

COL783 Assignment 2

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Q1.

Part a:

Original image



Disk kernel of $r=5$. So size of kernel = 11×11 and convolved the image with mirror padding. $M = 250$, $N = 500$

After convolving with disk filter:

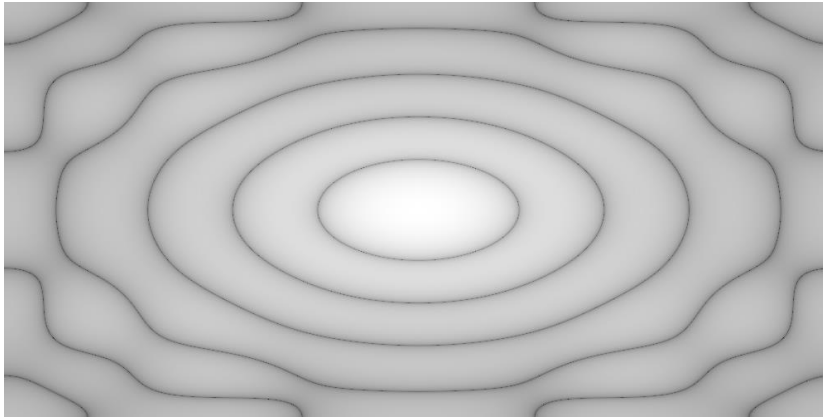


Part b:

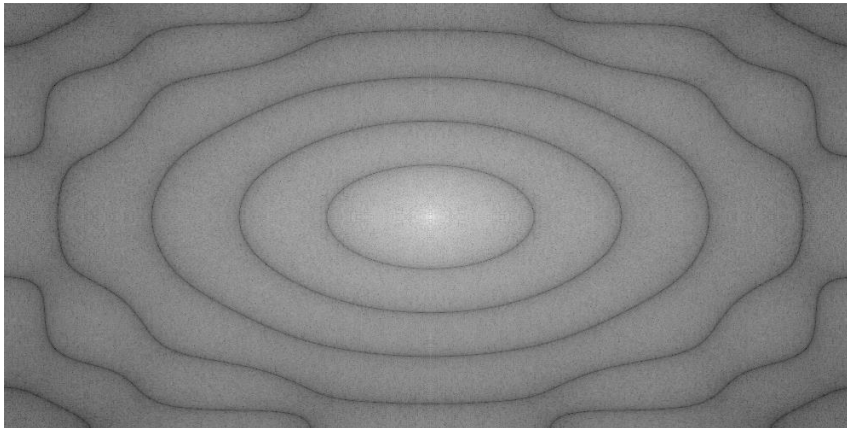
$R = 5$, kernel size = 11×11 , $M = 250$, $N = 500$

Mirror padding, padded image size $2M \times 2N = 500 \times 1000$

Shifted kernel:



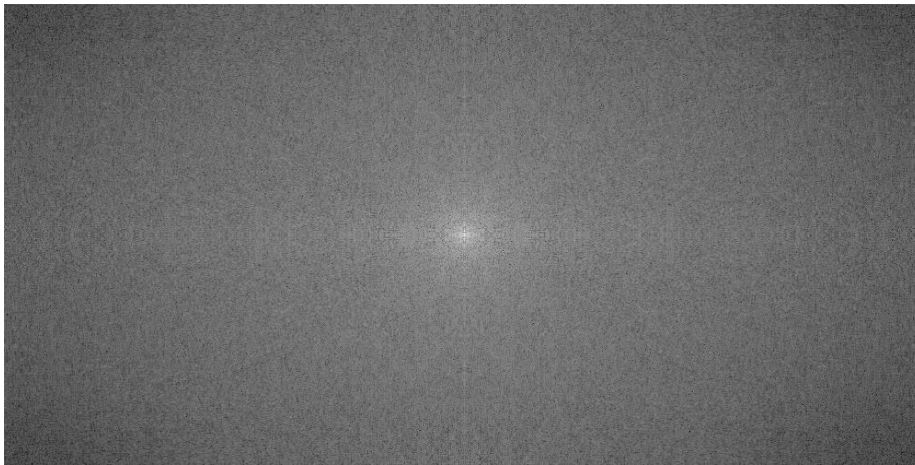
Filtered image in frequency domain:



Output of image filtered in frequency domain:



Shifted spectrum of original image:



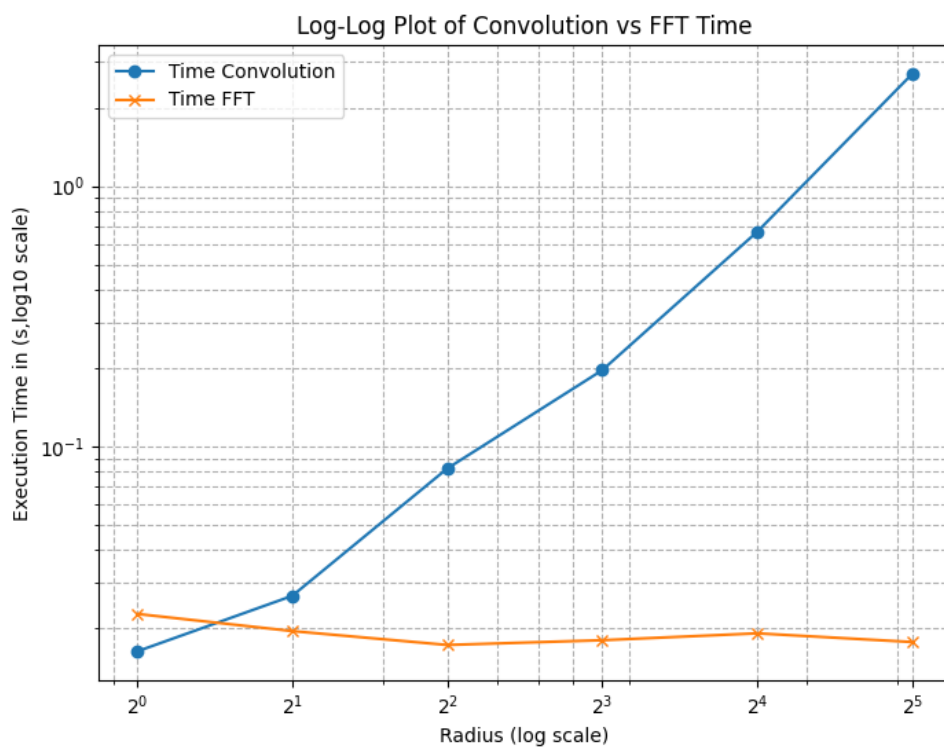
Part c:

The DFT of the blur kernel $h(x,y)$ appears elliptical, even though h is circularly symmetric, because it operates on a discrete grid with unequal image dimensions M and N .

When $M \neq N$ (i.e., for a rectangular image), the frequency domain is sampled at different rates along the two axes. This causes the spatial frequencies to be scaled unevenly in the horizontal and vertical directions, distorting the circular symmetry of the spatial domain into an elliptical shape in the frequency domain. The elliptical appearance is a result of the Fourier transform applying different scaling along the axes due to the varying image dimension.

Part d:

Plot of execution time vs radius for FFT and convolution in log-log scale.



Q2:

Part a:

Original image: 500 x 400



Image after resampling using bilinear interpolation



Part b:

The cutoff frequency is given by nyquist rate:

D0 of x = $1/2 \cdot S_x$ and D0 of y = $1/2 \cdot S_y$ where S_x, S_y are sampling rates along x and y axis.

Gaussian low pass filter is used for filtering the image.

$S_x = S_y = 0.2$

Rescaled image without filtering the original image.



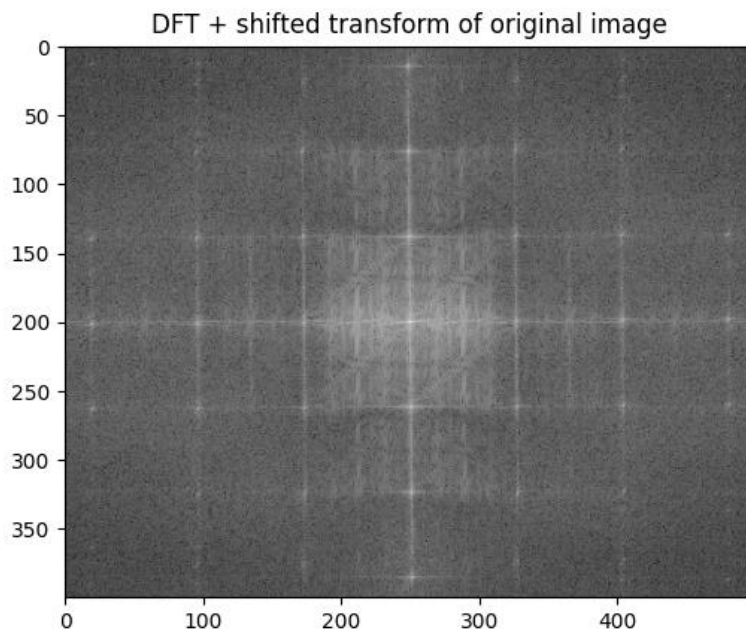
Rescaled image after filtering the original image.



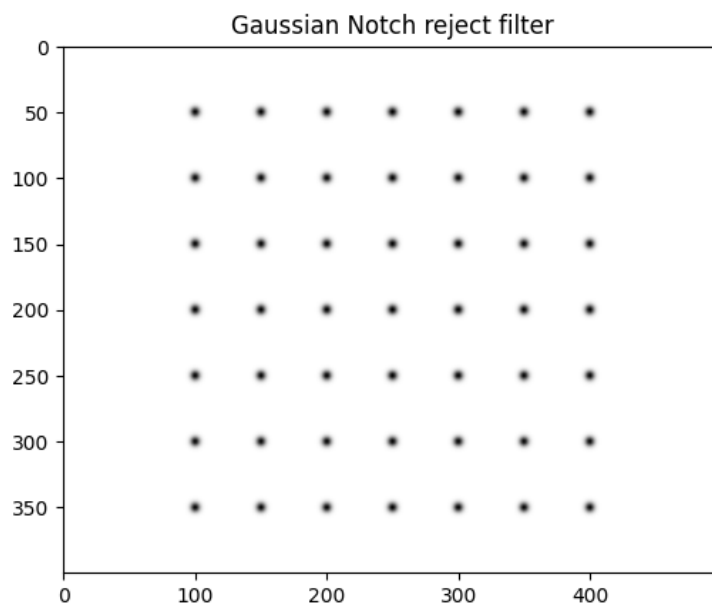
Part c:

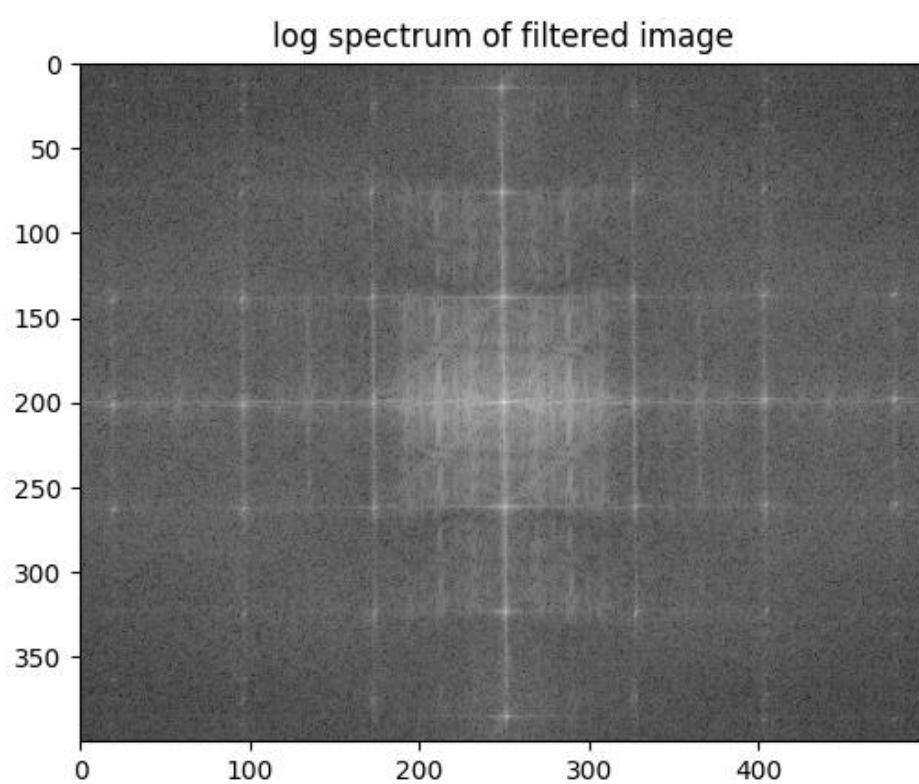
Original image:

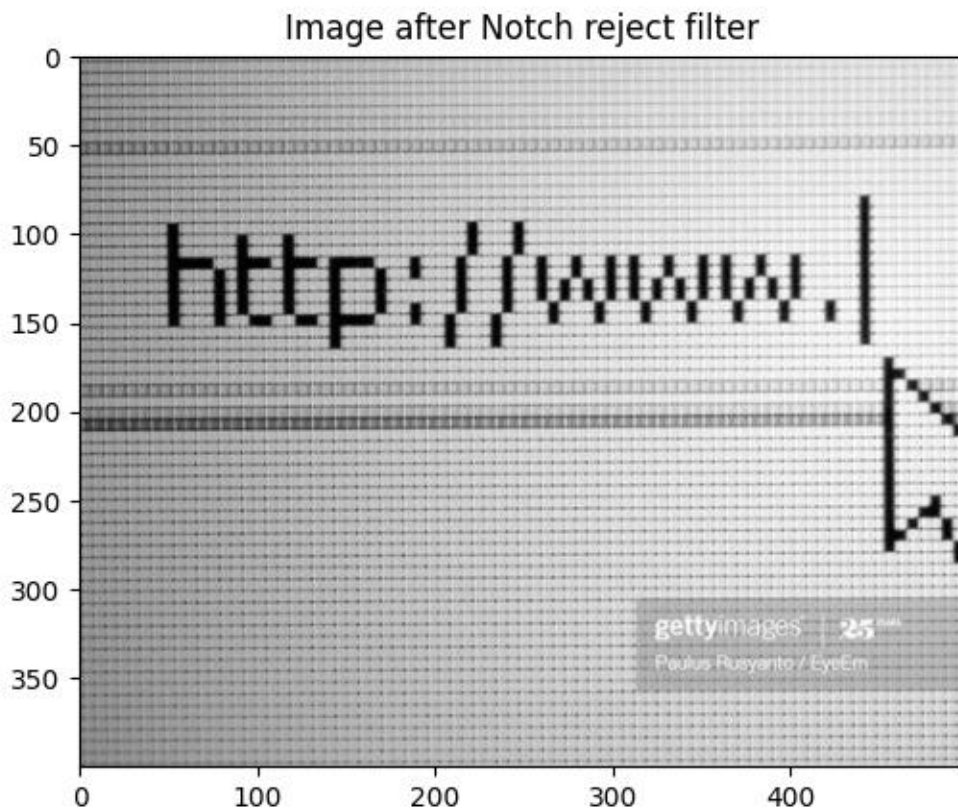




$D0 = 1/2 \times \text{scale factor for notch reject filter}$







Part d:

Algorithm for mean, variance, covariance of f in $O(MN(m+n))$ time:

Horizontal cumulative sum with window size m , C_x :

$$C_x(i,j) = \sum_{x=i-\frac{m}{2}}^{x=i+\frac{m}{2}} f(x,j) \quad 0 \leq i < M$$

Time complexity = MNm

Vertical cumulative sum C_y with window size n :

$$C_y(i,j) = \sum_{x=j-\frac{n}{2}}^{x=j+\frac{n}{2}} f(i,x) \quad 0 \leq j < N$$

Time complexity = MNn

First do horizontal sum and then vertical sum on it.

Using C_x and C_y computing mean for $f, f^2, f \cdot g$ takes $O(MN(m+n))$ time

Using the relations:

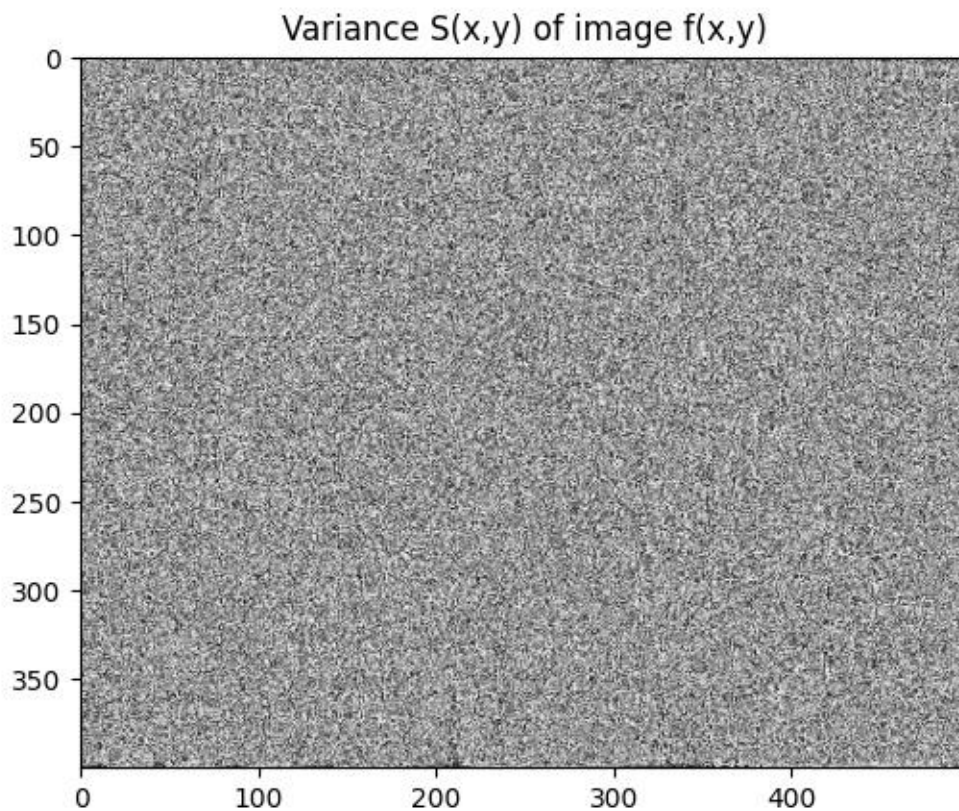
$$\text{Var}(X) = \text{Mean}(X^2) - \text{Mean}(X)^2$$

$$\text{Cov}(X,Y) = \text{Mean}(X,Y) - \text{Mean}(X) \cdot \text{Mean}(Y)$$

Var and Covariance takes $O(MN)$ time, assuming mean is computed.

$$\text{Overall time complexity} = O(MN(m+n)) + O(MN)$$

$$= O(MN(m+n))$$



Part E:

