

Number System

A number system used in digital electronics

a) Decimal

- The decimal number system is a type of number system that has a base value equal to 10.
- Decimal number system are represented by one ten symbols i.e 0 to 9.

b) Binary

- The binary number system is a type of number system that has a base value equal to 2 (0 and 1)
- A computer understand information composed of only 0 and 1. When we type some letters or words data is processed by the computer in the form of 0 and 1.

c) Hexadecimal

- The hexadecimal number system is a type of number system that has base value equal to 16
0 1 2 3 4 5 6 7 8 9 A B C D E F

d) Octal

- The octal number system is a type of number system that has base value equal

to 8.

Conversion in Number System.

① Binary to Decimal

$$\text{a) } 1011_2 \longrightarrow (11)_{10}$$

1	0	1	1	Binary numbers
2^3	2^2	2^1	2^0	Weight

$$= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

$$= 8 + 0 + 2 + 1$$

$$= 11$$

② $(10111)_2 \longrightarrow (23)_{10}$

1	0	1	1	1
2^4	2^3	2^2	2^1	2^0

$$= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

$$= 16 + 0 + 4 + 2 + 1$$

$$= 23$$

$$c) (1011.11)_2 \rightarrow (11.75)_{10}$$

1	0	1	1	.	1	1
2^3	2^2	2^1	2^0		2^{-1}	2^{-2}

$$= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^{-1} + 1 \times 2^{-2}$$

$$= 8 + 0 + 2 + \frac{1}{2} + \frac{1}{4} = 10.75$$

$$= 10 + 0.5 + 0.25 + 1$$

$$= 10.75.11.75$$

$$d) (101.01)_2 \rightarrow (5.25)_{10}$$

1	0	1	.	0	1	1
2^2	2^1	2^0		2^{-1}	2^{-2}	

$$= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 0 \times \frac{1}{2} + 1 \times \frac{1}{2^2}$$

$$= 4 + 0 + 1 + 0 + 0.25$$

$$= 5.25$$

ii) Octal to Decimal

a) $(14)_8 \rightarrow (12)_{10}$

Octal number	1	4	
Weight	8^1	8^0	

$$= 1 \times 8^1 + 4 \times 8^0$$

$$= 8 + 4 \times 1$$

$$= 8 + 4$$

$$= 12.$$

b) $(14.2)_8 \rightarrow (12.25)_{10}$

Octal number	1	4	.	2	
Weight	8^1	8^0	.	8^{-1}	

$$= 1 \times 8^1 + 4 \times 8^0 + 2 \times \frac{1}{8^1}$$

$$= 8 + 4 + 0.25$$

$$= 12.25$$

c) $(AE3)_{16} \rightarrow (2787)_{10}$

Hexadecimal	A	E	3	
Weight	16^2	16^1	16^0	

$$= A \times 16^2 + E \times 16^1 + 3 \times 16^0$$

$$= 10 \times 16^2 + 14 \times 16^1 + 3 \times 16^0$$

$$\begin{aligned}
 &= 10 \times 256 + 15 \times 16 + 3 \times 1 \\
 &= 2560 + 2240 + 3 \\
 &= 2787
 \end{aligned}$$

HexadeCimals are used in the following :-

- 1) To define locations in memory.
- 2) To define colours on webpage.
- 3) To represent media access control (MAC).address.
- 4) To display error messages.

Numericals.

1) $(10110)_2 \rightarrow (22)_{10}$

1	0	1	1	0
2^4	2^3	2^2	2^1	2^0

$$\begin{aligned}
 &= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\
 &= 16 + 0 + 4 + 2 + 0 \\
 &= 22.
 \end{aligned}$$

2) $(1011.011)_2 \rightarrow (11.375)_{10}$

1	0	1	1	.	0	1	1
2^{-3}	2^{-2}	2^{-1}	2^0	2^{-1}	2^{-2}	2^{-3}	

$$\begin{aligned}
 &= 1 \times 2^{-3} + 0 \times 2^{-2} + 1 \times 2^{-1} + 1 \times 2^0 + 0 \times \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} \\
 &= 0.125 + 0 + 2 + 1 + 0 + 0.25 + 0.125 \\
 &= 11.375
 \end{aligned}$$

$$3) (A4F \cdot 3)_{16} \rightarrow (2639 \cdot 1875)_{10}$$

A	4	F	3
16^2	16^1	16^0	16^{-1}

$$\begin{aligned}
 &= 10 \times 16^2 + 4 \times 16^1 + 15 \times 16^0 + 3 \times 16^{-1} \\
 &= 2560 + 64 + 15 + 3 \times 0.0625 \\
 &= 2560 + 64 + 15 + 0.1875 \\
 &= 2,639.1875
 \end{aligned}$$

Memory units.

$$1) 1 \text{ Nibble} = 4 \text{ bits.} \quad 10) 1 \text{ YB} = 1024 \text{ ZB.}$$

$$2) 1 \text{ byte} = 8 \text{ bits}$$

Exabyte (EB)

$$3) 1 \text{ KB} = 1024 \text{ bytes.}$$

Zettabyte (ZB)

Yottabyte (YB)

$$4) 1 \text{ MB} = 1024 \text{ KB.}$$

$$5) 1 \text{ GB} = 1024 \text{ MB}$$

$$6) 1 \text{ TB} = 1024 \text{ GB}$$

$$7) 1 \text{ PB} = 1024 \text{ TB}$$

$$8) 1 \text{ EB} = 1024 \text{ PB}$$

$$9) 1 \text{ ZB} = 1024 \text{ EB}$$

Any number system to binary system.

To convert any number system into binary number system the integer and the fractional parts of the number should be considered separately.

Q.1. $(13)_{10} \rightarrow (?)_2$

1	3
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$$\begin{array}{r} 2 | 13 \\ 2 | 6 \\ 2 | 3 \end{array} \quad \begin{array}{l} R \\ 1 \\ 0 \\ 1 \end{array}$$

$\therefore (13)_{10} \rightarrow (1101)_2$

Q.2. $(1101)_2 \rightarrow (?)_{10}$

1	1	0	$\frac{1}{2}$	Binary number
2^3	2^2	2^1	2^0	Weight

$$= 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$= 8 + 4 + 0 + 1$$

$$= 13$$

$\therefore (1101)_2 \rightarrow (13)_{10}$

$$\begin{array}{r} 2 | 13 \\ 2 | 6 \\ 2 | 3 \end{array} \quad \begin{array}{l} 1 \\ 0 \\ 1 \end{array}$$

Q.3. $(13.25)_{10} \rightarrow (?)_2$

$$(1101.01)_2$$

$$0.25 \times 2 = 0.5$$

1	1	0	1	.	0	1
2^3	2^2	2^1	2^0		2^{-1}	2^{-2}

$$0.5 \times 2 = 1$$

$$\begin{aligned}
 &= 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} \\
 &= 8 + 4 + 0 + 1 + 0 + 1 \times \frac{1}{4} \\
 &= 8 + 4 + 0 + 1 + 0 + 0.25 \\
 &= 13.25
 \end{aligned}$$

$$(1101.25)_2 \rightarrow (13.25)_{10}$$

$$\text{Q.N.4. } (16.73)_{10} \rightarrow (?)_2$$

$(10000.10111)_2$										$\frac{R}{2}$
LQ	0	0	0	1	0	1	1	1	1	0
2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
2	8	4	2	1	0	0	0	0	0	0
2	4	2	1	0	0	0	0	0	0	0
2	2	1	0	0	0	0	0	0	0	0

$$\begin{aligned}
 &= 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 + \\
 &\quad 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4} + 0.73 \times 2 = 1.46 \\
 &\quad 1 \times 2^{-5} \qquad \qquad \qquad 0.46 \times 2 = 0.92 \quad 0 \\
 &= 32 + 0 + 0 + 0 + 1 \times \frac{1}{2} + 0.73 + 0.46 + 0.84 + 0.68 = 1.46 + 0.92 + 0.84 + 0.68 = 4.8 \\
 &\quad 0.84 \times 2 = 1.68 \quad 1 \\
 &\quad 0.68 \times 2 = 1.3 \quad 1
 \end{aligned}$$

$$(16.73)_{10} \rightarrow (10000.10111)_2$$

Octal number to Binary number.

Q.1. $(540)_8 \rightarrow (?)_2$

421	421	421
5	4	0
101	100	000

$$(540)_8 \rightarrow (101\ 100\ 000)_2$$

Q.2. $(352)_8 \rightarrow (?)_2$

421	421	421
3	5	2
011	101	010

$$(352)_2 \rightarrow (011\ 101\ 010)_2$$

Q.3. $(352.563)_8 \rightarrow (?)_2$

421	421	421	421	421	421
3	5	2	5	6	3
011	101	010	101	110	011

$$\therefore (352.563)_8 \rightarrow (011101\ 010.101\ 110\ 011)_2$$

Hexadecimal to Binary System.

A = 10

B = 11

C = 12

D = 13

E = 14

F = 15

8421	8421	8421	
8	5	2	
1000	0101	0010	

$$(852)_{16} \rightarrow (1000 \ 0101 \ 0010)_2$$

Binary to Hexadecimal

8421

$$Q.1 (1000 \ 0101 \ 0010)_2 \rightarrow (?)_{16}$$

↓ ↓ ↓

8 5 2

$$\therefore (1000 \ 0101 \ 0010)_2 \rightarrow (852)_{16}$$

Octal

421

$$Q.2 (001010000011)_2 \rightarrow (?)_8$$

↓ ↓ ↓ ↓

1 2 0 3

[If there is only one number left at the beginning we should add 0 in front]

$$\therefore (001010000011)_2 \rightarrow (1203)_8$$

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Q.3. $(101\ 000\ 0011)_2 \rightarrow (?)_8$

~~421~~

$00(1\ 010\ 000\ 011\cdot 01)0$

↓ ↓ ↓ ↓ ↓
1 2 0 3 2

$$(001\ 010\ 000\ 011)_2 \rightarrow (1203.2)_8$$

Binary Operations.

Imp

$$\begin{array}{r} 1) 0 \\ + 0 \\ \hline 0 \end{array} \quad \begin{array}{r} 2) 0 \\ + 1 \\ \hline 1 \end{array} \quad \begin{array}{r} 3) 1 \\ + 0 \\ \hline 1 \end{array} \quad \begin{array}{r} 4) 1 \\ + 1 \\ \hline 10 \end{array}$$

5) $\begin{array}{r} 1 \\ + 1 \\ \hline 10 \end{array}$

carry

carry.

Binary Operations.

The automatic operations in digital computers are performed on Binary numbers only. The hexa-decimal representation is used only for entering information into the computers.

Use of complements to represent negative numbers.

Today, computer systems performed subtraction using complemented number. This is done because this is economical.

Imp

- a) 1's complement (opposite - 3rd col) $\begin{bmatrix} 0 \rightarrow 1 \\ 1 \rightarrow 0 \end{bmatrix}$

Q.N.1. Find 1's complement of $(1011)_2$.

$$\begin{array}{r} (1011)_2 \\ \downarrow \downarrow \downarrow \downarrow \\ 0100 \end{array}$$

1's complement of $(1011)_2 \rightarrow (0100)_2$.

b) 2's complement

- b) 2's complement. $\begin{bmatrix} \text{formula} \rightarrow 2's \text{ complement} \\ = 1's \text{ complement} + 1 \end{bmatrix}$

Q.N.2. Find 2's complement of $(1011)_2$.

$$(1011)_2$$

$\downarrow \downarrow \downarrow \downarrow$

$$0 - 1 0 0$$

$$1 0 1 0$$

$$0100$$

$$+ 1$$

$$\hline 0101$$

$\therefore 2's \text{ complement of } \cancel{1011} \rightarrow (1011)_2 \rightarrow (0101)_2$

Useful for checking answers in Exam

$$\begin{array}{r} 1 0 1 1 \\ \hline \end{array} \rightarrow \text{keep } 1 \text{ constant}$$

$$\downarrow \downarrow \downarrow$$

$$0 1 0 1$$

Subtraction

Small no - big no [case 1]

- There is no carry.
- take 2's complement of answer and put (-) negative sign.

big no - small no [case 2]

= There is always a carry.

- Remove carry.

Q.N.1)

Subtract 1011_2 from 1100_2 .

$$= (1100)_2 - (1011)_2$$

$$= (1100)_2 + \text{2's complement of } (1011)_2$$

$$= (1100)_2 + \begin{pmatrix} 0100 \\ + 1 \end{pmatrix}_2$$

$$= (1100)_2 + (0101)_2$$

$$= 0001.$$

$$\begin{array}{r} 1100 \\ + 0101 \\ \hline 1)0001 \end{array}$$

Remove .

Q.N.2. Find 1's and 2's complement of.

a) $(1010101)_2$

Solution:

1's complement of $(1010101)_2$ is $(0101010)_2$

2's complement of 1010101 is 0101010

$$\begin{array}{r} + 1 \\ \hline 010 \\ 0101011 \end{array}$$

b) $(0111000)_2$

Solution:

1's complement of (0111000) is $(1000111)_2$

2's complement of $(0111000)_2$ is $01110\ 1000111$

$$\begin{array}{r} + 1 \\ \hline 1001000 \end{array}$$

Q.N.1. $(1745.246)_8 \rightarrow (?)_{16}$

Solution:

Octal to Binary.

421

1	7	4	5	2	4	6
001	111	100	101	010	100	110

$$\therefore (1745.246)_8 \rightarrow (001\ 111\ 100\ 101\cdot 010\ 100)_2$$

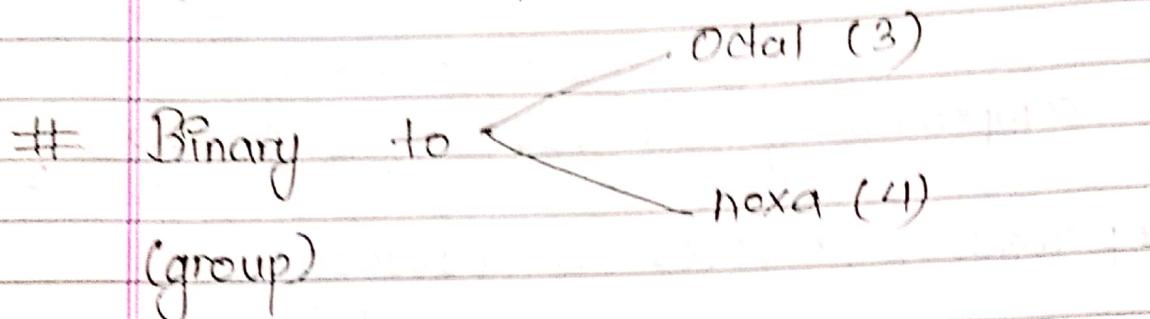
Binary to Hexadecimal

8421.

110

$$(001\ 111\ 100\ 101\cdot 010\ 100)_2 = (?)_{16}$$

8421.

(0011110010.010100)10(3E5.530)₁₆.Example

Q. 1. ~~(0011000000)₂ → (?)₈~~
421

~~0011000000~~
~~1 4 0~~

Q. 1. ~~(101100000.111000)₂ → (?)₁₆~~

Solution;

8421.

000101100000.11100000
↓ ↓ ↓ ↓ ↓
1 6 0 . E 0

(160.E0)₁₆.

$$Q.N.1 \quad 01011_2 - 11100_2$$

$$= 01011_2 + (2's \text{ complement of } 11100_2)$$

$$= 2's \text{ complement of } 11100_2$$

$$= 1's \text{ complement of } 11100_2 + 1$$

$$\begin{array}{r} 00011 \\ + 1 \\ \hline 00100 \end{array}$$

$$= 01011_2 + 00100_2$$

$$= 01111$$

$$\begin{array}{r} 01011 \\ + 00100 \\ \hline 01111 \end{array}$$

∴ Since, there is no carry. So, 2's complement of 01111_2 is 1's complement of $01111 + 1$.

$$10000$$

$$+ 1$$

$$\hline 10001$$

$$= -(10001)_2$$

Q.1. $(10101)_2 - (10111)_2$

= $(10101)_2 + (\text{2's complement of } 10111)_2$

= 2's complement of 10111_2

= 1's complement of $10111_2 + 1$

= 01000

+ 1

01001

= $(10101)_2 + (01001)_2$

= $(11110)_2$

$$\begin{array}{r} 1 \\ 10101 \\ + 01001 \\ \hline 11110 \end{array}$$

∴ Since, there is no carry so, 2's complement of $(11110)_2$ is 1's complement of $(11110 + 1)_2$.

00001

+ 1

00010

= $(00010)_2$

Q.N.2 $(10102)_2 - (00111)_2$

= $(10101)_2 + (\text{2's complement of } 00111)_2$

$2^{\text{'s complement}}$ of 00111_2

= $1^{\text{'s complement}}$ of $00111_2 + 1$

$$\begin{array}{r} \cancel{1}000_2 \\ + 1 \\ \hline 000\cancel{0}1 \end{array} \quad \begin{array}{r} 11000 \\ + 1 \\ \hline 11001 \end{array}$$

$$= (10101)_2 + (11001)_2$$

$$= (101110)_2$$

Since, there is carry. the answer is $(01110)_2$.
 carry

$$\text{Q.N.3. } (10011)_2 - (11100)_2$$

$$= (10011)_2 + (2^{\text{'s complement}} \text{ of } 11100_2)$$

$2^{\text{'s complement}}$ of 11100_2

= $1^{\text{'s complement}}$ of $11100_2 + 1$

$$\begin{array}{r} 000\cancel{1} \\ + 1 \\ \hline 00100 \end{array}$$

$$= (10011)_2 + (00100)_2$$

$$= (10111)_2$$

$$\begin{array}{r} 10011 \\ + 00100 \\ \hline 10111 \end{array}$$

Since, there is no carry. So, $2^{\text{'s complement}}$ of $(10111)_2$ is $1^{\text{'s complement}}$ of $(10111 + 1)_2$.

$$01000$$

$$+ 1$$

$$01001$$

$$= -(01001)_2$$