Bias Var

$\sigma^2 = E[(X-\mu)^2]$

$\hat{\sigma}^2 = \frac{1}{n}\sum_{k=1}^{n}(X_k - \mu)^2$ → don't know ↗

unbiased estimate of σ^2

$$\bar{S}^2 = \frac{1}{n}\sum_{k=1}^{n}(X_k - \bar{X})^2$$

$$= \frac{1}{n}\sum\left(X_k^2 + \bar{X}^2 - 2X_k\bar{X}\right)$$

$$= \frac{1}{n}\left[\sum X_k^2 + n\bar{X}^2 - 2\bar{X}(n\bar{X})\right]$$

$$= \frac{1}{n}\left\{\left(\sum X_k^2\right) - n\bar{X}^2\right\}$$

eg. $X_1 \ldots X_n$ — random sample $\quad EX_i = \mu \quad Var(X_i) = \sigma^2$

estimator

$$\text{Bias}(\hat{\theta}) = E[\hat{\theta}] - \theta$$

Bias tells you if the estimate is centred around the true value on average

Bias of the estimator

$$B(\bar{S}^2) = E[\bar{S}^2] - \sigma^2$$

$$E\bar{X}^2 = (E\bar{X})^2 + Var\bar{X} = \mu^2 + \frac{\sigma^2}{n}$$

$$E[\bar{S}^2] = \frac{1}{n}\left(\sum EX_k^2 - nE\bar{X}^2\right)$$

$$= \frac{1}{n}\left(n(\mu^2+\sigma^2) - n\left(\mu^2 + \frac{\sigma^2}{n}\right)\right)$$

$$= \frac{n-1}{n}\sigma^2$$

$$B(\bar{S}^2) = E[\bar{S}^2] - \sigma^2 = -\frac{\sigma^2}{n}$$

biased estimate of Variance

Bias $= 0$
Estimator is unbiased.

If I repeat a million times and average I would get the correct answer.

Sometimes you use a biased estimator with lower variance.

$$E[\bar{S}^2] = \frac{n-1}{n}\sigma^2$$

$\frac{n\bar{S}^2}{n-1} \rightarrow$ unbiased estimate of σ^2

$$S^2 = \frac{1}{n-1}\sum (X_k - \bar{X})^2 = \frac{1}{n-1}\left(\sum X_k^2 - n\bar{X}^2\right)$$

Sample Variance

$$Var(\hat{\theta}) = E\left[(\hat{\theta} - E[\hat{\theta}])^2\right]$$

How spread out estimators values are across different samples.

$$MSE(\hat{\theta}) = E\left[(\hat{\theta} - \theta)^2\right]$$

$$= \text{Bias}^2 + \text{Variance}.$$

$S = \sqrt{S^2}$

Sample std is a biased estimata of std

S - random $Var(S) > 0$

$$0 < Var(S) = ES^2 - (ES)^2 = \sigma^2 - (ES)^2$$

$$(ES)^2 < \sigma^2$$

$$ES < \sigma$$

S is a biased estimata for σ

$\hat{\theta} = h(X_1, X_2, \ldots, X_n) \rightarrow$ point estimata for θ

$$B(\hat{\theta}) = E[\hat{\theta}] - \theta$$

Desirable $B[\hat{\theta}] = 0$ $E[\hat{\theta}] = \theta$

eg) $X_1, X_2, \dots, X_n$ — random sample

$$\hat{\theta} = \underbrace{\bar{X}}_{\text{sample mean}} = \frac{X_1 + \dots + X_n}{n}$$

unbiased estimator

$\theta = EX_i$

$$B(\hat{\theta}) = E(\hat{\theta}) - \theta = E(\bar{X}) - EX_i = EX_i - EX_i = 0$$

But

say $\hat{\theta} = X_1$

$$B(\hat{\theta}) = E[X_1] - EX_i = 0$$

$$MSE(\hat{\theta}) = E[(\hat{\theta} - \theta)^2]$$

$\hat{\theta} = X_1$

$$MSE = E[(X_1 - EX_i)^2]$$

$$= E[(X_1 - EX_1)^2]$$

$\hat{\theta} = \bar{X}$

$$MSE = E[(\bar{X} - \theta)^2]$$

$$Var(\bar{X} - \theta) + [E(\bar{X} - \theta)]^2$$

θ constant $\to 0$

$= \sigma^2$

$= Var(\bar{X})$

$= \underbrace{\dfrac{\sigma^2}{n}}_{\text{Lower variance}}$

let $\hat{\theta}_1, \ldots, \hat{\theta}_n$ point estimators of θ

$\hat{\theta}_n$ is consistant if

$$\lim_{n \to \infty} P\left(|\hat{\theta}_n - \theta| \geq \varepsilon\right) = 0 \quad \forall \varepsilon > 0$$