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1.

a) $f(t) = 3 + 7t + t^2 + \delta(t)$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{3 + 7t + t^2 + \delta(t)\} = \frac{3}{s} + \frac{7}{s^2} + \frac{2}{s^3} + 1$$

$$F(s) = \frac{3}{s} + \frac{7}{s^2} + \frac{2}{s^3} + 1$$

$$F(s) = \frac{s^3 + 3s^2 + 7s + 2}{s^3}$$

b) $f(t) = t \cdot \cos 3t$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{-(-t)\cos 3t\}$$

$$-\frac{d}{ds}\left(\frac{s}{s^2+9}\right) = -\left(\frac{1 \cdot (s^2+9) - s \cdot 2s}{(s^2+3^2)^2}\right) = \left(\frac{s^2+9-2s^2}{(s^2+9)^2}\right)$$

$$F(s) = \frac{s^2-9}{(s^2+9)^2}$$

c) $$F(s) = \frac{1}{s(s+2)^2} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$\frac{A(s+2)^2 + Bs(s+2) + Cs}{s(s+2)^2}$$

$$1 = A(s+2)^2 + Bs(s+2) + Cs$$

$\boxed{\cdot\, S=0}$

$1 = A \cdot 2^2$

$A = 1/4$

$\boxed{\cdot\, S=-2}$

$1 = C \cdot -2$

$C = -1/2$

$\boxed{\cdot\, S=-1}$

$1 = A - B - C$

$1 = \frac{1}{4} - B + \frac{1}{2}$

$B = -\frac{1}{4}$

$$F(s) = \frac{1}{S(S+2)^2} = \frac{1}{4} \cdot \frac{1}{S} - \frac{1}{4} \cdot \frac{1}{S+2} - \frac{1}{2} \cdot \frac{1}{(S+2)^2}$$

$$\mathcal{L}^{-1}\{f(s)\} = \mathcal{L}^{-1}\{ 1/4 \cdot 1/s - 1/4 \cdot 1/s+2 - 1/2 \cdot 1/(s+2)^2 \}$$

$$\frac{1}{4} \cdot 1 - \frac{1}{4} e^{-2t} - \frac{1}{2} e^{-2t} \cdot t$$

$$f(t) = \frac{e^{-2t}}{4} \cdot \left(-2t + e^{2t} - 1\right)$$

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d) $F(s) = \dfrac{3s+2}{s(s+1)(s^2+4s+10)} = \dfrac{A}{s} + \dfrac{B}{s+1} + \dfrac{Cs+D}{s^2+4s+10}$

$\boxed{\cdot\, s=0}$ $2 = A \cdot 10$

$A = 1/5$

$\boxed{\cdot\, s=-1}$ $-1 = 3.-1+2 = B(-1)(1-4+10)$

$B = 1/7$

$\boxed{\cdot\, s=1}$ $3+2 = A.2(1+4+10) + B(1+4+10) + C.2 + D2$

$5 = 30A + 15B + 2C + 2D$

$D = \dfrac{-11}{7} - C$

$\boxed{\cdot\, s=-2}$ $3.-2+2 = A(-1)(4-8+10) + B(-2)(4-8+10) + C.4.-1 + D.-2.-1$

$-4 = -6A - 12B - 4C + 2D$

$-4 = \dfrac{-6}{5} + \dfrac{12}{7} - 4C + 2D$

$\dfrac{-140+42+60}{35} = -4C + 2D$

$\dfrac{-19}{35} - D = 2C$

$2C = \dfrac{19}{35} - \dfrac{11}{7} - C$ $\quad C = \dfrac{-12}{35}$

$D = \dfrac{-11}{7} - C$ $\quad D = \dfrac{-43}{35}$

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$$\frac{A}{S}+\frac{B}{S+1}+\frac{Cs+D}{S^2+4s+10}=\frac{1}{5}+\frac{1}{S}+\frac{1}{7}\cdot\frac{1}{S+1}+\frac{-\frac{12}{35}s-\frac{43}{35}}{S^2+4s+10}$$

$$\frac{1}{35}\left(\frac{7}{s}+\frac{5}{s+1}-\frac{12s-43}{s^2+4s+10}\right)$$

$$\frac{1}{35}\left(\frac{7}{s}+\frac{5}{s+1}-12\cdot\left(\frac{s+2}{(s+2)^2+\sqrt{6}^2}\right)-\frac{19}{\sqrt{6}}\cdot\frac{\sqrt{6}}{s(s+2)\sqrt{6}}\right)$$

Laplace inversa

$$\frac{1}{35}\left(7+5e^{-t}-12e^{-2t}\cos(\sqrt{6}t)-\frac{19}{\sqrt{6}}\cdot e^{-2t}\operatorname{sen}(\sqrt{6}t)\right)$$

$$\frac{1}{35}e^{-2t}\left(7e^{2t}+5e^{t}-12\cos(\sqrt{6}t)-\frac{19}{\sqrt{6}}\operatorname{sen}(\sqrt{6}t)\right)$$

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2.

a) $-6x_1(t) - 4\ddot{x}_1(t) - 2\dot{x}_1(t) + 2\dot{x}_2(t) = 0$

$4s^2X_1(s) + 2sX_1(t) + 6X_1(s) = 2sX_2(s)$

$X_1(s)[4s^2 + 2s + 6] = X_2(s)[2s]$

$-4\ddot{x}_2(t) - 4\dot{x}_2(t) - 6x_2(t) + 2\dot{x}_1 + 6x_3(t) + f(t) = 0$

$4s^2X_2(s) + 4sX_2(s) + 6X_2(s) - 2sX_1(s) - 6X_3(s) = F(s)$

$X_2(s)[4s^2 + 4s + 6] - X_1(s)[2s] - X_3(s)[6] = F(s)$

$-6x_3(t) - 4\ddot{x}_3(t) - 2\dot{x}_3 + 6\dot{x}_2(t) = 0$

$X_3[4s^2 + 2s + 6] = X_2(s)[6]$

$X_3(t)\cancel{[4s^2 + 2s + 6]} = \cancel{6}^{3}\dfrac{\cancel{[4s^2 + 2s + 6]}}{2s}X_1(s)$

$X_3(t) = \dfrac{3}{s}X_1(s)$

$X_1(s)\left[\dfrac{4s^2 + 2s + 6}{2s}\right]\left[4s^2 + 4s + 6\right] + X_1(s)[-2s] + X_1(s)\left[-\dfrac{3}{s}\right]6 = F(s)$

$X_1(s)\left[\dfrac{(2s^2 + s + 3)(4s^2 + 4s + 6) - 2s^2 - 18}{s}\right] = F(s)$

$\dfrac{X_1(s)}{F(s)} = \dfrac{s}{8s^4 + 8s^3 + 12s^2 + 4s^3 + 4s^2 + 6s + 12s^2 + 12s + 18 - 12s^2 - 18}$

$\dfrac{X_1}{F(s)} = \dfrac{s}{8s^4 + 12s^3 + 26s^2 + 18s}$

$\dfrac{X_3}{F(s)} = \dfrac{3}{s} \cdot \dfrac{X_1(s)}{F(s)}$ $\qquad \dfrac{X_3(s)}{F(s)} = \dfrac{s}{8s^4 + 12s^3 + 26s^2 + 18s}$

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b)

$$\begin{bmatrix} 0 \\ T(s) \end{bmatrix} = \begin{bmatrix} J_1 s^2 + D_1 s & -(D_2 s + K_1) \\ -(D_2 s + K_1) & J_2 s^2 + K_2 \end{bmatrix}$$

$$T_s \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 5s^2 + 8s & -(s+9) \\ -(s+9) & 3s^2+3 \end{bmatrix}$$

$$\theta_1(s) = \begin{bmatrix} 0 & -(s+9) \\ 1 & 3s^2+3 \end{bmatrix} \qquad \theta_2(s) = \begin{bmatrix} 5s^2+8s & 0 \\ -(s+9) & 1 \end{bmatrix}$$

$$\frac{|\theta_1(s)|}{|T(s)|} = \frac{0 - (-(s+9))}{(5s^2+8s)(3s^2+3) - ((-s-9)^2)} = \boxed{\frac{s+9}{15s^4 + 15s^2 + 24s^3 + 24s - s^2 - 18s - 81}}$$

$$\frac{|\theta_2(s)|}{|T(s)|} = \frac{5s^2 + 8s - 0}{15s^4 + 24s^3 + 16s^2 + 6s - 81} = \boxed{\frac{5s^2+8s}{15s^4 + 24s^3 + 16s^2 + 6s - 81}}$$

c) $T(s) = (J_1 s + K_1)\,\theta_1(s) + (J_2 s^2 + K_2)\,\theta_2(s) + (J_3 s^2 + D_3)\,\theta_3$

$$\frac{N_1}{N_2} = \frac{\theta_2}{\theta_1} \qquad \theta_1 = \frac{N_2}{N_1}\theta_2 = \frac{50}{5}\theta_2 = 10\,\theta_2$$

$$\frac{N_1}{N_3} = \frac{\theta_3}{\theta_1} \qquad \theta_3 = \frac{N_3}{N_1}\theta_1 = \frac{35}{5}(10\,\theta_2) = 50\,\theta_2$$

$$T(s) = (3s^2+3)\,10\,\theta_2(s) + (150s^2 + 300)\,\theta_2(s) + (100s^2 + 5000s)\,50\,\theta_2(s)$$

$$T(s) = [30s^2 + 30 + 150s^2 + 300 + 5000s^2 + 250000s]\,\theta_2(s)$$

$$\frac{\theta_2(s)}{T(s)} = \frac{1}{5180s^2 + 250000s + 330}$$

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3

a) $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{\omega n^2}{s(s+2\zeta\omega n)}}{1+\frac{\omega n^2}{s(s+2\zeta\omega n)}} = \frac{\omega n^2}{s^2+2\zeta\omega n . s+\omega n^2}$

$\boxed{\zeta = 0{,}8 \, , \, \omega n = 25 \text{ rad/s}}$

$\omega d = \omega n\sqrt{1-\zeta^2} = 25\sqrt{1-0{,}8^2} = 15 \text{ rad/s}$

$\sigma d = \zeta\omega n = 0{,}8 . 25\sqrt{1-0{,}8^2} = 20$

$T_r \cong \frac{1{,}8}{\omega n} \cong \frac{1{,}8}{25} \cong 0{,}072 \text{ seg}$

$T_p = \frac{\pi}{\omega d} = \frac{\pi}{20} \cong 0{,}157 \text{ seg}$

$T_s = \frac{4}{\zeta\omega n} = \frac{4}{\sigma d} = \frac{4}{20} = 0{,}2 \text{ seg}$

$M_p = e^{-\left(\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)} . 100\% = e^{-\left(\frac{0{,}8\pi}{\sqrt{0{,}36}}\right)} . 100\% = 1{,}516\%$

b) $\Sigma F = m . a(t)$

$F(s) - fv . sX(s) - KX(s) = Ms^2X(s)$
$F(s) = m . s^2X(s) + fv . X(s) + KX(s)$
$\frac{F(s)}{m} = X(s)\left[s^2 + \frac{fv}{ms} + K/m\right]$

$\frac{X(s)}{F(s)} = \frac{1/m}{s^2+\frac{fv}{m}s+K/m}$ $\quad \frac{X(s)}{F(s)} = \frac{0{,}2}{s^2+0{,}4s+4}$

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$\omega_n^2 = 4$

$\omega_n = 2 \ rad/s$

$2\xi\omega_n = 0,4$

$\xi = \frac{0,4}{2 \cdot 2} = 0,1$

$\sigma_d = \xi\omega_n = 0,1 \cdot 2 = 0,2$

$\omega_d = \omega_n\sqrt{1-\xi^2} = 2 \cdot \sqrt{0,99} \cong 1,989 \ rad/s$

$\theta = \cos^{-1}\xi = \cos^{-1} 0,1 \cong 84,26°$

$M_p = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)} \cdot 100\% = 72,92\%$

$T_r \cong \frac{1,8}{\omega_n} \cong \frac{1,8}{2} \cong 0,9s$

$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{1,989} = 1,579 s$

$T_s = \frac{4}{\sigma_d} = \frac{4}{0,2} = 20s$

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c)

$\boxed{\bullet\ \theta_1(s) = \theta_2(s)}$

$$T(s) - Js^2\theta_1(s) - Ds\,\theta_1(s) - K\theta_1(s) = 0$$

$$\frac{1}{2}T(s) = \theta_1(s)\left[s^2 + \tfrac{1}{2}s + \tfrac{1}{2}\right]$$

$$\frac{\theta_1(s)}{T(s)} = \frac{\theta_2(s)}{T(s)} = \frac{-\tfrac{1}{2}}{s^2 + \tfrac{1}{2}s + \tfrac{1}{2}}$$

$\omega_n^2 = 1/2 \qquad \omega_n = 0{,}707 \text{ rad/s}$

$2\zeta\omega_n = \frac{1}{2} \qquad \zeta = \frac{1}{2} \cdot \frac{1}{2 \cdot 0{,}707} = 0{,}354$

$\sigma_d = \zeta\omega_n = 0{,}354 \cdot 0{,}707 \cong 0{,}25$

$\omega_d = 0{,}707\sqrt{1 - 0{,}354^2} = 0{,}661 \text{ rad/s}$

$\theta = \cos^{-1}\zeta = \cos^{-1}0{,}354 = 69{,}3^\circ$

$M_p = e^{-\left(\frac{0{,}354\pi}{\sqrt{0{,}936}}\right)} \cdot 100\% = 30{,}443\%$

$T_r \cong \frac{1{,}8}{0{,}707} = 2{,}546\,s$

$T_p = \frac{\pi}{0{,}661} = 4{,}753\,s$

$T_s = \frac{4}{0{,}25} = 16\,s$

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4

a) $H_{(s)} = \dfrac{G(s)}{1+G(s)H(s)} = \dfrac{\dfrac{450}{s^4+3s^3+10s^2+30s+150}}{1+\dfrac{450}{s^5+3s^4+10s^3+30s^2+150s}} = \dfrac{450s}{s^5+3s^4+10s^3+30s^2+150s+450}$

$s^5+3s^4+10s^3+30s^2+150s+450=0$

$s^5+3s^4+10s^3+30s^2+150s+450 \;\; \underline{| s+3}$

$-s^5-3s^4 \qquad\qquad s^4+10s^2+150$

$0+10s^3+30s^2+150s+450$

$-10s^3-30s^2$

$0 \quad +150s+450$

$-150s-450$

0

$s+3=0 \quad \therefore s_0=-3$

$s^4+10s^2+150=0 \quad \therefore x^2+10x+150=0$

$s=\sqrt{-5\pm i5\sqrt{5}}$

3 polos reais negativos (2 pares conjugados)
2 polos reais positivos (1 par conjugado)

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b) $H_{(s)} = \dfrac{G(s)}{1-G(s)H(s)} = \dfrac{\dfrac{18}{s^5+s^4-7s^3-7s^2-18s}}{1-\dfrac{18}{s^5+s^4-7s^3-7s^2-18s}} = \dfrac{18}{s^5+s^4-7s^3-7s^2-18s-18}$

$(s+1)(s+3)(s-3)(s^2+2)=0$

SISTEMA INSTÁVEL

5.

a) $\boxed{\begin{matrix} f_v = 1{,}5 \\ T_s = 4 \\ T_p = 1 \end{matrix}}$

$\sum F(t) = m \cdot a(t)$

$f(t) - f_v \cdot v(t) - Kx(t) = m \cdot a(t)$

$m \cdot a(t) + f_v v(t) + Kx = f(t)$

$m\ddot{x} + f_v\dot{x} + Kx = f(t)$

$\ddot{x} + \frac{f_v}{m}\dot{x} + \frac{K}{m}x = f(t)$

$s^2 X(s) + \frac{f_v}{m}s\, X(s) + \frac{K}{m}X(s) = \frac{F(s)}{m}$

$\frac{X(s)}{F(s)} = \frac{1/m}{s^2 + \frac{f_v}{m}s + \frac{K}{m}}$

- $T_p = \frac{\pi}{\omega_d}$ $\quad \omega_d = \frac{\pi}{T_p} = \frac{\pi}{1}$
- $T_s \cong \frac{4}{\sigma_d}$ $\quad \sigma \cong 1$

$\omega_n^2 = \frac{K}{m}$

$K \cong 8{,}16$

- $\sigma_d = \frac{f_v}{m} \cdot \frac{1}{2} = \frac{1{,}5}{2m} = \frac{0{,}75}{m}$ $\quad m = 0{,}75\ Kg$
- $\sigma_d = \zeta \cdot \omega_n = \frac{f_v}{m} \cdot \frac{1}{2}$ $\quad \zeta = \frac{1}{\omega_n}$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$ $\quad \pi = \omega_n\sqrt{1 - \frac{1^2}{\omega_n^2}}$ $\quad \omega_n = \frac{\pi}{\sqrt{\frac{\omega_n^2 - 1}{\omega_n^2}}} \cong 3{,}3$

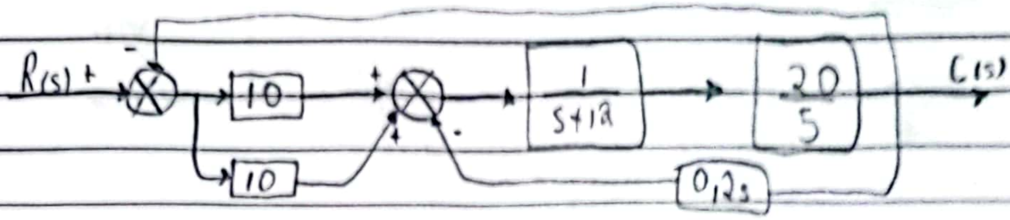
$\frac{f_v}{m} = 2\zeta\omega_n$ $\quad \zeta = \frac{1{,}5}{0{,}75} \cdot \frac{1}{2} \cdot \frac{1}{3{,}3} \cong 0{,}3$

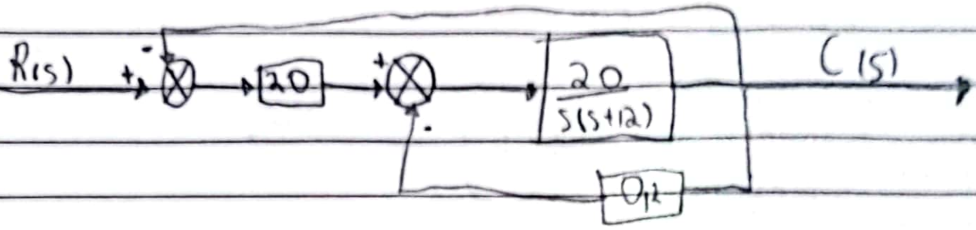
$\theta = \cos^{-1}\zeta \cong 72{,}5°$ $\quad T_r \cong \frac{1{,}8}{\omega_n} \cong 0{,}545$

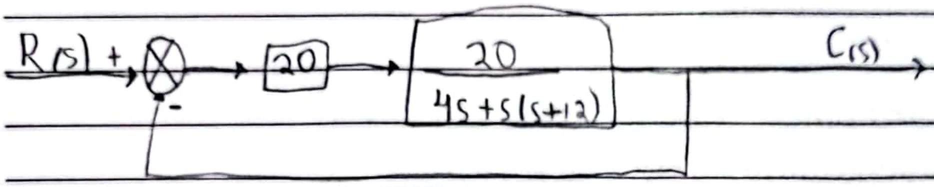
$M_p = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \cdot 100\% = 32{,}42\%$

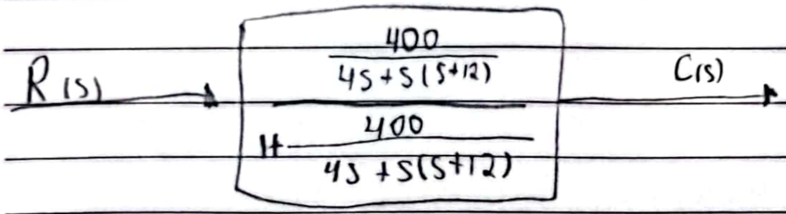
6.

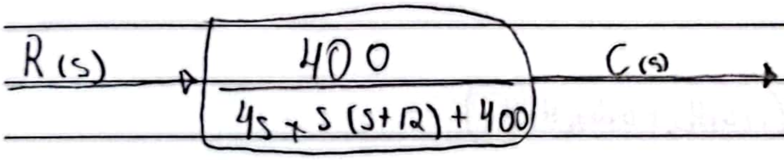
a)





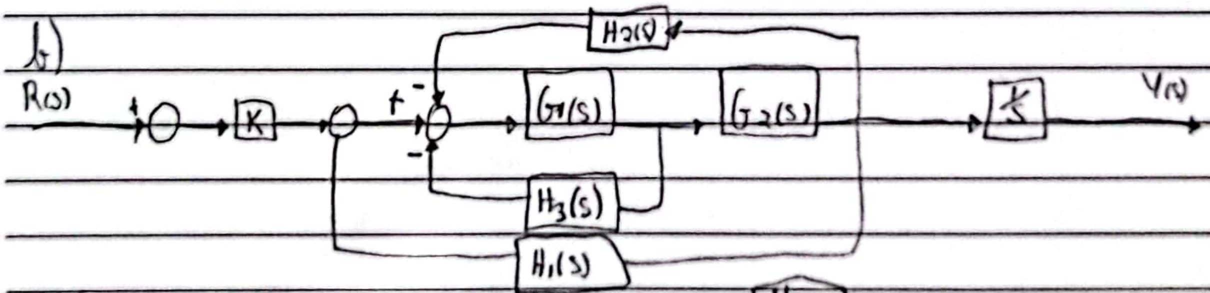


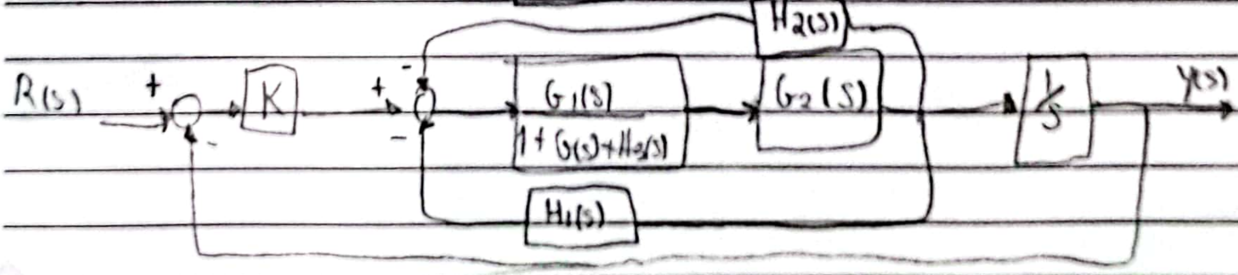


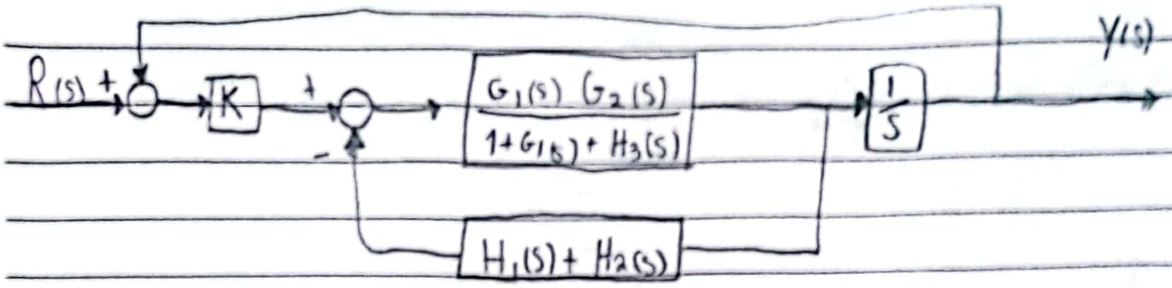


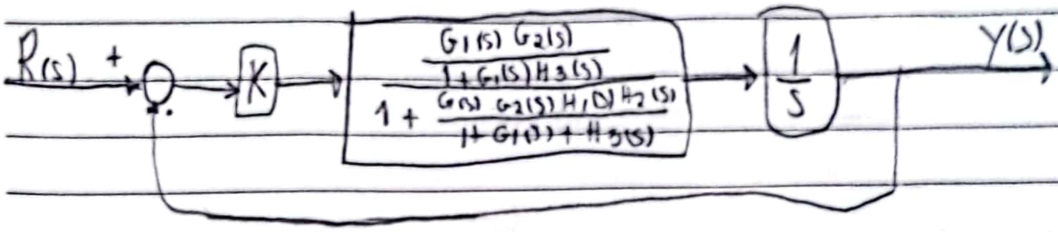
$$\frac{C(s)}{R(s)} = H(s) = \frac{400}{s^2+16s+400}$$

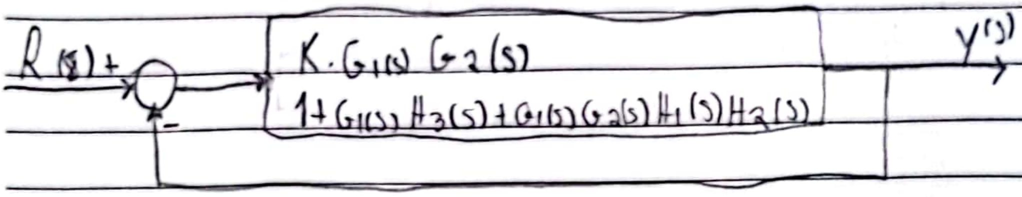
b)

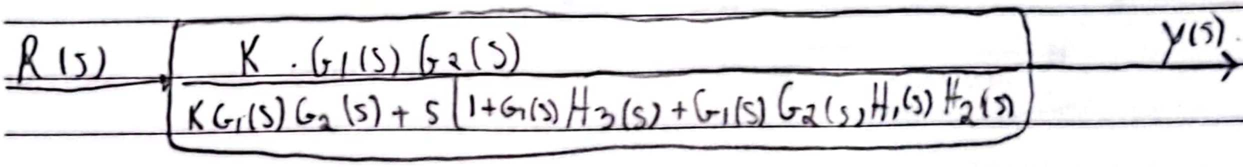












$$H(s)=\frac{Y(s)}{R(s)}=\frac{K\,G_1G_2}{K\,G_1\,G_2+s\left(1+G_1H_3+G_1G_2H_1H_2\right)}$$

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