Literature review on variable annuities

There are varieties of annuity contract, and one of it is variable annuity. Variable annuity is an annuity contract type, which the value can vary based on the performance of an underlying portfolio of sub accounts. And among the annuities, variable annuities does not provide a specific and guaranteed return unlike fixed annuities. Variable annuity consist of two elements; the amount retiree pay into the annuity, which is the principal, and the retiree annuity's underlying investments deliver on the principal over the course of time, which is the returns. Variable annuities should be considered long-term investments due to the limitations on withdrawals. Typically, the withdrawal will only be allowed once each year during the accumulation phase. However, if a withdrawal taken during the contract's surrender period, which can be as long as 15 years, generally insure will have to pay a surrender fee.

With variable annuities, buyers have options to take that can bring benefit, but also a risk of loss. By investing in a mutual fund there is a possibility of higher returns during the accumulation phase and a larger income during the pay-out phase. But there is also a possibility that the investment exposed with market risk and resulted in loss.

Typically, variable annuities offer main benefits to provide the needs of the retirees. Guaranteed minimum withdrawal benefit to provide a guaranteed income in any economic condition. Death benefit that will be given to the beneficiaries with fixed amount. Investment option, here is where insures are allowed to decide on how they design their portfolio according to their risk and investment interest.

There are different pricing methods that variable annuities used with different basing on how insurer calculate the annuity value. Guaranteed benefit base method calculates the annuity value based on the guaranteed minimum withdrawal benefit. Account value method calculate the annuity value based on the account balance. Net asset value method calculate the annuity value based on current value on the underlying investments. Other than this methods, variable annuities pricing based on mortality and expense risk charge which cover insurance operating cost, and also the investment management fees. Surrender and rider charges which is the withdrawing penalty if insure withdraw before maturity and the additional fee for optional features.

Variable annuities (VAs) are investment products that provide retirees with a combination of insurance and investment features. They offer a range of benefits designed to cater to the specific needs and risk appetites of retirees. This literature review examines the product design, risk management, and pricing methods employed in variable annuities.

Product Design:

VA product design is focused on meeting the unique needs of retirees. Key features include: Guaranteed Income: VAs often provide retirees with a guaranteed income stream for life, addressing their longevity risk and ensuring a steady cash flow during retirement, Investment Options: VAs offer a variety of investment options, such as equity funds, bond funds, and balanced portfolios. This allows retirees to tailor their investments based on their risk tolerance and financial goals and Flexibility: VAs may provide flexibility in terms of withdrawal options, allowing retirees to access their funds when needed while still maintaining the potential for growth.

Risk Management,

To address retirees' risk appetites, VAs incorporate various risk management features: Guaranteed Minimum Income Benefits (GMIB): GMIB riders assure retirees a minimum level of income regardless of market performance, protecting against downside risks and Guaranteed Death Benefits (GDB): GDB riders provide beneficiaries with a minimum death benefit, ensuring a certain level of inheritance for retirees' loved ones.

Pricing Methods:

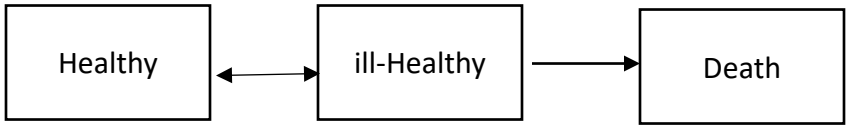
The pricing of variable annuities involves complex calculations, considering factors such as mortality rates, interest rates, and investment performance. Common pricing methods include: Actuarial Methods: Traditional actuarial methods utilize mortality tables, interest rate assumptions, and statistical modeling to determine the pricing of Vas, Market-Consistent Methods: These methods incorporate market-based inputs, such as current interest rates and market prices, to reflect the fair value of Vas, Stochastic Modeling: Stochastic modeling uses simulations to capture the uncertainty of future investment returns and mortality rates, providing a more dynamic approach to pricing.

Overall, variable annuities offer retirees a comprehensive solution by combining investment growth potential with insurance protection. The product design, risk management features, and pricing methods are designed to align with retirees' needs and risk appetites.

Methodology

Stochastic models are commonly used to estimate the prices of variable annuities. One common approach for pricing variable annuities is to use a stochastic model called a Monte Carlo. This involves simulating multiple possible future scenarios for the underlying investments and interest rate, and then calculating the expected pay-outs under each scenario. The expected payouts are then discounted to the present value.

Healthy-illhealth-death model is a model that commonly used in actuarial science to estimate mortality rates and life expectancies. This model is under the assumption that an individual can be only in 3 states which healthy, ill or dead. So there is only 3 transitions that we can derive based on this 3 states. Individual can move from healthy state to an ill state, from ill state to healthy, or from ill state to dead state. In this case, there are other assumptions we can hold on as well as the individual ages the probability of death also increase. To calculate the probability of each transitioning and the expected time spent in each state will include the Markov chain model.



Matriks transtition from diagram above is :
Probabilitas from healthy to healthy references is $P(Healthy \rightarrow healthy)$, Probabilitas from healthy to ill-healthy references $P(Healthy \rightarrow ill - healthy)$, Probabilitas from ill-healthy to healthy references $P(ill - ealthy \rightarrow healthy)$ and Probabilitas from ill-healthy to death references $P(ill - healthy \rightarrow death)$.

In this variable annuity (VA) with investment and mortality risks, retirees have a choice between two investment options: defensive (option 1) or growth (option 2). The investment returns at the start of the contract are given by $rt, 1 \in \{-0.01, 0.05\}$ for the defensive option and $rt, 2 \in \{-0.05, 0.2\}$ for the growth option. The fund value of an individual at time t, denoted as S_t, is calculated based on the chosen investment option and the previous fund value. Specifically, $S_t =$

$0.95S_t - 1(1 + r_{t,i})$, where i represents the investment choice (1 for defensive, 2 for growth) and S_0 is the initial deposit P.

A management fee of $0.05S_t - 1(1 + r_{t,i})$ is deducted from the fund value each period to cover costs associated with guarantees and administration expenses.There are three types of benefits that retirees may be entitled to: Yearly retirement benefit (B_t): At the end of each year, retirees receive a benefit equal to 3% of the fund value, i.e., $B_t = 0.03S_t$. The fund value is adjusted to reflect this deduction.

Benefits on ill-health ($\max(0.5_{\tau 1}, 0.5P)$): If an individual policyholder experiences ill-health for the first time at τ_1, they are eligible to receive benefits. The benefit amount is the maximum of 0.5 times the fund value at τ_1 or 0.5 times the initial deposit P. The fund value is adjusted to reflect this deduction.Benefits on death ($\max(S_{\tau 2}, 0.3P)$): In the event of an individual's death at time τ_2, beneficiaries receive benefits. The benefit amount is the maximum of the fund value at τ_2 or 0.3 times the initial deposit P.

To determine the final benefits for a retiree, you would need to calculate the fund value at each time period using the investment returns and management fees, adjust the fund value for retirement, ill-health, and death benefits as described above, and sum up the benefits over the relevant time periods.

Here is a pricing algorithm for a variable annuity for retirees at different ages:

1. Define the Variables:
    - Initial Deposit (P): The amount of money the retiree invests in the variable annuity.
    - Investment Options: Identify the available investment options and their corresponding returns, volatility, and correlation with other options.
    - Retirement Age (RA): The age at which the retiree plans to start receiving retirement benefits.
    - Life Expectancy (LE): The estimated remaining life expectancy of the retiree based on their age.
2. Calculate the Accumulation Phase:

- Determine the number of years until retirement (N = RA - Current Age).
- Generate a set of random scenarios for investment returns based on the chosen method (e.g., Monte Carlo simulation).
- For each scenario, calculate the growth of the annuity's fund value using the investment returns and the chosen investment options.
- Adjust the fund value by deducting any management fees or expenses associated with the annuity.

3. Calculate the Distribution Phase:
    - Determine the number of years the retiree is expected to receive retirement benefits (Distribution Years = LE - RA).
    - Generate a set of random scenarios for mortality rates based on actuarial data or other sources.
    - For each scenario, estimate the probability of survival at each year during the distribution phase.
    - Adjust the annuity's fund value for any deductions associated with retirement benefits or rider benefits (e.g., ill-health or death benefits).
4. Calculate the Present Value of Cash Flows:
    - Discount the future cash flows (retirement benefits) using an appropriate discount rate that reflects the time value of money and the risk associated with the annuity.
    - Sum the present values of all expected cash flows during the distribution phase.
5. Calculate the Annuity Price:
    - The annuity price is the present value of all expected cash flows during the distribution phase, discounted to the current age of the retiree.
    - Consider any additional factors or adjustments specific to the annuity product, such as minimum guarantees, fees, or optional riders.

Assumptions and Justifications:

- The pricing algorithm assumes that investment returns follow a certain distribution and that they are independent and identically distributed over time.

- The algorithm assumes that mortality rates are estimated based on actuarial data or other reliable sources and that they accurately reflect the probability of survival at each age.
- The chosen discount rate reflects the risk associated with the annuity and the time value of money. It may be determined based on market rates, risk-free rates, or other appropriate benchmarks.
- The algorithm assumes that the retiree will follow the prescribed investment options and hold the annuity until the end of the distribution phase.

It is important to note that the assumptions and justifications may vary depending on the specific context, data availability, and modeling approach used for pricing the variable annuity.

Result

The transition probability matrix for defensive investmen return :

$$P = \begin{pmatrix} 0.3806514 & 0.6193486 \\ 0.3806514 & 0.6193486 \end{pmatrix}$$

The transition probability matrix for growth investmen return :

$$P = \begin{pmatrix} 0.5127476 & 0.48725246 \\ 0.5507 & 0,4493 \end{pmatrix}$$

The transition probability matrix for healthy-illhealthy-death model :

Age 65-69 :

Health-Illhealth-Death probability transition matrix for age group 65-69

	Healthy	Ill-health	Death
Healthy	0.9079257	0.0747703	0.0173040
Ill-health	0.3278689	0.3387978	0.3333333
Death	0.0000000	0.0000000	1.0000000

Age 70-74 :

Health-Illhealth-Death probability transition matrix for age group 70-74

	Healthy	Ill-health	Death
Healthy	0.8960699	0.0858806	0.0180495
Ill-health	0.2745098	0.3355120	0.3899782
Death	0.0000000	0.0000000	1.0000000

Age 75-79 :

Health-Illhealth-Death probability transition matrix for age group 75-79

	Healthy	Ill-health	Death
Healthy	0.8753922	0.0999552	0.0246526
Ill-health	0.1794195	0.3720317	0.4485488
Death	0.0000000	0.0000000	1.0000000

Age 80-84:

Health-Illhealth-Death probability transition matrix for age group 80-84

	Healthy	Ill-health	Death
Healthy	0.8348891	0.1271879	0.0379230
Ill-health	0.1308901	0.3586387	0.5104712
Death	0.0000000	0.0000000	1.0000000

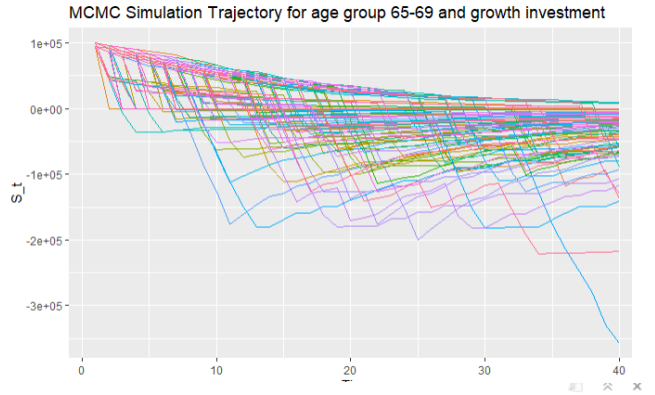
CALCULATION OF THE VA PRICE

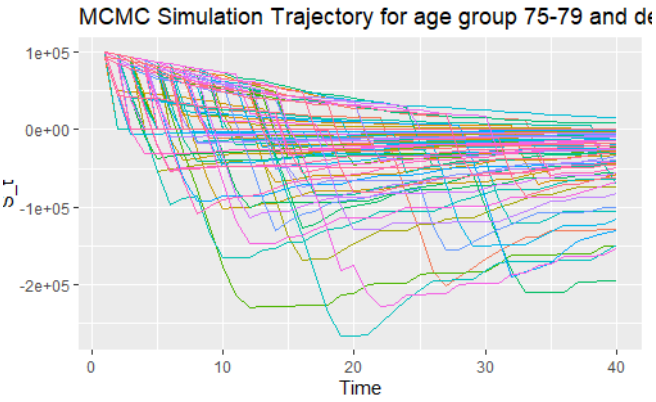
Misalkan jika diasumsikan bahwa jika dimulai dari starting investment fund (S_0) dengan menggunakan monte carlo solutiondengan

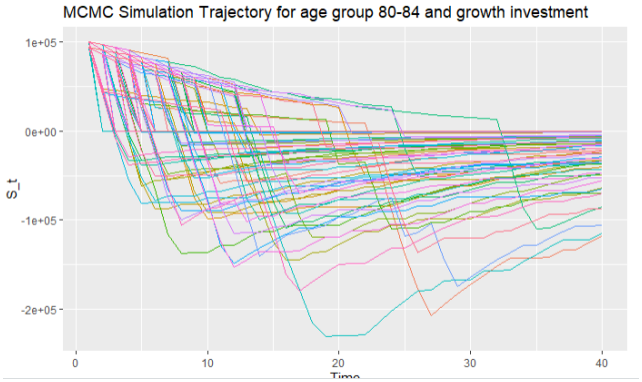
PV benefit based on each age group and investment group

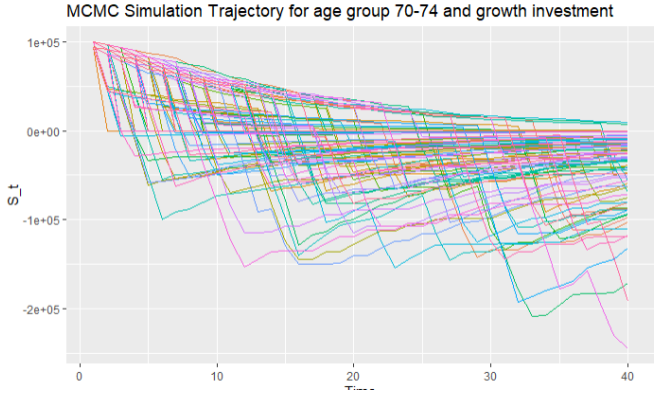
	65-69	70-74	75-79	80-84
Defensive	96587.36	100109.80	101576.2	106334.4
Growth	95317.85	99101.79	100489.4	105370.7

SUMMARY SENSITIVITY









REFERENCES

Mackay, A., Vachon, MC., & Cui, Z., 2022, *Analysis of Vix-Linked Fee Incenvtives in Variable Annuities Via Continuous-Time Markov Chain Approximation.*

Ganti, A. (2021). *Variable Annuity: Definition and How It Works, Vs. Fixed Annuity*. https://www.investopedia.com/terms/v/variableannuity.asp

Zhang, Y., 2016. Actuarial Modelling with Mixtures of Markov Chains. Electronic Thesis and Dissertation Reposity. Western University. https://ir.lib.uwo.ca/etd/4026.