\sqrt{16 + 9}$ the same as $\sqrt{16} + \sqrt{9}$? Mthunzi and Shaheeda write down their solutions as shown. Compare their solutions with your own.



Mthunzi

$\sqrt{16} + \sqrt{9} = 4 + 3$

$= 7$

Shaheeda

$\sqrt{16 + 9} = \sqrt{25}$

$= 5$

We know that $7 \neq 5$ so $\sqrt{16} + \sqrt{9} \neq \sqrt{16 + 9}$

Key ideas

- The root sign acts as a bracket to the expression under it.
- We need to add to our first and second rules:
  1. Simplify the operations inside the brackets and *under any root signs first.*
  2. Simplify all exponents. Work from left to right.
  3. Then do the multiplication and division operations. Work from left to right.
  4. Finally do the addition and subtraction. Work from left to right.

Worked example

How do we solve a problem like $\sqrt{65-16} \times 3^2 - (12 \div 4)$?

SOLUTION

$\sqrt{65-16} \times 3^2 - (12 \div 4)$
$= \sqrt{49} \times 3^2 - 3$
$= 7 \times 9 - 3$
$= 63 - 3$
$= 60$

Exercise 5.2 Roots and exponents

1. Evaluate the following pairs of expressions:
   a) i) $\sqrt{144+25}$ ii) $\sqrt{144}+\sqrt{25}$
   b) i) $\sqrt{169-25}$ ii) $\sqrt{169}-\sqrt{25}$
   c) Are any of the pairs equal? Explain.
2. Evaluate the following pairs of expressions:
   a) i) $\sqrt{9 \times 4}$ ii) $\sqrt{9} \times \sqrt{4}$
   b) i) $\sqrt[3]{64 \div 8}$ ii) $\sqrt[3]{64} \div \sqrt[3]{8}$
   c) Are any of the pairs equal? Explain.
3. Use your new rules to evaluate the following expressions:
   a) $10 \times \sqrt{16} - \sqrt{4}$ b) $10 \times (\sqrt{16} - \sqrt{4})$
   c) $(5 \times 2)^2 - \sqrt{61+3}$ d) $(4 \times 3)^2 - 9 \times \sqrt[3]{59+5}$
   e) $\sqrt[3]{125} \times 3^2 + 10$ f) $(2^4 - 12) \times (\sqrt[3]{27} + 7)$

4. For each of the following:
   i) write the expression in exponential form ii) calculate the answers.
   a) $3 \times 3 \times 3 + 2 \times 2$ b) $(4 \times 4 + 3 \times 3 \times 3 \times 3 + 3) \div (2 \times 2 \times 5)$
   c) $5 \times 5 \times 5 + 5 \times 5 \times 7 + 7 \times 7^2$
5. What is the difference between:
   a) 10^3 and 3×10 b) 6^4 and 4×6 c) 3^5 and 5×3.
6. Find the value:
   a) $5 + 5 + 5 + 5$ b) 5^4 c) $3 \times 5 + 2 \times 5$
   d) $5 \times 5 \times 5 \times 5$ e) 5×4

Summary

- If you have a calculator, check to see if it has $\sqrt{\ }$ and $\sqrt[3]{\ }$ keys. Use these to work out square and cube roots.
- The order of operations when working with exponents and roots:
  1. Simplify the operations inside the brackets and under any root signs first.
  2. Simplify all exponents. Work from left to right.
  3. Then do the multiplication and division operations. Work from left to right.
  4. Finally do the addition and subtraction. Work from left to right.

Check what you know

1. $\sqrt{16} - \sqrt{4}$ is equal to:
   a) $\sqrt{20}$ b) 4 c) $\sqrt{12}$ d) 2
2. $\sqrt{12}$ is equal to:
   a) $\sqrt{6} + \sqrt{6}$ b) 6 c) $\sqrt{3 \times 4}$ d) 4
3. Calculate:
   a) $3^2 + 4^2$ b) $5^3 - 10^2$ c) $10^2 \div 2^2$
   d) $5^3 - 9^2$ e) $8^2 \div 4^2$ f) $3^2 - 1^6 + 2^3$
4. Evaluate:
   a) $\sqrt{49} + \sqrt{9} + 4^2$ b) $(\sqrt{49} + \sqrt{9}) + 4^2$
   c) $\sqrt{25} + 2^2 \times (5^2 \div 5)$ d) $(6 \times 5)^2 \div 3^2 \times \sqrt[3]{25 + 2}$
   e) $\sqrt[3]{8} \times 4^2 + 18$ f) $(2^5 - 22) \times (\sqrt[3]{27} + 7)$