DAA TUTORIAL - 4

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Master Theorem

① if $f(n) = O(n^{\log_b a - \epsilon}), \epsilon > 0$, $T(n) = \Theta(n^{\log_b a})$

② if $f(n) = O(n^{\log_b a} \log^k n), k \geq 0$, $T(n) = \Theta(n^{\log_b a} \log^{k+1} n)$

③ if $f(n) = \Omega(n^{\log_b a} \log^k n)$; if $(n/k) \leq c f(n)$

1. $T(n) = 3T(n/2) + n^2$

$a = 3, b = 2$

$c = \log_2 3 = 1.5$

$n^2 > n^{1.5}$

$\Rightarrow T(n) = \Theta(n^2)$

2. $T(n) = 4T(n/2) + n^2$

$a = 4, b = 2$

$c = \log_2 4 = 2$

$f(n) = n^c \Rightarrow n^2 = n^2$

$T(n) = \Theta(n^2 \log n)$

3. $T(n) = T(n/2) + 2^n$

$a = 1, b = 2$

$c = \log_2 1 = 0$

$(n^c = 1)$ $2^n > 1 \Rightarrow T(n) = \Theta(2^n)$

4. $T(n) = 2^n T(n/2) + 2^n$

$a = 2^n, b = 2$ $c = \log_2 2^n$

$c = n \Rightarrow n^c = n^n$

$f(n) < n^c$ $T(n) = \Theta(n^n)$

5. $T(n) = 16T(n/4) + n$

$a = 16, b = 4$

$c = \log_2 4^2 = 2$

$f(n)$ & n^c : $n < n^2$

$T(n) = \theta(n^2)$

6. $T(n) = 2T(n/2) + n\log n$

$c = \log_2 2 = 1$

$f(n) > n^c$

$n\log n > n$

$\Rightarrow T(n) = \theta(n\log n)$

7. $T(n) = 2T(n/2) + n\log n$

$a = 2, b = 2$

$c = \log_2 2 = 1$

$\frac{n}{\log n} < n$

$f(n) < n^c$

$\Rightarrow T(n) = \theta(n)$

8. $T(n) = 2T(n/4) + n$

$a = 2, b = 4$

$c = \log_4 2 = 1/2 = 0.5$

$f(n) = n^c$

$n^{0.5} = n^{0.5}$

$\therefore T(n) = \theta(n^{0.5}\log n)$

(9) $T(n) = 0.5T(n/2) + 1/n$

$c = \log_2 1/2 = -1$

$n^{-1} = n^{-1}$

$\therefore T(n) = \theta(n^{-1}\log n)$

10. $T(n) = 16T(n/4) + n!$

$a = 16, b = 4$

$c = \log_4 16 = 2$

$n!$ & n^2 if $(n > 3) \Rightarrow n! > n^2$

$\Rightarrow T(n) = \Theta(n!)$

if $(n < 3) \Rightarrow n^2 > n!$

So, $T(n) = \Theta(n^2)$

11. $T(n) = 4T(n/2) + \log n$

$a = 4, \; b = 2$

$c = \log_2 4 = 2$

$\log n < n^2$

$T(n) = \Theta(n^2)$

12. $T(n) = \sqrt{n}\, T(n/2) + \log n$

$a = \sqrt{n}, \; b = 2$

$c = \log_2 n^{1/2}$

13. $T(n) = 3T(n/2) + n$

$a = 3, \; b = 2$

$c = \log_2 3 = 1.5$

$f(n) < n^c$

$n < n^{1.5}$

$T(n) = \Theta(n^{1.5})$

14. $T(n) = 3T(n/3) + \text{Sqrt}(n)$

$a = 3, \; b = 3$

$c = \log_b a = 1$

$f(n) = n^{1/2}$

$n^c = n^1 \Rightarrow f(n) < n^2$

$\therefore T(n) = \Theta(n)$

15. $T(n) = 4T(n/2) + cn$

$a = 4, \; b = 2$

$c = \log_2 2^2 = 2$

$f(n) < n^c$

$n < n^2$

$T(n) = \Theta(n^2)$

(16) $T(n) = 3T(n/3) + n/2$

$a=3, b=3$

$c = \log_b a = 1$

$f(n) = n/2$

$n^c = n$

$f(n) < n^c$

$T(n) = \Theta(n)$

(17) $T(n) = 3T(n/4) + n\log n$

$a=3, b=4$

$c = \log_4 3$

$f(n) = n\log n$

$n^c = n^{0.7}$

$f(n) > n^c$

$T(n) = \Theta(n \log n)$

(18) $T(n) = 6T(n/3) + n^2 \log n$

$a=6, b=3$

$c = \log_3 6 = 1.6$

$f(n) > n^c$

$T(n) = \Theta(n^2 \log n)$

(19) $T(n) = 4T(n/2) + n\log n$

$a=4, b=2$

$c = \log_2 4 = 2$

$f(n) = n\log n$

$n^c = n^2$

$n\log n < n^2$

$T(n) = \Theta(n^2)$

(20) $T(n) = 64T(n/8) - n^2 \log n$

$a=64, b=8$

$c = \log_8 64 = 2$

$f(n) > n^c$

$n\log n > n^2$

$T(n) = \Theta(n^2 \log n)$

(21) $T(n) = 7T(n/3) + n^2$

$a=7, b=3$

$c = \log_3 7 = 1.77$

$f(n) > n^c$

$n^2 > n^{1.77}$

$T(n) = \Theta(n^2)$

(22) $T(n) = T(n/2) + n(2 - \cos n)$

$a=1, b=2$

$\log_2^1 = 0$

$f(n) > n^c$

$n(2 - \cos n) > n^0$

$T(n) = \Theta(n(2 - \cos n))$