

$$x_1, \dots, x_n; x_i \sim \sum_{j=1}^K \pi_j \mathcal{N}(m, \sigma_j^2); m \sim \mathcal{N}(m_0, \sigma_0^2), \frac{1}{\sigma_j^2} \sim \Gamma(\alpha_j, \beta_j)$$

$$\begin{aligned} \text{a) } p(x, m, \sigma^2 | \alpha, \beta, m_0, \sigma_0^2, \pi) &= p(x | m, \sigma^2, \pi) p(m | m_0, \sigma_0^2) p(\sigma^2 | \alpha, \beta) = \\ &= \prod_{i=1}^n \left(\sum_{j=1}^K \pi_j \mathcal{N}(x_i | m, \sigma_j^2) \right) \mathcal{N}(m | m_0, \sigma_0^2) \prod_{j=1}^K \Gamma\left(\frac{1}{\sigma_j^2} | \alpha_j, \beta_j\right) \end{aligned}$$

$$\text{б) } p(m, \sigma^2 | x, \alpha, \beta, m_0, \sigma_0^2, \pi) = \frac{p(x, m, \sigma^2 | \alpha, \beta, m_0, \sigma_0^2, \pi)}{p(x | \alpha, \beta, m_0, \sigma_0^2)} \propto$$

$$p(x, m, \sigma^2 | \alpha, \beta, m_0, \sigma_0^2, \pi) - \text{нужно а)}$$

$$\begin{aligned} \text{в) } p(x, m, \sigma^2, z | \alpha, \beta, m_0, \sigma_0^2, \pi) &= \prod_{i=1}^n \left(\sum_{j=1}^K \pi_j \mathcal{N}(x_i | m, \sigma_j^2) \right)^{z_{ij}} \cdot \\ &\quad \prod_{j=1}^K \Gamma\left(\frac{1}{\sigma_j^2} | \alpha_j, \beta_j\right) \mathcal{N}(m | m_0, \sigma_0^2) \end{aligned}$$