MEC 361 Assignment

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Question 3.2

- Determine the torque T that causes a maximum shearing stress of 45 MPa in the hollow cylindrical steel shaft shown.
- Determine the maximum shearing stress caused by the same torque T in a solid cylindrical shaft of the same cross-sectional area.

<u>Solution</u>

- Finding the torque
  Given:

$$\tau = 45 \times 10^6 Pa$$

$$r_1 = 30mm = 0.03m$$

$$r_2 = 45mm = 0.024m$$

$$\tau = \frac{Tr}{J}$$

  Making T the subject of the formular

$$T = \frac{J\tau}{r} = \frac{\frac{\pi}{2} \times (0.045^4 - 0.03^4) \times 45 \times 10^6}{0.045} = 5.168 \times 10^3$$

- Assuming that it is a solid shaft and using the torque gotten from part A Find the cross sectional raduis

$$r = \sqrt{({r_1}^2 - {r_2}^2)} = \sqrt{(45^2 - 30^2)} = 33.541mm$$

  Now using this raduis as a the raduis for finding the torque, we have

$$\tau = \frac{Tr}{J} = \frac{T \times 2}{\pi r^3} = \frac{5.1689 \times 10^3 \times 2}{\pi 0.033541} = 87.2 \times 10^6 Pa$$

Question 3.6

- Determine the torque that can be applied to a solid shaft of $20 - mm$ diameter without exceeding an allowable shearing stress of 80 MPa.
- Solve part a, assuming that the solid shaft has been replaced by a hollow shaft of the same cross-sectional area and with an inner diameter equal to half of its outer diameter.

<u>Solution</u>

- Finding the torque of a solid shaft
  Given:

$$\tau = 80 \times 10^6 Pa$$

$$d_s = 20mm = 0.02m$$

$$\tau = \frac{Tr}{J}$$

  Making T the subject of the formular

$$T = \frac{J\tau}{r} = \frac{\frac{\pi}{32} \times (0.02^4) \times 80 \times 10^6}{\frac{0.02}{2}} = 125.7Nm$$

- Finding the torque of a hollow shaft using the same dimensions, we have
  Given:

$$\tau = 45 \times 10^6 Pa$$

$$r_1 = ?$$

$$r_2 = 0.5r_1$$

  Since according to the question, we are to assume that the cross sectional area of the solid shaft is the same to the hollow shaft, therefore we can say $A_H = A_S$

$$A_S = \pi \times \frac{{d_S}^2}{2} = \pi \times \frac{0.02}{2} = 0.0003141mm^2$$

$$A = \pi \times ({r_1}^2 - {r_2}^2) = \frac{\pi \times 3{r_1}^2}{4} = 0.0003141$$

  Solving for r_1, we have that

$$r_1 = 0.01154, dir_2 = 0.005772$$

$$T = \frac{J\tau}{r} = \frac{\frac{\pi}{2} \times (0.01154^4 - 0.005772^4) \times 80 \times 10^6}{0.01154} = 181.03Nm$$

Question 3.10

In order to reduce the total mass of the assembly of Prob. 3.9, a new design is being considered in which the diameter of shaft BC will be smaller. Determine the smallest diameter of shaft BC for which the maximum value of the shearing stress in the assembly will not increase.

<u>Solution</u>

- In Shaft AB
  Given:

$$d = 0.03m$$

$$T_a = 300Nm$$

$$r = \frac{d}{2} = 0.015m$$

The formular for solving this question $\tau = \frac{T \times r}{J}$

$$\tau = \frac{T \times r}{J} = \frac{300 \times 0.015}{\frac{\pi}{32} \times 0.03^4} = 56.58 \times 10^6 Nm$$

- In Shaft BC
  Given:

$$d = 0.046m$$

$$T_b = 400Nm$$

$$r = \frac{d}{2} = 0.023m$$

$$\tau = \frac{T \times r}{J} = \frac{700 \times 0.023}{\frac{\pi}{32} \times 0.046^4} = 20.92 \times 10^6 Nm$$

The highest shear stress is $56.58 \times 10^6 Nm$. Therefore we are going to use this as our shear stress. Making r subject of the formular we have that

$$r^3 = \frac{2 \times T}{T \times \pi}$$

$$r = \sqrt[3]{\frac{2 \times 700}{56.58 \times 10^6 \times \pi}} = 0.019896$$

$$d = 2r = 2 \times 0.019896 = 0.03979m$$

Question 3.13

Under normal operating conditions, the electric motor exerts a 12-kip.in. torque at E. Knowing that each shaft is solid, determine the maximum shearing stress in

- shaft BC
- shaft CD,

- shaft DE

Solution

Knowing that $1in$ diameter hole has been drilled into the beam, we have

- In Shaft BC, Given

$$r_2 = 0.5in, r_1 = \frac{1.75}{2} = 0.875in, T_{BC} = 3 \times 10^3 psi.in$$

$$\tau = \frac{T \times r}{J} = \frac{3 \times 10^3 \times 0.875}{\frac{\pi}{2} \times (0.875^4 - 0.5^4)} = 3.19 \times 10^3 psi.in$$

- In Shaft CD
  Given

$$r_2 = 0.5in, r_1 = \frac{2}{2} = 1in, T_{CD} = (4+3) \times 10^3 = 7 \times 10^3 psi.in$$

$$\tau = \frac{T \times r}{J} = \frac{7 \times 10^3 \times 1}{\frac{\pi}{2} \times (1^4 - 0.5^4)} = 4.75 \times 10^3 psi.in$$

- In Shaft DE
  Given

$$r_2 = 0.5in, r_1 = \frac{2.25}{2} = 1.125in, T_{DE} = (4+3+5) \times 10^3 = 12 \times 10^3 psi.in$$

$$\tau = \frac{T \times r}{J} = \frac{12 \times 10^3 \times 1.125}{\frac{\pi}{2} \times (1.125^4 - 0.5^4)} = 5.583 \times 10^3 psi.in$$

Question 3.18

The solid rod BC has a diameter of 30 mm and is made of an aluminum for which the allowable shearing stress is 25 MPa. Rod AB is hollow and has an outer diameter of 25 mm; it is made of a brass for which the allowable shearing stress is 50 MPa. Determine

- the largest inner diameter of rod AB for which the factor of safety is the same for each rod
- the largest torque that can be applied at A.

Solution

We are going to assume that the torque is the same

$$T_s = T_b = \text{Largest Torque}$$

Given:

$$T_s = 25 \times 10^6 Nm, r = 0.015m$$

$$T_s = \frac{\tau_s \times J}{r} = \frac{25 \times 10^6 \times \frac{\pi}{2} \times 0.015^4}{0.03} = 132.53Nm$$

Finding the inner diameter of the brass, we have

$$T_b = \frac{\tau_b \times \frac{\pi}{2} \times ({r_1}^4 - {r_2}^4)}{r_1}$$

Given:

$$T_b = 25 \times 10^6 Nm, r = 0.0125m$$

Making r_2 the subject of the formular, we have

$$r = \sqrt[4]{{r_1}^4 - \frac{2 \times T_b \times r_1}{\pi \times \tau}} = \sqrt[4]{0.0125^4 - \frac{2 \times 132.5 \times 0.0125}{\pi \times 50 \times 10^{-3}}} = 0.00759$$

$$d = 2r = 0.01518m$$

Question 3.32

For the aluminum shaft shown (G = 27 GPa), determine

- the torque T that causes an angle of twist of 4_o
- the angle of twist caused by the same torque T in a solid cylindrical shaft of the same length and cross-sectional area.

Solution

Given

$$\phi = 4^o = 0.06981rad, G = 27 \times 10^9 Pa, l = 1.25m, r_1 = 0.018m, r_2 = 0.012m$$

$$T = \frac{\phi JG}{L} = \frac{0.06981 \times \frac{\pi}{2} \times (0.018^4 - 0.012^4) \times 27 \times 10^9}{1.25} = 199.53Nm$$

Cross sectional areaof the solid cylinder equal to the cross sectional area of the hollow cylinder

$$A_c = A_h$$

$$A_h = \pi({r_1}^2 - {r_2}^2 = \pi(0.018^2 - 0.012^2) = 5.654 \times 10^{-4} m^2$$

$$A_c = 5.654 \times 10^{-4} m^2$$

$$\pi r^2 = 5.654 \times 10^{-4} m^2$$

Making r the subject of the formular, we have

$$r = \sqrt{\frac{5.654 \times 10^{-4}}{\pi}} = 0.0134m$$

The angle of twist is

$$\phi = \frac{TL}{JG} = \frac{199.53 \times 1.25}{\frac{\pi}{2} \times 0.0134^4 \times 27 \times 10^9} = 0.182rad$$

Question 3.36

The torques shown are exerted on pulleys B Problems , C, and D. Knowing that the entire shaft is made of aluminum (G = 27 GPa), determine the angle of twist between

- C and B
- D and B.

<u>Solution</u>

Given:

$$G = 27 \times 10^9 Pa, l = 1.25m, r_1 = 0.015m, T = 400Nm, L = 0.8m$$

Angle of twist at section BC:

$$\phi_{BC} = \frac{TL}{JG} = \frac{400 \times 0.8}{\frac{\pi}{2} \times 0.015^4 \times 27 \times 10^9} = 0.1490$$

Angle of twist at D and B

$$\phi_{BD} = \phi_{BC} + \phi_{CD}$$

Given:

$$G = 27{\times}10^9 Pa, l = 1.25m, r = 0.018m, T = 400Nm{-}900Nm = -500Nm, L = 1m$$

$$\phi_{BC} = \frac{TL}{JG} = \frac{-500 \times 1}{\frac{\pi}{2} \times 0.018^4 \times 27 \times 10^9} = -0.1123$$

$$\phi_{BD} = 0.182 + (-0.1123) = 0.0367rad$$

Question 3.40

The solid spindle AB has a diameter $d_s = 1.75in$and is made of a steel with$G = 11.2 \times 10^6 psi$ and $t_{all} = 12ksi$, while sleeve CD is made of a brass with $G = 5.6 \times 10^6 psi$ and $t_{all} = 7ksi$. Determine

- the largest torque T that can be applied at A if the given allowable stresses are not to be exceeded and if the angle of twist of sleeve CD is not to exceed 0.3758
- the corresponding angle through which end A rotates.

<u>Solution</u>

$$d_s = 1.75in, d_b = 3in, \phi = 0.375^o, L_{sleeve} = 8in$$

$$G_s = 11.2 \times 10^6 Psi, G_b = 5.6 \times 10^6 Psi$$

$$\tau_{all\,s} = 12ksi = 12 \times 10^3 Psi, \tau_{all\,b} = 12ksi = 7 \times 10^3 Psi$$

Torque based on shear stress of the spindle

$$\tau_{all} = \frac{Tr_s}{J_s} \Rightarrow T = \frac{\tau J_s}{r_s}$$

$$T = \frac{12 \times 10^3 \times \frac{\pi}{2} \times 0.875^4}{0.875} = 12628.11 Ibs.in$$

Torque based on sleeve

$$d_{B1} = 3in$$

$$d_{B2} = 3 - 2t = 3 - 2(0.25) = 2.5$$

$$r_{B1} = \frac{d_{B1}}{2} = 1.5in$$

$$r_{B2} = \frac{d_{B2}}{2} = 1.25in$$

$$J = \frac{\pi}{2}(1.5^4 - 1.25^4) = 4.1172$$

$$T = \frac{7 \times 10^3 \times 4.1172}{1.5} = 19213.6 lbs.in$$

$$\phi = 0.375^o = \frac{0.325}{180} = 6.545 \times 10^{-3}$$

$$\phi = \frac{TL}{GJ} \Rightarrow T = \frac{GJ\phi}{L} = \frac{5.6 \times 10^6 \times 4.1172 \times 6.545 \times 10^{-3}}{8} = 18862.93 lbs.in$$

$$L = 13, T = 12628.11$$

$$\phi = \frac{TL}{GJ} = \frac{12628.11 \times 13}{11.2 \times 10^6 \times 0.920} = 0.0147 rads$$

Question 3.44

For the gear train described in Prob. 3.43, determine the angle through which end A rotates when $T = 5lb.in, l = 2.4in, d = \frac{1}{16}in, G = 11.2 \times 10^6 psi$ and n= 2

Solution

Givens:

$$t = 5ib.in, l = 2.4in, c = \frac{1}{2}d = \frac{1}{32}in, G = 11.2{\times}10^6 psi, n = 2, J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(\frac{1}{32})^4 = 1.49803{\times}10^{-6} in^4$$

The formular is $\phi = \frac{Tl}{GJ}(1 + \frac{1}{n^2} + \frac{1}{n^4})$

$$\phi = \frac{5 \times 2.4}{11.2 \times 10^6 \times 1.49803 \times 10^{-6}}(1 + \frac{1}{4^2} + \frac{1}{16^4}) = 938.73 \times 10^{-3} rad$$

The angle through which end A rotates in degress is 53.8^o

Question 3.48

A hole is punched at A in a plastic sheet by applying a 600-N force P to end D of lever CD, which is rigidly attached to the solid cylindrical shaft BC. Design specifications require that the displacement of D should not exceed 15 mm from the time the punch first touches the plastic sheet to the time it actually penetrates it. Determine the required diameter of shaft BC if the shaft is made of a steel with $G = 77GPa$ and$t_{all} = 80MPa$.

<u>Solution</u>

$$T = rP = (0.3)(600) = 180N.m$$

$$\phi = \frac{\sigma}{r} = \frac{15}{300} = 0.005rad$$

$$\phi = \frac{TL}{GJ} = \frac{2TL}{\pi G r^4}$$

Making r^4 the subject of the formular, we have that

$$r^4 = \frac{2TL}{\pi \phi G} = \frac{2 \times 180 \times 0.5}{\pi \times 11 \times 10^9 \times 0.05} = 14.882 \times 10^{-9} m^4$$

$$r = 11.045 \times 10^{-3} m = 11.045m$$

$$d = 2r = 22.1mm$$

The shaft diameter based based on stress is

$$\tau = 80 \times 10^6 Pa$$

$$\tau = \frac{Tr}{J} = \frac{2T}{\pi r^3}$$

Making r the subject of the formular, we have

$$r = \sqrt[3]{\frac{2T}{\pi \tau}} = \frac{(2)(180)}{\pi(80 \times 10^6)} = \sqrt[3]{1.43239 \times 10^{-6} m^3} = 11.273 \times 10^{-3} m = 11.273mm$$

$$d = 2r = 22.5mm$$