Term 1

Unit 3 Ratio, rate and finance

In this unit you will:

- solve ratio problems where two or more quantities of the same kind are compared
- solve rate problems where two different kinds of quantities are compared
- solve financial problems involving whole numbers, percentages and decimal fractions.

Getting started Concentrate

Read the label on the bottle of juice concentrate. It gives you the instructions for mixing the juice. You want to mix a jug of juice.

1. How many parts of the jug of juice will be concentrate?
2. How many parts will be water?
3. How many parts are there all together?
4. What fraction of the juice will be concentrate?

If you said $\frac{2}{5}$, you are right! We can write the fraction $\frac{2}{5}$ as the ratio 2 : 5.

The ratio describes the relationship between the quantities of concentrate and juice. For every **2** parts of concentrate there are **5** parts of juice.

5. What fraction of the juice is water? Write this fraction as a ratio.
6. What does the ratio 2 : 3 represent?
7. You want to make ten litres of juice for a party. How many litres of concentrate will you need to buy?
8. A bottle of juice concentrate costs R17,50. How much will you need to budget for juice?

Activity 3.1 Ratio

A teacher asks a class of 48 learners if they play sport. He records the results in a table:

	Yes	No	TOTAL
Do you play sport?	32	16	48

The learners write about the results:

Anele writes:

The number of learners playing sport compared to the number of learners not playing sport is 32 to 16. I write this ratio as 32 : 16.

Grace writes:

The number of learners playing sport compared to the total number of learners in the class is 32 out of 48. I write this ratio as $\frac{32}{48}$ or 32 : 48.

1. Simplify the ratio 32 : 16.
2. Find an equivalent fraction to $\frac{32}{48}$
3. Write the fraction $\frac{32}{48}$ in its simplest form.
4. Now write the ratio 32 : 48 in its simplest form.
5. Write a ratio to compare the number of learners not playing sport to the total number of learners in the class.

Key ideas

- A ratio is a comparison of two similar quantities. We use the same units to measure the quantities. We can write ratios without including the units.
- We write ratios with the ' : ' symbol. Example: In a class of 20 boys and 30 girls, the ratio of boys to girls is 20 : 30. We can simplify this to 2 : 3. We read this as '2 is to 3'.
- We can also write ratios as fractions.

Worked example

SuperFood strawberry jam is made with fruit, sugar and water. The table below shows how much of each ingredient is in the jar.

	Small
Size of jar	200 g
Fruit	120 g
Sugar	20 g
Water	60 g

1. What fraction of the contents of the jar is a) fruit, b) sugar and c) water? Write the fraction in its simplest form.
2. Write the fraction of the contents of the jar that is a) fruit, b) sugar and c) water as a ratio.

SOLUTION

1. a) Fruit: $\frac{120\text{ g}}{200\text{ g}} = \frac{3}{5}$ b) Sugar: $\frac{20\text{ g}}{200\text{ g}} = \frac{1}{10}$ c) Water: $\frac{60\text{ g}}{200\text{ g}} = \frac{3}{10}$
2. a) 3 : 5 b) 1 : 10 c) 3 : 10

Exercise 3.1 Ratio

1. For your class, write the ratio of:
   a) boys to girls
   b) boys to the whole class
   c) girls to boys
   d) girls to the whole class.
2. Thabo kept a diary of his activities during the week. He then drew up a table to show an average day. Look at his table:

Activity	Time
Sleeping	8 hours
At school	7 hours
Doing homework	1 hour
Playing sport	2 hours
Watching TV	4 hours
Eating	2 hours

a) What is the ratio of time spent doing homework to the time spent at school?

b) For what fraction of the day does Thabo sleep?

c) Which three activities have time in the ratio 1 : 4?

d) What is the relationship between the time Thabo spends at school and the time he spends playing (excluding eating and sleeping)?

Activity 3.2 Rate

Ashley invites all her cousins to come to her birthday party at her house in Pinelands in Cape Town. Use the table below to discuss and answer the questions that follow.

Cousin	Travel from	Distance travelled	Time
Melanie	Perth, Australia	12 000 km	24 hours
Tamson	Centurion, SA	1 200 km	4 hours
Amy	Plumstead, SA	12 km	12 minutes
Simeon	Pinelands, SA	2 km	20 minutes

1. a) How do you think Melanie travelled to Ashley's party?

   b) How fast did Melanie travel, i.e. how many kilometres an hour?

2. Tamson also comes to the party by plane. How fast does her plane travel?

3. a) Compare how long Melanie and Tamson take to get to the party. Use a ratio.

   b) Simplify the ratio as much as possible. (*Hint:* Simplify by dividing both numbers by their highest common factor).

4. a) How do think Amy and Simeon got to the party?

   b) At what rate does each of them travel? (Remember to include the units).

Key ideas

- Melanie flies by aeroplane at 500 kilometres per hour to Ashley's party. This is the speed or rate at which she travelled. This tells us the relationship between the distance Melanie covered and how long it took her. We write this relationship as:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

- Fractions can represent the relationship between different things, e.g. between distance and time. We call this relationship a rate.
- A rate shows us the relationship between quantities that are not the same. For example, distance and time or rands and kilograms. Rates must always include units such as km/h or R/kg (say: rands per kilogram). The '/' is a symbol for saying 'per'.

Worked example

Star Supermarket sells apricot jam in three different size tins: small, medium and large. The table below shows the size and cost of each tin.

	Small	Medium	Large
Size of tin	200 g	450 g	1 kg
Cost of tin	R8,00	R9,00	R25,00

Which tin is the best value for money? Explain your answer.

SOLUTION

Small tin: 200 g for R8

$\frac{200}{8} = 25$ grams per rand

Medium tin: 450 g for R9

$\frac{450}{9} = 50$ grams per rand

Large tin: 1 kg for R25

$\frac{1\,000}{25} = 40$ grams per rand

The medium-sized tin is the best value for money. You get the most grams per rand from this tin.

Exercise 3.2 Rate

1. Complete the table:

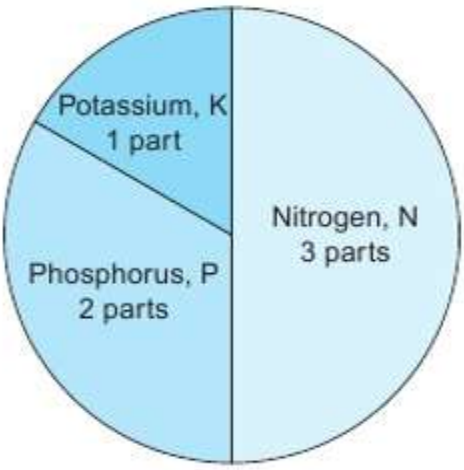
Distance (km) =	Speed (km/h) ×	Time (h)
a)	70	2
495	55	b)
c)	110	11
450	d)	9

2. Cape Town is 1 200 kilometres from Johannesburg.
   a) A taxi takes 15 hours to make the trip. How fast does it travel?
   b) How fast is it supposed to travel to get back in 10 hours?
   c) If the taxi only travels at 80 km/h for the first 10 hours of the journey, how far does it still have to go? If the trip takes 14 hours altogether, how much faster did the taxi travel for the last part of the journey?

Activity 3.3 Sharing a whole in a given ratio

A well-known ratio of the chemicals nitrogen (N), phosphorus (P) and potassium (K) in fertiliser, is the ratio N : P : K = 3 : 2 : 1.

1. How many parts are there in total in the mixture? Use the pie chart to help you.
2. What fraction of the mixture will be N?
3. You have 1 250 g of the N, P and K mixture. How many grams of N are there?
4. How many grams of P and how many grams of K are there in the 1 250 g mixture?



Key ideas

- The ratio 3 : 2 : 1 means that a total of $3 + 2 + 1 = 6$ parts make up the whole 1 250 g.
- In fraction form, this means that $\frac{3}{6}$ of 1 250 g will be N.
- Calculate: $\frac{3}{6}$ of $1\,250 = \frac{3}{6} \times \frac{1\,250}{1} = \frac{3\,750}{6} = 625$ g of N.
- The fraction $\frac{3}{6}$ tells us that 3 parts out of a total of 6 parts is N (nitrogen).
- When you share an amount in a given ratio, first write the ratio in fraction form. Then multiply the amount by the fractions in the ratio.

Exercise 3.3 Sharing wholes

1. The girl : boy ratio in a class of 40 learners is 3 : 2.
   a) Work out the number of boys and girls in the class.
   b) Check your answer. Add the number of boys and the number of girls. Do you get 40?
2. An old man died. He left 90 cows for his three daughters. The community has to share the cows between the eldest daughter, the second eldest daughter and the youngest daughter in the ratio 5 : 3 : 1. How many cows will each daughter get?

Activity 3.4 Managing a school musical – Calculating income

Jaylow High School musical a hit!

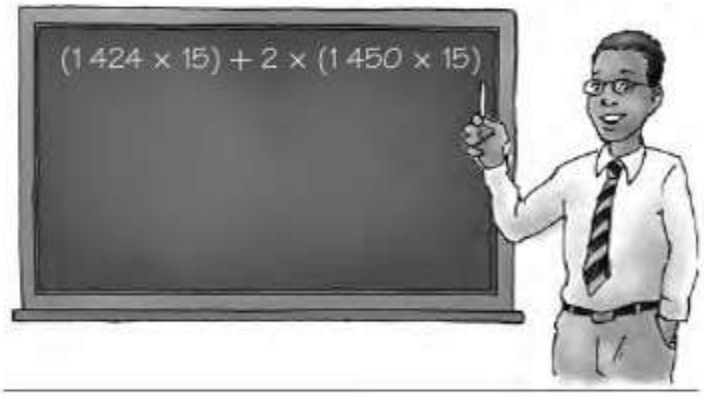
Jaylow High School staged a musical this week to celebrate its tenth anniversary. For months before the event, the learners and teachers worked hard to create a perfect performance. They certainly succeeded! The musical was staged for a full week, with eight shows.

There are 1 450 tickets available for each show. The tickets cost R15 each.

RECORD OF TICKET SALES

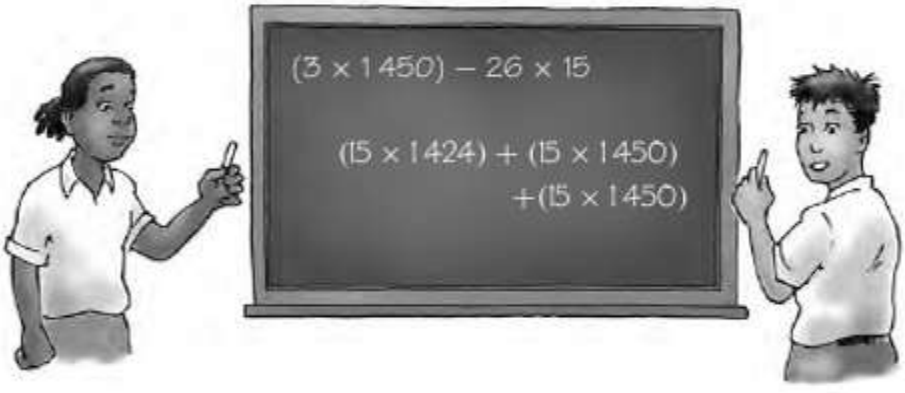
Mon 27 May:	49 tickets not sold	**Tues 28 May:**	13 tickets not sold
Wed 29 May:	sold out	**Thurs 30 May:**	28 tickets not sold
Fri 31 May:	sold out	**Sat. morning:**	26 tickets not sold
Sat. afternoon:	sold out	**Sat. evening:**	sold out

1. Mr Rezandt is the Head of Finance at Jaylow High. He writes an expression to calculate the income from ticket sales on the Saturday:



Calculate the income from ticket sales on the Saturday. Use Mr Rezandt's expression.

2. Two learners wrote different expressions to calculate the income from ticket sales on the Saturday:



a) Do the learners and Mr Rezandt all get the same answer?

b) Another learner wrote:

$15 \times (1\,426) + 1\,450 + 1\,450$

Is this expression correct?

3. Donald wrote the following expression to calculate the income from ticket sales on the Saturday:

$15 \times 1\,450 \times 3 - 26$

a) Calculate Donald's answer. Use the rules for multiple operations.

b) Explain why Donald did not get the same answer as the others.

c) Rewrite Donald's expression so that he also gets an answer of R64 860.

4. a) Write an expression to calculate the total income from ticket sales for all eight performances of the musical.

b) Calculate the total income for all eight performances of the musical.

Activity 3.5 Calculating profit or loss

1. Mr Rezandt wants to calculate the school's total profit. The profit will be the amount left once the school has covered all the expenses from the total income. Mr Rezandt has to deduct the total cost of hiring the auditorium. He also has to deduct the cost of props from the ticket sales.

Money paid out:
Props – R5 250
Hire of auditorium
Mon to Thurs = R380 per night
Fri = R580
Sat = R1 400

a) Write a suitable expression to calculate the total profit made by the school, then calculate the total profit of the musical.

b) Compare your expression to your friend's. Did you get the same answer?

2. If only 67 tickets were sold for each show:

a) What would the total income from ticket sales for all eight performances of the musical have been?

b) By how much would the expenses be greater than the total income? This amount will tell us what the school's loss would have been.

Key ideas

- Profit means financial gain. In particular, profit means the difference between the amount earned (income) and the amount spent in buying, operating or producing something (expenses). We say that:
  Profit = total income − expenses.
- When our income is less than our expenses, we have made a financial loss instead of a gain.

Exercise 3.4 Calculating income, profit and loss

Supa Supermarket ordered 750 two-litre bottles of milk for the weekend. They paid the supplier R6 000 for the milk. They sell the milk at R14 per bottle.

1. Work out the profit. Use a calculator.
2. Mrs Moonsamy bought R238 worth of milk for the school's feeding scheme. How many litres did she buy? (Be careful – litres, not bottles!)
3. On the Sunday evening, Supa Supermarket reduced the price of the remaining 23 bottles of milk. Mr Isaacs bought seven bottles of milk, a tub of margarine at R11 and a piece of cheese for R15. He paid with a R100 note. He got R39 change.
   a) What was the total cost of Mr Isaacs' groceries?
   b) What was the reduced price of one bottle of milk?
   c) What loss did the supermarket make on the reduced bottles of milk?
4. Calculate the total profit made by the supermarket selling milk, after reducing the remaining stock.

Key ideas

A budget is a financial plan. It shows our expected income and expenses for a period of time.

Activity 3.6 Budgeting

Sam wants to save up to travel home for the holidays. The table below shows the budget Sam makes of his weekly expenses:

	Expenses	Income
Weekly salary		R1 150
Rent	R385	
Food	R420	
Cellphone	R65	
Electricity	R40	
Transport	R130	
TOTAL		

1. a) Calculate Sam's total weekly expenses.
   b) How much does Sam save each week?
2. It will cost Sam R880 to travel home for the holidays. Sam needs a further R3 000 for presents for his family. He also needs R2 500 to spend while he is at home.
   a) How many weeks will it take for Sam to save up for his holiday?
   b) Will Sam be able to afford to take a three-week holiday each year? Explain.
   c) How much does Sam need to save each week in order to afford his holiday? Round your answer off to the nearest rand.

Exercise 3.5 Working with budgets

Nokuthula is making biscuits to sell at a food market. Look at her budget in the table alongside. Answer the questions.

1. Nokuthula makes a batch of 100 biscuits. What are her total expenses?
2. If Nokuthula sells all the biscuits at R3 each, what is her profit?
3. If Nokuthula only sells half of the biscuits, what is her profit?
4. How many biscuits does Nokuthula have to sell before she starts making a profit?

For 100 biscuits	Expenses	Income
@ R3 each		
Butter	R27	
Flour	R9	
Castor sugar	R6	
Sweets to decorate	R20	
Icing sugar	R7	
TOTAL		

Activity 3.7 Accounts, loans and simple interest

Zoliswa borrows R5 000 from the bank in the form of a loan. The bank says that Zoliswa must pay the money back over a period of six years. She has to pay simple interest of R750 per year.

Interest is the amount of money that you pay to borrow money. Simple interest is interest calculated as a set amount to be added to the original amount borrowed or loaned each year.

1. a) How much interest will Zoliswa have to pay over the period of the loan?
   b) How much money will Zoliswa have to pay back altogether?
   c) How many months are there in the time period of the loan?
   d) How much money will Zoliswa have to pay each month to pay back the total amount of money she owes? Round off to the nearest cent.

2. Zoliswa buys some new clothes on her account. Read her statement and answer the questions.

Best FASHION Bargains

STATEMENT	Zoliswa Manqina	
Jeans		R199
Casual top		R59
Hoodie		R79
TOTAL		R337
Payment within 30 days	R337	
Late payment	R40 per month added	
Payment plan – 3 months	R145 per month	

   a) If Zoliswa is late with her payment by two months, how much will she end up paying for the clothes?
   b) If Zoliswa takes the three-month payment plan, how much will she end up paying for the clothes?
   c) How much money can Zoliswa save by paying her account straight away instead of taking the three-month payment plan?
   d) Zoliswa's mother gives her a discount voucher for Best Fashion Bargains. The voucher says that Zoliswa can get a R30 discount if she pays straight away. How much would Zoliswa pay?

Key ideas

- When you work with money, round your answer off to the nearest cent.
- Interest is the amount of money that you pay to borrow money.
- Simple interest is interest calculated only on the original amount you borrowed.
- A discount is an amount of money that is subtracted from the usual amount.

Exercise 3.6 Accounts, interest and savings

1. Cyril opens a savings account at the bank. He deposits R3 000. Each year he earns simple interest of R360.
   a) How much money will Cyril have in his account after five years?
   b) How many years will it take for his money to double?
2. Charlene borrows R2 500. Her simple interest charge is R250 per year. She has to pay back her loan over three years. What are Charlene's monthly instalments?
3. Devide's yearly school fees are R1 428. The school offers a discount of R108 if fees are paid over three months.
   a) What is the monthly amount if school fees are paid over a year?
   b) What is the monthly amount if school fees are paid over three months?

Summary

- We can write ratios and rates as fractions.
- Ratios show the relationship between quantities with the same units.
- Rates show the relationship between quantities with different units.
- We can write ratios as two numbers separated by a colon without units.
- Rates are a single value. We always write rates with units.
- We always simplify ratios to the smallest whole numbers.
- We make use of the following two equations for problems of distance, speed and time:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{distance} = \text{speed} \times \text{time}$$

- When you share an amount in a given ratio, first write the ratio in fraction form. Then multiply the amount by the fractions in the ratio.

- A budget is a financial plan. It shows our expected income and expenses for a period of time.
- Profit means financial gain.
- When our income is less than our expenses, we have made a financial loss.
- Interest is the amount of money that you pay to borrow money.
- Simple interest is interest calculated only on the original amount.
- A discount is an amount of money that is subtracted from the given amount.

Check what you know

1. The ratio of women engineers to men engineers in a construction company is 2 : 7.
   a) There are six women engineers. How many men engineers are there in the company?
   b) How many engineers are there in the company altogether?
   c) What fraction of the total number of engineers are women?
   d) The company decides to improve their gender equality. The company wants to change the ratio of women engineers to men engineers to 2 : 5. The company cannot afford to employ more than 28 engineers in total. When the company achieves this ratio, how many women engineers and men engineers would they have?
2. The Cape Argus Pick 'n Pay Cycle Race is 110 kilometres long. How fast must I cycle to complete the race in five hours?
   a) I average 25 km/h. How long will I take to finish?
   b) 1 000 people complete the race. They get cola to drink at the end. They drink 250 litres of cola altogether. How much cola is that per person?
3. Together, the ages of a father, son and grandson add up to 100. Their ages are in the ratio 13 : 6 : 1. How old are they?
4. Mrs Khumalo has a budget of R1 200 per week for groceries at a pre-school. Each day she buys:
   - 5 loaves of bread @ R9 each
   - 10 litres of milk @ R14 per two-litre bottle
   - 2 kilograms of cheese @ R41 per kilogram
   - 1 pocket of oranges @ R26

   a) Write an expression and calculate how much Mrs Khumalo's groceries costs each day.

b) Write an expression and calculate how much money would be left over at the end of each week.

5. Look at the table below. It shows the weekly budget for running the pre-school.

Weekly budget	Expenses	Income
Pre-school fees		R48 per child per week
Food	R1 200	
Materials	R200	
Two assistants	R1 500	
TOTAL		

a) What are the expenses involved in running the pre-school for one week?

b) Each of the 60 children at the pre-school pay R48 per week to attend. What profit does Mrs Khumalo make running the pre-school?

c) Mrs Khumalo wants to pay herself a salary of R1 000 per week. She also has to cover her expenses. How much does she have to charge per child per week?

6. Jackson borrows R15 500. He pays simple interest of R1 860 per year. He has to pay back the loan over ten years.

a) How much interest will Jackson have to pay over the period of the loan?

b) How much money will Jackson have to pay back altogether?

c) How many months are there in the time period of the loan?

d) Jackson wants to pay back the total amount of money he owes. How much money will Jackson have to pay back each month?