凸优化第 14 周作业

1 SDP Relaxation

Problem 1 (Least Square on A Sphere) Consider the optimization problem as follow

$$\begin{aligned} \min \quad & \|Ax - b\|_2^2 \\ \text{s.t.} \quad & \|x\|_2 = 1, \end{aligned} \tag{1.1}$$

where $A \in \mathbb{R}^{m\times n}$ and $b \in \mathbb{R}^m$.

1. Write down the SDP relaxation for the original optimization problem (1.1).
2. Does the SDP relaxation provides the exact value of the original optimization problem (1.1)?

Problem 2 (Max Complete Subgraph Problem) Consider a undirected graph $G = (V, E)$, where V is the set of vertices, and $E \subseteq V \times V$ represents the connected relationship among the vertices, $(i, j) \in E \Leftrightarrow i$ and j are connected. A subgraph of G is *complete* if all vertices in the subgraph are connected. Define $\alpha(G)$ as the optimal value of the optimization below

$$\begin{aligned} \max \quad & \sum_{i\in V} x_i \\ \text{s.t.} \quad & x_i x_j = 0 \text{ if } (i,j) \notin E \\ & x_i \in \{0, 1\}, \forall i \in V. \end{aligned} \tag{1.2}$$

1. Prove that $\alpha(G)$ is the size of the max complete subgraph of G.
2. Write down a SDP relaxation for the problem (1.2).

2 作业说明

1. 请大家在网络学堂作业窗口提交 pdf 版本或者在下次上课前把纸质作业放在讲台上。
2. 请大家在截止日期前提交作业。
3. 本次作业满分 25 分。