MEC 361 Assignment

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Question 2.1

An 80-m-long wire of 5-mm diameter is made of a steel with E = 200 GPa and an ultimate tensile strength of 400 MPa. If a factor of safety of 3.2 is desired, determine

- the largest allowable tension in the wire
- the corresponding elongation of the wire

<u>Solution</u>

Given:

$$L = 80m$$

$$E = 200GPa$$

$$d = 5mm$$

$$n = 3.2$$

$$Formular : E = \frac{\sigma}{A}$$

$$\text{Allowable stress, } \sigma_{allow} = \frac{yieldstress}{N}$$

$$\sigma_{alllow} = \frac{400 \times 10^6}{3.2} = 125MPa$$

To find the extension,

$$e = \frac{\sigma}{E} = \frac{400 \times 10^6}{200 \times 10^9} = 2 \times 10^{-3}$$

The value for the extension has no unit

Question 2.9

An aluminum control rod must stretch 0.08 in. when a 500-lb tensile load is applied to it. Knowing that $\sigma_{ult} = 22ksi$ and $E = 10.1 \times 10^6 psi$, determine the smallest diameter and shortest length that can be selected for the rod.

<u>Solution</u>

Given:

$$\delta = 0.08in$$

$$\sigma = 22 \times 10^6$$

$$E = 10.1 \times 10^6 psi$$

$$P = 500lb$$

For the diameter:

$$A = \frac{\pi}{4} d^2$$

$$\sigma = \frac{P}{A}$$

Therefore

$$A = \frac{P}{\sigma} = \frac{500}{22 \times 10^6} = 0.0227 in^2$$

making d the subject of the formular, we get

$$d = \sqrt{\frac{4 \times 2.27 \times 10^{-2}}{\pi}} = 0.1701 in$$

For the shortest length, we have

$$E = \frac{\sigma}{e}, e = \frac{\delta}{L}$$

Therefore the equation above transcribe to

$$E = \frac{\sigma l}{\delta}$$

Making L the subject of the formular,

$$l = \frac{E\delta}{\sigma} = \frac{10.1 \times 10^6 \times 0.08}{22 \times 10^3} = 36.72 in$$

Question 2.15

A 4-ft section of aluminum pipe of cross-sectional area $1.75 in^2$ rests on a fixed support at A. The $\frac{5}{8}$-in.-diameter steel rod BC hangsfrom a rigid bar that rests on the top of the pipe at B. Knowing that the modulus of elasticity is 29×10^6 psi for steel and 10.4×10^6 psi for aluminum, determine the deflection of point C when a 15-kip force is applied at C.

Solution

Given:

Diameter of the Steel rod $d_s = \frac{5}{8} in$

Length of the aluminium rod, $L_A = 4ft = 48in$

Length of the steel rod, $L_S = 7ft = 84in$

Cross sectional Area of the alunimium, $A_A = 1.75 in^2$

Modulus of elasticity for aluminum, $E_A = 10.4 \times 10^6$

$$\text{Modulus of elaticity for steel. } E_S = 29 \times 10^6$$

$$\text{The force that } P = 15 \times 10^3$$

we will now find the cross cestional area of steel

$$A_S = \frac{\pi}{4} d_S = \frac{\pi}{4} \times (\frac{5}{8})^2 = 0.3068 in^2$$

Deflection at c :

$$\delta_c = \delta_A + \delta_S$$

$$\delta_A = \frac{PL_A}{A_A E_A} = \frac{15 \times 10^3 \times 48}{1.75 \times 10.4 \times 10^6} = 0.03956 in$$

$$\delta_S = \frac{PL_S}{A_S E_S} = \frac{15 \times 10^3 \times 84}{6.3068 \times 29 \times 10^6} = 0.14162 in$$

$$\delta_c = \delta_A + \delta_S$$

$$\delta_c = 0.03956 + 0.14162 = 0.181 in$$

Question 2.19

Both portions of the rod ABC are made of an aluminum for which $E = 70GPa$. Knowing that the magnitude of P is 4 kN, determine

1. the value of Q so that the deflection at A is zero
2. the corresponding deflection of B

Solution

Given:

$$E = 70GPa = 70 \times 10^9 Pa$$

$$P = 4KN = 4 \times 10^3 N$$

$$d_{DB} = 20mm.d_{BC} = 6mm.d_{AB} = 0.4mm.d_{B2} = 0.5m.$$

Calculating the area of different parts of aluminum,

$$A_{AB} = \frac{\pi}{4} d^2 = \frac{\pi}{4} 20 \times 10^{-3^2} = 3.24 \times 10^{-4} m^2$$

$$A_{BC} = \frac{\pi}{4} d^2 = \frac{\pi}{4} 60 \times 10^{-3^2} = 2.827 \times 10^{-3} m^2$$

Using the method of superposition, we can assume that the forces acting at different points in the metal, sum up to zero.
Now for there to be zero deflection at A,

$$\delta_{AB} = -\delta_{BC}$$

At section AB,

$$\delta_{AB} = \frac{PL_{AB}}{A_{AB}E} = \frac{4 \times 10^3 \times 0.4}{3.14 \times 10^{-4} \times 70 \times 10^9} = 7.279 \times 10^{-6}$$

At section BC

$$\delta_{BC} = \frac{(P-Q)L_{BC}}{A_{BC}E} = \frac{(P-Q) \times 0.5}{2.827 \times 10^{-3} \times 70 \times 10^9}$$

$$= \frac{(P-Q)0.5}{2.125 \times 10^{-3}} = (P-Q)2.52 \times 10^{-9}$$

Following the statement we made for a zero deflection at A,

$$7.275 \times 10^{-5} = -(P-Q)2.52 \times 10^{-9}$$

$$7.275 \times 10^{-5} = -(1.010 \times 10^{-5}) + Q(2.52 \times 10^{-9})$$

$$8.285 \times 10^{-5} = (Q)2.52 \times 10^{-9}$$

$$Q = \frac{8.285 \times 10^{-5}}{2.52 \times 10^{-9}} = 32.97 \times 10^3 N$$

For Deflection to occur at B,

$$\delta_{AB} = \delta_B = 7.275 \times 10^{-5}$$

Question 2.26

The length of the $\frac{3}{32}in.$-diameter steel wire CD has been adjusted so that with no load applied, a gap of $\frac{1}{16}in.$ exists between the end B of the rigid beam ACB and a contact point E. Knowing that$E = 29 \times 10^6$ psi, determinewhere a 50-lb block should be placed on the beam in order to cause contact between B and E.

<u>Solution</u>

Given:

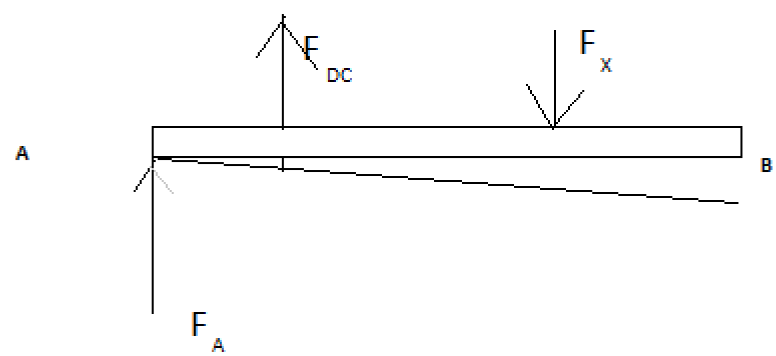
$$\text{Length of string,} L_s = 12.5in$$

$$\text{diameter of string,} d_s = \tfrac{3}{32}in$$

$$E_S = 29 \times 10^6$$

$$P = 50lb$$

Drawing a free body diagram of the syste, we have



The Free Body Diagram of the question

Taking moment about point A, we have

$$M_A = 4f_s - 50(20 - x) = 0$$

but f_s =?, However

$$\delta_s = \frac{f_s L_s}{A_s E}. \text{ This implies that } f_s = \frac{\delta_s A_s E}{L_s}$$

$$\delta_s = 4\theta$$

where 4 is th edistance fro the string to the point A
θ is the angle covered by the beam

$$\theta = \frac{\frac{1}{16}}{20} = 3.125 \times 10^{-3} rads$$

$$\delta_s = 4 \times 3.235 \times 10^{-2} = 0.065in$$

$$A_s = \frac{\pi}{4} \times (\frac{3}{32})^2 = 6.903 \times 10^{-3} in^2$$

$$F = \frac{0.0125 \times 6.903 \times 10^{-3} \times 29 \times 10^6}{12.5} = 200187 lbs$$

Hence,

$$4f_s - 50(20 - x) = 0$$

$$(4 \times 200.187) - 1000 + 50x = 0$$

$$800.748 - 1000 + 50x = 0$$

$$50x = 199252$$

$$x = \frac{199.252}{50}$$

$$x = 3.985 in$$

Question 2.35

A 4-ft concrete post is reinforced with four steel bars, each with a $\frac{3}{4} in$ diameter. Knowing that $E_s = 29 \times 10^6 psi$ and $E_c = 3.6 \times 10^6 psi$, determine the normal stresses in the steel and in the concrete when a 150-kip axial centric force P is applied to the post.

Solution

$$P = P_c + P_s$$

deformation of a steel bar and that of a concrete are equal, hence

$$\delta_c = \delta_s$$

$$\frac{P_c L_c}{A_c E_c} = \frac{P_s L_s}{A_s E_s}$$

making P_c the subject of the formular, we have

$$P_c = \frac{P_s A_c E_c}{A_s E_s}$$

$$A_s = 4 \times \frac{\pi}{4} \times (\frac{3}{4})^2 = 1.767 in^2$$

$$A_s = 8_2 - A_s = 64 - 1.767 = 62.233 in^2$$

Since $P_c = \frac{P_s A_c E_c}{A_s E_s}$

$$P = P_c + P_s$$

becomes

$$P = \frac{P_s A_c E_c}{A_s E_s} + P_s$$

This then becomes

$$P = P_s \frac{A_c E_c}{A_s E_s} + 1$$

$$P = 150Kips = 180 \times 10^3 lbs$$

$$150 \times 10^3 = P_s(\frac{62.233 \times 3.6 \times 10^6}{1.767 \times 29 \times 10^6} + 1)$$

$$150 \times 10^3 = P_s(4.372 + 1)$$

$$P_s = \frac{150 \times 10^3}{5.372}$$

$$P_s = 27922.56lbs$$

Therefore $P_c = 50000 - 27922.56 = 122077.44$ Normal Stress:

$$\sigma_c = \frac{P_c}{A_c} = \frac{122077.434}{62.233} = 1961.619psi$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{27922.56}{1767} = 15802.24psi$$

Question 2.41

<u>Solution</u>

The total deformation of the bar is

$$\delta = \delta_{AD} + \delta E = 0$$

Where δ_{AD} is as a result of the forces 60KN and 40KN and δ_E is as a result of the reaction at E

$$\delta_{AD} = \frac{P_1 L_1}{A_B E_B} + \frac{P_2 L_2}{A_B E_B} + \frac{P_3 L_3}{A_S E_S} + \frac{P_4 L_4}{A_S E_S}$$

$$P_1 = 0, P_2 = P_3 = 40KN = 40 \times 10^3, p_4 = 100 \times 10^3$$

$$A_1 = A_2 = \frac{\pi}{4} \times (30 \times 10^{-3})^2 = 7.069 \times 10^{-4}$$

$$A_3 = A_4 = \frac{\pi}{4} \times (40 \times 10^{-3})^2 = 1.257 \times 10^{-4}$$

$$l_1 = l_2 = 100mm = 0.1$$

$$l_3 = 120mm = 0.12$$

$$l_4 = 180mm = 0.18$$

$$E_s = 200 \times 10^9, E_b = 205 \times 10^{-3}$$

$$\delta_{AD} = 0 + \frac{40 \times 0.1 \times 10^3}{7.069 \times 10^{-3} \times 105 \times 10^9} + \frac{40 \times 10^3 \times 0.12}{1.259 \times 10^{-3} 200 \times 10^4} + \frac{100 \times 10^3 \times 0.18}{1.257 \times 10^{-3} \times 200 \times 10^4}$$

$$= (5.384 \times 10^{-5}) + (1.909 \times 10^{-5}) + (7.16 \times 10^{-5}) = 14.458 \times 10^{-5} m$$

$$\delta_E = \frac{R_E \times 0.2}{7.069 \times 10^{-3} \times 105 \times 10^9} + \frac{R_E \times 0.3}{1.259 \times 10^{-3} 200 \times 10^4}$$

$$= (6.695 \times 10^{-9} R_E) + (1.193 \times 10^{-3} R_E) = 3.886 \times 10^{-9} R_E$$

Equating δ_E and δ_{AD}

$$\delta_E = \delta_{AD}$$

$$14.458 \times 10^{-5} = 3.886 \times 10^{-9} R_E$$

$$R_E = \frac{14.458 \times 10^{-5}}{3.886 \times 10^{-9}} = 37186.21 = 37.19 KN$$

For reaction at A, R_A we have

$$R_E + R_A = (40 \times 10^3) + (60 \times 10^3)$$

$$R_A = (100 \times 10^3) = 37186.21$$

$$= 62813.786 N$$

Deflection at point C

$$\delta_C = \delta_{AB} + \delta_{BC}$$

$$\delta_C = \frac{R_A L_4}{A_B E_B} + \frac{P_A L_3}{A_S E_S}$$

Where $P = R_A - 60 \times 10^{-3}$

$$\frac{62813.786 \times 0.18}{1.257 \times 10^{-3} \times 200 \times 10^9} + \frac{(62813.786 - 60000) \times 0.12}{1.259 \times 10^{-3} 200 \times 10^4}$$

$$(4.497 \times 10^{-5}) + (1.34 \times 10^{-6}) = 4.63 \times 10^{-5} m$$

Question 2.58

Solution

Given

$$\sigma = -11ksi = -11 \times 10^3$$

$$T_1 = 75^o F$$

$$Z_b = 14in, L_{AC} = 18in$$

$$A_B = 2.4in^2$$

$$E_B = 15 \times 10^6 psi$$

$$\alpha_B = 15 \times 10^{-6} F^{-1}$$

$$A_{AC} = 2.8in^2$$

$$E_{AC} = 10.6 \times 10^6 psi$$

$$\alpha_{AC} = 12.9 \times 10^{-6} F^{-1}$$

$$\text{Pressure}, P = -\sigma_{AL} A_{AL} = -(11 \times 10^3)(2.8) = -30800N$$

$$p_1 = P_2 = P$$

deformation as a result of force, P

$$F_P = \frac{P_1 L_B}{A_B E_B} + \frac{P_2 L_{AL}}{A_{AL} E_{AL}}$$

$$= \frac{-30800 \times 14}{2.4 \times 15 \times 10^6} + \frac{-30800 \times 18}{2.8 \times 10.6 \times 10^6}$$

$$= -0.012 - 0.0187 = -0.0307in$$

Deformation due to the thermal stress,

$$f_T = \delta_P + \delta = 0.0307 + 0.02 = 0.0507$$

$$\text{But } F_T = \alpha_B \triangle T L_B + \alpha_{AL} \triangle T L_{AL}$$

$$= \triangle T(\alpha_B L_B + \alpha_{AL} L_{AL})$$

$$\triangle T(1.2 \times 10^{-6} \times 14 + 12.9 \times 10^{-6} \times 18)$$

$$= \triangle T(4.002 \times 10^{-4})$$

$$\text{Equating it} \delta_T = 0.0507, \text{ we have}$$

$$\triangle T(4.002 \times 20^{-4}) = 0.0507$$

$$\triangle T = \frac{0.0507}{4.002 \times 10^{-4}} = 126.69$$

T_2, new temperature for stress of $-11ksi$ to be experienced by aluminium = $\triangle T + T_1$

$$= 126.69 + 75$$

$$T_2 = 201.68^o F$$

$$F_A = L_{AL}\alpha_{AL}(\triangle T) - \frac{PL_{AL}}{E_{AL}A_{AL}}$$

$$= (18)(2.9 \times 10^{-6})(26.6) = \frac{30800 \times 18}{10.6 \times 10^6 \times 2.8} = 10.712 \times 10^{-3}$$

$$L_{exact} = 18 + 10.712 \times 10^{-3} = 18.0107in$$

Question 2.65

Solution

$$length, L = 2.5m$$

$$Outsidediameter, d_0 = 300mm = 0.3m$$

$$thickness, t = 15mm = 0.015m$$

$$E = 200GPa = 200 \times 10^9 Pa$$

$$v = 0.30$$

$$P = 700KN = 700 \times 10^3 N$$

- Change in length of the pipe,δ

$$\delta = \frac{-PL}{AE}$$

$$Area, A = \frac{\pi}{4} \times ({d_0}^2 - {d_1}^2)$$

$$d_1 = d_0 - 2t = 0.3 - 2(0.15) = 0.27$$

$$A = \frac{\pi}{4}(0.3^2 - 0.27^2) = 0.0134m^2$$

$$\delta = \frac{(700 \times 10^3)(2.5)}{(0.0134)(200 \times 10^9)} = -6.53 \times 10^{-4}$$

- Change in the outer diameter $\triangle d_0$

$$\triangle d_0 = E_{lateral} \times d_0$$

but

$$E_{lateral} = -\nu E_{axial}$$

$$E_{axial} = \frac{\delta}{L} = \frac{-6.53 \times 10^{-4}}{2.5} = -2.612 \times 10^{-4}$$

$$E_{lateral} = -(0.30)(-2.612 \times 10^{-4}) = 7.836 \times 10^{-6}$$

$$\triangle d_0 = 7.836 \times 10^{-5} \times 0.3 = 2.3508 \times 10^{-5} m$$

- Change in thickness, $\triangle t$

$$\triangle = E_{lateral} \times t_0 = 7.836 \times 10^{-5} \times 0.015 = 11.1754 \times 10^{-6} m$$

Question 2.81

Solution

$$G = 12MPa = 12 \times 10^6$$

$$L - 100mm = 0.1m$$

$$P = 45KN = 45 \times 10^3 N$$

$$a = ?, b = ?$$

$$\tau = 1.4MPa = 1.4 \times 10^6$$

$$\delta = 5mm = 5 \times 10^{-3} m$$

$$F = \frac{1}{2}P = \frac{45 \times 10^3}{2} = 22.5 \times 10^3 N$$

Finding b, bc = A but A= ?

$$\tau = \frac{F}{A} \Rightarrow A = \frac{F}{\tau} = \frac{22.5 \times 10^3}{1.4 \times 10^6} = 0.0161 m^2$$

$$bc = A \Rightarrow b = \frac{A}{C} = \frac{0.0161}{0.1} = 0.1607m$$

Finding a,

$$r = \frac{\delta}{a} \Rightarrow a = \frac{\delta}{r}$$

$$r = \frac{\tau}{G} = \frac{1.4 \times 10^6}{12 \times 10^6} = 0.1167$$

subsitituting into $a = \frac{\delta}{r}$

$$a = \frac{5 \times 10^{-3}}{0.1167} = 0.0428m$$