Group Assignment 2

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Solution

$$L = 80m$$

$$d = 5mm5 \times 10^{-3}m$$

$$E = 200GPa = 200 \times 10^9$$

$$\sigma_{uth} = 400MPa = 400 \times 10^6$$

$$n = 3.2$$

- $$\sigma_{uth} = \frac{P}{A}$$

  $$P = \sigma_{allow} \times A$$

  but $\sigma_{allow} = \frac{\sigma_{uth}}{n} = \frac{400 \times 10^6}{3.2} = 1.25 \times 10^6 Pa$

  $$P_{ult} = 125 \times 10^6 \times A$$

  $$Area, A = \frac{\pi}{4} \times 5 \times 10^{-3^2} = 1.963 \times 10^{-5}$$

  $$P_{ult} = 125 \times 10^6 \times 1.963 \times 10^{-5}$$

  $$p = 2453.26N$$

- Elongation of the wire, δ

  $$\delta = \frac{PL}{AE} = \frac{2453.75 \times 80}{200 \times 10^9 \times 1.963 \times 10^{-5}}$$

  $$= 0.05m$$

Question 2.9

Solution

$$\delta = 0.08in$$

$$P = 500lb(tensile)$$

$$\sigma_{ult} = 22ksi = 22 \times 10^3 psi$$

$$E = 10.1 \times 10^6 psi$$

smallest diameter, d=?

Shortest Length, l = ?

but $E = \frac{\delta}{L}$

Make L subject of the formular

$$L = \frac{\delta}{E} - - - - - - - - - - - -equ(1)$$

$$E = \frac{\sigma}{e}.Therefore, e = \frac{\sigma}{E}$$

$$E = \frac{22 \times 10^3}{10.1 \times 10^6} = 2.18 \times 10^{-3}$$

Substituting the value of E into the equation 1

$$L = \frac{0.08}{2.18 \times 10^{-3}} = 36.7in$$

Smallest Diameter d_1 ,

$$A = \frac{\pi}{4}d^2$$

Therefore, $d = 2 \times \sqrt{\frac{A}{\pi}}$

but A is unknown. So to find A,

$$\delta = \frac{PL}{AE} = \frac{500 \times 36.7}{A \times 10.1 \times 10^6} = 0.08$$

Therefore, A will be

$$A = \frac{500 \times 36.7}{0.08 \times 10.1 \times 10^6} = 0.0227$$

$$d = \sqrt{\frac{0.0227}{\pi}} = 0.17in$$

Question 2.15

Solution

Given:

$$L_{BA} = 4ft = 4 \times 12 = 48in$$

$$L_{AC} = 3ft = 3 \times 12 = 36in$$

$$L_{BC} = 7ft = 7 \times 12 = 84in$$

$$A_{BA} = 1.75in_2$$

$$d_{BC} = \frac{5}{8} in$$

$$E_s = 29 \times 10^6 psi, E_D = 10.4 \times 10^6 psi$$

$$p = 15Kip = 15 \times 10^3 lb$$

Deflection at point c, $\delta_c = \delta_{BC} + \delta_{BA}$

$$\delta_{BC} = \frac{L_{BC} P}{A_{BC} E_s}$$

$$A_{BC} = \frac{\pi}{4} \times (\frac{5}{8})^2 = 0.307 in^2$$

$$\delta_{BC} = \frac{15 \times 10^3 \times 84}{0.307 \times 29 \times 10^6} = 0.1415 in$$

$$\delta_{BA} = \frac{PL_{BA}}{A_{BA} E_D} = \frac{15 \times 10^3 \times 48}{1.75 \times 10.4 \times 10^6} = 0.03956$$

$$\delta = 0.1415 + 0.03956 = 0.18106 in$$

Question 2.19

Solution

Given:

$$E = 70GPa = 70 \times 10^9 Pa$$

$$P = 4KN = 4 \times 10^3 N$$

$$d_{DB} = 20mm.d_{BC} = 6mm.d_{AB} = 0.4mm.d_{B2} = 0.5m.$$

For deflection at A to be 0,

$$\delta_{AB} = \delta_{BC}$$

$$A_{AB} = \frac{\pi}{4} d^2 = \frac{\pi}{4} 20 \times 10^{-3^2} = 3.24 \times 10^{-4} m^2$$

$$A_{BC} = \frac{\pi}{4} d^2 = \frac{\pi}{4} 60 \times 10^{-3^2} = 2.827 \times 10^{-3} m^2$$

$$\delta_{AB} = \frac{PL_{AB}}{A_{AB} E} = \frac{4 \times 10^3 \times 0.4}{3.14 \times 10^{-4} \times 70 \times 10^9} = 7.279 \times 10^{-6}$$

$$\delta_{BC} = \frac{(Q-P)L_{BC}}{A_{BC} E} = \frac{(Q-P) \times 0.5}{2.827 \times 10^{-3} \times 70 \times 10^9}$$

$$= \frac{(Q-4000)0.5}{2.125 \times 10^{-3} \times 70 \times 10^9} = \frac{0.5Q - 2000}{1.9789 \times 10^4}$$

Since,

$$\delta_{AB} = \delta_{BC}$$

then

$$7.279 \times 10^{-6} = \frac{0.5Q - 2000}{19789 \times 10^{-4}}$$

$$0.5Q - 2000 = 7.279 \times 19789 \times 10^{4} = 1440.44$$

$$0.5Q = 1440.44 + 2000$$

$$0.5Q = 3440.44$$

$$q = \frac{3440.44}{0.5} = 6880.88N$$

Corresponding deflection at A

$$\delta_{BC} = \delta_{AC} = 7.279 \times 10^{-6}$$

Question 2.26

Solution

Given:

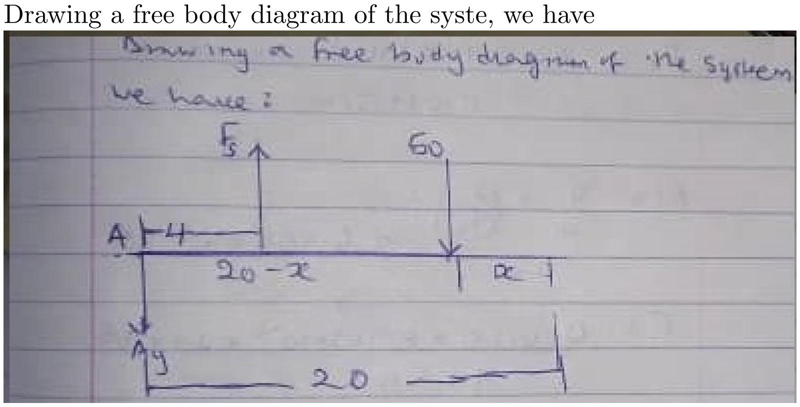
$$\text{Length of string,} L_s = 12.5in$$

$$\text{diameter of string,} d_s = \frac{3}{32}in$$

$$E_S = 29 \times 10^6$$

$$P = 50lb$$

Drawing a free body diagram of the syste, we have



Taking moment about point A, we have

$$M_A = 4f_s - 50(20 - x) = 0$$

but f_s =?, However

$$\delta_s = \frac{f_s L_s}{A_s E}. \text{ This implies that } f_s = \frac{\delta_s A_s E}{L_s}$$

$$\delta_s = 4\theta$$

where 4 is th edistance fro the string to the point A
θ is the angle covered by the beam

$$\theta = \frac{\frac{1}{16}}{20} = 3.125 \times 10^{-3} rads$$

$$\delta_s = 4 \times 3.235 \times 10^{-2} = 0.065in$$

$$A_s = \frac{\pi}{4} \times (\frac{3}{32})^2 = 6.903 \times 10^{-3} in^2$$

$$F = \frac{0.0125 \times 6.903 \times 10^{-3} \times 29 \times 10^6}{12.5} = 200187lbs$$

Hence,

$$4f_s - 50(20 - x) = 0$$

$$(4 \times 200.187) - 1000 + 50x = 0$$

$$800.748 - 1000 + 50x = 0$$

$$50x = 199252$$

$$x = \frac{199.252}{50}$$

$$x = 3.985in$$

Question 2.35

Solution

$$P = P_c + P_s$$

deformation of a steel bar and that of a concrete are equal, hence

$$\delta_c = \delta_s$$

$$\frac{P_c L_c}{A_c E_c} = \frac{P_s L_s}{A_s E_s}$$

making P_c the subject of the formular, we have

$$P_c = \frac{P_s A_c E_c}{A_s E_s}$$

$$A_s = 4 \times \frac{\pi}{4} \times (\frac{3}{4})^2 = 1.767in^2$$

$$A_s = 8_2 - A_s = 64 - 1.767 = 62.233in^2$$

Since $P_c = \frac{P_s A_c E_c}{A_s E_s}$

$$P = P_c + P_s$$

becomes

$$P = \frac{P_s A_c E_c}{A_s E_s} + P_s$$

This then becomes

$$P = P_s \frac{A_c E_c}{A_s E_s} + 1$$

$$P = 150Kips = 180 \times 10^3 lbs$$

$$150 \times 10^3 = P_s(\frac{62.233 \times 3.6 \times 10^6}{1.767 \times 29 \times 10^6} + 1)$$

$$150 \times 10^3 = P_s(4.372 + 1)$$

$$P_s = \frac{150 \times 10^3}{5.372}$$

$$P_s = 27922.56lbs$$

Therefore $P_c = 50000 - 27922.56 = 122077.44$ Normal Stress:

$$\sigma_c = \frac{P_c}{A_c} = \frac{122077.434}{62.233} = 1961.619psi$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{27922.56}{1767} = 15802.24psi$$

Question 2.41

Solution

Considering the reaction at E redundant and realsing the bar from that support and considering the reaction, R_E is an unknown lend,

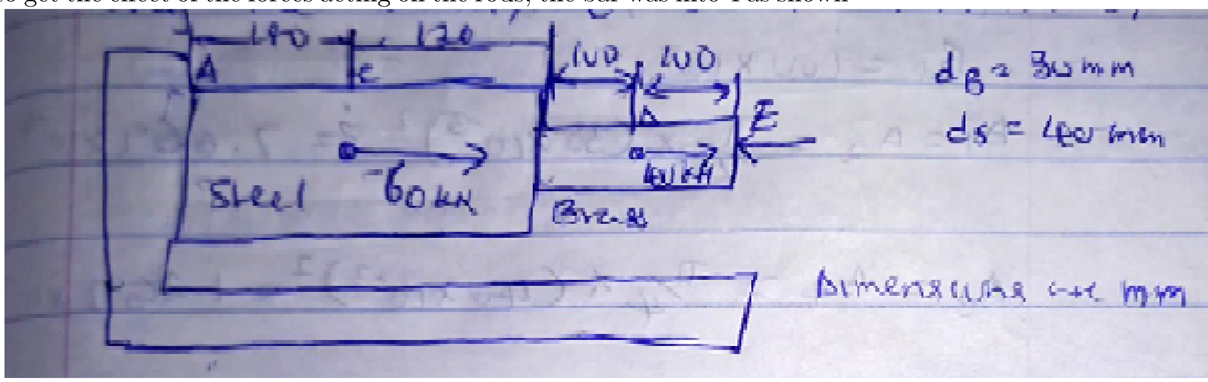
From The above, the total deformation of the bar is

$$\delta = \delta_{AD} + \delta E = 0$$

Where δ_{AD} is as a result of the forces 60KN and 40KN and δ_E is as a result of the reaction at E

$$\delta_{AD} = \frac{P_1 L_1}{A_B E_B} + \frac{P_2 L_2}{A_B E_B} + \frac{P_3 L_3}{A_S E_S} + \frac{P_4 L_4}{A_S E_S}$$

Note to get the effect of the forces acting on the rods, the bar was into 4 as shown below



$$P_1 = 0, P_2 = P_3 = 40KN = 40 \times 10^3, p_4 = 100 \times 10^3$$

$$A_1 = A_2 = \frac{\pi}{4} \times (30 \times 10^{-3})^2 = 7.069 \times 10^{-4}$$

$$A_3 = A_4 = \frac{\pi}{4} \times (40 \times 10^{-3})^2 = 1.257 \times 10^{-4}$$

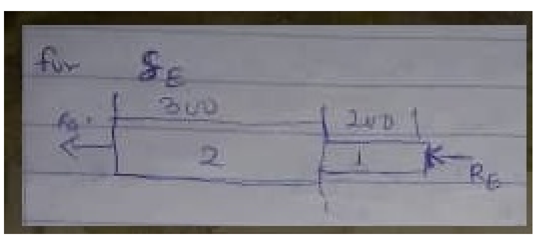
$$l_1 = l_2 = 100mm = 0.1$$

$$l_3 = 120mm = 0.12$$

$$l_4 = 180mm = 0.18$$

$$E_s = 200 \times 10^9, E_b = 205 \times 10^{-3}$$

$$\delta_{AD} = 0 + \frac{40 \times 0.1 \times 10^3}{7.069 \times 10^{-3} \times 105 \times 10^9} + \frac{40 \times 10^3 \times 0.12}{1.259 \times 10^{-3} 200 \times 10^4} + \frac{100 \times 10^3 \times 0.18}{1.257 \times 10^{-3} \times 200 \times 10^4}$$



$$= (5.384 \times 10^{-5}) + (1.909 \times 10^{-5}) + (7.16 \times 10^{-5}) = 14.458 \times 10^{-5} m$$

$$\delta_E = \frac{R_E \times 0.2}{7.069 \times 10^{-3} \times 105 \times 10^9} + \frac{R_E \times 0.3}{1.259 \times 10^{-3} 200 \times 10^4}$$

$$= (6.695 \times 10^{-9} R_E) + (1.193 \times 10^{-3} R_E) = 3.886 \times 10^{-9} R_E$$

Equating δ_E and δ_{AD}

$$\delta_E = \delta_{AD}$$

$$14.458 \times 10^{-5} = 3.886 \times 10^{-9} R_E$$

$$R_E = \frac{14.458 \times 10^{-5}}{3.886 \times 10^{-9}} = 37186.21 = 37.19KN$$

For reaction at A, R_A we have

$$R_E + R_A = (40 \times 10^3) + (60 \times 10^3)$$

$$R_A = (100 \times 10^3) = 37186.21$$

$$= 62813.786N$$

Deflection at point C

$$\delta_C = \delta_{AB} + \delta_{BC}$$

$$\delta_C = \frac{R_A L_4}{A_B E_B} + \frac{P_A L_3}{A_S E_S}$$

$$\text{Where } P = R_A - 60 \times 10^{-3}$$

$$\frac{62813.786 \times 0.18}{1.257 \times 10^{-3} \times 200 \times 10^9} + \frac{(62813.786 - 60000) \times 0.12}{1.259 \times 10^{-3} 200 \times 10^4}$$

$$(4.497 \times 10^{-5}) + (1.34 \times 10^{-6}) = 4.63 \times 10^{-5} m$$

Question 2.58

Solution

Given

$$\sigma = -11ksi = -11 \times 10^3$$

$$T_1 = 75^o F$$

$$Z_b = 14in, L_{AC} = 18in$$

$$A_B = 2.4in^2$$

$$E_B = 15 \times 10^6 psi$$

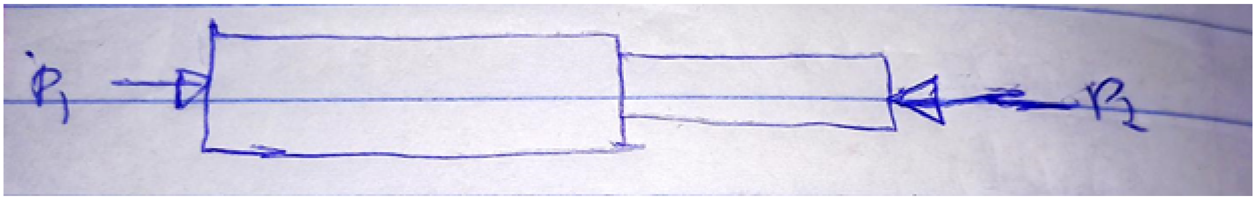
$$\alpha_B = 15 \times 10^{-6} F^{-1}$$

$$A_{AC} = 2.8in^2$$

$$E_{AC} = 10.6 \times 10^6 psi$$

$$\alpha_{AC} = 12.9 \times 10^{-6} F^{-1}$$

$$\text{Pressure,} P = -\sigma_{AL} A_{AL} = -(11 \times 10^3)(2.8) = -30800N$$



$$p_1 = P_2 = P$$

deformation as a result of force, P

$$F_P = \frac{P_1 L_B}{A_B E_B} + \frac{P_2 L_{AL}}{A_{AL} E_{AL}}$$
$$= \frac{-30800 \times 14}{2.4 \times 15 \times 10^6} + \frac{-30800 \times 18}{2.8 \times 10.6 \times 10^6}$$
$$= -0.012 - 0.0187 = -0.0307in$$

This shows that the force F_1 and F_2 are compressive forces and the deformation is compression.
Hence, $\delta_P = 0.0307$ Deformation due to the thermal stress,

$$f_T = \delta_P + \delta = 0.0307 + 0.02 = 0.0507$$

$$\text{But } F_T = \alpha_B \triangle T L_B + \alpha_{AL} \triangle T L_{AL}$$

$$= \triangle T(\alpha_B L_B + \alpha_{AL} L_{AL})$$
$$\triangle T(1.2 \times 10^{-6} \times 14 + 12.9 \times 10^{-6} \times 18)$$
$$= \triangle T(4.002 \times 10^{-4})$$

Equating it$\delta_T = 0.0507$, we have

$$\triangle T(4.002 \times 20^{-4}) = 0.0507$$

$$\triangle T = \frac{0.0507}{4.002 \times 10^{-4}} = 126.69$$

T_2, new temperature for stress of $-11ksi$ to be experienced by aluminium = $\triangle T + T_1$

$$= 126.69 + 75$$
$$T_2 = 201.68^o F$$

Corresponding exact length of aluminum bar

$$F_A = L_{AL}\alpha_{AL}(\triangle T) - \frac{PL_{AL}}{E_{AL}A_{AL}}$$
$$= (18)(2.9 \times 10^{-6})(26.6) = \frac{30800 \times 18}{10.6 \times 10^6 \times 2.8} = 10.712 \times 10^{-3}$$
$$L_{exact} = 18 + 10.712 \times 10^{-3} = 18.0107in$$

Question 2.65

Solution

$$length, L = 2.5m$$

$$Outsidediameter, d_0 = 300mm = 0.3m$$

$$thickness, t = 15mm = 0.015m$$

$$E = 200GPa = 200 \times 10^9 Pa$$

$$v = 0.30$$

$$P = 700KN = 700 \times 10^3 N$$

- Change in length of the pipe,δ

$$\delta = \frac{-PL}{AE}$$

$$Area, A = \frac{\pi}{4} \times ({d_0}^2 - {d_1}^2)$$

$$d_1 = d_0 - 2t = 0.3 - 2(0.15) = 0.27$$

$$A = \frac{\pi}{4}(0.3^2 - 0.27^2) = 0.0134m^2$$

$$\delta = \frac{(700 \times 10^3)(2.5)}{(0.0134)(200 \times 10^9)} = -6.53 \times 10^{-4}$$

- Change in the outer diameter $\triangle d_0$

$$\triangle d_0 = E_{lateral} \times d_0$$

but

$$E_{lateral} = -\nu E_{axial}$$

$$E_{axial} = \frac{\delta}{L} = \frac{-6.53 \times 10^{-4}}{2.5} = -2.612 \times 10^{-4}$$

$$E_{lateral} = -(0.30)(-2.612 \times 10^{-4}) = 7.836 \times 10^{-6}$$

$$\triangle d_0 = 7.836 \times 10^{-5} \times 0.3 = 2.3508 \times 10^{-5} m$$

- Change in thickness, $\triangle t$

$$\triangle = E_{lateral} \times t_0 = 7.836 \times 10^{-5} \times 0.015 = 11.1754 \times 10^{-6} m$$

Question 2.81

<u>solution</u>

$$G = 12MPa = 12 \times 10^6$$

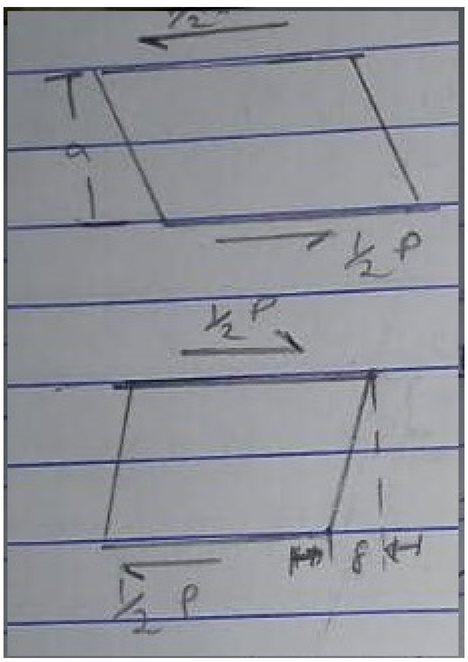
$$L - 100mm = 0.1m$$

$$P = 45KN = 45 \times 10^3 N$$

$$a = ?, b = ?$$

$$\tau = 1.4MPa = 1.4 \times 10^6$$

$$\delta = 5mm = 5 \times 10^{-3} m$$



$$F = \frac{1}{2}P = \frac{45 \times 10^3}{2} = 22.5 \times 10^3 N$$

Finding b, bc = A but A= ? However

$$\tau = \frac{F}{A} \Rightarrow A = \frac{F}{\tau} = \frac{22.5 \times 10^3}{1.4 \times 10^6} = 0.0161m^2$$

$$bc = A \Rightarrow b = \frac{A}{C} = \frac{0.0161}{0.1} = 0.1607m$$

Finding a,

$$r = \frac{\delta}{a} \Rightarrow a = \frac{\delta}{r}$$

But r is an unknown, However,

$$r = \frac{\tau}{G} = \frac{1.4 \times 10^6}{12 \times 10^6} = 0.1167$$

subsitituting into $a = \frac{\delta}{r}$

$$a = \frac{5 \times 10^{-3}}{0.1167} = 0.0428m$$