Rumors with Personality:

Differential and Agent-Based Models of Information Spread through Networks

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Abstract

We devised the "ISTK" model to approximate the spread of viral information—a *rumor*—through a given (social) network. For our first version of the model, we used a set of ordinary differential equations to assess the spread of a rumor in face-to-face interactions in a homogenous population. Our second model translated this system into an equivalent stochastic agent-based model, but also incorporated a network that encodes the relationship between individuals. Our third model considered *features*: demographic information that characterizes individuals in our representative population. We also generated a feature vector for the rumor, its *personality*, to simulate the targeting of viral information. Our results showed that incorporating the structure of a network alters the dynamics of a rumor's spread, but preserves steady states. However, the addition of a feature vector greatly influenced the rumor's spread through the network; the rumor's ability to spread across the network was positively correlated with its "similarity" to the individuals. This final agent-based, feature-vector ISTK model provides a more realistic mechanism to account for social behaviors, thus permitting more precise study of the dynamics of rumor spread through networks.

1 Introduction

1.1 Background

A rumor is defined as a "proposition for belief of topical reference disseminated without official verification," [12] a notion that lends itself quite well to the imagination of applied mathematicians. The mathematics of rumor spread is somewhat explored, beginning with a variation of the SIR epidemic model by Daley and Kendall in 1965. This model's assumptions of homogeneous interactions and its lack of well-defined parameters likely caused "the superficial similarity between rumors and epidemics to break down on closer scrutiny" [4]. Nonetheless, the similarities of knowing and spreading a rumor, and having and spreading a disease, share parallels that only deviate in some of the intricacies of their mechanism. In both, an "infected" individual in a network desires to (or inadvertently spreads) their "condition." With disease, one of the mechanisms of suppressing spread is vaccination; with rumors it is an individual's eventual boredom and desire for novel information.

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More recent models of rumor spread in a population examined the dynamics through randomized networks [11], examining the rumor transmission in exponentially distributed networks [15], and the time of rumor spread given contacts of the initial spreader in a regular network [7]. There is increasing emphasis on the structure of networks themselves and how this affects model dynamics [17,18,23,26]. Many of these models derive their structure and dynamics from complex yet internally homogeneous simulations of how individuals interact with rumors. However, with the advent of social media and their massive networks that allow sharing of information on a large scale, more analyses focus on frequently-found behaviors on these platforms. Simulations of rumor spread through social media are becoming increasingly realistic by including "forgetting" mechanisms typical of social media [24], comparing time of rumor spread in random networks as compared to structured networks [13], methods for combating rumor spread in social networks [21], and examining rumor spread on gaming networks [9].

1.2 The ISTK Model

In this paper we consider a rumor spread model with four categories of individuals: the "ignorant" individuals, those who have never heard the rumor; the "spreaders," those that have heard the rumor and are actively spreading it; the "stiflers," those who have heard the rumor and actively suppress further transmission (either because they now consider the rumor old news, or they never believed the rumor in the first place); and finally the "knowledgeable" population, those who have heard, but have subsequently forgotten the rumor. The rumor initializes in only a small fraction of the population, and spreads as the individuals interact. The ignorant, spreader, and stifler populations were presented in the Daley-Kendall (DK) model [4], but it had been postulated that a distinct "knowledgeable" population was necessary [24,25]. Assuming otherwise presumes that the attitude of an individual who has forgotten a rumor is identical to the behavior of an individual who had not yet heard the rumor. We account for this distinction with the addition of the knowledgeable population to the DK model. We dub this updated model the Ignorant, Spreader, Stifler, and Knowledgeable (ISTK) model.

We use three variations of this model: one differential, and two agent-based. The differential ISTK model simulates a homogenous group of people, and has no awareness of the concept of individuals; it simply "moves" proportions of the group of people from one population to another over time. The first agent-based model, the "simple model," simulates individuals through several iterations (rounds) over time. The model incorporates a network that represents the connections between individuals, which we based off of a dataset of Facebook friends [14].

In the feature-vector model, we further consider how a rumor might be targeted towards certain individuals. Instead of assuming that every individual is equally likely to spread a rumor, we assumed that certain characteristics of the rumor and of the individual affect the likelihood of the individual to spread the rumor. This idea is supported by evidence that suggests that people are more likely to believe information that comes from others with similar values [8]. Therefore, we decided to use demographics (e.g. education level, gender, and language) of the individuals, which was included in our dataset, as their "features." While an individual's response to viral information is certainly more complex, our underlying assumption is simply that a rumor *can* be targeted towards an individual. Demographic information seemed like a reasonable way to equip individuals and rumors with these features. We call this feature vector the "personality" of the rumor. Moreover, we can compute a notion of "similarity" between the feature vector of the rumor and that of an individual. In this way, the more similar an individual and rumor were, the more likely an individual would spread the rumor. This effect is also suggested by the theory of confirmation bias, insofar as we are more likely to accept information that confirms our previous beliefs [22].

1.3 ISTK Model Equations

We denote I, S, T, and K to represent the total Ignorant, Spreader, Stifler, and Knowledgeable populations respectively. The total size of the population is represented as N, in that $N = I + S + T + K$ (n.b. we assume no one dies or is born, so N stays constant). We also use several parameters as follows:

- the "credibility" of the rumor, c, expressed as a probability that an Ignorant believes the Spreader
- $(1 - c)$ the complement of c, equivalent to being incredulous of the rumor
- l, the chance per day of interaction, which is the complement of the overall probability that an individual does not talk with a single Spreader: it is computed by a set of Bernoulli trials with success probability ρ where $\rho = 1 - \frac{S}{N}$ and number of trials to be τ (i.e. $l = 1 - \rho^{\tau}$)
- d, the number of days after which an individual forgets a rumor
- α_1, the loss of novelty of the rumor
- α_2, the chance that the Spreader becomes a Stifler upon interacting with a Stifler

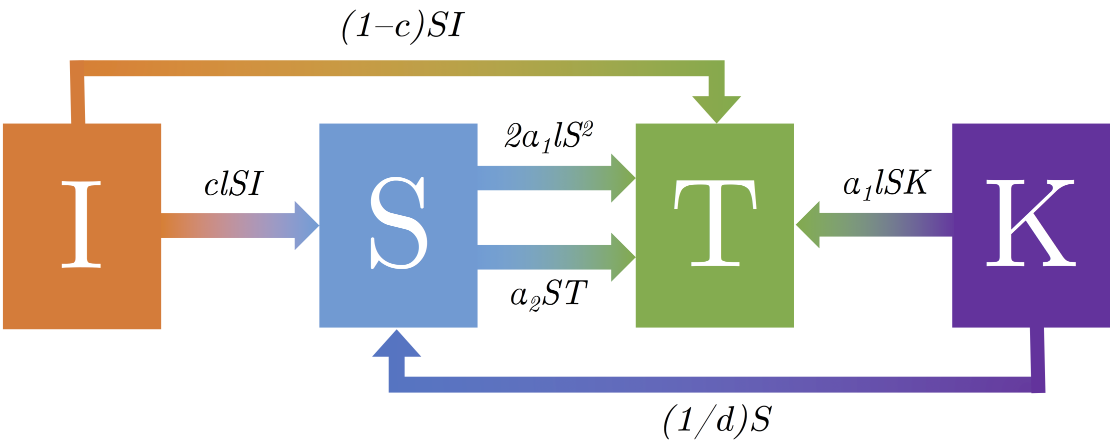

Figure 1: The ISTK Model as a flowchart

Figure 1 represents how individuals can move from one population to another. The base of each arrow represents the population an individual moves from, while the head represents the population an individual moves to. The labels for each arrow express how we compute the "chance" per day that an individual can move from one population to another. For example, the arrow that connects the I and T population with the label $(1 - c)SI$ reads: the chance an Ignorant becomes a Stifler is proportionate to their incredulity, the chance per day of interacting with a Spreader, and the size of the Spreader and Ignorant populations.

The model can be surmised by four differential equations, representing the size of each population over time, as follows:

$$\frac{dI}{dt} = -clSI - (1 - c)lSI \tag{1}$$

In Equation 1, the first term describes the interaction between an Ignorant and a Spreader, which moves individuals to the Spreader population depending on the credibility of the rumor. The second term of Equation 1 accounts for being "incredulous" of the rumor, which moves Ignorants to Stiflers. (n.b. simplifying this equation to $(-lSI)$ does not allow us to distinguish the credulous (c) and incredulous $(1 - c)$ group of people.)

$$\frac{dS}{dt} = clSI - \frac{1}{d}S - 2\alpha_1 lS^2 - \alpha_2 ST \tag{2}$$

The first term of Equation 2 is the addition of members from the Ignorant class who believed the rumor. The second term $\frac{1}{d}$, the inverse of the number of days it takes to forget the rumor, represents the chance per day to forget the rumor. The third term of Equation 2 accounts for two Spreaders who interact with each other and foster disinterest in the rumor (since the rumor has lost its novelty). When these two Spreaders interact (S^2), we account for the chance that each Spreader could become a Stifler by multiplying the term by 2. The final term represents the disillusionment power of a Stifler when interacting with a Spreader.

$$\frac{dT}{dt} = 2\alpha_1 lS^2 + \alpha_1 lSK + \alpha_2 ST + (1-c)lSI \tag{3}$$

In Equation 3, the first term describes the transition from Spreader to Stifler. The second term describes the population of Knowledgeable individuals who become a Stifler, as described in Equation 2. The third and fourth terms of Equation 3 describe the addition of members to the Stifler population from the Spreader and Ignorant populations, respectively.

$$\frac{dK}{dt} = \frac{1}{d}S - \alpha_1 lSK \tag{4}$$

Finally, Equation 4 describes the individuals in the Spreader class who forget the rumor and become Knowledgeable; and the population which loses the novelty of the rumor and become Stiflers.

In our model, we do not consider the interaction between a Stifler and an Ignorant because neither one has a reason to broach the subject of a rumor (the Ignorant because they do not know and the Stifler because they no longer care). Moreover, by a similar logic, when a Knowledgeable and Stifler interact, there is no change in populations.

Even more than the addition of the Knowledgeable population, our model differs from the Daley-Kendall (DK) model[4] by adding the parameter c. This parameter allows us to avoid the assumption that every individual believes the rumor upon hearing it. Furthermore, the DK model permits individuals to return to the Ignorant class, which cannot happen in our model. Ignorant individuals are individuals who have truly never heard the rumor.

2 Differential Model

2.1 Modeling Method

Initially, we solved the differential model in order to compare a continuous model to a stochastic agent-based model. We examined how parameters based on face-to-face interaction had an impact on rumor spread versus interaction over a network (described by an adjacency matrix). Specifically, we used parameters for a model on consumer goods, in order to examine how long it took the rumor to reach a significant proportion (90%) of the population and the effect on the amount of time until steady states were reached with perturbations in initial parameters.

2.2 Choosing Parameters

The parameters necessary to estimate were *credibility*, the *loss of novelty*, and the *number of close interactions* for an individual. Using consumer statistics on perceptions of reliability of information from different sources, we initially estimated the credibility $c = 2.8/7$[10]. Estimations in number of close contacts varied from 12–26 people per day, varying based on age[2,6,16]. We took the average number of close contacts to be 22 (i.e. $\tau = 22$, which determines our parameter of interaction l). Although the differential model itself does not change with the medium of Facebook, the meaning of τ changes.

Instead of 22 close interactions per day, we selected an appropriate analog, in that we assumed the average individual reads approximately 22 posts per day. We estimated the value representing loss of novelty at $\alpha_1 = .01$, and $\alpha_2 = .02$, since the spreaders will have a stronger effect on the stiflers. In all cases, these were the "baseline" parameters, and were only modified for the Feature vector model, in which case parameters c and α_1 were based on the features vectors of agents, and rumors. Sensitivity analyses for c, and δ were run on the interquartile ranges of the studies on which they were based. α_1 was an approximated variable, so we simply run as small of an α_1 variable that the differential model was capable of processing, up to an α_1 value of .25.

3 Agent-Based Models

3.1 Simple Model Method

In order to incorporate a network into our model, we constructed an agent-based model (a useful stochastic technique for modeling dynamics with graphs) [20]. We discretized the data we used for the differential model, in essence using our parameterized proportions as the probabilities that certain individuals would move between populations. Using data from the Facebook network, we allowed individuals to communicate only with those to whom they are connected. Since agent-based models are based on probabilities, and are inherently non-deterministic, we essentially have to perform many "trials" of the model. Each trial consists of initializing a set of "agents" into one of the four populations: Ignorant, Spreader, Stifler, or Knowledgeable. In each trial, there are several time-steps, at which point each agent has a "turn." At each turn, an agent can interact with other agents, and move from one population to another. The rules the define what an agent can or cannot do on a turn are described by the ISTK model. For example, in the differential ISTK model, an Ignorant becomes a Spreader by the term $clSI$. Translating this term to the agent-based model: lSI represents the chance that an Ignorant and Spreader interact (as characterized by the network), and c represents the credibility of the rumor expressed as a probability.

3.2 Choosing Parameters

We performed 400 distinct trials, where each trial constituted 22 days. In each trial, each agent began as ignorant, except for a randomly selected subset of the population who started as spreaders. The chance of becoming a spreader was set at 5% distributed randomly (i.e. without considering the network). Because it is a large population (4 039 individuals), each trial would have had around 202 spreaders, but the actual number of spreaders varies from trial to trial. Additionally, it was possible for all of the spreaders to be concentrated in a subnetwork or a "pocket of friends."

To begin each day, every user was assigned an amount of time logged in by picking randomly from a normal distribution with a mean of 23 minutes and a standard deviation of 4 minutes, bounded above 0. We made the assumption that the majority of people will probably be logged on during an 8 hour period of the day; therefore, we only modeled 480 minutes per day, within which the users select their logon time (n.b. users also could not log on in the last 23 minutes of the day, as 23 minutes a day was set as the mean browsing time). Each user had a probability of 14/365 to "post" in a given day. Then, users were assigned a "time" (from a uniform random distribution) at which they made their post. This occurred on each of the 22 days that constituted a trial.

Each "day," after determining the logon time, posting order, and post time, the simulation of rumor spread began. Every minute, each user could "view" posts written at that minute from people to whom they were directly connected. Users were also capped at reading 10 posts a minute. If a poster was a spreader, they had chance $\delta = \frac{1}{d} = \frac{1}{22*480}$ of forgetting the rumor. Then, based off of the probabilities in Table 1, the state of each person was immediately recalculated. Therefore, if a person changed state

Reader State \ Poster State	I	S	T	K
I	—	$\mathbb{P}(S) = c = 0.8$ $\mathbb{P}(T) = 1 - c = 0.2$	—	—
S	—	$\mathbb{P}(T) = \alpha_1 = 0.01$	$\mathbb{P}(T) = \alpha_2 = 0.02$	—
T	—	—	—	—
K	—	$\mathbb{P}(T) = \alpha_1 = 0.01$	—	—

Table 1: Next reader state for possible interactions between reader and poster. $\mathbb{P}(X)$ denotes probability that reader changes to class X

at a particular minute within a day, then that person would interact as that state with other users in every minute after that. Finally, after 22 days, the trial ended.

3.3 Feature-Vector Model Method

This model followed a similar logic as the preceding agent-based model. However, the different interactions accounted for the similarity between two agents or the similarity between an agent and the rumor. First, a baseline feature space of dimension $D = 195$ was taken as a subsample from the Facebook dataset[14]. Each feature corresponds to some data from the original Facebook profiles, like language, identified gender, etc. Although we use demographic data, we only assume that it is possible to target rumors towards certain individuals based on features; this could be easily extended to other types of data, e.g. interests, favorite music, personality traits etc. Each of our features is boolean, although our original dataset included "N/A" if the value is unknown. To move around this problem, any "N/As" for a given feature were filled in randomly with some probability p, where $p = \frac{x_t}{x_f + x_t}$, x_t is the number of *true* values there were for a particular feature across the population, and x_f is the corresponding number of *false* values. The rumor itself was also initialized with a randomly-generated feature vector. Feature vectors permit us to investigate how a targeted rumor affects its spread in a network. We constructed several rumors with different feature vectors, and also included a "most similar" rumor feature vector (created by rounding every p to 0 or 1 for each feature) and a "most dissimilar" rumor feature vector (the logical complement of the "most similar" rumor vector).

Next, pairwise angular similarity $S_{p,r}$ was taken between the two interacting agents, poster and reader. This is incidentally equivalent to the cosine value of the angle between the vectors, which is nicely contained between 0 and 1, where 1 is the most similar. This is given by

$$S_{p,r} = \frac{\vec{\mathbf{v_r}} \cdot \vec{\mathbf{v_p}}}{||\vec{\mathbf{v_r}}||||\vec{\mathbf{v_p}}||}$$

where the poster has feature vector $\vec{\mathbf{v_p}}$ and the reader has feature vector $\vec{\mathbf{v_r}}$.

Angular similarity between the feature and the reader was also determined, where

$$F_r = \frac{\vec{\mathbf{v_r}} \cdot \vec{\mathbf{f}}}{||\vec{\mathbf{v_r}}||||\vec{\mathbf{f}}||}$$

$\vec{\mathbf{f}}$ is the feature vector of the particular rumor.

We also determined a "baseline" $b = 0.5$, which is the "influence" of an original parameter, and where $1 - b = 0.5$ represents the influence of the interaction of feature vectors. This baseline determined how much each parameter was affected by the similarity scores of feature vectors, and guaranteed the values would be at least half of the original model values. The simple agent-based model was run again, with $b * c + (1 - (b * c)) * F_k$ substituted for c and $b * \alpha + (1 - (b * \alpha)) * S_{p,r}$ substituted for the respective α values and agents i and j.

We tested 86 different feature vectors, with 100 trials each. In addition for our simulated "most similar" rumor and the "most dissimilar" rumor, we ran 300 repetitions with each rumor, of the stochastic agent based model, with the same population.

4 Results

4.1 Differential Model

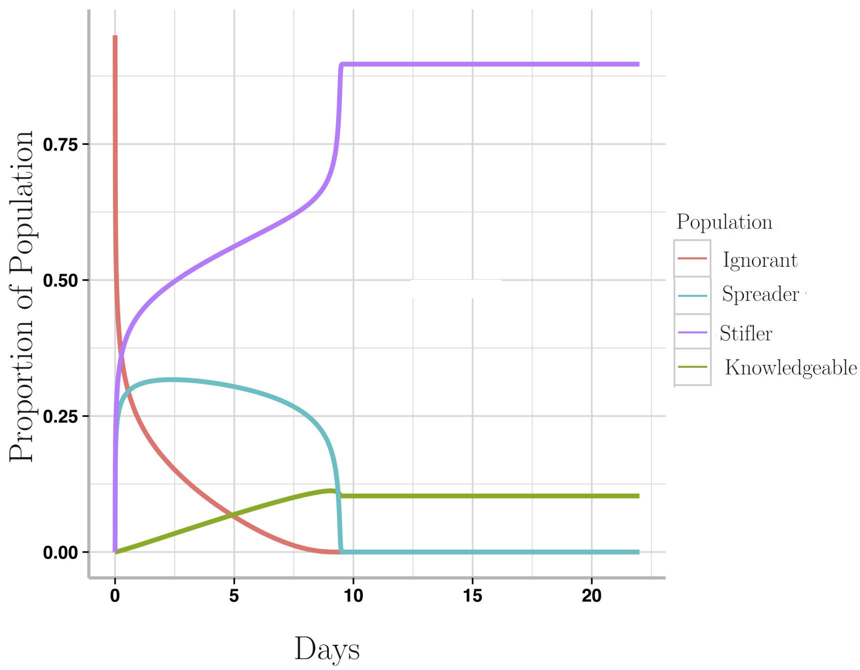

Figure 2: Differential model proportions of the population in each class over 22 days.

For the differential model, as is demonstrated in Figure 2, the spreader and ignorant populations become negligible by the end of the 22 days, and the knowledgeable and stifler populations stabilize above 0. The ignorant population declines, as the spreader population initially grows, and then declines as the stifler population grows. In the differential model, essentially all individuals learn about the rumor. Varying the parameters impacts how quickly the population hears of the rumor, but not the ignorant and spreader populations.

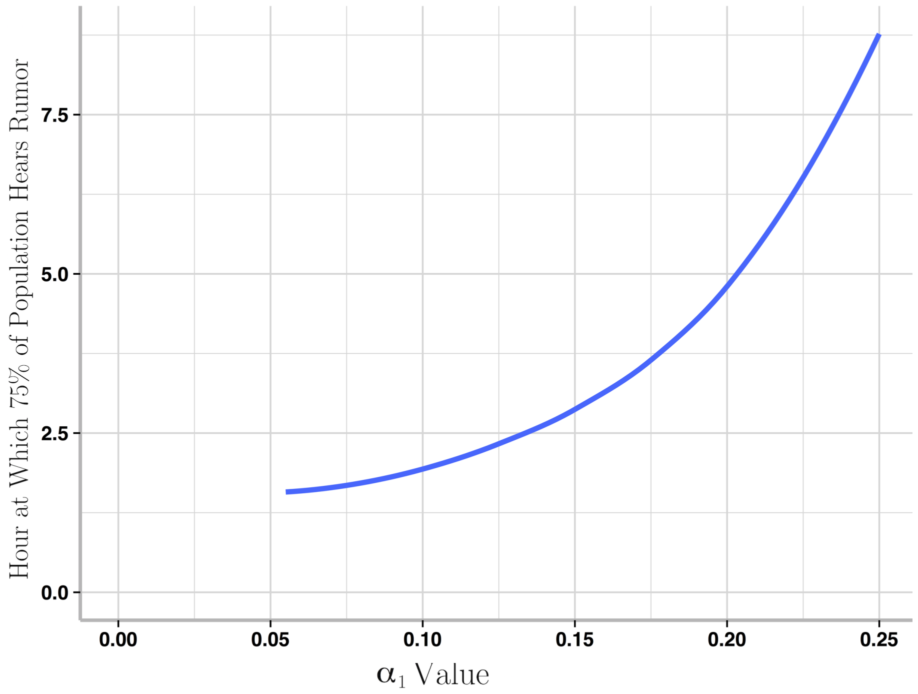

Figure 3: Sensitivity analysis of parameter α_1 (loss of novelty) in the differential model, n.b. α_2 always equaled $2 * \alpha_1$

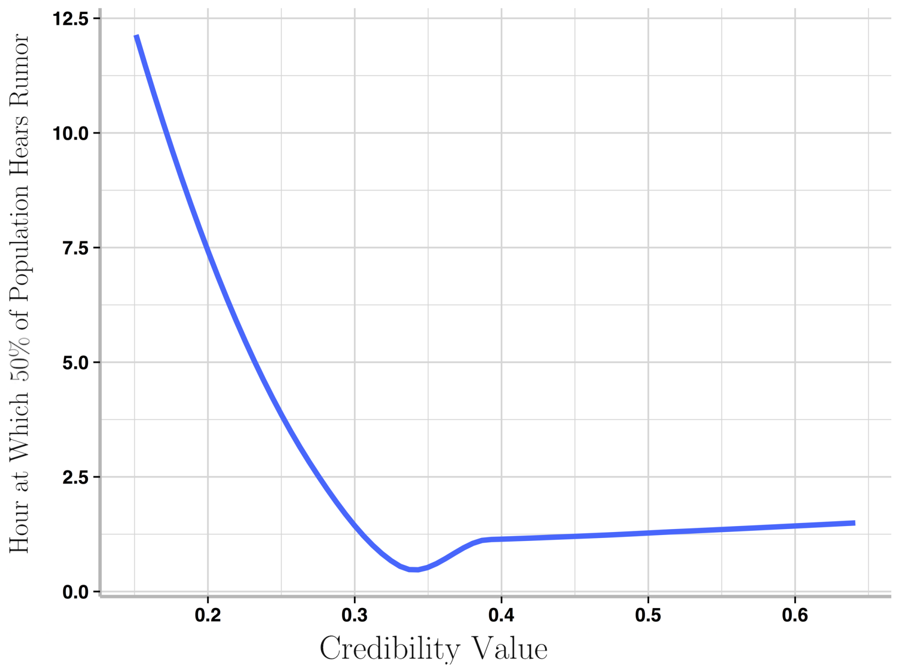

Figure 4: Sensitivity analysis of parameter c (credibility) in the differential model.

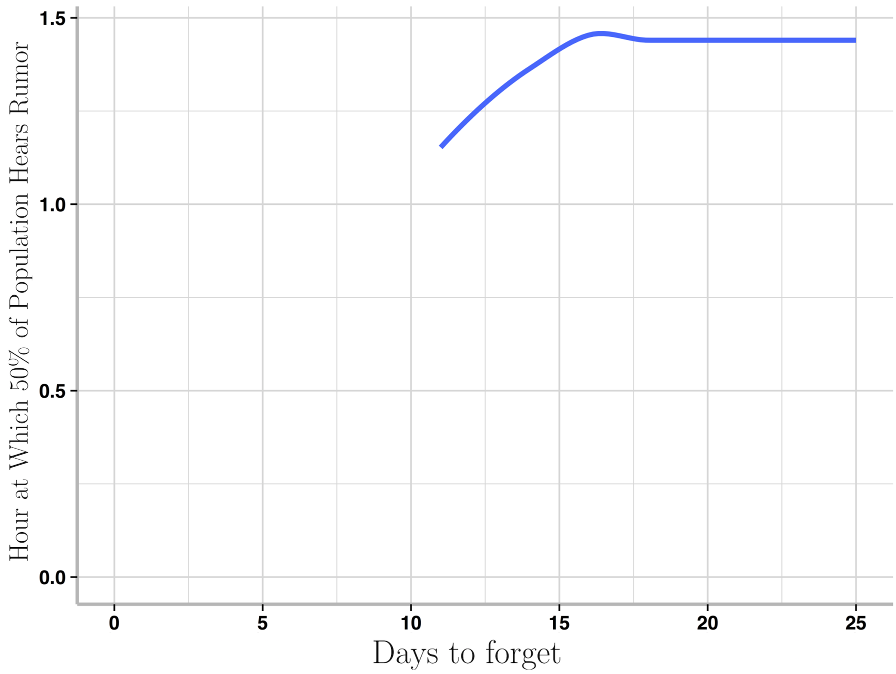

Figure 5: Results of the sensitivity analysis of parameter d (average days to forget) in the differential model.

Looking at Figures 3, 4, and 5, we see can see how changing parameters in the differential model influences the speed at which the rumor travels. The sensitivity analyses indicate that the rumor travels exponentially slower with increasing α values, while traveling exponentially faster with increasing credibility values. The rumor was not particularly sensitive to d, the average days to forget it, but to our surprise, the rumor required more time to travel as d *increased*. This effect is probably due to the fact that a shorter d caused faster movement between populations.

4.2 Simple, Agent-Based Model

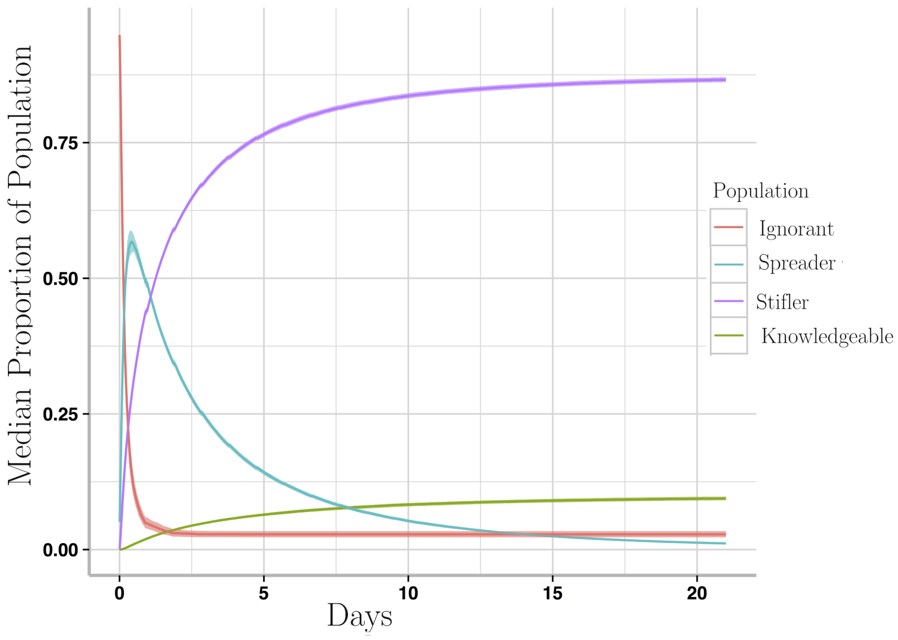

Figure 6: Results of the agent-based model. Solid line indicates median proportion of population across the 400 trials; shadow indicates IQR.

Analogous to (Figure 2) for the differential model, we see the average results of the simple, agent-based model. Interestingly, the dynamics of the rumor's spread are quite different: the slope of the population graphs are more gradual, rather than moving sharply, as in the differential model.

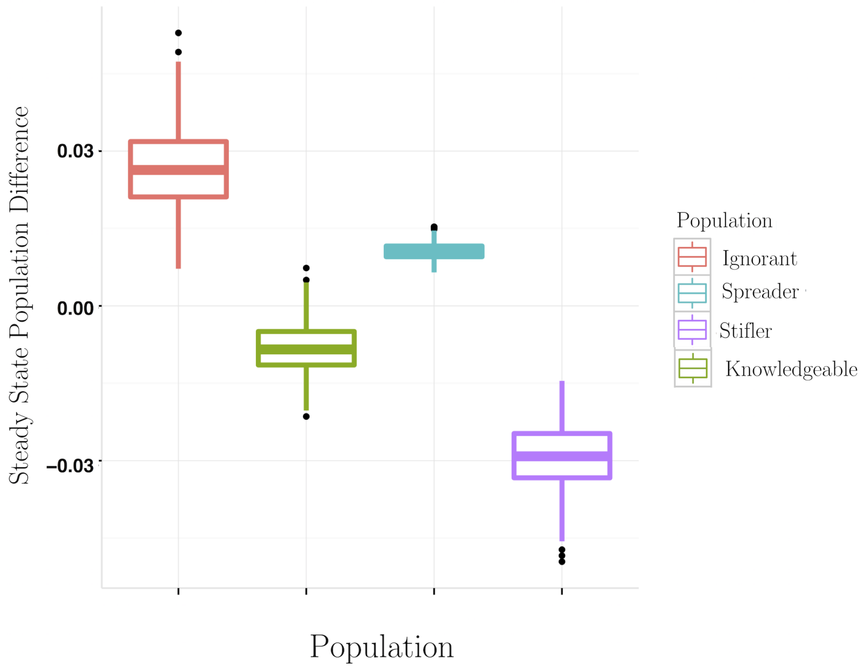

Figure 7: Box-and-whisker plot comparing the steady states in the differential model and simple, agent-based model.

With Figure 7, we can directly compare the steady states of the differential model from Figure 2 and the distribution of steady states of the simple, agent-based model from from Figure 6. Interestingly, the steady states remain similar, even though the differential model is deterministic, and the simple model incorporates a network. Both models seem to confirm that eventually, the entire population will be exposed to the rumor, though (as is expected in a stochastic process) the simple model contains very small pockets of "ignorance."

4.3 Feature-Vector, Agent-Based Model

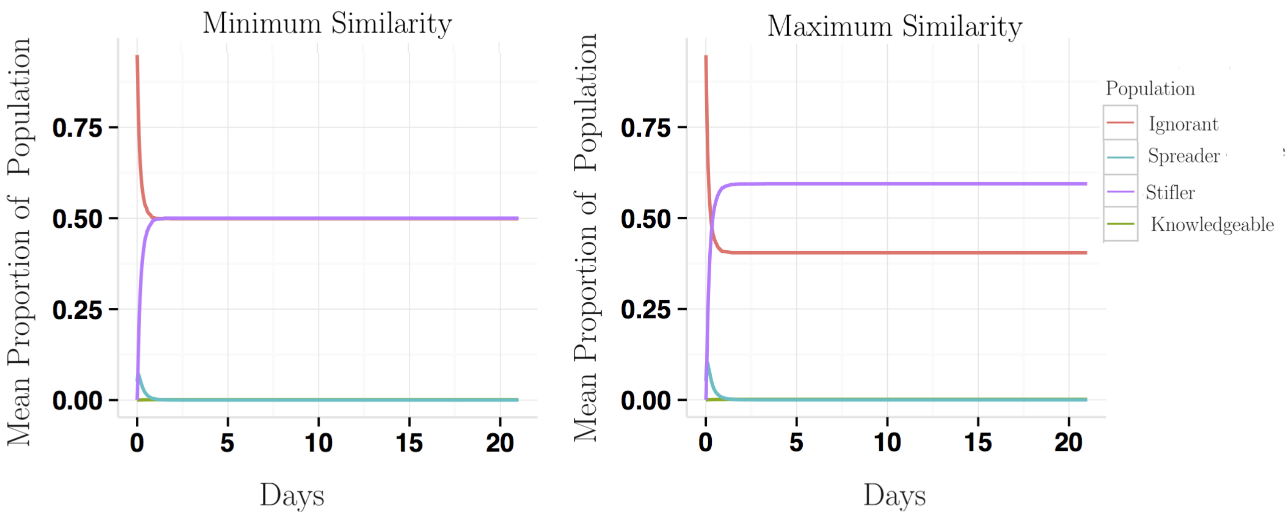

Figure 8: Results of the most and least similar feature vectors to the population in the agent-based model (average across 300 trials).

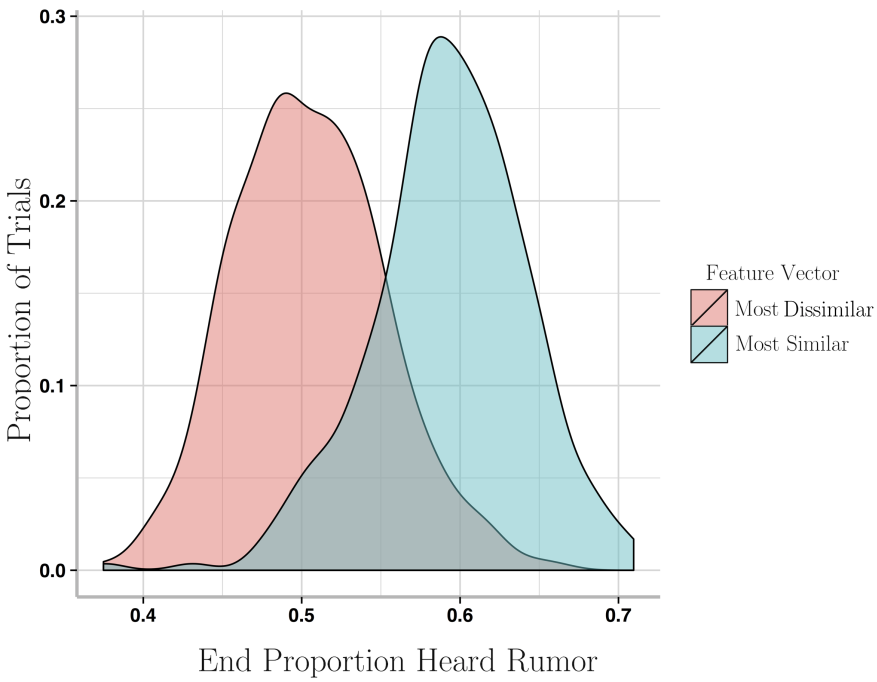

Figure 9: Density of the proportion of the population who heard the rumor after 22 days with the most and least similar rumors (300 trials).

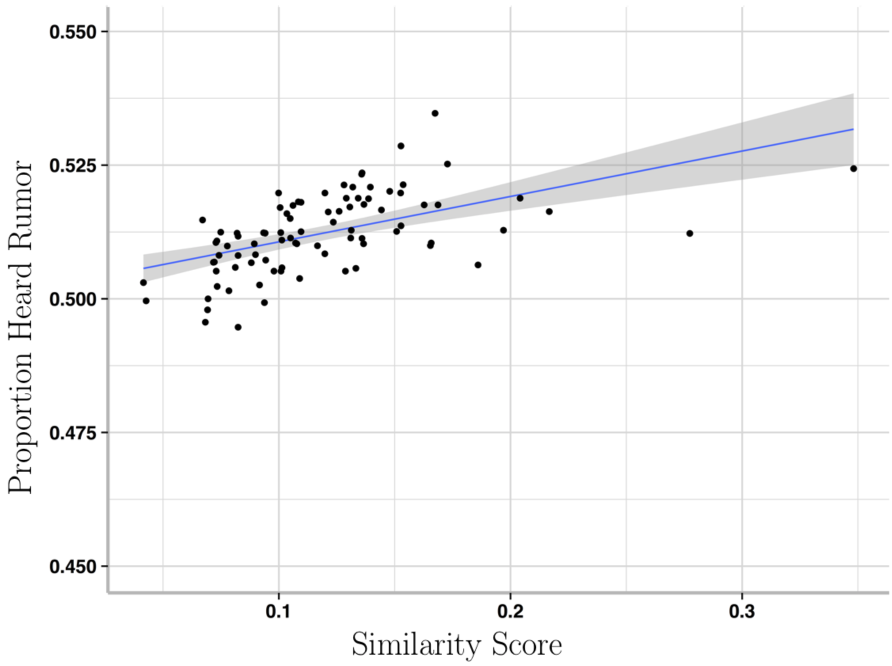

Figure 10: Linear model of the relationship between the final percentage of the population heard rumor and average similarity score of the feature vector ($r = 0.538$). Shading designates the 95% confidence interval.

When the similarity score is added, even the most similar rumor dies out, as seen in Figure 8. This plot is identical to Figure 2 and Figure 6, but is the stochastic agent-based case. However, the average similarity score of a rumor with the population does affect the spread (Figure 9). The most similar rumor to the population spreads to more of the population than does the least similar. The trend from the other feature vectors supports this claim, as demonstrated by Figure 10. The predictive power of the similarity in predicting the number of individuals who heard the rumor is decent where the bulk of the data lies.

5 Discussion

There was relatively little difference between the end states of the differential and simple agent-based models, despite the fact that the former aggregates the population and the latter provides more granularity. As previously noted by Chierchetti et al.[3], in a fully-connected network with push-pull interactions, a rumor will spread to the majority of the population with high probability. We came to similar conclusions as this previous study's findings, though our model had a fully-connected network and assumed different interactions: individuals only had the opportunity to interact with the same individuals at every time step, as opposed to choosing new "partners" each time[3]. In the agent-based model, not every individual learned about the rumor, and the addition of some structured social network delayed rumor spread. That is to say, rumors must diffuse through a complex network in order to have large-scale effects on the population. Thus, the curves are less dramatic, change more gradually, and there is no guarantee every individual will hear the rumor: by the end of the 22 days essentially none of the population remains ignorant in the differential model, whereas in the agent-based model 2.8% of the population remains ignorant. However, the trajectories of the two models are qualitatively similar, suggesting that the agent-based model tends to a vanishing of an ignorant population, save for a small connected subnetwork. Just like the claim so well supported in push models, eventually there is a high probability all individuals will hear the rumor[1,19].

The incorporation of feature vectors in the agent-based model changes the overall spread of the rumor, since spread of the rumor is a factor of both the similarity of individuals to each other, and similarity of the rumor to each individual. As indicated by our results, even where the rumor spreads, individuals become stiflers so quickly that the rumor dies out before reaching a large proportion of the population. Perhaps, if the individuals in our network cluster based on features, it would explain how a rumor could die out trying to navigate several disjoint subnetworks. This behavior is familiar to anyone who has been on social media, and had friends who relentlessly post stories that bear no significance to their personal beliefs or preferences. We speculate that in a community with many highly-similar individuals, one could much more easily engineer a rumor to spread through the whole network. Since the simple, agent-based model still spread the rumor to (essentially) every individual, the topology of the network is probably less important in preventing rumor spread. Inoculation against hearing the rumor seems more a factor of the general dissimilarity of individuals to each other in the population. We suspect that the inevitable “death” of our rumors may be due to a population of individuals with a great variety of different feature vectors.

6 Conclusions

Since the rumor tends to spread rapidly at the start of the simulation (resulting in a corresponding boost in the stifler population), these results inspire the consideration of different network configurations. A rumor spreads rather more quickly in preferentially connected real-world graphs than in common theoretical mathematical graphs [5], however, In our case, even the “best-performing” rumor—one that maximized spread—still died out. It would seem that our diverse interests and demographics prevent global conquest by any one rumor. That being said, it might be possible to construct a network topology that depended on the features, which might promote better spread, or even that would saturate the network. Alternatively, it might be possible to place strategic “blockers”: individuals who inhibited the spread of rumors between subnetworks. In all, it would seem as though our model—in part thanks to the advent of increased computing power for simulations—can begin to unravel the nuances and intricacies of information spread through a social network. By arriving at a model that uses feature vectors and graphs, we have greater control and specificity in looking at the spread of viral information, possibly leading us to mathematically “perfect” viral information.

References

[1] S. Angelopoulos, B. Doerr, A. Huber, and K. Panagiotou. Tight bounds for quasirandom rumor spreading. *Electron J Combined*, 2009.

[2] L. Cahill, R. J. Haier, J. Fallon, M. T. Alkire, C. Tang, D. Keator, J. Wu, and J. L. McGaugh. Amygdala activity at encoding correlated with long-term, free recall of emotional information. *Proceedings of the National Academy of Sciences*, 93(15):8016–8021, 1996.

[3] F. Chierichetti, S. Lattanzi, and A. Panconesi. Almost tight bounds for rumour spreading with conductance. *Proceedings of the forty-second ACM symposium on Theory of computing*, pages 399–408, 2010.

[4] D. J. Daley and D. G. Kendall. Stochastic rumors. *IMA Journal of Applied Mathematics*, 1(1): 42–55, December 1965.

[5] B. Doerr, M. Fouz, and T. Friedrich. Why rumors spread so quickly in social networks. *Communications of the ACM*, 55(6):70–75, June 2012.

[6] W. J. Edmunds, G. Kafatos, J. Wallinga, and J. R. Mossong. Mixing patterns and the spread of close-contact infectious diseases. *Emerging Themes in Epidemiology*, 3(1):10, 2006. ISSN 1742-7622. doi: 10.1186/1742-7622-3-10.

[7] N. Fountoulakis and K. Panagiotou. Rumor spreading on random regular graphs and expanders. *ArXiv e-prints*, Feb. 2010.

[8] N. A. Gillespie and L. Mann. Transformational leadership and shared values: The building blocks of trust. *Journal of Managerial Psychology*, 19(6):588–607, 2004.

[9] A. Grabowski, N. Kruszewska, and R. A. Kosinski. Properties of on-line social systems. *European Physical Journal B*, 66(1):107–113, November 2008.

[10] M. A. Kamins, V. S. Folkes, and L. Perner. Consumer responses to rumors: Good news, bad news. *Journal of Consumer Psychology*, 6(2):165–187, 1997. ISSN 1057-7408.

[11] R. Karp, C. Schindelhauer, S. Shenker, and B. V́ocking. Randomized rumor spreading. *Foundations of Computer Science*, Proceedings of the 41st Annual Symposium on Foundations of Computer Science:565–574, November 2000.

[12] R. H. Knapp. A psychology of rumor. *Public Opinion Quarterly*, pages 22–27, 1944.

[13] D. Liu and X. Chen. Rumor propagation in online social networks like twitter – a simulation study. *Multimedia Information Networking and Security (MINES) Third International Conference*, November 2011.

[14] J. Mcauley and J. Leskovec. Discovering social circles in ego networks. *ACM Trans. Knowl. Discov. Data*, 8(1):4:1–4:28, Feb. 2014. ISSN 1556-4681. doi: 10.1145/2556612.

[15] Y. Moreno, M. Nekovee, and A. Pacheco. Dynamics of rumoring spread in complex networks. *Physics Review E*, 69, June 2004.

[16] J. Mossong, N. Hens, M. Jit, P. Beutels, K. Auranen, R. Mikolajczyk, M. Massari, S. Salmaso, G. S. Tomba, J. Wallinga, J. Heijne, M. Sadkowska-Todys, M. Rosinska, and W. J. Edmunds. Social contacts and mixing patterns relevant to the spread of infectious diseases. *PLoS Med*, 5(3):e74, 03 2008. doi: 10.1371/journal.pmed.0050074.

[17] L. Pellis, F. Ball, and P. Trapman. Reproduction numbers for epidemic models with households and other social structures. i. definition and calculation of r0. *Mathematical Biosciences*, 235(1): 85–97, January 2012.

[18] L. Pellis, S. Spencer, and T. House. Real-time growth rate for general stochastic sir epidemics on unclustered networks. *Mathematical Biosciences*, 265:65–81, July 2015.

[19] B. Pittel. On spreading a rumor. *SIAM Journal on Applied Mathematics*, 47(1):213–223, 1987.

[20] T. A. Snijders, G. G. van de Bunt, and C. E. Steglich. Introduction to stochastic actor-based models for network dynamics. *Social Networks*, 32(1):44–60, 2010. ISSN 0378-8733. doi: 10.1016/j.socnet.2009.02.004. Dynamics of Social Networks.

[21] R. Tripathy, A. Bagchi, and S. Metha. A study of rumor control stategies on social networks. *CIKM '10 Proceedings of the 19th ACM international conference on information and Knowledge manegment*, pages 1817–1820, 2010.

[22] P. C. Wason. On the failure to eliminate hypotheses in a conceptual task. *Quarterly Journal of Experimental Psychology*, 12(3):129–140, 1960. doi: 10.1080/17470216008416717.

[23] X. Zhang, G. Sun, Y. Zhu, J. Ma, and Z. Jin. Epidemic dynamics on semi-directed complex networks. *Mathematical Biosciences*, 246(2):242–251, December 2013.

[24] L. Zhao, Z. Wang, J. Cheng, Y. Chen, J. Wang, and W. Huang. Rumor spreading model with consideration of forgetting mechanism: A case of online blogging livejournal. *Physica A: Statistical Mechanics and its Applications*, 390(13):2619–2625, July 2011.

[25] L. Zhao, J. Wang, Y. Chen, Q. Wang, J. Cheng, and H. Cui. Sihr rumor spreading model in social networks. *Physica A: Statistical Mechanics and its Applications*, 391(7):2444–2453, April 2012.

[26] J. Zhou, Z. Liu, and B. Li. Influence of network structure on rumor propagation. *Physics Letters A*, 368:458–463, January 2007.