Oxygen

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Executive Summary

The aim of the present report id to build a best fit model that predicts Oxygen intake rates using the fitness dataset from Rawlings (1998), which contain measurements of seven variables obtained from 31 men.

The initial Linear Regression Model to be implemented will have six explanatory variables **(e.g. Age, Weight, RunTime, RestPulse, RunPulse and MaxPulse)** that will help explain or predict the behaviour of the response variable (Oxygen).

Starting from a general model which includes all the variables, and through the analyses of each variable and respective interactions, one aims to produce a best fit linear model.

In order to decide which variables better interact with the dependent variable y (Oxygen) some statistical methods (e.g. Bootstrap, Randomisation) will be performed in order to decide the composition of the Final Model.

The aim is to exclude variables that present, essentially, the same information about response avoiding this way collinearity. The focus of the whole report is to find a balance and have a good set of covariates within our Oxygen model.

In order to help the reader and to facilitate understanding of the model development several visual elements (plots) will be added in the report.

In order to guarantee that the model is being developed within reasonable, several assumptions will be assessed along the way (Linearity, Homoscedasticity, Independence, Normality).

A final Model, or the best fitted model will be presented.

Introduction

The present report aims to build a model that predicts Oxygen intake rates (a measure of aerobic fitness) supported on a series of measurements. The fitness dataset from Rawlings (1998) contains measurements of the following seven variables obtained from 31 men:

```
* Age: Age in years;
* Weight: Weight in kg;
* Oxygen: Oxygen intake rate, ml per kg body weight per minute;
* RunTime: time to run 1.5 miles in minutes;
* RestPulse: heart rate while resting;
* RunPulse: heart rate at end of run;
* MaxPulse: maximum heart rate recorded while running;
```

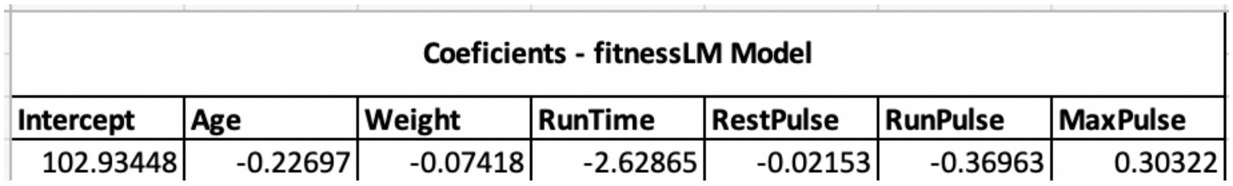
From the data set fitness.csv a linear model (predicting Oxygen) will be developed. The bootstrapping function used to provide confidence intervals came from an original function provided by Donovan (2018), which was improved at a later stage.

The report uses R 3.5.1 software (R Core Team, 2018). It was produced a linear model which was fitted in each analysis and the bootstrap used to generate confidence intervals for each of the covariates of interest. We aim to exclude variables that present, essentially, the same information about response avoiding this way collinearity.

The reasonability of the assumptions on which the model is based were assessed:

```
1. Linearity
2. Homoscedasticity
3. Independence
4. Normality
```

Bootstrap methods were used in order draw conclusion to hypothesis tests in regards to the significance of the relationships between the response and the parameter estimates. If the confidence interval contains zero, one fails to reject the null hypothesis, and if it does not contain zero, one can reject the null hypothesis [7].



Coeficients - fitnessLM Model						
Intercept	Age	Weight	RunTime	RestPulse	RunPulse	MaxPulse
102.93448	-0.22697	-0.07418	-2.62865	-0.02153	-0.36963	0.30322

Figure 1: fitnessLM

Exploratory Findings

Fitness Data Set

Based on our fitness data set, we are going to implement a Linear Regression Model in which explanatory variables (e.g. Age, Weight, RunTime, RestPulse, RunPulse and MaxPulse) will help explain or predict the behaviour of the response variable (Oxygen). The model is specified as follows:

```r
fitness <- read.csv("data/fitness.csv", header = T)
head(fitness)
```

```
##   Age Weight Oxygen RunTime RestPulse RunPulse MaxPulse
## 1  44  89.47 44.609   11.37        62      178      182
## 2  40  75.07 45.313   10.07        62      185      185
## 3  44  85.84 54.297    8.65        45      156      168
## 4  42  68.15 59.571    8.17        40      166      172
## 5  38  89.02 49.874    9.22        55      178      180
## 6  47  77.45 44.811   11.63        58      176      176
```

The Initial Model

```
##
## Call:
## lm(formula = Oxygen ~ Age + Weight + RunTime + RestPulse + RunPulse +
##     MaxPulse, data = fitness)
##
## Coefficients:
## (Intercept)          Age       Weight      RunTime    RestPulse
##   102.93448     -0.22697     -0.07418     -2.62865     -0.02153
##    RunPulse     MaxPulse
##    -0.36963      0.30322
```

A first approach to the relationship between the variables within our **fitnessLM** model present the following results:

- (Intercept) = 102.93448. This value represents the value of the intercept β_0, when all other β are zero. Therefore, y = β_0 = Oxygen = 102.93448.
- RunTime = -2.62865. Meaning that everytime RunTime increases by 1 unit, the Oxygen level decreases by 2.6.

In order to foresee how our fitnessLM model is behaving, it can be produced a **summary(fitnessLM)** of the model, given the variables.

```
##
## Call:
## lm(formula = Oxygen ~ Age + Weight + RunTime + RestPulse + RunPulse +
##     MaxPulse, data = fitness)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.4026 -0.8991  0.0706  1.0496  5.3847
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 102.93448   12.40326   8.299 1.64e-08 ***
## Age          -0.22697    0.09984  -2.273  0.03224 *
## Weight       -0.07418    0.05459  -1.359  0.18687
## RunTime      -2.62865    0.38456  -6.835 4.54e-07 ***
## RestPulse    -0.02153    0.06605  -0.326  0.74725
## RunPulse     -0.36963    0.11985  -3.084  0.00508 **
## MaxPulse      0.30322    0.13650   2.221  0.03601 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.317 on 24 degrees of freedom
## Multiple R-squared:  0.8487, Adjusted R-squared:  0.8108
## F-statistic: 22.43 on 6 and 24 DF,  p-value: 9.715e-09
```

The **Residual Standard Error (2.317)** gives us an idea of how far the Oxygen levels are from the fitted model.

- Multiple R-Squared = 0.8487. Almost 85% of the variation in Oxygen can be explained by our model.
- p-value = 9.715e-09. This value is extremely small, smaller than 0.05. Therefore, we reject the Null Hypothesis (H0) which assumes that all the model coefficients (Betas) are zero(0).
- Pr (>|t|) gives us the p-value for the t-test. In this case, all the values which are below 0.05 are of interest to our model as it can be improved by those.
- In this particular case, and looking at the value of Weight (0.18687), it can be observed that it is greater than 0.05. Therefore, it fails to reject the Null Hypothesis (H0)) which is based on the assumption that all the coefficients Beta are equal to zero. In this scenario, having to remove a variable from the model, the variable Weight would be, for instance, one of the possibilities.

Collinearity

Collinearity, simply, can be seen as the correlation between the predictor or independent variables, in a way such that they express a linear relationship in a regression model [9]. It can be said that when two variables are highly correlated, then there is collinearity. In fact, at this stage what we are looking at is to exclude variables that offer, essentially, the same information about response, i.e., we want to avoid collinearity. Therefore, we will use **VIF** to calculate variance-inflation and generalized variance-inflation factors **(GVIF)** for linear, generalized linear, and other models [9].

```
##       Age    Weight   RunTime RestPulse  RunPulse  MaxPulse
##  1.512836  1.155329  1.590868  1.415589  8.437274  8.743848
```

```
##       Age    Weight   RunTime RestPulse  RunPulse  MaxPulse
##     FALSE     FALSE     FALSE     FALSE      TRUE      TRUE
```

In this case, looking at the values presented by **RunPulse (8.437273)** and **MaxPulse (8.743848)**, one can easily realise that they have very similar values, therefore are correlated. These variables offer, essentially, the same information. So, the variable with the highest value **(MaxPulse)** can be omitted.

The new model less MaxPulse

The following is the updated model. The **fitnessLM** with the variable **MaxPulse** removed. We now have the new fit for the original model.

```
##
## Call:
## lm(formula = Oxygen ~ Age + Weight + RunTime + RestPulse + RunPulse,
##     data = fitness)
##
## Coefficients:
## (Intercept)          Age       Weight      RunTime    RestPulse
##   116.48761     -0.28528     -0.05184     -2.70392     -0.02711
##    RunPulse
##    -0.12628
```

Final Model

The main objective here, and within the whole report, is to find a balance and have a good set of covariates within our Oxygen model. If we look only at a few set of variables we are likely to be disregarding valuable information. On the other way around, if we include, in our model, both essential and non-essential variables, the standard error, confidence interval and p-values tend to be larger [9]. In order to determine the **finalModel**, it will be used the function step(), which follows the **Akaike Information Criterion (AIC)** to select the model, based in the following rule: **the lower the AIC the better the model**.

```
## Start:  AIC=61.96
## Oxygen ~ Age + Weight + RunTime + RestPulse + RunPulse
##
##             Df Sum of Sq    RSS    AIC
## - RestPulse  1     0.905 156.23 60.139
## - Weight     1     5.011 160.34 60.943
## <none>                   155.33 61.958
## - RunPulse   1    36.204 191.53 66.453
## - Age        1    47.085 202.41 68.166
## - RunTime    1   267.469 422.80 91.000
##
## Step:  AIC=60.14
## Oxygen ~ Age + Weight + RunTime + RunPulse
##
##            Df Sum of Sq    RSS    AIC
## - Weight    1      4.60 160.83 59.037
## <none>                  156.23 60.139
## - RunPulse  1     38.88 195.11 65.027
## - Age       1     46.54 202.77 66.221
## - RunTime   1    346.82 503.06 94.388
##
## Step:  AIC=59.04
## Oxygen ~ Age + RunTime + RunPulse
##
##            Df Sum of Sq    RSS    AIC
## <none>                  160.83 59.037
## - RunPulse  1     39.89 200.72 63.905
## - Age       1     42.29 203.12 64.274
## - RunTime   1    370.44 531.27 94.080
```

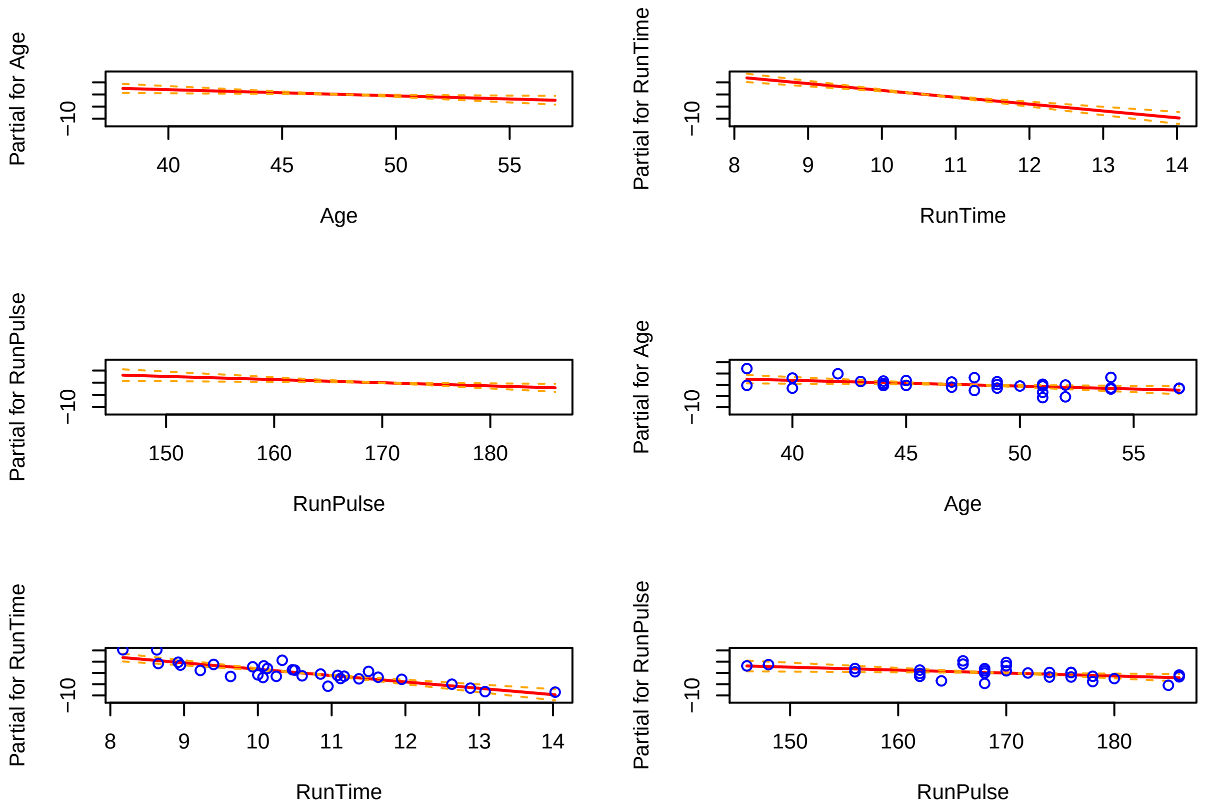
We have now reached the final model **(finalLM)** which contain the variables **RunPulse**, **Age** and **RunTime**.

With a starting AIC of 61.96 on the first step we have now come up with an **AIC value of (59.04)**. Removing either RunPulse, or Age, or RunTime would result in a much higher AIC. Therefore it is wiser to leave the model as it is and assume it as the better fitted model.

Checking Model Assumptions

Assessing Linearity

At this stage, partial residual plots, both with and without interaction, are appropriate to check the linearity for each term within the working model.



Looking at the plots (slopes) one can infer that the variables **Age** and **RunPulse** are similar and close to zero. When it come to the variable **RunPulse** it presents a steeper slope. The steeper the slope is, the better the variable fits within the model. The objective here, as said before, is to include variables that fit the model. In fact what we are doing is [9]:

- including variables with strong relationships between x and y (variables already considered within the model);
- see if these offer new information about y;
- looking at the slope of the line (regression coefficient);
- verify the amount of scatter along the line. The least scatter, the more important.
- identify large residuals.

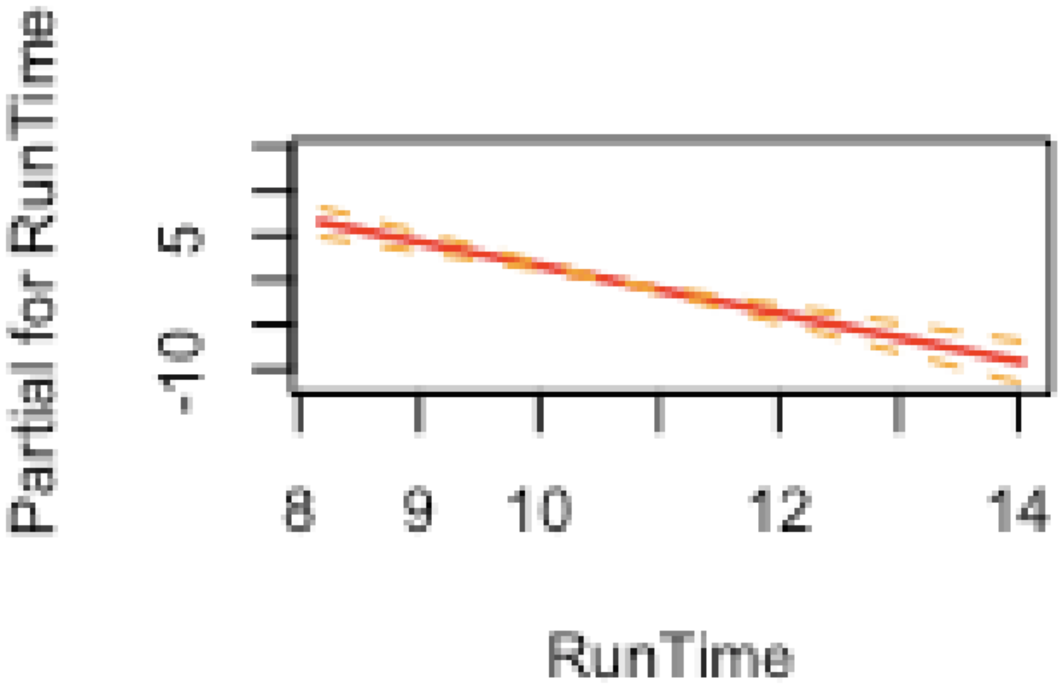


Figure 2: partial RunTime

As it can be observed in Figure 3, the covariate RunTime gives us more information about the selected model. It has not only a steeper line but very little scatter along the line, which leads us to point **RunTime** as probably the most important variable within the model, at this stage.

Partial Residual Plots

At this stage we have add on the residuals to model and once again **RunTime** seems to present the best fit to the model as the residuals tend to be more concentrated along the slope line.

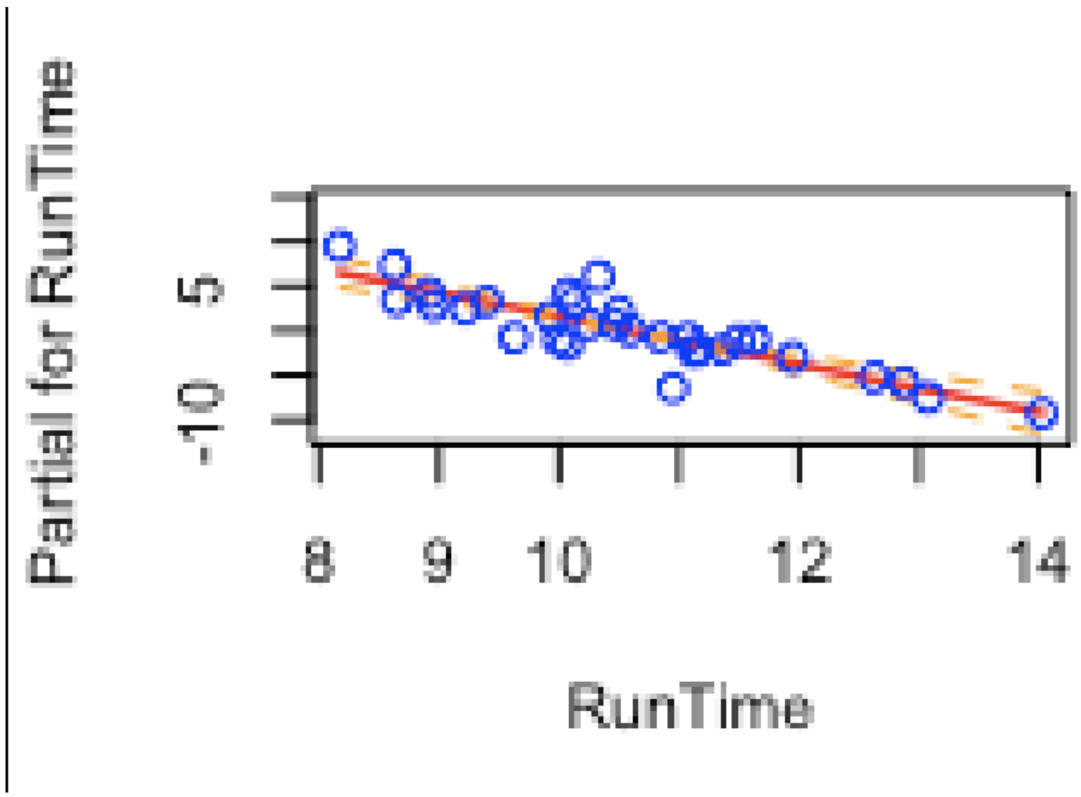
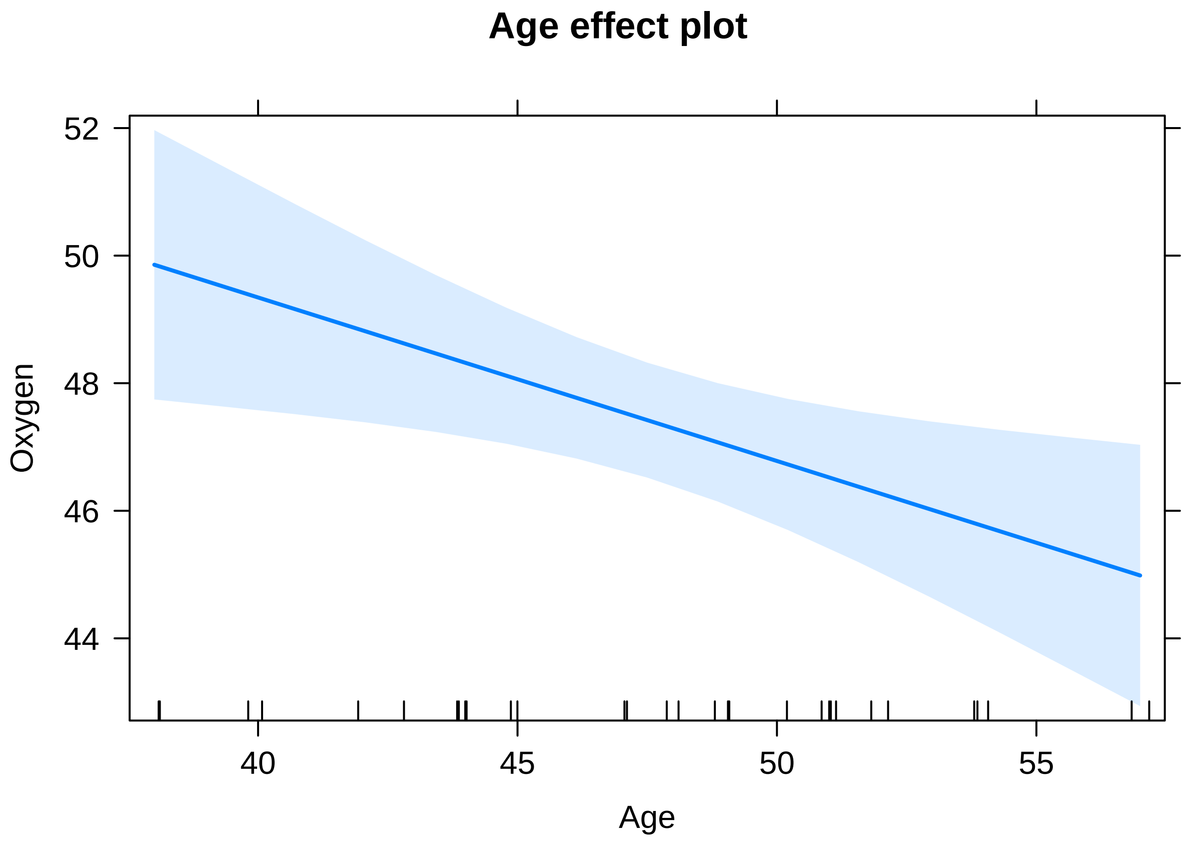
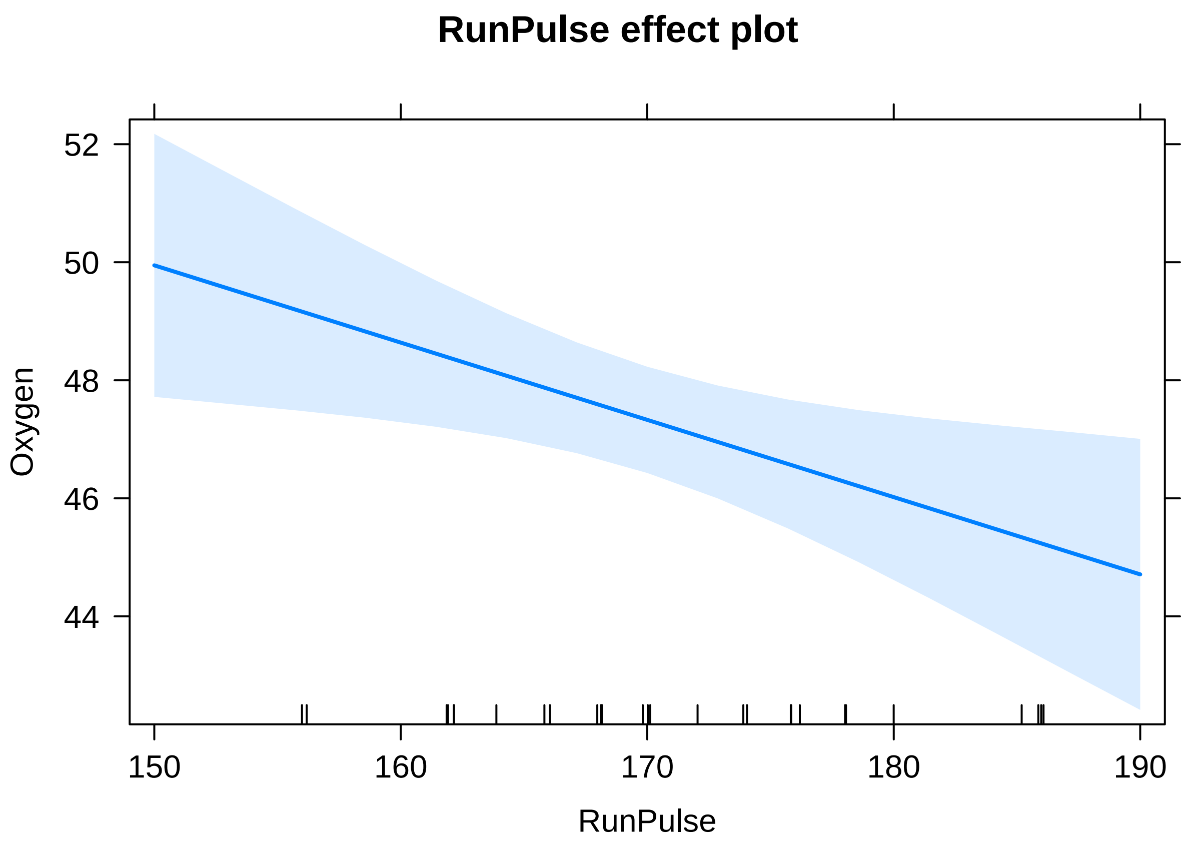
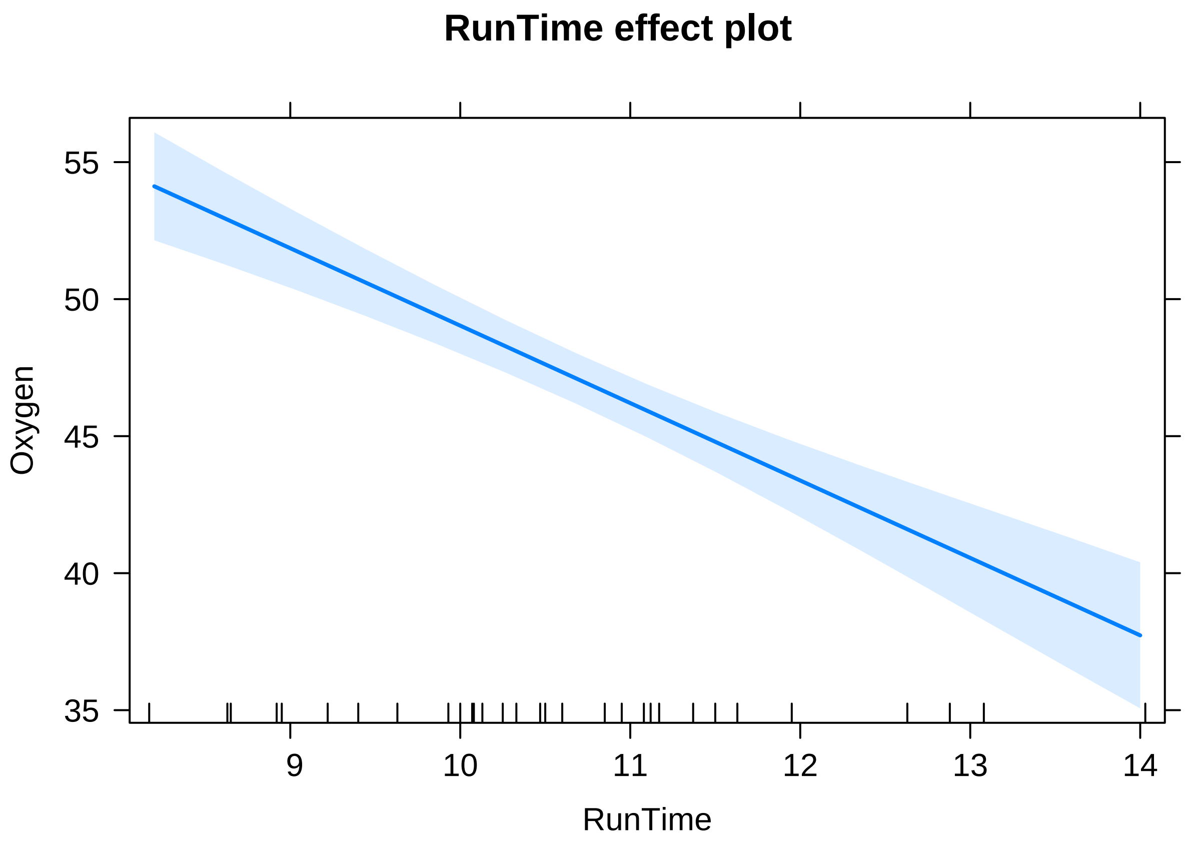


Figure 3: partial RunTime with interactions



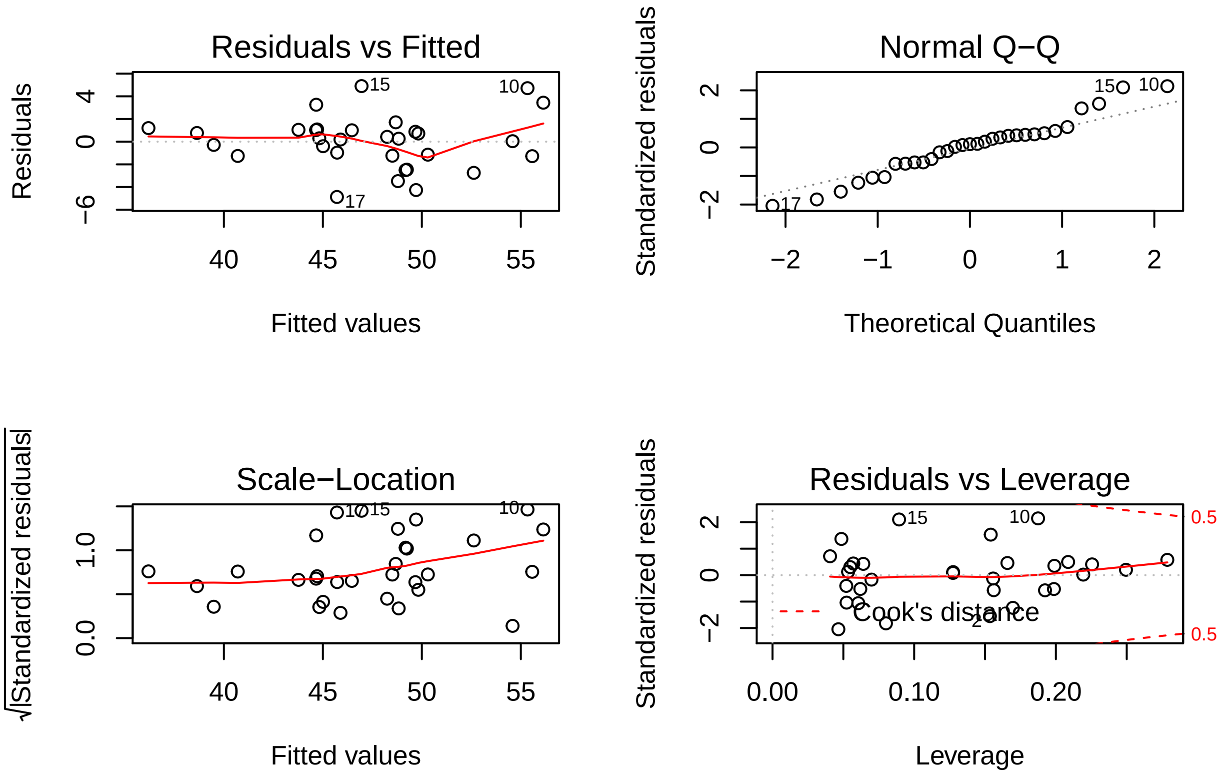




Looking at **RunTime** plot, compared with **Age** and **RunPulse**, one can notice a higher convergence of points to the central line (no outliers) and the extremes are narrower than the other two variables in analysis. The smother the line, the better the variable contribution to the model. Once again **RunTime** has proven to be a higher performing contributor to the model.

Assessing Constant Variance - Homoscedasticity

The finalModel Plot



"Constant error variance can be checked visually using residual plots" [9, p.21]. Therefore, looking at the plots, namelly the **Scale-Location plot** one can observe that the residuals show roughly equal spread across the range of fitted values (the red line is almost straight and the residual points are spread along the line). This way, the constant variance assumption **(homoscedasticity)** can also be validated.

Assessing Independence

Since we have assumed from the very beggining that relationship amongst each covariates and the response were linear, we also need to verify another model assumption (Independence). In order to verify the independence we will use the Durbin-Watson test, *"which computes residual autocorrelations and generalized Durbin-Watson statistics and their bootstrapped p-values"* [12]

```
## [1] 1.95975
```

Following the results and considering that our Null Hypothesis H0 is the correlation of the errors equal to zero, with a resulting p_value > 0.05, one can infer that it failed to reject the Null Hypothesis and that the independence assumption is verified.

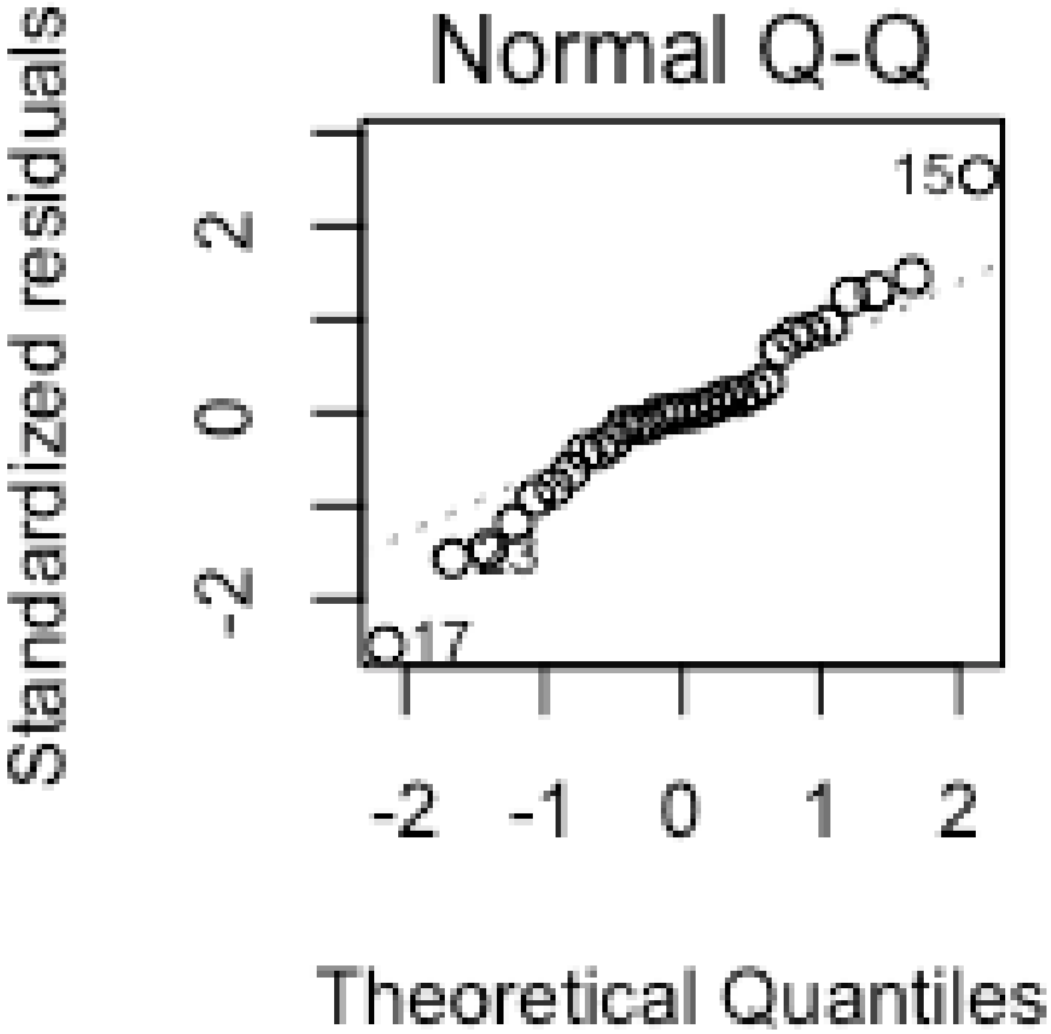


Figure 4: QQ Normal

Normality

QQ Normal

In order to check normality, as it can be checked visually, we will be using a quantile-quantile (QQ) plot.

As it can be seen we have residual values that can be plotted against a hypothetical sample (quantiles) from a N(0, 1) normal distribution. Has we obtained a straight line with scatter, we can infer that the normality assumption is checked [6].

Shapiro-Test

It was also performed the **shapiro.test** to the **finalModel** resulting in a p-value (0.4492) < 0.05. Therefore it failed to reject the NULL hypothesis (H0), which is "the residuals are normally distributed".

This way, the normality assumption for the model is also checked and confirmed.

```
##
##  Shapiro-Wilk normality test
##
## data:  resid(finalModel)
## W = 0.96735, p-value = 0.4492
```

Bootstrapping

The general technique used to estimate *"unknown quantities associated with statistical models"* [11].

Often the bootstrap is used to find:

1. standard errors for estimators,
2. confidence intervals for unknown parameters or
3. p values for test statistics under a null hypothesis.

"Thus the bootstrap is typically used to estimate quantities associated with the sampling distribution of estimators and test statistics"[11].

The aim of this section is to create a function for bootstrapping algorithm to use inside other functions and to provide confidence intervals for all terms in the **finalModel** [10].

```
##                 2.5%        97.5%
## intercept 95.5168648 129.14672417
## Age       -0.4626130  -0.06158671
## RunTime   -3.4458204  -2.21485130
## RunPulse  -0.2047215  -0.06372288
```

Bootstrap methods were used in order draw conclusion to hypothesis tests in regards to the significance of the relationships between the response and the parameter estimates. If the confidence interval contains zero, one fails to reject the null hypothesis, and if it does not contain zero, one can reject the null hypothesis [7].

So, base in this assumption and looking at the output results for the variables **Age [-0.4631321, -0.05926544]**, **RunTime [-3.4440874, -2.19519845]** and **RunPulse [-0.2040866, -0.06207323]** one can observe that there is no ZERO(0) within the Confidence Interval, for each of the variables. Therefore, we can reject the NULL hypothesis (H0), which was having all the beta coefficients (b0=b1=bn) equal to zero.

Randomisation

Randomisation tests are closer to more traditional parametric tests than are bootstrapping procedures. The usual goal is to test some null hypothesis, although that null is distinctly different from what it would be with a parametric test [3].

Randomisation testes allow us to compute confidence limits and look at distributions of outcomes.

```
##        intercept       Age     RunTime  RunPulse
## [1,] 0.003996004 0.2257742 0.001998002 0.2197802
```

Looking at the results for the covariates **Age** and **RunTime**, they present a higher p-value (>0.05) than the covariate **RunTime**, under the NULL hypothesis (H0 = there is no correlation between the response variable and the covariates). For a p_value < 0.05 there is a real difference between the covariate and the response variable. This leads us to conclude that the covariate **RunTime** is the most important covariate for this particular model.

Conclusions

The initial Linear Regression Model was implemented with six explanatory variables **(e.g. Age, Weight, RunTime, RestPulse, RunPulse and MaxPulse)** in order to help explain or predict the behaviour of the response variable (Oxygen).

In order to determine the **finalModel**, along side with methods like Bootstrap and Randomisation, the function step () was utilised. It follows the **Akaike Information Criterion (AIC)** to select the model and ended up helping in the selection of the final model, or model with the lowest AIC.

On the path to the Final Model, or the best fit model, variables like **RestPulse, Weight, MaxPulse** were dropped down and the final model ended up containing the variables *Age, RunPulse and RunTime* as the final covariates interacting with the response variable Oxygen.

This leads us to conclude that the covariate **RunTime** is the most important covariate for this particular model.

Finally, these model estimates are computer generated and not always the chosen model has the "perfet" covariates. Sometimes it is necessary to look at other aspects of correlation between covariates and decide differently.

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Appendices

Appendix 1 - Code Part I - Data Exploration & Start Model

```
library(stats)
library(tidyverse)
library(ggplot2)
library(car)
library(effects)
library(doParallel)
library(parallel)

fitness <- read.csv(“data/fitness.csv”, header = T)
head(fitness)

fitnessLM <- lm(Oxygen ~ Age + Weight + RunTime + RestPulse + RunPulse + MaxPulse, data = fitness)
fitnessLM

summary (fitnessLM)

vif(fitnessLM)
vif(fitnessLM) > 5

fitnessLM <- lm(Oxygen ~ Age + Weight + RunTime + RestPulse + RunPulse, data = fitness)
fitnessLM

finalModel <- step(fitnessLM)

par(mfrow = c(3,2))
termplot(finalModel, se = T)
termplot(finalModel, se = T, partial.resid = TRUE, col.res = ‘blue’)

plot(effect(“Age”, finalModel, rug = TRUE))

plot(effect(“RunPulse”, finalModel, rug = TRUE))

plot(effect(“RunTime”, finalModel, rug = TRUE))

par(mfrow = c(2,2))
plot(finalModel)

durbinWatsonTest(resid(finalModel))

par(mfrow = c(2,2))
plot(finalModel)

shapiro.test(resid(finalModel))
```

Appendix 2 - Code Part Two - Bootstrapping

Create a function for bootstrapping algorithm to use inside other functions

```
bootLM <- function(samples, inputData, index){

bootData <- inputData[samples[, index], ]
Xmat <- bootData[, -1]
Ymat <- bootData[, 1]
beta <- solve(t(Xmat)%%Xmat)%%t(Xmat)%*%Ymat
return(beta)
}

lmBoot_par <- function(inputData, nBoot){

nObs <- nrow(inputData)

sampleData <- as.matrix(cbind(inputData[, 1], 1, inputData[, -1]))

nCores <- detectCores()
myClust <- makeCluster(nCores - 1, type = “PSOCK”)
registerDoParallel(myClust)

bootSamples <- matrix(sample(1:nrow(inputData), nObs * nBoot, replace = T),
nrow = nObs, ncol = nBoot)

bootResults <- matrix(NA, nBoot, ncol(sampleData[, -1]))
bootResults <- parSapply(myClust, 1:nBoot, bootLM, inputData = sampleData,
samples = bootSamples)

stopCluster(myClust)

return(t(bootResults))
}

filteredData <- fitness %>% select(Oxygen, Age, RunTime, RunPulse)
bootRes <- lmBoot_par(filteredData, 1e4)

bootResCI <- matrix(NA, ncol(bootRes), 2, byrow = T)
for(i in 1:ncol(bootRes)){
bootResCI[i, ] <- quantile(bootRes[, i], probs = c(0.025, 0.975))
}
colnames(bootResCI) <- c(‘2.5%’, ‘97.5%’)
rownames(bootResCI) <- c(‘intercept’, ‘Age’, ‘RunTime’, ‘RunPulse’)
bootResCI
```

Appendix 3 - Code Part Three - Randomisation

```
randFunc <- function(nRand){
set.seed(180029941)
fitness <- read.csv(“data/fitness.csv”, header = T)
estimatedCoef <- coef(lm(Oxygen ~ Age + RunTime + RunPulse, data = fitness))

simResults <- matrix(NA, nRand + 1, 4)
simData <- fitness

for(i in 2:(nRand + 1)) {
simDataOxygen < −sample(fitnessOxygen, nrow(fitness), replace = F)
simLm <- lm(Oxygen ~ Age + RunTime + RunPulse, data = simData)
simResults[i, ] <- coef(simLm)
}

simResults[1, ] <- estimatedCoef
colnames(simResults) <- c(‘intercept’, ‘Age’, ‘RunTime’, ‘RunPulse’)

simPvalues <- matrix(NA, 1, 4)
for(i in 1:4){
locEst <- c(1, rep(0, 999))
locEst <- locEst[order(simResults[, i])]
k <- which(locEst == 1)
simPvalues[i] <- min(k / (nRand + 1), 1 - k / (nRand + 1)) * 2
}
colnames(simPvalues) <- c(‘intercept’, ‘Age’, ‘RunTime’, ‘RunPulse’)
return(simPvalues)
}
randFunc(1000)
```