18/01/25

→ DFT: $X[n] = \sum_{k=0}^{N-1} C_k e^{j 2\pi \frac{k}{N} n}$ → LTI → $y[n] = \sum_{k=0}^{N-1} H\left(\frac{k}{N}\right) C_k e^{j \frac{2\pi k}{N} n}$

$C_k \downarrow$

$\frac{1}{N} X[k]$

↳ Leads to the idea of filtering

→ But for compression, it is not necessarily the most optimal

$$x[n] = \sum_{0}^{K-1} b_K \, C_K[n] \quad , n = 0, 1, \ldots, N-1$$

if most info captured in few b_K

ie most $b_K \simeq 0$,

$x[n]$: N values

significant b_K: L values save factor $\frac{N}{L}$

∴ JPEG, MP3, MPEG, etc

→ Discrete Cosine Transform

$$x[n] = \frac{1}{N} x^c[0] + \frac{1}{N} \sum_{K=1}^{N-1} 2 x^c[K] \left(\cos 2\pi \left(\frac{K+1/2}{2N} \right) n \right)$$

$n = 0, 1, \ldots, N-1$

$b_0 = \frac{x^c[0]}{N}$ $\quad c_0[n] = 1$ $\quad b_K = \frac{1}{N} 2 x^c[K]$, $c_K[n] = \cos 2\pi \left(\frac{K+1/2}{2N} \right) n$

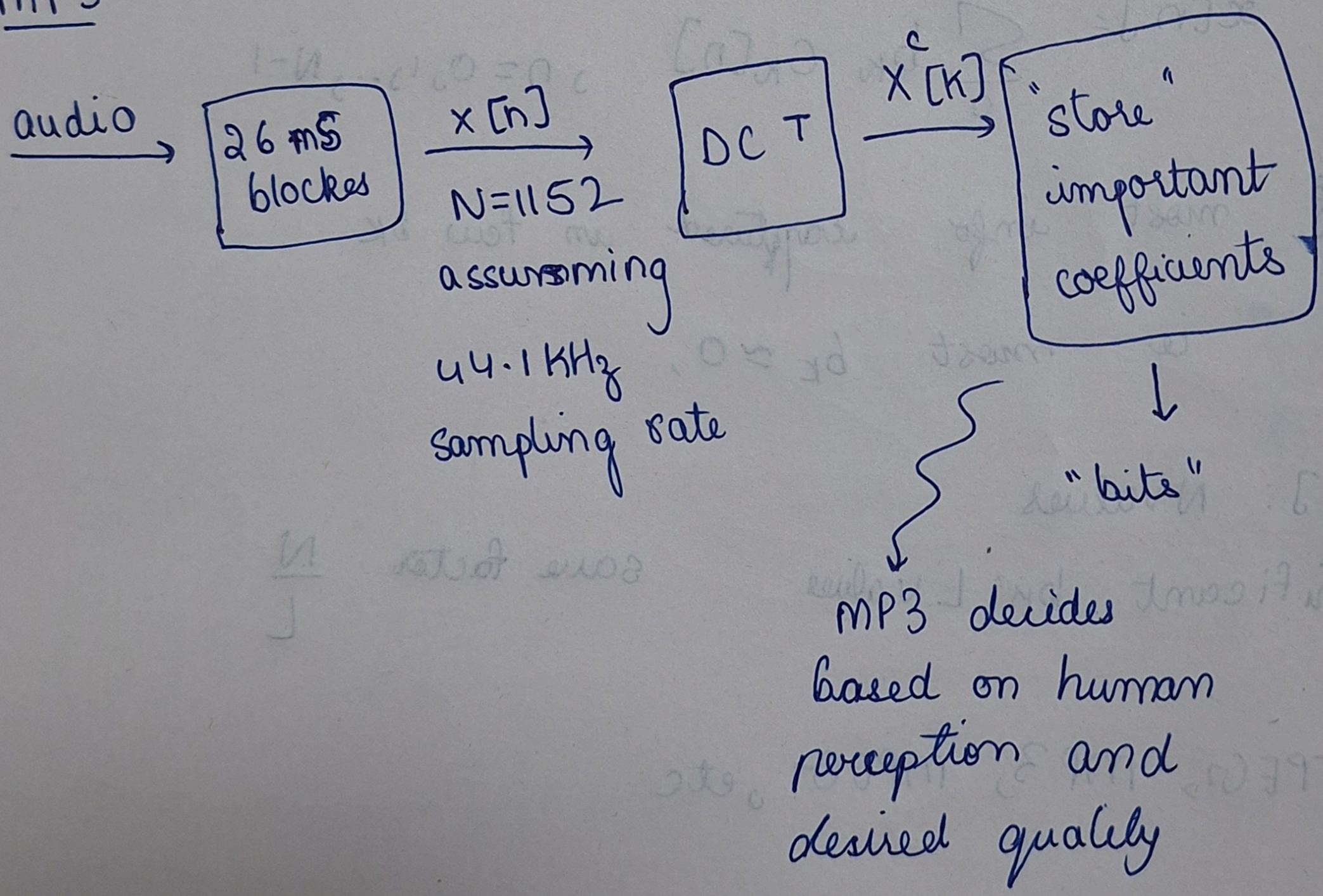
$x^c[k]$ are real!

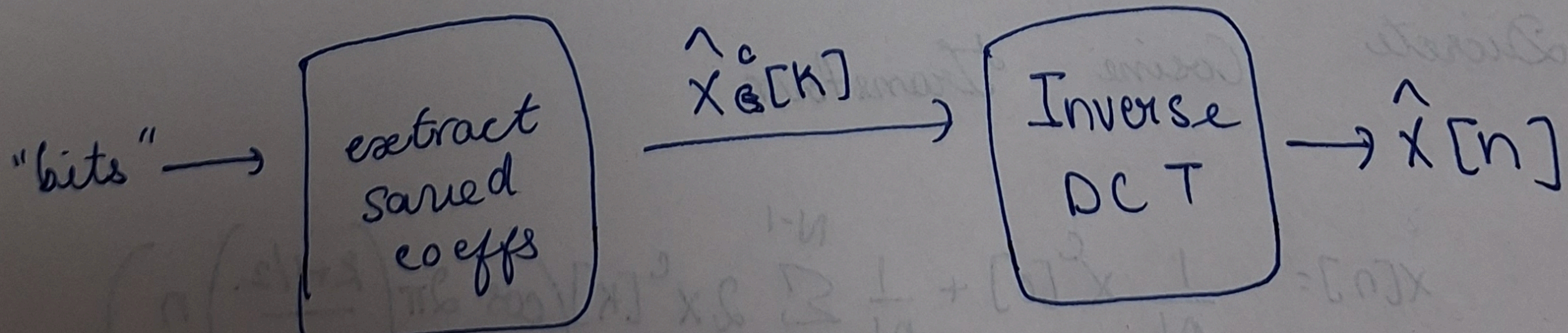
$$\cos(2\pi \hat{f} n) \rightarrow \boxed{LTI} \rightarrow |H(\hat{f})| \cos\left(2\pi \hat{f} n + \phi(\hat{f})\right)$$

phase factor

not an Eigenfunction of LTI systemes

mp3





→ Image Compression : JPEG

Consider a black and white image w/ resolution 1000 × 1000. Each pixel 8 bits.

∴ 8,000,000 bits per image

Consider 30 fps, 3 seconds

3 × 30 × 8,000,000 = 720,000,000 bits

→ JPEG

DCT → Quantization → Zig Zag Scan → RLE & DPCM → Entropy Coding

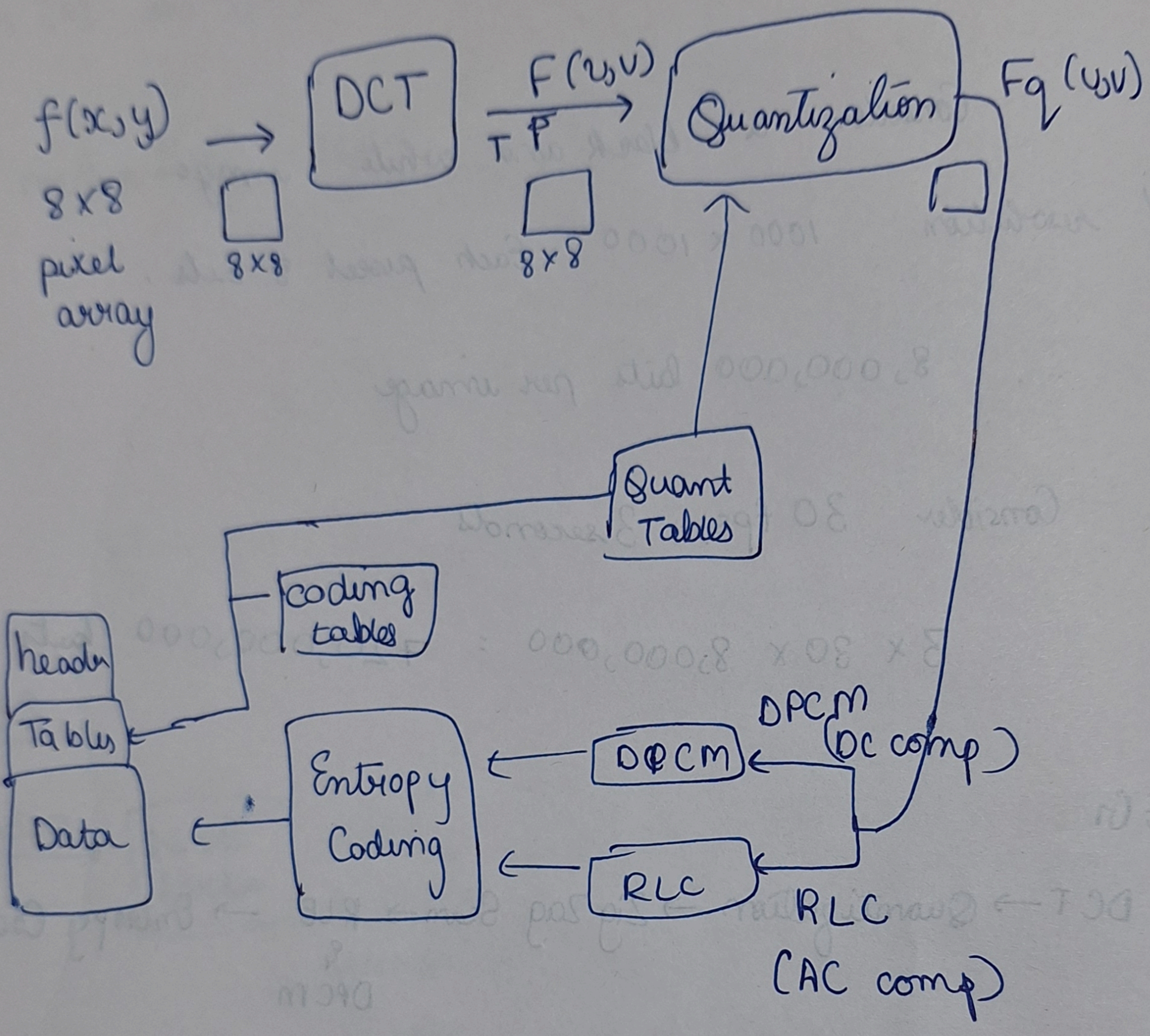
Modes

Sequential / Lossless / Progressive / Hierarchical.

→ Compression ratio of lossless methods not high enough.

→ JPEG - transform coding

- lower spatial frequency components contain more information


f(x,y)
8×8
pixel
array
8×8
DCT
F(u,v)
T F
8×8
Quantization
Fq(u,v)
Quant
Tables
coding
tables
header
Tables
Data
Entropy
Coding
DPCM
DPCM
(DC comp)
RLC
RLC
(AC comp)

2D DCT

$$x[m,n] = \frac{1}{N} \sum_{k=0}^{N-1} \sum_{\ell=0}^{N-1} a_k a_\ell X^c[k,\ell] \; C_{k,\ell}[m,n]$$

$$\cos\left[2\pi\left(\frac{\ell}{2N}\right)\left(n+1/2\right)\right] \cos\left[2\pi\left(\frac{k}{2N}\right)\left(m+1/2\right)\right]$$

$$a_i = \begin{cases} 1 & i=0 \\ \sqrt{2} & \text{otherwise} \end{cases}$$

$$X[m,n] = \sum_{k=0}^{N-1} \sum_{\ell=0}^{N-1} b_{k,\ell} \; C_{k,\ell}[m,n]$$

weighted sum of "basis images"

$C_{k,\ell}[m,n]$: Product of vertical $\left(\frac{k}{2N} \text{ freq}\right)$ and horizontal $\left(\text{freq } \frac{\ell}{2N}\right)$ cosine