Muhammad Dayyan Hussain Khan

101423382

Assignment 2: Modelling and Simulation of a DC Motor Drive

Aalto University

School of Electrical Engineering

ELEC-E8405

Report

30 October 2023

Contents

2 Testing the model

Rated voltage u_N	120 V
Rated current i_N	20 A
Rated speed n_N	3 000 r/min
Rated torque τ_N	7 Nm
Resistance R	0.5 Ω
Inductance L	2.5 mH
Flux factor k	0.35 Vs
Total moment of inertia J	0.001 kgm^2

Figure 1. Rating and parameters of the DC motor.

The following state equations are taken as a starting point for a simulation model of the permanent-magnet DC motor:

$$L\frac{di}{dt} = u - Ri - k\omega_M$$

$$J\frac{d\omega_M}{dt} = ki - \tau_L$$

1. Simulate the sequence corresponding to Figure 7. Modify the plotting script so that the per-unit current and the per-unit speed are plotted (use their rated values as base values and do not normalize time). Show this result in your report. Remember to change the axis labels. Explain why there is a very large peak in the current after the voltage step is applied.
2. Using the analytical motor model, calculate the values for the current i and the rotor speed ω_M in the steady state, when the voltage $u = u_N$ and the load torque $\tau_L = \tau_N$. Compare these values to your simulation results.
3. Limit the rising rate of the voltage to 120 V/0.1 s using the Rate Limiter block, see Figure 8. Place this block between the voltage step and the motor model. Simulate the model and show the results in your report. Briefly comment on the current and speed responses.

1)

As can be seen in figure 1, there is a very large peak in the current response after the voltage is applied. This is due to the sudden change in voltage (the step input), thus inducing a large current in the rotor windings. The model used to produce the plots in figure 1 can be seen in figure 2.

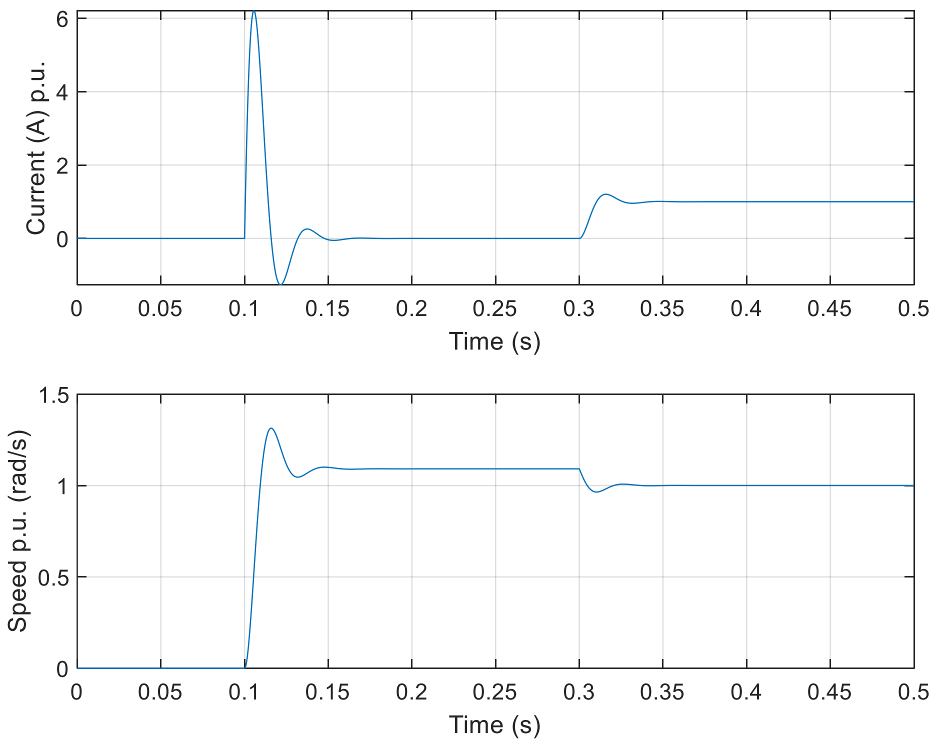

Figure 1. Plot of the simulated permanent-magnet DC motor's speed and current responses to the rated voltage step (120 V) at t = 0.1 s and rated torque step at t = 0.3 s.

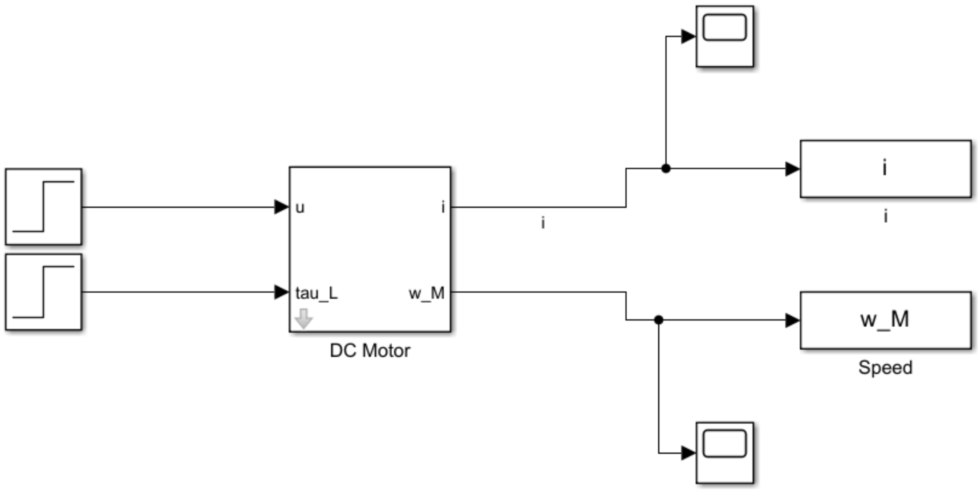

Figure 2. Simulink model of permanent-magnet DC motor used to produce plots in figure 1.

2)

$$u = u_N = 120\ V$$

$$\tau_L = \tau_N = 7\ Nm$$

$$i = ?$$

$$\omega_N = ?$$

$$k_f = 0.35\ Vs$$

$$n_N = 3000\ r/\min = 314.16\ rad/s$$

$$u = k_f \omega_M$$

$$\tau_M = k_f i$$

$$p_M = \tau_M \omega_M = ui$$

∴

$$\omega_M = \frac{u}{k_f} = \frac{120}{0.35} = 342.85 \frac{rad}{s}$$

∴

$$i = \frac{\tau_M}{k_f} = \frac{7}{0.35} = 20\ A$$

∴

Here it can be seen that the calculated values are very similar to those obtained from the simulation. The main difference lies, however, in the differences in rotor speeds in the steady state.

3)

As can be seen in figure 2 the effect of the addition of the rate limiter block to the motor model's current and speed responses is significant. The reduced voltage rise time induces a significantly smaller current at t = 0.1 s which settles to zero when the motor speed reaches its steady state before load torque is applied. A natural consequence of this is that the motor's speed response reaches its steady state after it has responded to the rate limited step input, more slowly. The updated model can be seen in figure 3.

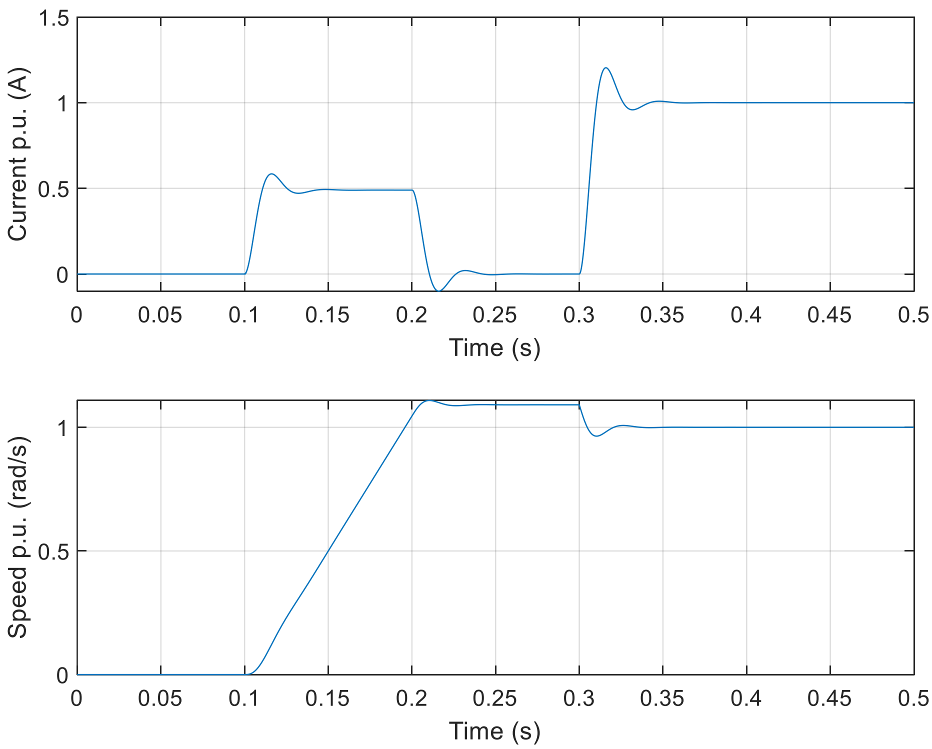

Figure 3. Plot of the simulated permanent-magnet DC motor's speed and

current responses to the rated voltage step (120 V) at t = 0.1 s and rated torque step at t = 0.3 s. In this instance, the rising rate of the voltage has been limited.

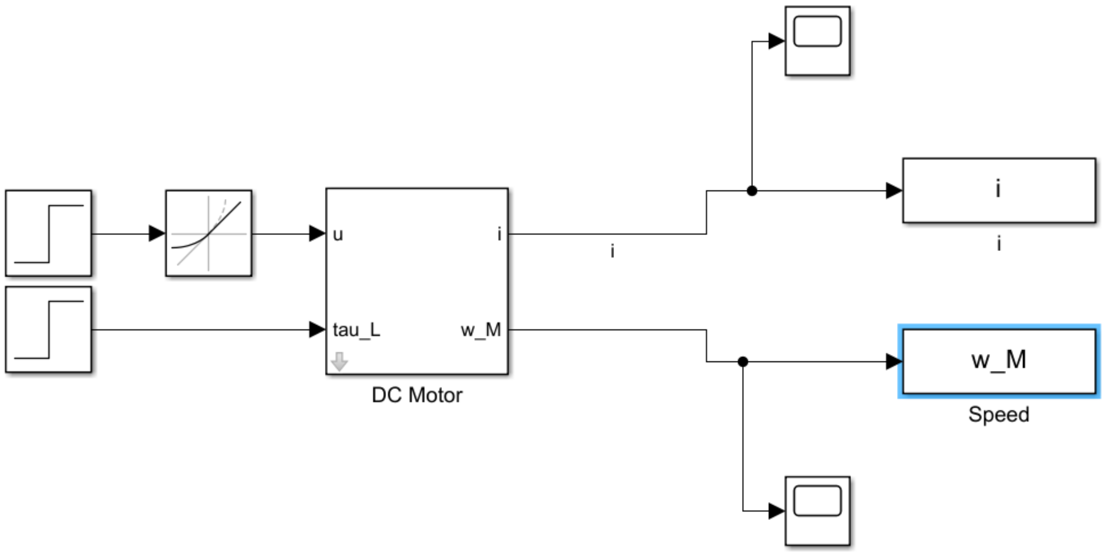

Figure 4. Simulink model of permanent-magnet DC motor used to produce plots in figure 2. Note the addition of the rate limiter block before the motor's voltage input.

3 DC-DC Converter and Unipolar PWM

4. Augment your simulation model with unipolar PWM and converter models. Your model should function similarly to the model in Figure 10(b). Simulate the model and show the results in your report. Briefly comment on differences compared to the previous simulation, where an ideal voltage source was assumed. Submit this version of your simulation model to MyCourses.
5. Plot the waveforms of the actual current `i` and the synchronously sampled current `i_k` in the same subplot. Show also the waveform of the voltage `u`. You can plot the results using the following script:

```
subplot(2,1,1)
plot(i.time,i.data); grid on; hold on;
stairs(i_k.time,i_k.data,'r');  % Discrete signal
axis([0.15 0.1504 9 11]);        % Controls axis scaling
xlabel('Time (s)'); ylabel('Current (A)');
subplot(2,1,2)
plot(u.time,u.data); grid on;
axis([0.15 0.1504 -10 150]);
xlabel('Time (s)'); ylabel('Voltage (V)');
```

   Show the results in your report and briefly comment on them.

$$u = (q_a - q_b)U_{dc}$$

4)

Compared to the previous simulation, where an ideal voltage source was assumed, the motor's current response exhibits an oscillating behaviour which is clearly apparent at the onset of its various steady states during the simulation. This is naturally due to the effect of the pulse-width modulation produced by the addition of the unipolar PWM, and can be seen more clearly in figure 5, where the output voltage from the DC-DC converter has been plotted against the simulation time. From the plot, its frequency has been calculated to be 10000 Hz Figure 6 depicts the motor model's current and speed responses after the addition of unipolar PWM and DC-DC converter.

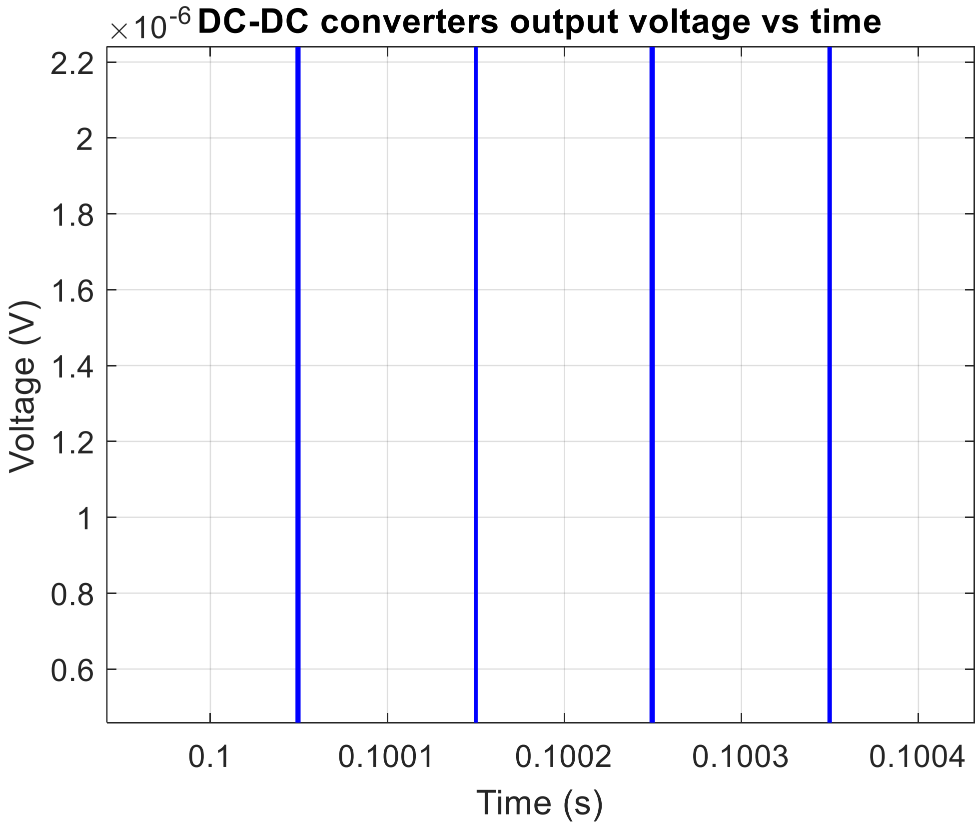

Figure 5. Plot of DC-DC converter's output voltage vs time, where PWM instigated switching frequency is apparent (10000 Hz).

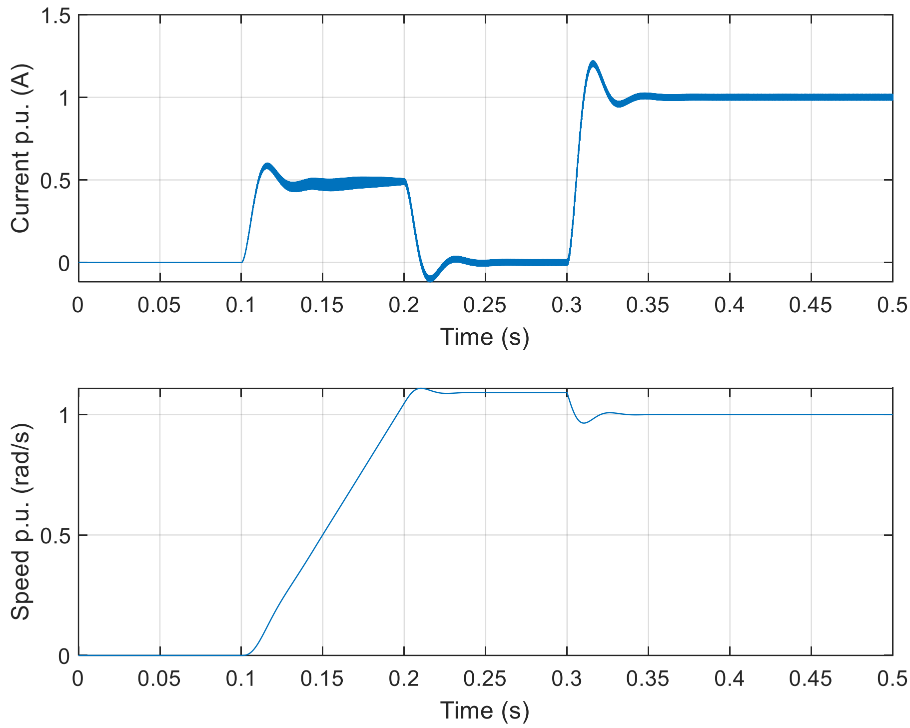

Figure 6. Plot of the simulated permanent-magnet DC motor's speed and current responses to the rated voltage step (120 V) at t = 0.1 s and rated torque step at t = 0.3 s. In this instance, the rising rate of the voltage has been limited and a unipolar PWM and DC-DC converter is included in the model.

5)

As can be seen in figure 7, relative to the motor model's current response, the synchronously sampled current response is an average devoid of oscillation, owing to the discrete zero-order hold performed on the current response. The current achieves its peak value at the end of the 'on' phase of the switching, i.e., at the instant before the switch is disconnected from the positive potential of the DC bus. Conversely, the current achieves its minimum value at the instant before the switch is connect to the positive potential of the DC bus. The updated model can be seen in figure 8.

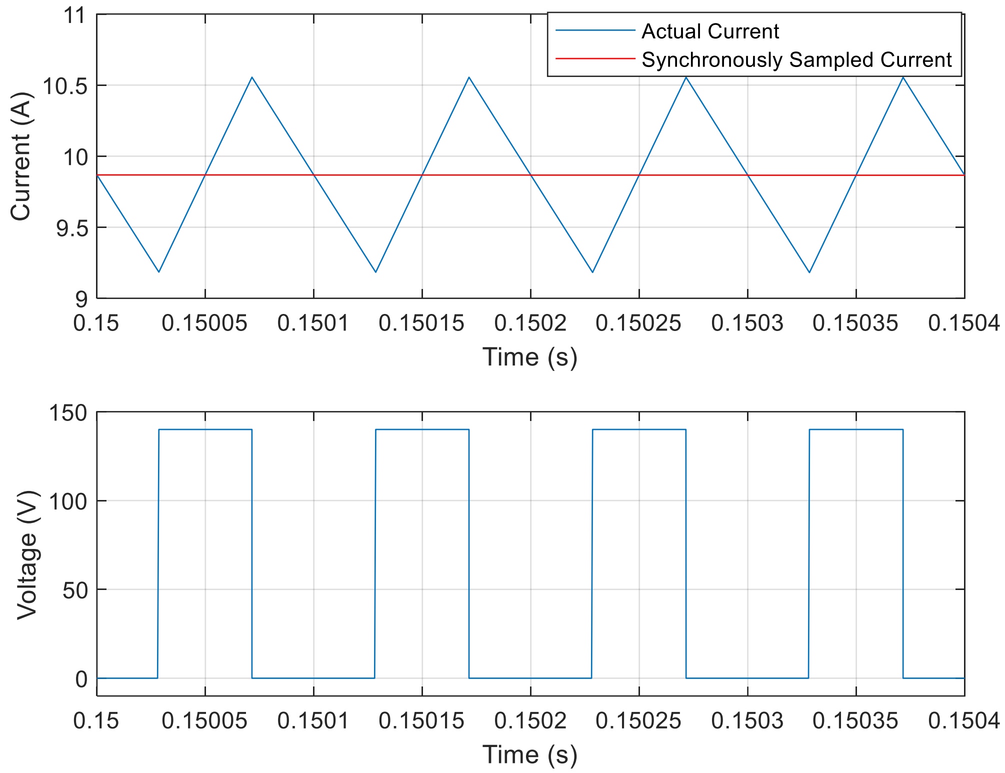

Figure 7. Plot of simulated permanent magnet DC motor's current response and synchronously sampled current response, after the addition of unipolar PWM and DC-DC converter. Plot of DC-DC converter's output voltage is also included, where switching frequency is clearly apparent.

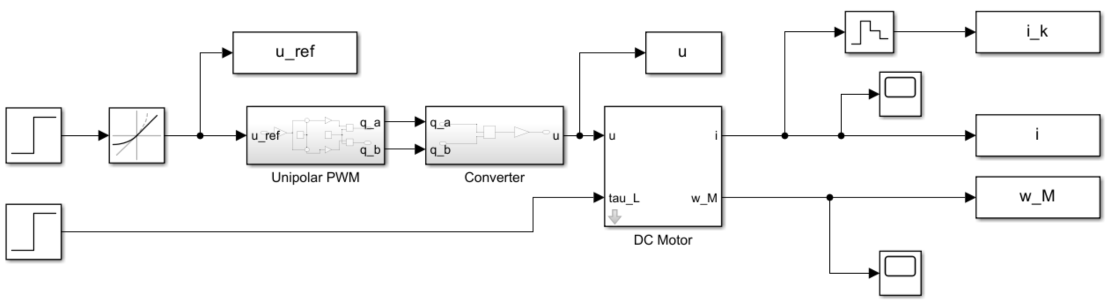

Figure 8. Simulink model of permanent-magnet DC motor used to produce plots in figure 7. Note the addition of the unipolar PWM and DC-DC converter subsystems.

4 Cascade control

> 6. Calculate the theoretical rise time of the torque and compare it to the simulated rise time.

Figure 9 depicts a simple model of the cascade control system of a DC motor.

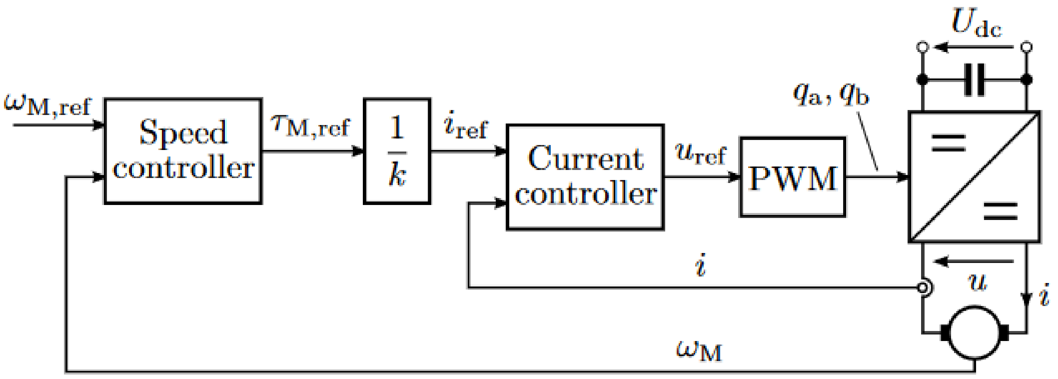

Figure 9. Cascade control system of the DC motor

6)

The current rise time is simply the time constant of the plant transfer function of the electrical subsystem of the DC motor with cascade control. This transfer function is:

$$\frac{1}{sL + R}$$

The time constant $T = \frac{L}{R} = \frac{2.5 \cdot 10^{-3}}{0.5} = 0.005$. Theoretical rise time of the torque is therefore $0.005\ s$, since the torque is proportional to the current according to the following mathematical relationship:

$$\tau_M = ki$$

The simulated motor torque response as a function of time can be seen in figure 10. The effect of the 2DOF PI current controller with anti-windup is apparent, when compared to the 100 Hz square wave torque reference. The script used to calculate the simulated torque rise time can be seen in figure 11. Simulated rise time is therefore,

$$0.00198\ s$$

and the difference between the two values (theoretical and simulated) is due to the effect of the proportional gain component of the PI controller reducing the rise time.

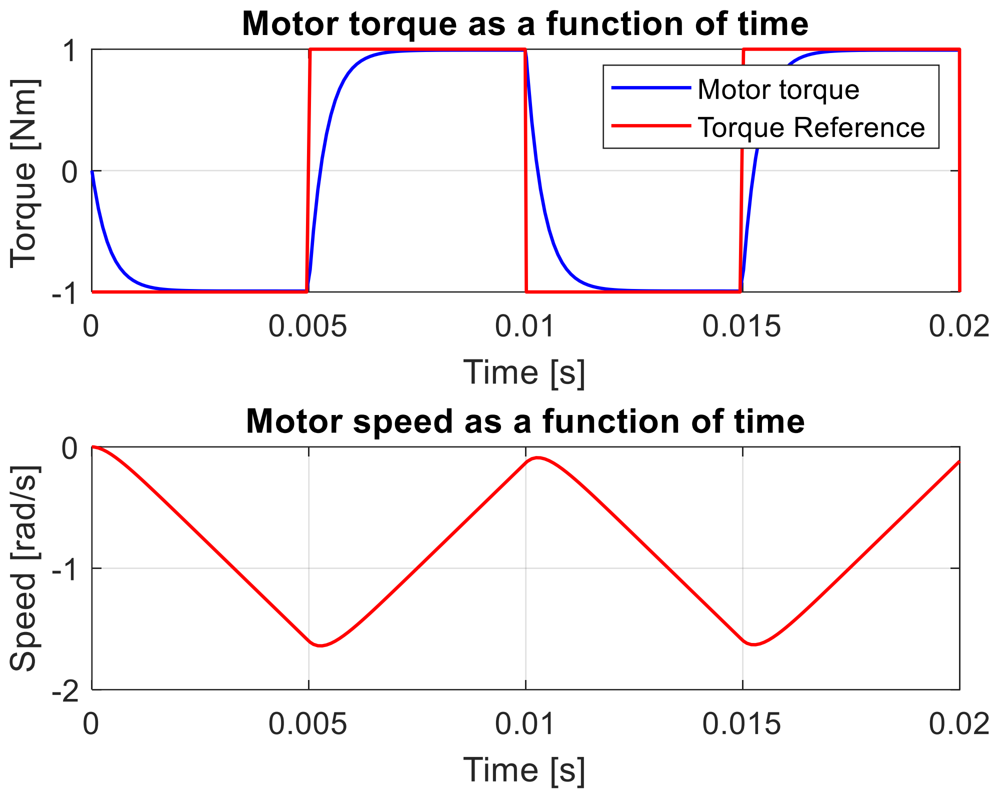

Figure 10. Plot of simulated DC motor's torque and speed responses as a function of time, with the torque reference overlayed with the torque response.

```
>> S = stepinfo(Current.time,Current.data);
>> S

S =

  struct with fields:

        RiseTime: 1.9782
   TransientTime: 0.9923
    SettlingTime: 0.9923
     SettlingMin: 0.0181
     SettlingMax: 0.0200
       Overshoot: 0
      Undershoot: 0
            Peak: 0.0200
        PeakTime: 0.9888
```

Figure 11. Script used to determine motor's simulated torque rise time.

7. Tune the speed controller of your simulation model for the closed-loop bandwidth $\alpha_s = \alpha_c/10$. Test your model using the square-wave speed reference, whose amplitude is 160 rad/s and frequency is 4 Hz. Generate the rated load torque step at $t = 0.3$ s. Show results of this simulation in your report. Show also the figures describing the main level of your simulation model and the implemented speed controller. Submit this version of your Simulink model to MyCourses (including your initialization script).
8. This problem aims to illustrate the robustness of the closed-loop control scheme against parameter errors. Generally, resistances depend on temperature (about 0.4%/K) and inductances may vary due to the magnetic saturation. Change the actual resistance R in the motor model to 150% of its original value and the actual inductance L to 70% of its original value, but do not change the values in the control system. Simulate the model. Show the results and comment on them in your report. After this problem, restore the parameter values back to their original values.
9. This problem aims to illustrate the importance of the anti-windup scheme. Remove the anti-windup in the speed controller (but do not remove the saturation of the controller output). Show results of your simulation and comment on them. After this problem, restore the anti-windup method back to the original form.
10. Parametrize the speed controller so that it becomes a regular (1DOF) PI controller, while keeping the closed-loop poles the same. Furthermore, parametrize the speed controller so that it becomes the proportional controller, while keeping the same reference-tracking performance as that of the original 2DOF PI controller. For both cases, show the simulation results and briefly comment on them.

7)

Results of the simulation of the DC motor model with 2DOF PI speed controller added can be seen in figure 12. Rated load torque step of 7 Nm is applied at t = 0.3 s, as can be seen from the figure. Motor current remains elevated after the load torque has been applied. Motor speed is reduced momentarily after the load torque has been applied and then stabilises. The 2DOF PI speed controller is depicted in figure 13. The main level of the simulation model with implemented speed controller is depicted in figure 14.

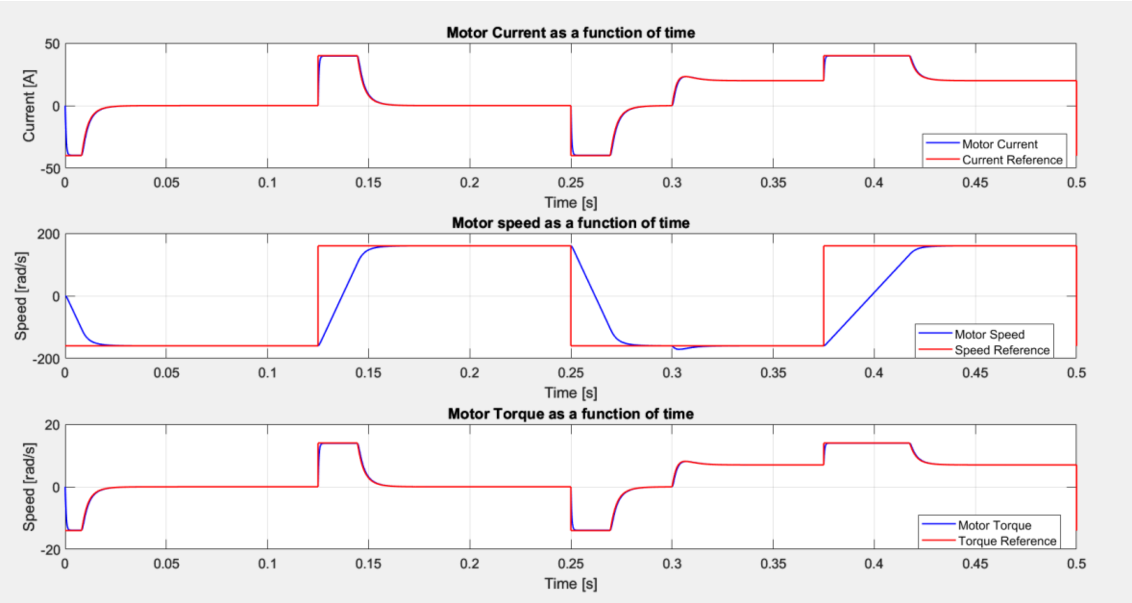

Figure 12. Plot of simulated DC motor's current, torque, and speed responses as a function of time, with the respective reference values overlayed on the same plot.

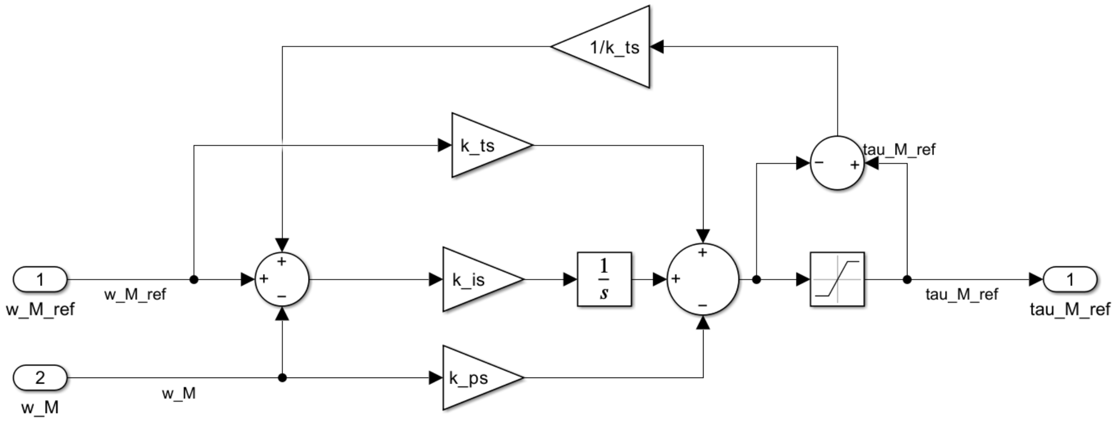

Figure 13. 2DOF PI speed controller as modelled for permanent magnet DC motor model with cascade control.

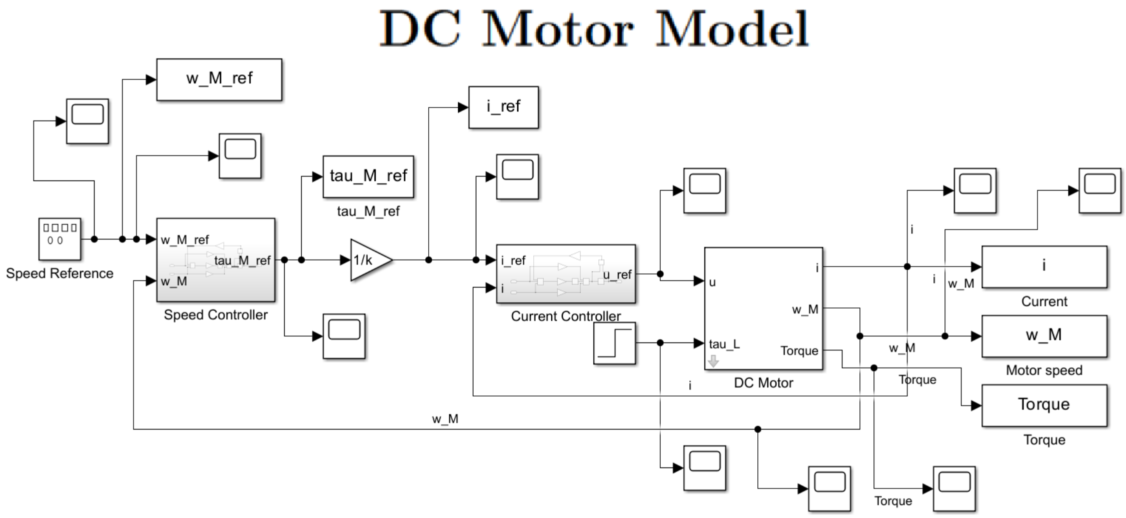

Figure 14. Main level of permanent magnet DC motor simulation model with implemented speed and current controllers.

8)

Depicted below are the simulated motor model's current and speed responses when the resistance in the motor model has been changed to 150 % of its original value and inductance has been changed to 70 % of its original value. As can be seen in the figure, the closed loop control scheme exhibits a distinct robustness, for despite the changes to the motor's resistance and inductance parameters, the current and speed responses remain almost identical.

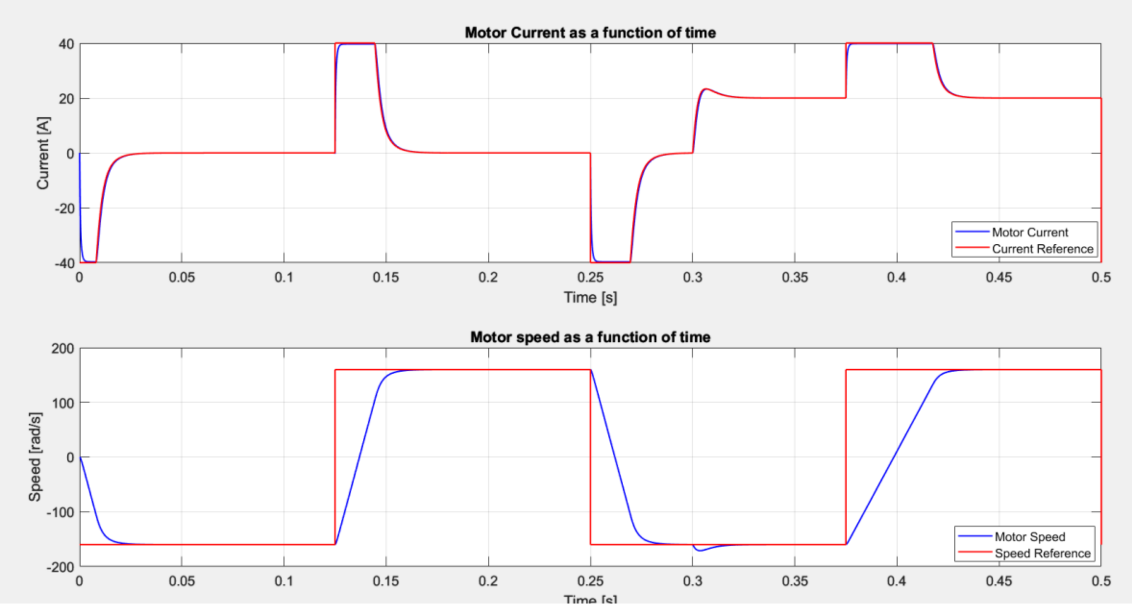

Figure 15. Plot of simulated DC motor's current and speed responses as a function of time, with their respective reference values overlayed on the same plot. Motor's resistance has been changed to 150 % of its original value and motor's inductance has been changed to 50 % of its original value.

9)

Figure 15 depicts simulated model's current and speed responses after the anti-windup functionality had been removed from the speed controller. As can be seen in the figure, whilst the current response still follows the current reference accurately, the speed response is clearly affected by the windup phenomena leading to large overshoot from the reference value due to the integral state continuing to accumulate.

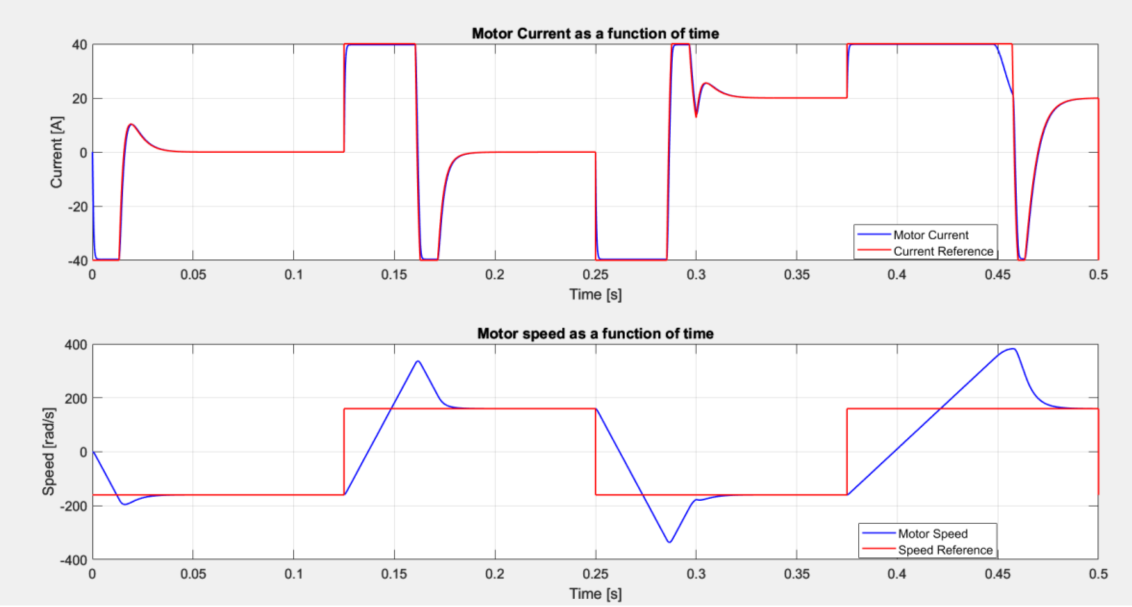

Figure 15. Plot of simulated DC motor's torque and speed responses as a function of time, with their respective reference values overlayed on the same plot. Speed controller's anti-windup functionality has been removed.

10)

Figure 16 depicts the simulated model's current and speed responses after the speed controller had been parametrised such that it becomes a regular 1DOF PI controller. This was achieved by setting the gain, $k_t = k_p$, whilst the gain k_i remains the same as in the 2DOF PI controller's case i.e., $k_i = \alpha_s^2 \hat{J}$. As can be seen in the figure, the speed controller's reference step response leads to speed response overshoot, due to the reference tracking zero no longer matching with the pole.

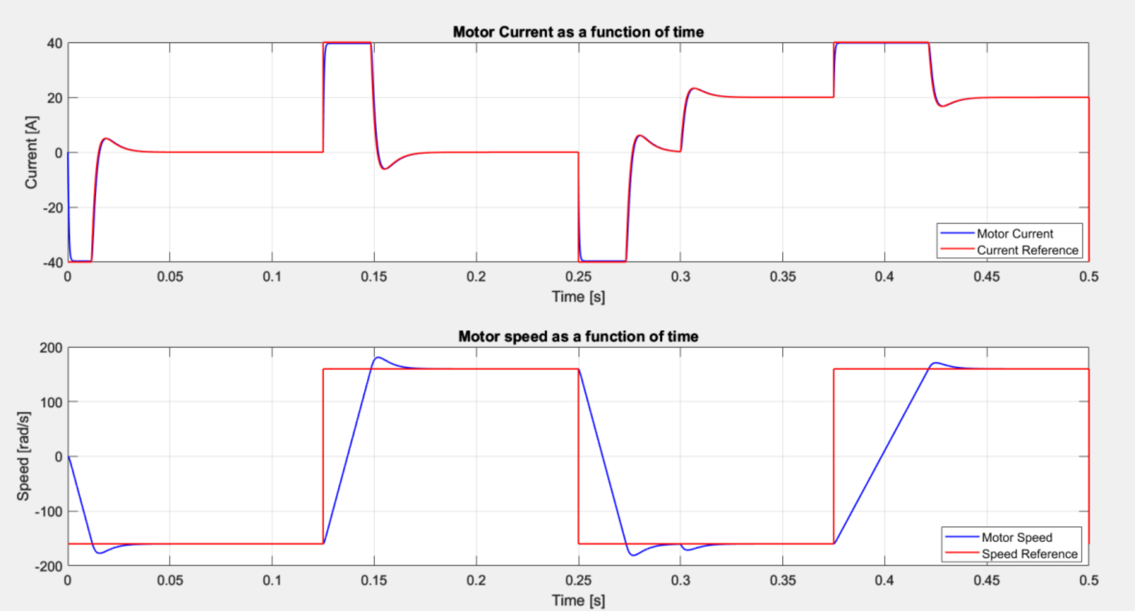

Figure 16. Plot of simulated DC motor's current and speed responses as a function of time, with their respective reference values overlayed on the same plot. Speed controller had been parametrised such that the gain $k_t = k_p$.

Figure 17 depicts the simulated model's current and speed responses after the speed controller had been parametrised such that it becomes a regular P controller. This was achieved by setting the gain, $k_t = k_p$, whilst the gain k_i is set to zero. As can be seen in the figure, the parametrised speed controller's reference tracking performance is the same as the original 2DOF PI controller, however, its disturbance rejection is far worse. After the load torque is applied at $t = 0.3s$, the speed response exhibits load dependent permanent steady state error, which is to be expected.

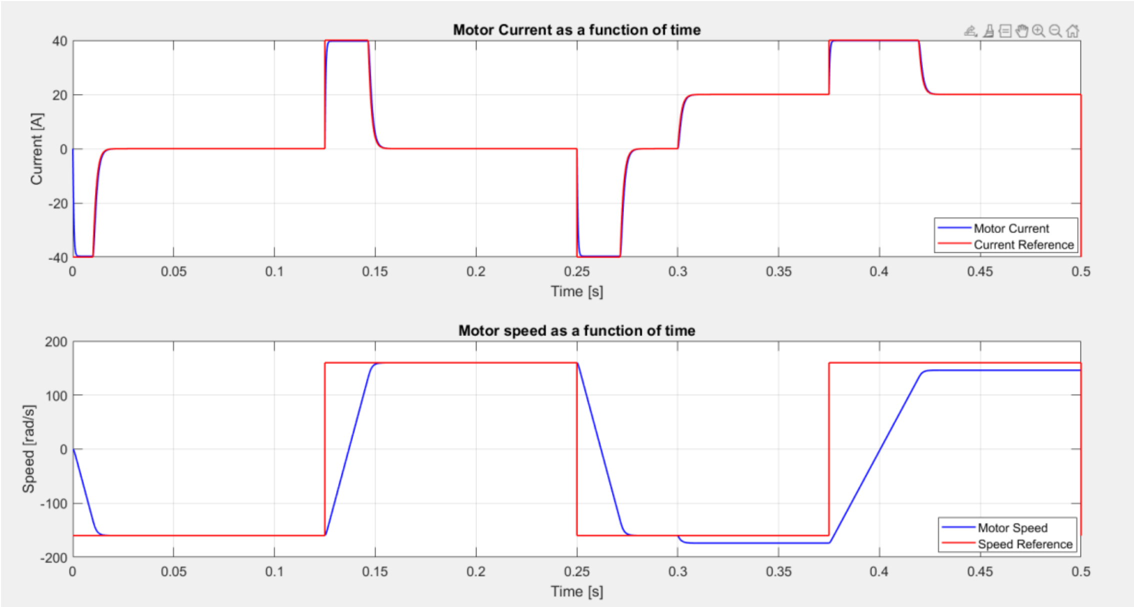

Figure 17. Plot of simulated DC motor's current and speed responses as a function of time, with their respective reference values overlayed on the same plot. Speed controller had been parametrised such that the gain $k_t = k_p$ and $k_i = 0$.