Aluno: Matheus Ferreira Gomes
matrícula: 11621ECP007

1. a) $\mathcal{L}\{u(t)\} = \frac{1}{s}$
b) $\mathcal{L}\{t\,u(t)\} = \frac{1}{s^2}$
c) $\mathcal{L}\{\operatorname{sen} wt\, u(t)\} = \frac{w}{s^2+w^2}$
d) $\mathcal{L}\{\cos wt\; u(t)\} = \frac{s}{s^2+w^2}$

2 a) $L\{e^{-at} \operatorname{sen} wt\, u(t)\}$

$$L\{e^{-at} \cdot f(t)\} = F(s+a)$$

$$L\{f(t) = \frac{w}{s^2+w^2}$$

$$L\{e^{-at} \cdot \operatorname{sen}(wt) \cdot u(t)\} = \frac{w}{(s+a)^2+w^2}$$

b) $L\{e^{-at} \cos(wt) \cdot u(t)\} = \frac{s+a}{(s+a)^2+w^2}$

c) $L\{t^3\, u(t)\} = \frac{3!}{s^4} = \frac{6}{s^4}$

8) $\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + y = \frac{d^3x}{dt^3} + 4\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x$

$$L\{\dddot{y} + 3\ddot{y} + 5\dot{y} + y = \dddot{x} + 4\ddot{x} + 6\dot{x} + 8x\}$$

$$s^3 Y(s) + 3s^2 Y(s) + 5s\,Y(s) + Y(s) = s^3 X(s) + 4s\,X(s) + 8X(s)$$
$$Y(s)(s^3 + 3s^2 + 5s + 1) = X(s)(s^3 + 4s^2 + 6s + 8)$$
$$\frac{Y(s)}{X(s)} = \frac{s^3 + 4s^2 + 6s + 8}{s^3 + 3s^2 + 5s + 1}$$

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9a) $\frac{X(s)}{F(s)} = \frac{7}{s^2+5s+10}$

$L\{7(F(s)) = X(s)(s^2+5s+10)\}$

$7f(x) = \frac{d^2}{dt}x + 5\frac{dx}{dt} + 10x$

$f(x) = \frac{1}{7}\left(\frac{d^2}{dt^2} + 5\frac{dx}{dt} + 10x\right)$

b) $\frac{X(s)}{F(s)} = \frac{15}{(s+10)(s+11)}$

$15 = X(s) + (s+10) + (s+11)$

$L\{F(s) = \frac{X(s)}{15}(s^2+21s+110)\}$

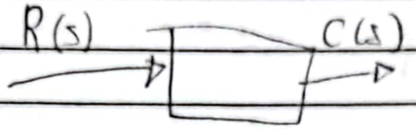
$f(s) = \frac{1}{15}\cdot\left(\frac{d^2x}{dt^2} + 21\frac{dx}{dt} + 110x\right)$

c) $\frac{X(s)}{F(s)} = \frac{s+3}{s^3+11s^2+12s+18}$

$L^{-1}\{(s+3)F(s) = X(s)(s^3+11s^2+12s+18)\}$

$\frac{df(t)}{dt} + 3f(t) = \frac{d^3X}{dt} + 11\frac{d^2x}{dt} + 12\frac{dt}{dx} + 18X$

10)



$$\frac{C(s)}{R(s)} = \frac{s^5+2s^4+4s^3+s^2+4}{s^6+7s^5+3s^4+2s^3+s^2+s}$$

$$\mathcal{L}^{-1}\{C(s)\,(s^6+7s^5+3s^4+2s^3+s^2+s) = R(s)\,(s^5+2s^4+4s^3+s^2+4)\}$$

$$\frac{d^6c(t)}{dt^6} + 7\frac{d^5c(t)}{dt^5} + 3\frac{d^4c(t)}{dt^4} + 2\frac{d^3c}{dt^3} + \frac{d^2c(t)}{dt^2} + s\,c(t)$$

$$\frac{d^5r(t)}{dt^5} + \frac{2d^4r(t)}{dt^4} + 4\frac{d^3r(t)}{dt^3} + \frac{d^2r(t)}{dt} + 4r(t)$$

11) $$\frac{C(s)}{R(s)} = \frac{s^4+3s^3+2s^2+s+1}{s^5+4s^4+3s^3+2s^2+3s+2} \; ; \; r(t) = 3t^3$$

$$\mathcal{L}^{-1}\{C(s)\,(s^5+4s^4+3s^3+2s^2+3s+2) = R(s)\,(s^4+3s^3+2s^2+s+1)\}$$

$$r(t) = 3t^3 \rightarrow \frac{dr}{dt} = 9t^2 \rightarrow \frac{d^2r}{dt^2} = 18t$$

$$\frac{d^3r}{dt^3} = 18 \qquad \frac{d^4t}{dt^4} = 18\,\delta(t) \approx 0$$

$$\frac{d^5c}{dt^5} + \frac{4d^4c}{dt^4} + \frac{3d^3c}{dt^3} + \frac{2d^2c}{dt} + \frac{3dc}{dt} + 2c = \delta(t) + (3t^3 + 9t^2 + 36t + 54)\,u(t)$$

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12) $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 5x = 1$; $x(0) = 1; x(0) = -1$

$\mathcal{L}\{ x(t) + 4x(t) + 5x(t) = 1 \cdot u(t) \}$

$s^2 \mathcal{L}\{x(t)\} - s x(0) - s x(0) + 4[s \cdot \mathcal{L}\{x(t)\} - x(0)] + 6\mathcal{L}\{x(t)\}$

$s^2 X(s) - s.1 - (-1) + 4[s\ X(s) - 1] + 6X(s) = 1/s$

$s^2 X(s) - s + 1 + 4s\ X(s) - 4 + 6\ X(s) = 1/s$

$X(s)(s^2 + 4s + 6) = \frac{1}{s} + s + 3$

$X(s) = \frac{1}{(s(s^2+4s+6))} + \frac{s}{(s^2+4s+6} + \frac{3}{s^2+4s+6}$

3.2 | b)

$f(t) = 3 + 7t + t^2 + \delta(t)$

$\mathcal{L}\{f(t)\} = F(s) = \mathcal{L}\{3\} + 7\mathcal{L}\{t\} + \mathcal{L}\{t^2\} + \mathcal{L}\{\delta(t)\}$

$F(s) = 3 \cdot \frac{1}{s} + 7 \cdot \frac{1}{s^2} + \frac{2!}{s^3} + 1$

$F(s) = \frac{s^3 + 3s^2 + 7s + 2}{s^3}$

d) $f(t) = (t+1)^2$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{t^2 + 2t + 1\}$

$F(s) = \frac{2!}{s^3} + 2\frac{1}{s^2} + \frac{1}{s}$

$F(s) = \frac{2 + 2s + s^2}{s^3}$

e) $f(t) = \operatorname{senh}(t)$

$\mathcal{L}\{f(t)\} = \mathcal{L}\left\{\frac{e^t - e^{-t}}{2}\right\}$

$F(s) = \frac{1}{s^2 - 1}$

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3.3 a) $\mathcal{L}\{f(t)\} = \mathcal{L}\{3\cos 6t\}$

$$F(s) = 3\left(\frac{s}{s^2+6^2}\right)$$

b) $\mathcal{L}\{f(t)\} = \mathcal{L}\{\operatorname{sen}(2t) + 2\cos(2t) + e^{-t}\operatorname{sen}(2t)\}$

$$F(s) = \frac{2}{s^2+2^2} + 2\left(\frac{s}{s^2+2^2}\right) + \frac{2}{(s+1)^2+2^2}$$

c) $\mathcal{L}\{f(t)\} = \mathcal{L}\{t^2 + e^{-2t}\operatorname{sen} 3t\}$

$$F(s) = \frac{2}{s^3} + \frac{3}{(s+2)^2+3^2}$$

3.5 a) $\mathcal{L}\{f(t)\} = \mathcal{L}\{\operatorname{sen} t \cdot \operatorname{sen} 3t\}$

$$F(s) = \frac{1}{2}\left[\mathcal{L}\{\cos 2t\} - \mathcal{L}\{\cos 4t\}\right]$$

$$F(s) = \frac{1}{2}\left[\frac{s(s+16) - s(s^2+4)}{(s^2+4)(s+16)}\right]$$

$$F(s) = \frac{6s}{(s^2+4)(s^2+16)}$$

b) $\mathcal{L}\{f(t)\} = \mathcal{L}\{\operatorname{sen}^2 t + 3\cos^2 t\}$

$$F(s) = \mathcal{L}\{1 + 2\cos^2 t\}$$

$$F(s) = 2 \cdot \frac{1}{s} + 4\frac{s}{s^2+2^2} = \frac{2}{s} + \frac{s}{s^2+4}$$

c) $L\{f(t)\} = L\{\operatorname{sen} t / t\}$

$$\begin{cases} L\{g(t)\} = G(s) \\ L\left\{\dfrac{g(t)}{t}\right\} = \displaystyle\int_s^{\infty} G(s)\,ds \end{cases}$$

$$\lim_{t \to 0} \left\{\frac{\operatorname{sen}(t)}{t}\right\} = \frac{0}{0} \xrightarrow{LH} \lim_{t \to 0} \left\{\frac{\cos(t)}{1}\right\} = 1$$

$$L\left\{\frac{\operatorname{sen}(t)}{t}\right\} = \int_s^{\infty} \left(\frac{1}{s^2+1}\right) ds \quad \begin{cases} s = tg\,\theta \\ ds = \sec^2\theta\, d\theta \end{cases}$$

$$F(s) = \left[tg^{-1} s\right]_s^{\infty}$$

$$F(s) = \frac{\pi}{2} - tg^{-1} s$$

$$F(s) = tg^{-1}\left(\frac{1}{s}\right)$$

3.7 a) $L^{-1}\{F(s)\} = L^{-1}\left\{\dfrac{2}{s(s+2)}\right\}$

$$\frac{2}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

$$2 = A(s+2) + B(s)$$

$s = 0:\ 2 = A \cdot 2 + 0 \Rightarrow A = 1$

$s = -2:\ 2 = 0 + B(-2) \rightarrow B = -1$

$$L^{-1}\{F(s)\} = L^{-1}\left\{\frac{1}{s} - \frac{1}{s+2}\right\}$$

$$f(t) = 1(t) - e^{-2t} \cdot 1(t)$$

$$f(t) = 1 - e^{-2t}$$

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d) $L^{-1}\{F(s)\} = L^{-1}\left\{\dfrac{3s^2+9s+12}{(s+2)(s^2+5s+11)}\right\}$

$$\frac{3s^2+9s+12}{(s+2)(s^2+5s+11)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+5s+11}$$

$s=-2: \quad 12-18+12 = A(4-10-11)+0$

$$6 = 5A$$
$$A = 6/5$$

$$9s^2+3s+\left(\frac{6}{5}\right) = Bs^2+(2B+C)s+2C$$

$B = \dfrac{9}{5}$; $C = -\dfrac{3}{5}$; $2B+C=3 \rightarrow \dfrac{18}{5} - \dfrac{3}{5} = \dfrac{15}{5} = 3$

$$L^{-1}\{F(s)\} = L^{-1}\left\{\frac{6}{5}\cdot\frac{1}{s+2} + \frac{\frac{9}{5}s - \frac{3}{5}}{s^2+5s+11}\right\}$$

$$f(t) = \frac{6}{5}e^{-2} + \frac{1}{5}L^{-1}\left\{\frac{9s-3}{s^2+5s+11}\right\}$$

$$f(t) = 6\frac{e^{-2t}}{5} + \frac{9}{5}\left[e^{-\frac{5}{2}t}\cdot\cos\left(\frac{\sqrt{19}}{2}t\right) - \frac{13}{15}\cdot e^{-\frac{5}{2}t}\cdot\sin\left(\frac{\sqrt{19}}{2}\cdot t\right)\right]$$

$$f(t) = \frac{6e^{-2t}}{5} + \frac{9}{5}e^{-\frac{5}{2}t}\left[\cos\left(\frac{\sqrt{19}}{2}t\right) - \frac{13}{15}\cdot\sin\left(\frac{\sqrt{19}t}{2}\right)\right]$$

e) $L^{-1}\{F(s)\} = L^{-1}\left\{\frac{1}{s^2+4}\right\}$

$L^{-1}\{F(s)\} = \frac{1}{2} L^{-1}\left\{\frac{2}{s^2+2^2}\right\}$

$f(t) = \frac{1}{2} \cdot \operatorname{sen}(2t)$

f) $L^{-1}\{F(s)\} = L^{-1}\left\{\frac{2(s+2)}{(s+1)(s^2+4)}\right\}$

$f(t) = 2 \cdot L^{-1}\left\{\frac{s+2}{(s+1)(s^2+4)}\right\}$

$\frac{s+2}{(s+1)(s^2+4)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+4}$

$s+2 = A(s^2+4) + (Bs+c)(s+1)$

$s=1: \; 1 = A(1+4) + 0 \rightarrow \boxed{A = 1/5}$

$s+2 = \frac{1}{5}(s^2+4) + (Bs+c)(s+1)$

$B = -\frac{1}{5} \qquad c = 6/5 \qquad B+C \overset{?}{=} 1 \rightarrow -\frac{1}{5} + \frac{6}{5} \overset{?}{=} 1 \rightarrow 1 = 1$

$f(t) = 2 \cdot L^{-1}\left\{\frac{1}{5} \cdot \frac{1}{s+1} + \frac{-\frac{1}{5}s + \frac{6}{5}}{s^2+4}\right\}$

$f(t) = \frac{2}{5}\left[e^{-t} + 3 \cdot \operatorname{sen}(2t) - \cos(2t)\right]$

$i) \mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{4}{s^4+4}\right\}$

$$\frac{4}{s^4+4} = \frac{4}{(s^2+2s+2)(s^2-2s+2)} = \frac{As+B}{s^2+2s+2} + \frac{Cs+D}{s^2-2s+2}$$

$$4 = (As+B)(s^2-2s+2) + (Cs+D)(s^2+2s+2)$$

$$4 = As^3 - 2As^2 + 2As + Bs^2 - 2Bs + 2B + Cs^3 + 2Cs^2 + 2Cs + Ds^2 + 2Ds + 2D$$

$\cdot\ A+C=0 \rightarrow A=-C;\ 2B+2D=4 \rightarrow B+D=2 \rightarrow D=2-B$

$-2A+B+2(-A)+D=0$
$-4A+B+(2-B)=0$
$-4A+2=0$

$-4B=-4 \rightarrow \boxed{B=1};\ \boxed{D=1}$

$$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{\frac{1}{2}s+1}{s^2+2s+2} + \frac{-\frac{1}{2}s+1}{s^2-2s+2}\right\}$$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)+1^2} - \frac{1}{2}\left(\frac{s+1}{(s+1)^2+1^2} - \frac{1}{(s^2+1)^2+1^2}\right) \frac{-s-1}{(s-1)^2+1} + \frac{1}{2}\left(\frac{s-1}{(s-1)^2+1} + \frac{1}{(s^2+1)}\right)\right\}$$

$$f(t) = \frac{1}{2}e^{-t}\cos t + \frac{1}{2}e^{-t}\operatorname{sen} t - \frac{1}{2}e^{t}\cos t + \frac{1}{2}e^{t}\operatorname{sen} t$$

$$f(t) = \frac{1}{2}\left[\cos t\,(e^{-t} - e^{t}) + \operatorname{sen} t\,(e^{-t} + e^{t})\right]$$

~~$f = \operatorname{sen}(t) \cdot \cosh(t) - \cos(t) \cdot \operatorname{senh}(t)$~~

j) $F(s) = \frac{e^{-s}}{s^2}$

$L\{f(t-a) \cdot u(t-a)\} = e^{-as} \cdot L\{f(t)\}$

$F(s) = e^{-s} \cdot \frac{1}{s^2}$

$F(s) = L\{f(t-a) \cdot u(t-a)\}$

$f(t) = t \rightarrow f(t-1) = t-1$

$L^{-1}\{F(s)\} = (t-1) \cdot u(t-1)$

$\frac{1}{s^2} = L\{f(t)\}$

$L^{-1}\{\frac{1}{s^2}\} = f(t)$

$f(t) = t$

3.9 a)

$\ddot{y}(t) + \dot{y}(t) + 3y(t) = 0 ; y(0) = 1 ; \dot{y}(0) = 2$

$L\{\ddot{y}(t) + \dot{y}(t) + 3y(t) = 0\}$

$s^2 Y(s) - s \cdot 1 - 2 + s Y(s) - 1 + 3 Y(s) = 0$

$Y(s) = \frac{s}{s^2+s+3} + \frac{3}{s^2+s+3}$

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$L^{-1}\{Y(s) = \frac{5}{(s+\frac{1}{2})^2+\frac{11}{4}} + \frac{3}{(s+\frac{1}{2})^2+\frac{11}{4}}$

$y(t) = e^{-\frac{t}{2}} \cdot \cos\left(\frac{\sqrt{11}\,t}{2}\right) + \frac{5\sqrt{11}}{2\cdot 2} \cdot e^{\frac{t}{2}} \cdot \sin\left(\frac{\sqrt{11}\,t}{2}\right)$

$y(t) = e^{-\frac{t}{2}}\left[\cos\left(\frac{\sqrt{11}\,t}{2}\right) + \frac{5\sqrt{11}}{4}\sin\left(\frac{\sqrt{11}\,t}{2}\right)\right]$

f) $\ddot{y}(t) - 2\dot{y}(t) + 4y(t) = 0,\ y(0)=1,\ \dot{y}=2$

$L\{\ddot{y}(t) - 2\dot{y}(t) + y(t) = 0\}$

$[s^2Y(s) - s\,y(0) - \dot{y}(0)] - 2[sY(s) - y(0)] + 4Y(s) = 0$

$Y(s)(s^2 - 2s + 4) - s(1) + (-2+2) = 0$

$Y(s) = \frac{s}{s^2-2s+4}$

$Y(s) = \frac{s}{(s-1)^2+3} \rightarrow Y(s) = \frac{s-1}{(s-1)^2+3} + \frac{1}{(s-1)^2+3}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

$L^{-1}\{Y(s) = \frac{s-1}{(s-1)^2+3} + \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{(s-1)^2+3}$

$y(t) = e^{t}\cdot\cos(\sqrt{3}\,t) + \frac{1}{\sqrt{3}}\cdot e^{t}\cdot\sin(\sqrt{3}\,t)$

e) $\ddot{y}(t) + 2\dot{y}(t) = e^{t}$; $y(0) = 1$, $\dot{y}(0) = 2$

$\mathcal{L}\{\ddot{y}(t) + 2\dot{y}(t) = e^{t}\}$

$$\left[s^2 Y(s) - s\,y(0) - y(0)\right] + 2\left[s\,Y(s) - y(0)\right] = \frac{1}{s-1}$$

$$Y(s)\,(s^2 + 2s) = \frac{1}{s-1} + s + 4$$

$$Y(s) = \frac{1}{(s-1)s(s+2)} + \frac{1}{s+2} + \frac{4}{s(s+2)}$$

$$\frac{1}{(s-1)\,s(s+2)} = \frac{A}{s-1} + \frac{B}{s} + \frac{C}{s+2}$$

$$1 = A(s+2)s + B(s-1)(s+2) + C(s-1)s$$

$s=0$: $0A + B(-1)(2) + 0C = 1 \rightarrow B = -1/2$

$s=1$: $A(3) + 0B + 0C = 1 \quad A = 1/3$

$s=-2$: $0A + 0B + C(-3)(-2) = 1 \rightarrow C = 1/6$

$$\frac{1}{(s-1)s(s+2)} = \frac{1}{3}\cdot\frac{1}{s-1} + \left(\frac{-1}{2}\right)\frac{1}{s} + \frac{1}{6}\cdot\frac{1}{s+2}$$

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$$Y(s) = \left(\frac{1}{3}\cdot\frac{1}{s-1} - \frac{1}{2}\cdot\frac{1}{s} + \frac{1}{6}\cdot\frac{1}{s+2}\right) + \frac{1}{s+2} + 2\cdot\frac{2}{s(s+2)}$$

$$\mathcal{L}^{-1}\left\{ Y(s) = \left(\frac{1}{3}\cdot\frac{1}{s-1}\right) - \left(\frac{1}{2}\cdot\frac{1}{s}\right) + \left(\frac{7}{6}\cdot\frac{1}{s+2}\right) + \left(2\cdot\frac{2}{s(s+2)}\right)\right.$$

$$y(t) = \frac{1}{3}\cdot e^{t} - \frac{1}{2}\cdot 1(t) + \frac{7}{6}\cdot e^{-2t} + 2\left(1 - e^{-2t}\right)$$

$$y(t) = \frac{e^{t}}{3} - \frac{5}{6}e^{-2t} + \frac{3}{2}$$

f) $\ddot{y}(t) + y(t) = t;\ y(0) = 1,\ \dot{y}(0) = -1$

$$\mathcal{L}\{\ddot{y}(t) + y(t) = t\}$$

$$\left[s^2\,Y(s) - s\,y(0) - \dot{y}(0)\right] + Y(s) = \frac{1}{s^2}$$

$$Y(s)(s^2+1) - s\cdot 1 - (-1) = \frac{1}{s^2}$$

$$Y(s) = \frac{1}{s^2(s^2+1)} + \frac{s}{s^2+1} - \frac{1}{s^2+1}$$

$$\frac{1}{s^2(s^2+1)} = \frac{A_0}{s^2} + \frac{A_1}{s} + \frac{Bs+C}{s^2+1}$$

$$A_0(s^2+1) + A_1(s^2+1)\cdot s + (Bs+C)s^2 = 1$$

$$s=0:\ A_0\cdot 1 + A_1\cdot 0 + 0 = 1 \rightarrow \boxed{A_0 = 1}$$

$$1(s^2+1) + A_1 s^3 + A_1 s + B s^3 + C s^2 = x$$

$$s^3(A_1+B) + s^2(C) + s(A_1) = -s^2$$

$$\begin{cases} A_1 + B = 0 \therefore B = 0 \\ C = -1 \\ A_1 = 0 \end{cases}$$

$$\frac{1}{s^2(s^2+1)} = \frac{1}{s^2} + \frac{\cancel{0}}{s} + \cancel{0s} + \frac{-1}{s^2+1}$$

$$Y(s) = \left(\frac{1}{s^2} - \frac{1}{s^2+1}\right) + \frac{s}{s^2+1} - \frac{1}{s^2+1}$$

$$\mathcal{L}^{-1}\left\{ Y(s) = \frac{1}{s^2} - 2 \cdot \frac{1}{s^2+1} + \frac{s}{s^2+1} \right\}$$

$$y(t) = t - 2\,\text{sen}(t) + \cos(t)$$