

Unit 2 Multiples and factors

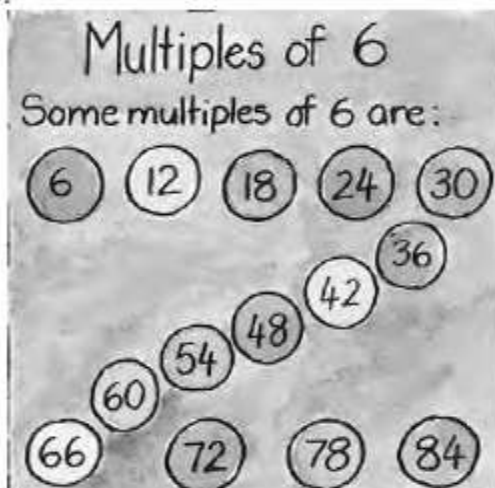
In this unit you will:

- recognise, name and find factors, prime factors and the highest common factor (HCF) of a group of numbers
- recognise, name and find multiples and the lowest common multiple (LCM) of a group of numbers.

Getting started Multiples, factors and prime numbers

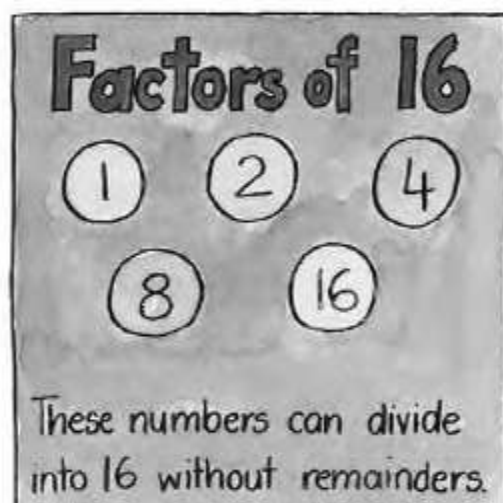
1. The picture shows the first 14 multiples of 6.

- Write down the first ten multiples of 9.
- Write down the numbers that are multiples of both 6 and 9.
- Explain what a multiple is.



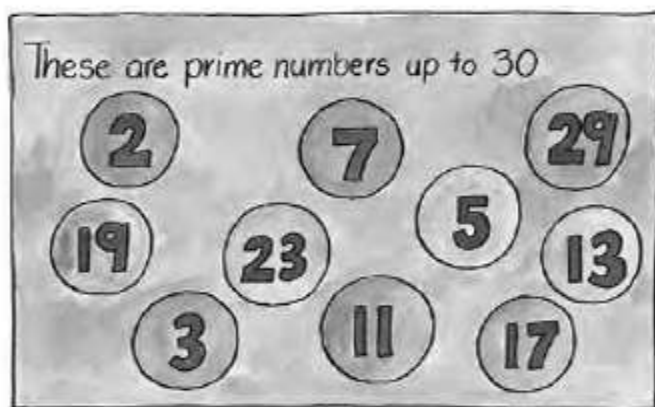
2. This picture shows the factors of 16.

- Write down the factors of 12.
- Write down the factors that are common to both 12 and 16.
- Explain what a factor is.



3. This picture shows prime numbers up to 30.

- a) Write down all the factors for the prime numbers in the picture.
- b) Write down all the prime numbers between 30 and 50.
- c) Explain what a prime number is.
- d) Which of the following



34 29 76 57 99 23

4. List the factors of the following numbers:

- a) 20 b) 36 c) 64 d) 98

Activity 2.1

Finding factors of 2-digit and 3-digit whole numbers

Ntsiki and Thobeka want to find all the factors of 128. Which method do you prefer?

Ntsiki writes:

I want to find the factors of 128. I will look for all the pairs of numbers that multiply to give 128.

$1 \times 128 = 128$; $2 \times 64 = 128$; $4 \times 32 = 128$; $8 \times 16 = 128$

All the numbers in the pairs will be the factors.

Thobeka writes:

I want to find the factors of 128. I will find all the numbers that divide into 128 without leaving a remainder.

I can divide 128 by 1: $128 \div 1 = 128$ and $128 \div 128 = 1$. So 1 and 128 are factors of 128.

I can divide 128 by 2: $128 \div 2 = 64$ and $128 \div 64 = 2$. So 2 and 64 are factors of 128.

I can divide 128 by 4: $128 \div 4 = 32$. So 4 and 32 are factors.

I can divide 128 by 8: $128 \div 8 = 16$. So 8 and 16 are factors.

Ntsiki and Thobeka list the factors of 128 as: 1; 2; 4; 8; 16; 32; 64; 128.

Key ideas

- A factor of a whole number divides into that number without leaving a remainder. Example: The factors of 12 are: 1, 2, 3, 4, 6, 12.
- When we arrange the factors from smallest to biggest, a pattern forms showing the pairs of factors that are multiplied to give the number. Example:



Exercise 2.1 Factors

1. Find all the factors of the following numbers.
a) 36 b) 21 c) 120 d) 200 e) 921 f) 60
2. Fill in the missing factors.
a) The factors of 24 are 1; 2; ; 4; ; 8; ; 24
b) The factors of 144 are ; ; ; ; ; ; ; 12; 16; 18; 24; 36; 48; 72; 144

Activity 2.2 Finding common factors and the HCF

Answer the questions.

1. You can see the factors and the *common factors* of 28 and 42.
Factors of 28: 1; 2; 4; 7; 14; 28
Factors of 42: 1; 2; 3; 4; 6; 7; 14; 21; 42
Common factors of 28 and 42: 1; 2; 7; 14.
a) What do we mean by 'the common factors of 28 and 42'?
b) Which of the common factors of 28 and 42 is the biggest or highest?
2. a) Find the common factors of 180 and 675.
b) Which of the common factors is the highest common factor?

Key ideas

- Common factors of two numbers are the factors that are the same for both.
- A common factor of three whole numbers must be a factor of all three.
- We write the highest common factor as HCF.
- 1 is a common factor for all numbers. We do not list it as a common factor.

Worked example

1. Find the common factors of 8, 12 and 20.
2. Find the HCF of 8, 12 and 20.

SOLUTION

1. Factors of 8: 1; 2; 4; 8
Factors of 12: 1; 2; 3; 4; 6; 12
Factors of 20: 1; 2; 4; 5; 10; 20
The common factors of 8, 12 and 20 are 2 and 4.
2. The **highest** common factor of 8, 12 and 20 is 4.

Exercise 2.2 Factors and the HCF

1. Find the highest common factor (HCF) of the following:
a) 36 and 120 b) 120 and 200 c) 21, 75 and 921
2. a) What are the common factors of 156 and 64?
b) What is the highest common factor of 156 and 64?
3. Two whole numbers multiply to give 72. What whole numbers can they be? List all possible answers.

Activity 2.3 Multiples and the Lowest Common Multiple

1. We know that:
 - *Multiples* of 200 are 200, 400, 600, 800, and so on.
 - *Multiples* of 150 are 150, 300, 450, 600, and so on.
 - a) What will the next five multiples of 200 and 150 be? Write them down.
 - b) Which multiples of 200 and 150 are *common multiples*? Common multiples means multiples that are the same for both 200 and 150.
 - c) Which one is the smallest or *lowest common multiple*?
2.
 - a) Write down the first six multiples of 6, 8 and 12.
 - b) List the common multiples.
 - c) Which one is the lowest common multiple?

Key ideas

- A multiple of a number is $1 \times$ the number or $2 \times$ the number or $3 \times$ the number, or $\dots 97 \times$ the number, etc. Example: 48 is a multiple of 6 because 48 is 8×6 .
- Common multiples of two or more numbers are the multiples that are the same for the numbers. Example: A common multiple of 2, 4 and 5 is 20.
- We write the lowest common multiple as LCM.

Worked example

1. Find the first five multiples of 120, 150 and 300.
2. Find the LCM of 120, 150 and 300.

SOLUTION

1. Multiples of 120: 120, 240, 360, 480, 600
Multiples of 150: 150, 300, 450, 600, 750
Multiples of 300: 300, 600, 900, 1 200, 1 500
2. The lowest common multiple of 120, 150 and 300 is 600.

Exercise 2.3

Multiples and the LCM

1. List the first 10 multiples of the following numbers:
a) 9 b) 10 c) 12 d) 15 e) 20 f) 25
2. Find the lowest common multiple (LCM) of the following groups of numbers:
a) 10 and 15 b) 3 and 12 c) 3 and 5 d) 2, 3 and 8
e) 2, 3 and 4 f) 3, 6 and 9 g) 15 and 30 h) 20 and 25
3. Is 104 a multiple of 13 or 4 or both? Explain your answer.
4. a) Find two whole numbers that will be multiples of both 11 and 15.
b) Find the LCM of 11 and 15.

Activity 2.4

Whole numbers as products of prime factors

Express 12 as a product of its prime factors.

The factors of 12 are 1; 2; 3; 4; 6; 12.

Of these factors, only 2 and 3 are prime numbers.

$2 \times 3 = 6$ but if we multiply by 2 again we get 12.

We can write $12 = 2 \times 2 \times 3$.

We say that 2 and 3 are the prime factors of 12.

- Express 625 as a product of its prime factors.
- Compare your answer to Nandipha's and Francina's answers.

Nandipha's answer:

$$625 \div 5 = 125$$

$$125 \div 5 = 25$$

$$25 \div 5 = 5$$

$$5 \div 5 = 1$$

$$625 = 5$$

$$5 \times 5 \times 5 \times 5 = 625$$

Francina's answer:

$$5 \mid 625$$

$$5 \mid 125$$

$$5 \mid 25$$

$$5 \mid 5$$

$$1$$

$$625 = 5 \times 5 \times 5 \times 5$$

Whose method do you prefer?

Key ideas

- A prime number has only two factors – the number itself and 1.
- We can write any number as the product of its prime factors. Sometimes we may need to use the factor more than once.
- Numbers that have more than two factors are not prime numbers. We call them *composite numbers*. For example, 4, 6, 8, 9, etc. are composite numbers.

Exercise 2.4 Prime factors

- Draw a 10 by 10 grid and fill it with numbers 1 to 100. Circle all the prime numbers. What digits do most prime numbers end in?
- Which of the following are prime numbers?
a) 241 b) 295 c) 148 d) 133 e) 269 f) 121
- Look at the table on the next page. It shows the factors of whole numbers between 30 and 40.
a) Complete columns B and C of the table.
b) Can you write any whole number as a product of its prime factors?

Number	A. Factors	B. Prime factors	C. Product of prime factors
30	1; 2; 3; 5; 6; 10; 15; 30	2; 3; 5	$30 = 2 \times 3 \times 5$
31	1; 31	31	$31 = 31$
32	1; 2; 4; 8; 16; 32	2	$32 = 2 \times 2 \times 2 \times 2 \times 2$
33	1; 3; 11; 33		
34	1; 2; 17; 34		
35	1; 5; 7; 35		
36	1; 2; 3; 4; 9; 6; 12; 18; 36		

4. Write the following numbers as products of their prime factors.

- a) 220 b) 100 c) 210 d) 126 e) 924 f) 105

Activity 2.5

Using the product of prime factors to find the HCF

To find the highest common factor of two or more numbers, we list all their factors to see which are common so that we can choose the highest one. For example, to find the highest common factor of 120, 300 and 900:

Factors of 120 1; 2; 3; 4; 5; 6; 8; 10; 12; 15; 20; 24; 30; 40; 60; 120

Factors of 300 1; 2; 3; 4; 5; 6; 10; 12; 15; 20; 25; 30; 50; 60; 75; 100; 150; 300

We can see that 1; 2; 3; 4; 5; 6; 10; 12; 15; 20; 30 and 60 are all common factors. The HCF is 60.

Now we will use the product of prime factors to find the HCF.

Step 1: First write the numbers as products of their prime factors.

Step 2: Circle the common prime factors.

$$120 = 2 \times 2 \times 2 \times 3 \times 5$$

$$300 = 2 \times 2 \times 3 \times 5 \times 5$$

$$900 = 2 \times 2 \times 3 \times 3 \times 5 \times 5$$

Step 3: Multiply the **common** prime factors of the three numbers to find the HCF.

Step 4: The common prime factors are 2, 2, 3 and 5.

$$\text{The HCF} = 2 \times 2 \times 3 \times 5 = 60.$$

Exercise 2.5 Finding the HCF through prime factors

- Find the highest common factor (HCF) of the following groups of numbers:
a) 220 and 100 b) 210 and 220 c) 105 and 210
d) 126 and 924 e) 220 and 924 f) 126, 210 and 924
- Priscilla is making up parcels of clothes for Aids orphans. She has 16 jerseys and 24 shirts. She wants each of the bundles to have the same number of jerseys and the same number of shirts. Using the HCF, find the maximum number of parcels that Priscilla can make up.

Summary

Multiples and factors

- The factors of a whole number are the numbers that can divide into it without leaving a remainder.
- A common factor of three whole numbers must be a factor of all three whole numbers. We call the highest common factor the HCF.
- A multiple of a number is $1 \times$ the number, or $2 \times$ the number, or $3 \times$ the number, etc.
- Common multiples of two numbers are the multiples that are the same for the two numbers. We call the lowest common multiple the LCM.
- A prime number has only two factors – the number itself and 1.
- We call numbers that have more than two factors *composite numbers*.
- We do not consider the number 1 a prime number. This is because it only has one factor, being 1.
- We can write any number as the product of its prime factors. Sometimes we may need to use a factor more than once.
- We can use the product of prime factors to find the HCF.

Check what you know

- Find all the factors of the following numbers:
a) 18 b) 24 c) 28 d) 104 e) 256
- a) Find three common factors of 140, 350 and 105.
b) What is the highest common factor of these three numbers?
- List the first ten multiples of the following numbers:
a) 18 b) 20 c) 25 d) 35 e) 40 f) 100

4. Find the lowest common multiple (LCM) of the following groups of numbers:
a) 2 and 5 b) 20 and 100 c) 9 and 5 d) 3, 5 and 10
5. Write the following as products of their prime numbers:
a) 252 b) 350 c) 88 d) 264 e) 396
6. Find the highest common factor (HCF) of the following groups of numbers.
a) 252 and 350 b) 88 and 264 c) 264 and 396
d) 252 and 900 e) 396, 252 and 900 f) 88, 264 and 900
7. Fay-yaadh is using two different strings of flashing lights to decorate his house. One string flashes every 9 seconds. The other string flashes every 15 seconds. When Fay-yaadh switches on both strings, they both flash at the same time. Work out how long it will be before they flash at the same time again. Use the LCM.

