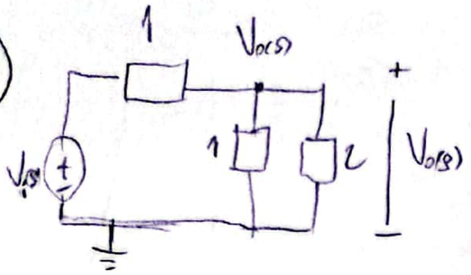
MATHEUS FERREIRA GOMES

11621ECP007

SEMANA 4

17)



Nó 1 ENTRADA $\Rightarrow \dfrac{V_i(s) - V_o(s)}{1}$

SAEM $\Rightarrow \dfrac{V_o}{s} + \dfrac{V_o}{2}$

$$V_i(s) - V_o(s) = \frac{V_o}{s} + \frac{V_o}{2}$$

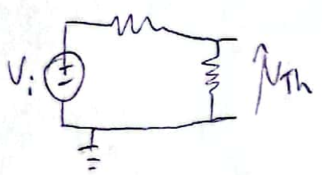
$$V_i(s) = V_o(s)\left(\frac{1}{s} + \frac{1}{2} + 1\right)$$

$$\frac{V_i(s)}{V_o(s)} = \frac{2s}{3s+2}$$

$$G(s) = \frac{V_o(s)}{V_i(s)}$$

$$G(s) = \frac{2s}{3s+2}$$

VTh

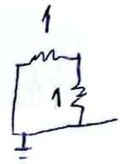


$$V_i(t) - 2.i = 0$$

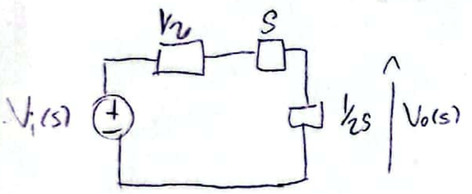
$$i = \frac{V_i(t)}{2}$$

$$V_{Th} = \frac{V_i(t)}{2}$$

RTh

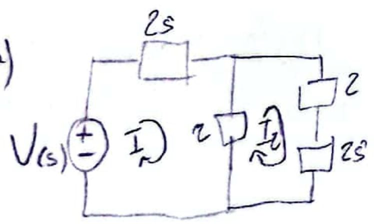


$$R_{Th} = R_{eq} = \frac{1}{2}\,\Omega$$



$$V_o(s) = \frac{V_i(s)}{2} \cdot \frac{\frac{1}{2s}}{\frac{1}{2} + s + \frac{1}{2s}} = V_i(s) \cdot \frac{1}{\cancel{4s}} \cdot \frac{\cancel{2s}}{s + 2s^2 + 1} = \frac{1}{4s^2 + 2s + 2} \Rightarrow G(s) = \frac{1}{4s^2 + 2s + 2}$$

18/a)



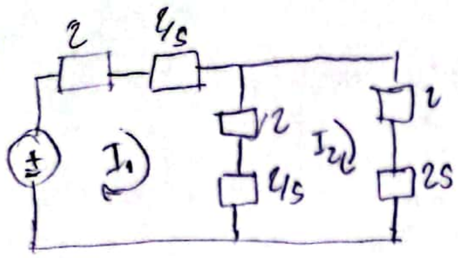
$$V_i(s) = 2s \cdot I_2$$

$$\begin{bmatrix} V(s) \\ 0 \end{bmatrix} = \begin{bmatrix} 2s+2 & -2 \\ -2 & 4+2s \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$I_2 = \frac{\begin{bmatrix} 2s+2 & V(s) \\ -2 & 0 \end{bmatrix}}{\begin{bmatrix} 2s+2 & -2 \\ -2 & 4s+2 \end{bmatrix}} \Rightarrow I_2 = \frac{2V(s)}{(2s+2)(4s+2) - 4} = \frac{2V(s)}{8s^2 + 4s + 8s + \cancel{4} - \cancel{4}} = \frac{2V(s)}{2s(4s+6)} \Rightarrow$$

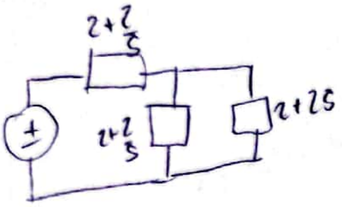
$$\Rightarrow \underbrace{I_2 \cdot 2s}_{V_L(s)} = \frac{2V(s)}{2(2s+3)} \Rightarrow V_L(s) = \frac{V(s)}{2s+3} \Rightarrow G(s) = \frac{1}{2s+3}$$

b)



$$G(s) = \frac{V_L(s)}{V(s)}$$

$$V_L(s) = 2s \cdot I_2$$



$$\begin{bmatrix} V(s) \\ 0 \end{bmatrix} = \begin{bmatrix} 4+\frac{4}{s} & -2-\frac{2}{s} \\ -\left(2+\frac{2}{s}\right) & 4+\frac{2}{s}+2s \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} \frac{4s+4}{s} & -\frac{(2s+2)}{s} \\ -\frac{(2s+2)}{s} & \frac{2s^2+4s+2}{s} \end{bmatrix} = \frac{1}{s^2} \cdot \left[(4s+4)(2s^2+4s+2) - (2s+2)^2\right] =$$

$$= \frac{1}{s^2}\left[8s^3+16s^2+\cancel{8s}+8s^2+16s+8-(4s^2+\cancel{8s}+4)\right] =$$

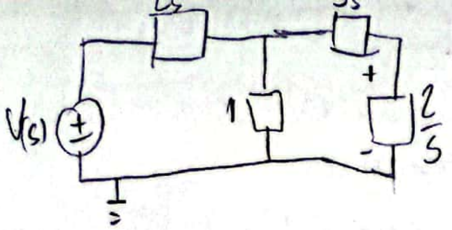
$$= \frac{8s^3+120s^2+16s+4}{s^2}$$

$$I_2 = \frac{\begin{bmatrix} \frac{4s+4}{s} & V(s) \\ -\frac{2s+2}{s} & 0 \end{bmatrix}}{\Delta} = -\frac{\frac{V(s)\cdot(2s+2)}{\cancel{s}}}{\frac{8s^3+120s^2+16s+4}{s^{\cancel{2}}}} = \frac{-V(s)\cdot s(2s+2)}{8s^3+120s^2+16s+4}$$

$$V(L) = I_2 \cdot 2s$$

$$I_2 2s = \frac{-V(s)s(2s+2)\cdot 2s}{8s^3+120s^2+16s+4} \Rightarrow G(s) = \frac{-4s^3-4s^2}{8s^3+120s^2+16s+4}$$

13) a)



$$\begin{bmatrix} V(s) \\ 0 \end{bmatrix} = \begin{bmatrix} 2s+1 & -1 \\ -1 & 3s+1+\frac{2}{s} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 2s+1 & -1 \\ -1 & 3s+1+\frac{2}{s} \end{bmatrix} \Rightarrow \Delta = (2s+1)\left(3s+1+\frac{2}{s}\right) - 1 = 6s^2+2s+4+3s+\cancel{1}+\frac{2}{s}-\cancel{1} =$$

$$= \frac{6s^3+5s^2+4s+2}{s}$$

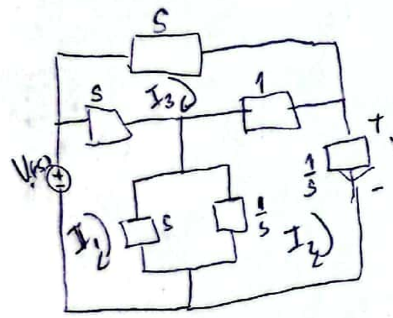
$$V_o(s) = 3s \cdot I_2$$

$$I_2 = \frac{V_o(s)}{3s}$$

$$I_2 = \frac{\begin{bmatrix} 2s+1 & V_i(s) \\ -1 & 0 \end{bmatrix}}{\Delta} \Rightarrow I_2 = V_i(s) \cdot \frac{1}{\Delta} = V_i(s) \cdot \frac{s}{6s^3+5s^2+4s+2} \Rightarrow \frac{V_o(s)}{3s} = V_i(s) \cdot \frac{s}{6s^3+5s^2+4s+2}$$

$$G(s) = \frac{3s^2}{6s^3+5s^2+4s+2}$$

b)



$$\text{I MALHA 1} \quad V_i(s) = \left(1s + \frac{s}{s^2+1}\right) \cdot I_1 - 1s \cdot I_3 - \frac{s}{s^2+1} I_2$$

$$\text{II MALHA 2} \quad 0 = \left(1 + \frac{1}{s} + \frac{s}{s^2+1}\right) I_2 - \frac{s}{s^2+1} I_1 - 1 \cdot I_3$$

$$\text{III MALHA 3} \quad 0 = (2s+1) I_3 - sI_1 - I_2$$

$$\rightarrow I_3 = \frac{sI_1 + I_2}{2s+1}$$

III Em II

$$\frac{sI_1 + I_2}{2s+1} = \left[\frac{(s^3+s)+(s^2+1)s^2}{s^3+s}\right] I_2 - \frac{s}{s^2+1} I_1$$

$$\frac{sI_1 + I_2}{2s+1} = \left(\frac{s^3+2s^2+s+1}{s^3+s}\right) I_2 - \frac{s}{s^2+1} I_1$$

$$\frac{s(s^3+s)I_1 + (2s+1)(s^3+s)I_2}{\cancel{(2s+1)(s^2+1)s}} = \frac{(s^3+2s^2+s+1)(2s+1)I_2 - s(2s+1) \cdot sI_1}{\cancel{(2s+1)(s^2+1) \cdot s}} \Rightarrow$$

$$\Rightarrow (s^4+s^2)I_1 + (\cancel{2s^4}+2s^2+s^3+s)I_2 = (\cancel{2s^4}+4s^3+2s^2+2s+s^3+2s^2+s+1)I_2 - (2s^3+s^2)I_1 \Rightarrow$$

$$\Rightarrow I_1 = \frac{4s^3+2s^2+2s+1}{s^4+2s^3+2s^2} I_2 \quad \Rightarrow I_3 = \frac{s}{2s+1} \cdot \frac{4s^3+2s^2+2s+1}{s(s^3+2s^2+2s)} \cdot I_2 + \frac{I_2}{2s+1}$$

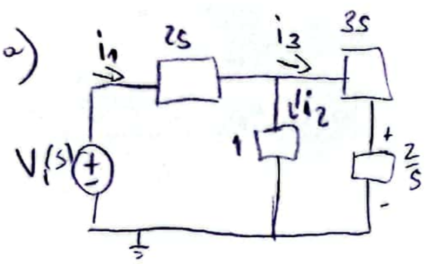
$$I_3 = I_2\left[\frac{(4s^3+2s^2+2s+1)+(s^3+2s^2+2s)}{(2s+1)(s^3+2s^2+2s)}\right] = I_2\cdot\frac{(5s^3+4s^2+4s+1)}{(2s+1)(s^3+2s^2+2s)}$$

$V_o(s)$ → COLOCA EVIDÊNCIA

MALHA FORA: $V_{i(s)} = \frac{1}{s}I_2 + sI_3 \Rightarrow V_i(s) = \frac{1}{s}I_2 + s\,I_2\frac{(5s^3+4s^2+4s+1)}{s(2s^3+5s^2+6s+2)}$

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{2s^3+5s^2+6s+2}{5s^4+6s^3+9s^2+7}$$

20) a)




$i_1 = i_2 + i_3$ • As impedâncias são inversamente proporcionais às correntes, quando estas se dividem

$$\uparrow\ \frac{3s+\frac{2}{s}}{1} \Leftrightarrow \downarrow\ \frac{i_3}{i_2} \rightarrow \frac{\frac{3s^2+2}{s}}{1} = \frac{i_2}{i_3} \rightarrow \frac{3s^2+2}{s} = \frac{i_2}{i_3}$$

$$i_1 = i_3\frac{(3s^2+2)}{s} + i_3 \Rightarrow i_1 = i_3\frac{(3s^2+s+2)}{s}$$

$$V_i(s) + 2s\cdot i_1 + V_o(s) = i_2 + V_o(s) \qquad V_o(s) = 3s\cdot i_3$$

$$V_i(s) + 2\cancel{s}\cdot i_3\frac{(\cancel{3s^2}+s+\cancel{2})}{\cancel{s}} + V_o(s) = i_3\frac{(\cancel{3s^2}+\cancel{2})}{s}$$

$$V_i(s) + 2s\cdot\frac{i_3 s}{3} + V_o(s) = 0$$

$$3V_i(s) = (-V_o(s) - 2s\,i_3)\cdot 3$$

$V_o(s)$

$$3\,V_i(s) = -5\,V_o(s)$$

$$G(s) = -\frac{3}{5}$$

26) $G(s) = \dfrac{X_2(s)}{F(s)}$ $\quad \begin{bmatrix} 0 \\ 0 \\ F(s) \end{bmatrix} = \begin{bmatrix} 8s^2+6s & -4s & 0 \\ -4s & 4s+2 & -2 \\ 0 & -2 & 2 \end{bmatrix} \cdot \begin{bmatrix} X_1(s) \\ X_2(s) \\ X_3(s) \end{bmatrix}$

$$\Delta = (8s^2+6s)(4s+2).2 + 0 + 0 - 0 - 4s.4s.2 - (8s^2+6s).2.2 =$$

$$= (32s^3+24s^2+16s^2+12s).2 - 32s^2 - 24s = 64s^3+80s^2-32s^2-32s^2 = 16(4s^3+s^2)$$

$$X_2 = \frac{\begin{bmatrix} 8s^2+6s & 0 & 0 \\ -4s & 0 & -2 \\ 0 & F(s) & -2 \end{bmatrix}}{\Delta} \Rightarrow X_2 = \frac{-(8s^2+6s)\,F(s).(-2)}{16(4s^3+s^2)} = \frac{\cancel{4s}(4s+3).F(s)}{\cancel{16s}_{4}(4s^2+s)}$$

$$X_2 = \frac{(4s+3)\,F(s)}{16s^2+4s}$$

$$G(s) = \frac{X_2(s)}{F(s)} = \frac{4s+3}{16s^2+4s}$$

27) $\begin{bmatrix} 0 \\ F(s) \end{bmatrix} = \begin{bmatrix} s^2+6s+9 & -3s-5 \\ -3s-5 & 2s^2+5s+5 \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$

$$\Delta = (s^2+6s+9)(2s^2+5s+5) - (3s+5)(3s+5) = 2s^4+5s^3+5s^2+12s^3+30s^2+\cancel{30s}+18s^2+45s+45-9s^2-\cancel{30s}$$

$$= 2s^4+17s^3+44s^2+45s+20$$

$$X_1 = \frac{\begin{bmatrix} 0 & -(3s+5) \\ F(s) & 2s^2+5s+5 \end{bmatrix}}{\Delta} \Rightarrow X_1 = \frac{F(s).(3s+5)}{\Delta}$$

$$G(s) = \frac{X_1(s)}{F(s)} = \frac{3s+5}{2s^4+17s^3+44s^2+45s+20}$$

28) $G(s) = \dfrac{X_3(s)}{F(s)}$ $\quad \begin{bmatrix} 0 \\ F(s) \\ 0 \end{bmatrix} = \begin{bmatrix} 4s^2+2s+6 & -2s & 0 \\ -2s & 4s^2+4s+6 & -6 \\ 0 & -6 & 4s^2+2s+6 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\Delta = (4s^2+2s+6)(4s^2+2s+6)(4s^2+4s+6)+0+0-0-4s^2(4s^2+2s+6)-36(4s^2+2s+6)$$
$$= (4s^2+2s+6)\left[(4s^2+2s+6)(4s^2+4s+6)-4s^2-36\right] = (4s^2+2s+6).4.(4s^4+6s^3+13s^2+3s)$$

$$X_3 = \frac{\begin{bmatrix} 4s^2+2s+6 & -2s & 0 \\ -2s & 4s^2+4s+6 & F(s) \\ 0 & -6 & 0 \end{bmatrix}}{\Delta} \qquad X_3 = \frac{-(8s)^3 . F(s) . \cancel{(4s^2+2s+6)}}{\cancel{(4s^2+2s+6)} . \cancel{4}_{2} (4s^4+6s^3+14s^2+9s+1)}$$

$$G(s) = \frac{X_3(s)}{F(s)} = \frac{3}{2(4s^4+6s^3+13s^2+9s)}$$

29) a) $G(s) = \dfrac{X_3(s)}{F(s)}$ $\quad \begin{bmatrix} F(s) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4s^2+8s+5 & -8s & -5 \\ -8s & 4s^2+16s & -4s \\ -5 & -4s & 4s+5 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\Delta = (4s^2+8s+5)(4s^2+16s)(4s+5)-160s^2-160s^2-25(4s^2+16s)-64s^2(4s+5)-16s^2(4s^2+8s+5)$$

$$X_3 = \begin{bmatrix} 4s^2+8s+5 & -8s & F(s) \\ -8s & 4s^2+16s & 0 \\ -5 & -4s & 0 \end{bmatrix} = \frac{32s^2 F(s) + 5(4s^2+16s)F(s)}{\Delta}$$

$$\Delta = (4s^2+16s).(16s^3+32s^2+20s+20s^2+40s+25)-320s^2-25(4s^2+16s)-256s^3-320s^3-16s^2(4s^2+8s+5)$$

$$\Delta = (4s^2+16s)(16s^3+52s^2+60s)-256s^3-640s^2-64s^4-128s^3-80s^2$$

$$\Delta = (4s^2+16s)(16s^3+52s^2+60s)-384s^3-720s^2-64s^4$$

$$\Delta = (4s^2+16s)(16s^3+52s^2+60s)-16s^2(4s^2+24s+4s)$$

$$\Delta = 4s\left[(s+4)(16s^3+52s^2+60s) - 4s(4s^2+24s+4s)\right]$$

$$\Delta = 16s^2\left[4s^3+13s^2+15s+16s^2+52s+60-4s^2-24s-4s\right]$$

$$\Delta = 16s^2(4s^3+25s^2+43s+15)$$

$$X_3 = \frac{4s(13s+20).F(s)}{16s^2(4s^3+25s^2+43s+15)} \qquad G(s) = \frac{13s+20}{4s(4s^3+25s^2+43s+15)}$$

b) $G(s) = \dfrac{X_3(s)}{F(s)}$

$$\begin{bmatrix} 0 \\ F(s) \\ 0 \end{bmatrix} = \begin{bmatrix} 8s^3+4s+16 & -(4s+1) & -1s \\ -(4s+1) & 3s^2+20s+1 & -16s \\ -1s & -16s & 16s+15 \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

$$\Delta = (8s^2+4s+16)(3s^2+20s+1)(16s+15) - 2.240s(4s+1) - 22s(3s^2+20s+1) - (4s+1)^2(16s+15)$$
$$-256s^2(8s^2+4s+16)$$

$$X_3 = \frac{\begin{bmatrix} 8s^2+4s+16 & -(4s+1) & 0 \\ -(4s+1) & 3s^2+20s+1 & F(s) \\ -1s & -16s & 0 \end{bmatrix}}{\Delta} = \frac{1s(4s+1)F(s) + 16s(8s^2+4s+16)F(s)}{\Delta}$$

$$\Delta = (8s^2+4s+16)\left[(48s^3+365s^2+316s+15) - 256s^2\right] - 480s(4s+1) - 22s(3s^2+20s+1) - (4s+1)$$
$$= (8s^2+4s+16)(48s^3+109s^2+316s+15) - (4s+1)(480s+64s^2+76s+15) - 22s(3s^2+20s+1)$$
$$= (8s^2+4s+16)(48s^3+109s^2+316s+15) - (4s+1)(64s^2+556s+15) - 22s(3s^2+20s+1)$$
$$= (8s^2+4s+16)(48s^3+109s^2+316s+15) - 1s\left[(172s^3+148s^2+4s+43s^2+37s+1)+(45s^2+300s+15)\right] - 4s^2 - s$$
$$= (8s^2+4s+16)(48s^3+109s^2+316s+15) - 4s^2 - s - 15\left[172s^3+281s^2+341s+16\right]$$
$$= 384s^5+872s^4+2528s^3+120s^2+192s^4+436s^3+1264s^2+60s+768s^3+1744s^2+5056s+\cancel{240}-4s^2-s$$
$$-2580s^3-4215s^2-5115s-\cancel{240}$$

$$\Delta = 384s^5+1064s^4+1152s^3-1093s^2 0s.$$

$$X_3 = \frac{F(s)\left[(60s+15)+(128s^3+64s^2+256s)\right]}{\Delta}$$

$$G(s) = \frac{X_3(s)}{F(s)} = \frac{128s^3+64s^2+316s+15}{384s^5+1064s^4+1152s^3-1031s^2}$$

32) a) $\theta_1(5s^2+9s+9)-\theta_2(1s+9)=0$

$\theta_2(3s^2+s+12)-\theta_1(s+9)=T(s)$

b)

$\theta_1(J_1s^2+K_1)-\theta_{12}(K_1)=T(s)$

$-\theta_1(K_1) \quad +\theta_{12}(D_1s+K_1)-\theta_2(D_1s)=0$

$-\theta_{12}(D_1s) \quad +\theta_2(D_1s+J_2s^2+K_2)-\theta_{23}(K_2)=0$

$-\theta_2(K_2)+\theta_{23}[D_2s+(K_2+K_3)]\theta_3(D_2s+K_3)=0$

$-\theta_{23}(D_2s+K_3)+\theta_3(J_3s^2+D_2s+K_3)=0$

33) $\begin{bmatrix} T(s) \\ 0 \end{bmatrix} = \begin{bmatrix} s^2+2s+1 & -(s+1) \\ -(s+1) & 2s+1 \end{bmatrix} \cdot \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$ $\quad G(s) = \frac{\theta_2(s)}{T(s)}$

$\Delta = (s^2+2s+1)(2s+1)-(s+1)^2 = 2s^3+4s^2+\cancel{2s}+\cancel{s^2}+2s+\cancel{1}-\cancel{s^2}-\cancel{2s}-\cancel{1}$

$\Delta = 2s^3+4s^2+2s$

$$\theta_2 = \frac{\begin{bmatrix} s^2+2s+1 & T(s) \\ -(s+1) & 0 \end{bmatrix}}{2s^3+4s^2+2s} = \frac{-T(s).(-1).(s+1)}{2s^3+4s^2+2s} \Rightarrow G(s) = \frac{\theta_2}{T(s)} = \frac{s+1}{2s^3+4s^2+2s}$$

34) $G(s) = \dfrac{\theta_1(s)}{T(s)}$

$$\begin{bmatrix} T(s) \\ 0 \end{bmatrix} = \begin{bmatrix} s^2+s+1 & -(s+1) \\ -(s+1) & s^2+s+2 \end{bmatrix} \cdot \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

$$\Delta = (s^2+s+1)(s^2+s+2) - (s+1)^2 = s^4+s^3+2s^2+s^3+s^2+\cancel{2s}+\cancel{s^2}+s+2\cancel{-s^2}\cancel{-2s}-1$$

$$\Delta = s^4+2s^3+3s^2+s+1$$

$$\theta_1 = \frac{\begin{vmatrix} T(s) & -(s+1) \\ 0 & s^2+s+2 \end{vmatrix}}{\Delta} \Rightarrow \theta_1 = \frac{T(s).(s^2+s+2)}{s^4+2s^3+3s^2+s+1} \Rightarrow G(s) = \boxed{\frac{s^2+s+2}{s^4+2s^3+3s^2+s+1}}$$