MEC 331 Assignment 3

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Problem 3.2

<u>Solution</u>

- Finding the torque
  Given:

$$\tau = 45MPa = 45 \times 10^6 Pa$$

$$r = 30mm = 0.03m$$

$$R = 45mm = 0.024m$$

$$\tau = \frac{Tr}{J}$$

  Making T the subject of the formular

$$T = \frac{J\tau}{r} = \frac{\frac{\pi}{2} \times (0.045^4 - 0.03^4) \times 45 \times 10^6}{0.045} = 5.168 \times 10^3$$

- For solid shaft with the same cross sectional area, $A_s = A_h$

$$A_s = \pi(R^2 - r^2) = \pi((25 \times 10^{-3})^2 - (30 \times 10^{-3})^2) = 3.5343 \times 10^{-3} m^2$$

  But $A_s = \pi r^2$

$$r = \sqrt{\frac{A_s}{\pi}} = \sqrt{\frac{3.5343 \times 10^{-3}}{\pi}} = \sqrt{1.1225 \times 10^{-3}} = 0.0335m$$

$$\tau_{max} = \frac{Tr}{J} = \frac{T \times 2}{\pi r^3} = \frac{5.1689 \times 10^3 \times 2}{\pi 0.033541} = 87.2 \times 10^6 Pa$$

Problem 3.6, Page 154

<u>Solution</u>

- Given:

$$\tau_{allow} = 80 \times 10^6 Pa$$

$$d_s = 20mm = 0.02m$$

$$r = \frac{20}{2} = 10mm10 \times 10^{-3} m$$

$$\tau = \frac{Tr}{J}$$

  Making T the subject of the formular

$$T = \frac{J\tau}{r} = \frac{\frac{\pi}{32} \times (0.02^4) \times 80 \times 10^6}{\frac{0.02}{2}} = 125.664Nm$$

- Assuming it becomes a hollow cylinder, $A_s = A_h$
  Given:

$$\tau = 45 \times 10^6 Pa$$

$$r_1 = ?$$

$$r_2 = 0.5r_1$$

$$A_s = A_h$$

$$\pi r^2 = \pi({r_2}^2 - {r_1}^2)$$

$$(10 \times 10^{-3})^2 = ({r_2}^2 - {r_1}^2)$$

$$10 \times 10^{-4} = ({r_2}^2 - {r_1}^2) - - - - - - - - - - - - - equ(1)$$

but inner diameter equals half of the outer diameter. This implies that $r_1 = \frac{r_2}{2}$ Substituting it into equ(1) above, we get

$$10 \times 10^{-4} = ({r_2}^2 - \frac{{r_2}^2}{2})$$

$$10 \times 10^{-4} = ({r_2}^2 - \frac{{r_2}^2}{4})$$

$$10 \times 10^{-4} = \frac{3{r_2}^2}{4}$$

Making r_2 subject of the formular, we have

$$r_2 = \sqrt{\frac{1 \times 10^{-4} \times 4}{3}} = \sqrt{1.333 \times 10^{-4}} = 0.01155m$$

$$r_1 = 0.01154, r_2 = 0.005772$$

$$T = \frac{J\tau}{r} = \frac{\frac{\pi}{2} \times (0.01154^4 - 0.005772^4) \times 80 \times 10^6}{0.01154} = 181.03Nm$$

Problem 3.10

Solution

- In Shaft AB
  Given:

$$d = 0.03m$$

$$T_{AB} = 300Nm$$

$$r = \frac{d}{2} = 0.015m$$

The formular for solving this question $\tau_{max} = \frac{T \times r}{J}$

$$\tau_{max} = \frac{T \times r}{J} = \frac{300 \times 0.015}{\frac{\pi}{32} \times 0.03^4} = 56.58 \times 10^6 Nm$$

- In Shaft BC
  Given:

$$d = 0.046m$$
$$T_{BC} = 400Nm + 300Nm = 700Nm$$
$$r = \frac{d}{2} = 0.023m$$

$$\tau = \frac{T \times r}{J} = \frac{700 \times 0.023}{\frac{\pi}{32} \times 0.046^4} = 20.92 \times 10^6 Nm$$

The highest shear stress is $56.58 \times 10^6 Nm$ occurs in portion AB. Reduce the diameter of BC to provide the same stress

$$T_{br} = 700Nm, \tau_{max} = \frac{T_r}{J} = \frac{2T}{\pi r^3}$$

Making r subject of the formular we have that

$$r^3 = \frac{2 \times T}{T \times \pi}$$
$$r = \sqrt[3]{\frac{2 \times 700}{56.58 \times 10^6 \times \pi}} = 0.019896$$
$$d = 2r = 2 \times 0.019896 = 0.03979m$$

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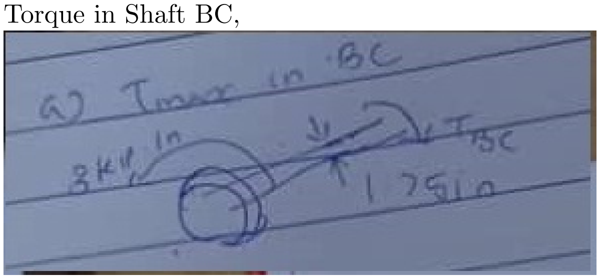
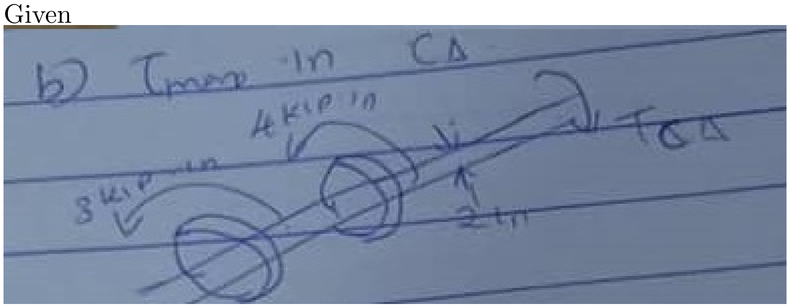
<u>Solution</u>

- Torque in Shaft BC,



Given

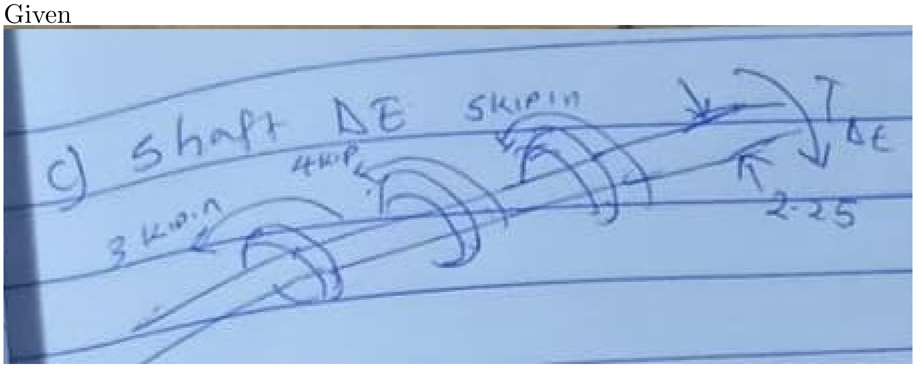
$$r_2 = 0.5in, r_1 = \frac{1.75}{2} = 0.875in, T_{BC} = 3 \times 10^3 lbs.in$$
$$\tau_{max} = \frac{T \times r}{J} = \frac{3 \times 10^3 \times 0.875}{\frac{\pi}{2} \times (0.875^4 - 0.5^4)} = 3.19 \times 10^3 lbs.in$$

- Torque in Shaft CD
  Given
  

  From the free body diagram, we have

$$r_2 = 0.5in, r_1 = \frac{2}{2} = 1in, T_{CD} = (4+3) \times 10^3 = 7 \times 10^3 psi.in$$

$$\tau_{max} = \frac{T \times r}{J} = \frac{7 \times 10^3 \times 1}{\frac{\pi}{2} \times (1^4 - 0.5^4)} = 4.75 \times 10^3 psi.in$$

- In Shaft DE
  Given
  

$$r_2 = 0.5in, r_1 = \frac{2.25}{2} = 1.125in, T_{DE} = (4+3+5) \times 10^3 = 12 \times 10^3 psi.in$$

$$\tau_{max} = \frac{T \times r}{J} = \frac{12 \times 10^3 \times 1.125}{\frac{\pi}{2} \times (1.125^4 - 0.5^4)} = 5583.36 psi.in$$

Problem 3.18

Solution

Since the factor of safety is the same for each of them,

$$T_s = T_b = \text{Largest Torque}$$

Given:

$$T_s = 25 \times 10^6 Nm, r = 0.015m$$

$$T_s = \frac{\tau_{allow} \times J}{r} = \frac{25 \times 10^6 \times \frac{\pi}{2} \times 0.015^4}{0.03} = 132.53Nm$$

Finding the inner diameter of the brass, we have

$$T_b = \frac{\tau_b \times \frac{\pi}{2} \times ({r_1}^4 - {r_2}^4)}{r_1}$$

Given:

$$T_b = 25 \times 10^6 Nm, r = 0.0125m$$

$$r = \sqrt[4]{{r_1}^4 - \frac{2 \times T_b \times r_1}{\pi \times \tau}} = \sqrt[4]{0.0125^4 - \frac{2 \times 132.5 \times 0.0125}{\pi \times 50 \times 10^{-3}}} = 0.00759$$

$$d = 2r = 0.01518m$$

The largest Torque is $T_{allow} = 132.5Nm$

Problem 3.32

Solution

Given

$$\phi = 4^o = 0.06981rad, G = 27 \times 10^9 Pa, l = 1.25m, r_1 = 0.018m, r_2 = 0.012m$$

$$T = \frac{\phi JG}{L} = \frac{0.06981 \times \frac{\pi}{2} \times (0.018^4 - 0.012^4) \times 27 \times 10^9}{1.25} = 199.53Nm$$

Matching areas. Hence,

$$A_c = A_h$$

$$\pi r^2 = \pi({r_2}^2 - {r_1}^2)$$

$$r^2 = ({r_2}^2 - {r_1}^2)$$

Making r the subject of the formular, we have

$$r^2 = (1.8 \times {10^{-3}}^2 - 2 \times {10^{-3}}^2)$$

$$r = \sqrt{\frac{5.654 \times 10^{-4}}{\pi}} = 0.0134m$$

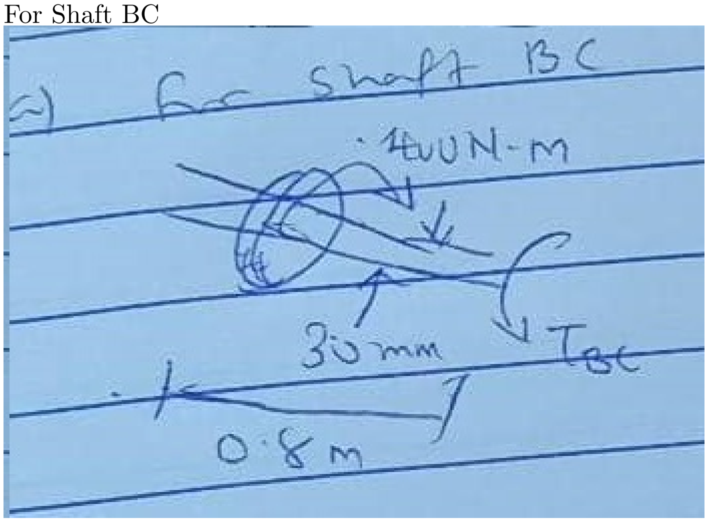
The angle of twist is

$$\phi = \frac{TL}{JG} = \frac{199.53 \times 1.25}{\frac{\pi}{2} \times 0.0134^4 \times 27 \times 10^9} = 0.1825rad$$

Problem 3.36

Solution

For Shaft BC



Given:

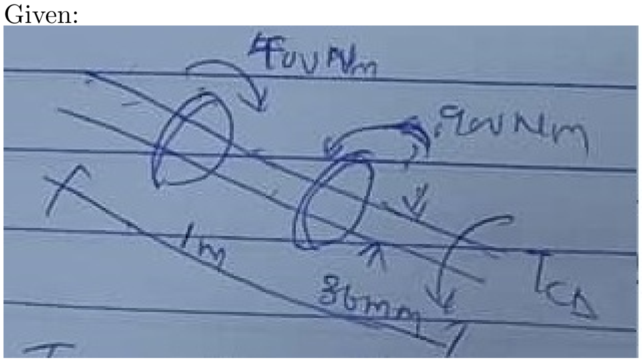
$$G = 27 \times 10^9 Pa, l = 1.25m, r_1 = 0.015m, T_{BC} = 400Nm, L = 0.8m$$

$$\phi_{BC} = \frac{T_{BC}L}{JG} = \frac{400 \times 0.8}{\frac{\pi}{2} \times 0.015^4 \times 27 \times 10^9} = 0.1490$$

at D and B

$$\phi_{BD} = \phi_{BC} + \phi_{CD}$$

Given:



$$G = 27{\times}10^9 Pa, l = 1.25m, r = 0.018m, T_{CD} = 400Nm{-}900Nm = -500Nm, L_{CD} = 1m$$

$$\phi_{BC} = \frac{T_{CD}L_{CD}}{JG} = \frac{-500 \times 1}{\frac{\pi}{2} \times 0.018^4 \times 27 \times 10^9} = -0.1123$$

$$\phi_{BD} = 0.182 + (-0.1123) = 0.0367rad$$

Problem 3.40

Solution

$$d_s = 1.75in, d_b = 3in, \phi = 0.375^o, L_{sleeve} = 8in$$

$$G_s = 11.2 \times 10^6 Psi, G_b = 5.6 \times 10^6 Psi$$

$$\tau_{all\,s} = 12ksi = 12 \times 10^3 Psi, \tau_{all\,b} = 12ksi = 7 \times 10^3 Psi$$

Torque based on shear stress of the spindle

$$\tau_{all} = \frac{Tr_s}{J_s} \Rightarrow T = \frac{\tau J_s}{r_s}$$

$$T = \frac{12 \times 10^3 \times \frac{\pi}{2} \times 0.875^4}{0.875} = 12628.11 Ibs.in$$

Torque based on sleeve

$$d_{B1} = 3in$$

$$d_{B2} = 3 - 2t = 3 - 2(0.25) = 2.5$$

$$r_{B1} = \frac{d_{B1}}{2} = 1.5in$$

$$r_{B2} = \frac{d_{B2}}{2} = 1.25in$$

$$J = \frac{\pi}{2}(1.5^4 - 1.25^4) = 4.1172$$

$$T = \frac{7 \times 10^3 \times 4.1172}{1.5} = 19213.6 lbs.in$$

Torque based on angle of rotation of sleeve

$$\phi = 0.375^o = \frac{0.325}{180} = 6.545 \times 10^{-3}$$

$$\phi = \frac{TL}{GJ} \Rightarrow T = \frac{GJ\phi}{L} = \frac{5.6 \times 10^6 \times 4.1172 \times 6.545 \times 10^{-3}}{8} = 18862.93 lbs.in$$

The largest torque that can be applied not exceed the stresses and the angle of halt $= 12628.11 ibs.in$

$$L = 13, T = 12628.11$$

$$\phi = \frac{TL}{GJ} = \frac{12628.11 \times 13}{11.2 \times 10^6 \times 0.920} = 0.0147 rads$$

Problem 3.44

Solution

Givens:

$$t = 5ib.in, l = 2.4in, c = \frac{1}{2}d = \frac{1}{32}in, G = 11.2{\times}10^6 psi, n = 2, J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(\frac{1}{32})^4 = 1.49803{\times}10^{-6} in^4$$

The formular is $\phi = \frac{Tl}{GJ}(1 + \frac{1}{n^2} + \frac{1}{n^4})$

$$\phi = \frac{5 \times 2.4}{11.2 \times 10^6 \times 1.49803 \times 10^{-6}}(1 + \frac{1}{4^2} + \frac{1}{16^4}) = 938.73 \times 10^{-3} rad$$

The angle through which end A rotates in degress is 53.8^o

Problem 3.48

Solution

$$T = rP = (0.3)(600) = 180N.m$$

Shaft of diameter based on displacement limit, we have based on the angle of displacement,

$$\phi = \frac{\sigma}{r} = \frac{15}{300} = 0.005rad$$

$$\phi = \frac{TL}{GJ} = \frac{2TL}{\pi G r^4}$$

$$r^4 = \frac{2TL}{\pi \phi G} = \frac{2 \times 180 \times 0.5}{\pi \times 11 \times 10^9 \times 0.05} = 14.882 \times 10^{-9} m^4$$

$$r = 11.045 \times 10^{-3} m = 11.045m$$

$$d = 2r = 22.1mm$$

Shaft diameter based on stress

$$\tau = 80 \times 10^6 Pa$$

$$\tau = \frac{Tr}{J} = \frac{2T}{\pi r^3}$$

$$r = \sqrt[3]{\frac{2T}{\pi\tau}} = \frac{(2)(180)}{\pi(80 \times 10^6)} = \sqrt[3]{1.43239 \times 10^{-6} m^3} = 11.273{\times}10^{-3} m = 11.273mm$$

$$d = 2r = 22.5mm$$

Use the larger value to meet both limits