Лекция 9. Задание 2 номер 5

$$\int_0^{\frac{1}{4}} \frac{dx}{x \ln x}$$

$$\lim_{\varepsilon \to 0} \int_{0+\varepsilon}^{\frac{1}{4}} \frac{dx}{x \ln(x)} = \lim_{\varepsilon \to 0} \int_{0+\varepsilon}^{\frac{1}{4}} \frac{1}{x \ln(x)} dx \ominus \left| \begin{array}{l} u = \ln(x) \\ du = d\ln(x) \\ du = \frac{1}{x} dx \\ dx = x\,du \end{array} \right.$$

$$\ominus \lim_{\varepsilon \to 0} \int_{0+\varepsilon}^{\frac{1}{4}} \frac{1}{x u} x\,du = \lim_{\varepsilon \to 0} \int_{0+\varepsilon}^{\frac{1}{4}} \frac{1}{u} du = \left(\ln|u| + C\right)\Big|_{0+\varepsilon}^{\frac{1}{4}} \ominus$$

$$\ominus \lim_{\varepsilon \to 0} \left(\ln|\ln(x)| + C\right)\Big|_{0}^{\frac{1}{4}} = \ln\left|\ln\left(\tfrac{1}{4}\right)\right| - \ln|\ln(\varepsilon)| \ominus$$

$$\ominus \ln\left|\ln\left(\tfrac{1}{4}\right)\right| + \infty$$

Ответ: несобственный интеграл расходится

Лекция 9. Задание 2, номер 4

$$\int_0^1 \ln(x)\,dx$$

$$\int_0^1 \ln(x)\,dx = \lim_{\varepsilon\to 0}\int_{0+\varepsilon}^1 \underbrace{\ln(x)}_{u}\cdot\underbrace{1\,dx}_{dv} \quad \Bigg| \begin{matrix} u=\ln(x) & du=\frac{1}{x}dx \\ dv=1dx & v=x \end{matrix}$$

Используем формулу $\int u\,dv = uv - \int v\,du$

$$\ln(x)\cdot x-\int x\cdot\frac{1}{x}dx = \ln(x)\cdot x-\int 1dx = \lim_{\varepsilon\to 0}\left(\ln(x)\cdot x - x + C\right)\Big|_{0+\varepsilon}^{1}$$

$$=\left(\ln(1)\cdot 1-1\right)-\left((-\infty)\cdot\varepsilon-\varepsilon\right) = \boxed{-1}$$ ответ

Лекция 9, Задание 1 номер 5

$$\int_0^{+\infty} x\cos x\,dx$$

$$\int_0^{+\infty} x\cos x\,dx = \lim_{b\to+\infty}\int_0^b \underbrace{x}_{u}\cdot\underbrace{\cos x}_{dv} \;=\; \left|\begin{array}{ll} u = x & du = dx \\ dv = \cos dx & v = \int \cos dx = \sin x \end{array}\right.$$

Используем формулу $\int u\,dv = uv - \int v\,du$

$$= \lim_{b\to+\infty}\left(x\cdot\sin x\Big|_0^b - \int_0^b \sin x\,dx\right) = \lim_{b\to+\infty}\left(\left(b\cdot\sin b - 0 + \cos x\right)\Big|_0^b\right) =$$

$$= \lim_{b\to+\infty}\left(b\cdot\sin b + \cos b - 1\right)$$

Ответ: несобственный интеграл расходится

№1.3

$$\int_0^{+\infty} x e^{-x^2} dx = \lim_{a \to +\infty} \int_0^a x e^{-x^2} dx = \lim_{a \to \infty} \left(-\frac{1}{2} \int_0^a e^{t} dt \right) =$$

$-x^2 = t$

$dx = -\frac{1}{2x} dt$

$$\lim_{a \to +\infty} \left(-\frac{1}{2} e^{t} \Big|_0^a \right) = \lim_{a \to +\infty} \left(-\frac{1}{2} e^{-x^2} \Big|_0^a \right) =$$

$$\lim_{a \to +\infty} \left(-\frac{1}{2e^a} \right) + \lim_{a \to +\infty} \left(\frac{1}{2e^0} \right) = \frac{1}{2}$$

Задание 2 п3.

$$\int_0^3 \left(\frac{dx}{\sqrt{9-x^2}}\right) = \lim_{a\to 3}\left(\int_0^a \frac{dx}{\sqrt{9-x^2}}\right) = \lim_{a\to 3}\left(\arcsin\left(\frac{x}{3}\right)\Big|_0^a\right) =$$

$$= \lim_{a\to 3}\left(\arcsin\left(\frac{a}{3}\right) - \arcsin\left(\frac{0}{3}\right)\right) = \arcsin\left(\frac{3}{3}\right) = \frac{\pi}{2}$$

Ответ: $\frac{\pi}{2}$

Задание N1.4

$$\int_2^{+\infty} \frac{dx}{x^2+x-2} = \lim_{a\to+\infty}\left(\int_2^a \frac{dx}{x^2+x-2}\right) = \lim_{a\to+\infty}\left(\int_2^a \frac{1}{x^2+2x-x-2}dx\right) =$$

$$= \lim_{a\to+\infty}\left(\int_2^a \left(\frac{dx}{x(x+2)-(x+2)}\right)\right) = \lim_{a\to+\infty}\left(\int_2^a \left(\frac{A}{x+2}+\frac{B}{x-1}\right)dx\right) =$$

$$\frac{1}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$$

$$1 = A(x-1) + B(x+2)$$

$$1 = Ax - A + Bx + 2B$$

$$1 = x(A+B) + (2B-A)$$

$$\begin{cases} 1 = -A + 2B \\ 0 = A + B \end{cases}$$

$$1 = 3B \qquad 0 = A + \frac{1}{3}$$

$$B = \frac{1}{3} \qquad A = -\frac{1}{3}$$

$$= \lim_{a\to\infty}\left(-\frac{1}{3}\int_2^a \frac{dx}{x+2} + \frac{1}{3}\int_2^a \frac{dx}{x-1}\right) = \lim_{a\to\infty}\left(\left(-\frac{1}{3}\ln(|x+2|) + \frac{1}{3}\ln(|x-1|)\right)\Big|_2^a\right)$$

$$= \lim_{a\to+\infty}\left(\left(-\frac{1}{3}\ln(|a+2|) + \frac{1}{3}\ln(|a-1|)\right) - \left(-\frac{1}{3}\ln(|2+2|) + \frac{1}{3}\ln(|2-1|)\right)\right) =$$

$$\lim_{a\to+\infty}\left(-\frac{1}{3}\ln(|a+2|) + \frac{1}{3}\ln(|a-1|) + \frac{2}{3}\ln(2)\right) = \frac{2}{3}\ln 2$$