Практическое задание №7.
Вариант 1.

1.1

$y = \ln \sin x$

$$l = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{1 + (\ln \sin x)'^2}\, dx$$

$$(\ln \sin x)' = \frac{1}{\sin x} \cos x = \operatorname{ctg} x$$

$$\int \sqrt{1 + \operatorname{ctg}^2 x}\, dx = \int \sqrt{\frac{1}{\sin^2 x}}\, dx =$$

$$= \int \frac{1}{|\sin x|} dx = \int \frac{1}{\sin x} dx$$

$$\boxed{\sin x > 0 \quad \text{при} \quad \frac{\pi}{3} < x < \frac{\pi}{2}}$$

$$\int \frac{1}{\sin x} dx = \int \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} =$$

$$= \frac{2}{2} \int \frac{du}{\cos u \sin u}$$

$$\boxed{\text{пусть } \frac{x}{2} = u, \quad x = 2u, \quad dx = 2du}$$

$$\int \frac{du}{\cos u \sin u} = \int \frac{du}{\cos^2 u \cdot \operatorname{tg} u} =$$

$$= \int \frac{\cos^2 u}{t \cdot \cos^2 u} dt = \int \frac{1}{t} dt$$

$$\boxed{\begin{array}{l} \frac{\sin u}{\cos u} \cdot \cos^2 u = \cos u \cdot \sin u \\ \operatorname{tg} u = t \qquad dt = \frac{1}{\cos^2 u} du \end{array}}$$

$$\int \frac{1}{t} dt = \ln |t| + C = \ln |\operatorname{tg} u| + C =$$

$$= \ln \left|\operatorname{tg} \frac{x}{2}\right| + C$$

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{\sin x} dx = \ln \operatorname{tg} \frac{x}{2} \Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}} =$$

$= \ln tg\frac{\pi}{4} + \ln tg\frac{\pi}{3} = \ln\frac{1}{\frac{1}{\sqrt{3}}} = \ln\sqrt{3} =$

$= \frac{1}{2}\ln 3$

$\boxed{2.5}$

$x = \cos^2 t$

$0 \le t \le \frac{\pi}{2}$

$y = \sin^2 t$

$$l = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$$

$$l = \int_0^{\frac{\pi}{2}} \sqrt{(-2\cos t\sin t)^2 + (2\sin t\cos t)^2}\, dt =$$

$$= 2\sqrt{2}\int_0^{\frac{\pi}{2}} |\cos t\sin t|\, dt = 2\sqrt{2}\int_0^{\frac{\pi}{2}} -\cos t\, d(\cos t) =$$

$\cos t \ge 0$	при	$0 \le t \le \frac{\pi}{2}$
$\sin t \ge 0$	при	$0 \le t \le \frac{\pi}{2}$

$$= 2\sqrt{2}\left(-\frac{1}{2}\cos^2 t\right)\Big|_0^{\frac{\pi}{2}} = -2\sqrt{2}\left(\frac{1}{2}\underbrace{\cos^2\frac{\pi}{2}}_{=0} -\right.$$

$$\left.- \frac{1}{2}\underbrace{\cos^2 0}_{=1}\right) = -2\sqrt{2}\cdot\left(-\frac{1}{2}\right) = \sqrt{2}$$

$x = 8\sin t + 6\cos t$

$y = 6\sin t - 8\cos t$, $0 \le t \le \frac{\pi}{2}$

$$l = \int_{T_1}^{T_2} \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$$

$$l = \int_0^{\frac{\pi}{2}} \sqrt{64\cos^2 t - 96\cos t\sin t + 36\sin^2 t + 64\sin^2 t + 96\sin t\cos t + 36\cos^2 t}\, dt = \int_0^{\frac{\pi}{2}} \sqrt{100(\cos^2 t + \sin^2 t)}\, dt$$

По формуле Ньютона-Лейбница

$$\int_0^{\frac{\pi}{2}} 10\sqrt{\sin^2 t + \cos^2 t}\, dt = F(t)\Big|_0^{\frac{\pi}{2}} = F\left(\frac{\pi}{2}\right) - F(0)$$

$F(t) = \int 10\sqrt{\sin^2 t + \cos^2 t}\, dt = 10t + C$

$F\left(\frac{\pi}{2}\right) = 5\pi$

$F(0) = 0$

$l = 5\pi$

Задание 1 N3 Найти длину дуги.

$y = \frac{2}{5}\sqrt[4]{x} - \frac{2}{3}\sqrt[4]{x^3}$ $x > 0$. Между точками пересечения с O_x

$\frac{2}{5}\sqrt[4]{x} - \frac{2}{3}\sqrt[4]{x^3} = 0$

$\frac{2}{15}\sqrt[4]{x}\left(3x - 5\sqrt{|x|}\right) = 0$

$\sqrt[4]{x}\left(3x - 5\sqrt{|x|}\right) = 0$

$\sqrt[4]{x} = 0$ $3x - 5\sqrt{|x|} = 0$

$-5\sqrt{|x|} = -3x$

$25|x| = 9x^2$

т.к. $x > 0$, $25x - 9x^2 = 0$

$x = 0$ $x = \frac{25}{9}$

$x_1 = 0$ $x_2 = \frac{25}{9}$ - границы интегрирования

$y' = \left(\frac{2}{5}\cdot x^{\frac{5}{4}}\right)' - \left(\frac{2}{3}\cdot x^{\frac{3}{4}}\right)' = \frac{2}{5}\cdot\frac{5}{4}\cdot x^{\frac{1}{4}} - \frac{2}{3}\cdot\frac{3}{4}x^{-\frac{1}{4}} = \frac{1}{2}x^{\frac{1}{4}} - \frac{1}{2}x^{-\frac{1}{4}}$

$l = \int_0^{\frac{25}{9}} \sqrt{1 + \left(\frac{1}{2}\left(x^{\frac{1}{4}} - x^{-\frac{1}{4}}\right)\right)^2}\,dx = \int_0^{\frac{25}{9}} \sqrt{1 + \left(\frac{1}{2}x^{\frac{1}{4}} - \frac{1}{2}x^{-\frac{1}{4}}\right)^2}\,dx = \int_0^{\frac{25}{9}} \sqrt{1 + \left(\frac{1}{4}x^{\frac{1}{2}} - \frac{2x^{\frac{1}{4}}}{4x^{\frac{1}{4}}} + \frac{1}{4x^{\frac{1}{2}}}\right)}\,dx$

$= \int_0^{\frac{25}{9}} \sqrt{1 + \frac{1}{4}x^{\frac{1}{2}} - \frac{1}{2} + \frac{1}{4x^{\frac{1}{2}}}}\,dx = \int_0^{\frac{25}{9}} \sqrt{\frac{1}{4}x^{\frac{1}{2}} + \frac{1}{2} + \frac{1}{4x^{\frac{1}{2}}}}\,dx =$

$= \int_0^{\frac{25}{9}} \sqrt{\left(\frac{1}{2}\cdot\sqrt[4]{x} + \frac{1}{2\sqrt[4]{x}}\right)^2}\,dx = \int_0^{\frac{25}{9}} \frac{1}{2}\sqrt[4]{x} + \frac{1}{2\sqrt[4]{x}}\,dx = \int_0^{\frac{25}{9}} \frac{1}{2}\sqrt[4]{x}\,dx + \int_0^{\frac{25}{9}} \frac{1}{2\sqrt[4]{x}}\,dx =$

$= \frac{1}{2}\cdot\frac{4}{5}\cdot x\cdot\sqrt[4]{x}\Big|_0^{\frac{25}{9}} + \frac{1}{2}\cdot\frac{4}{3}\cdot x^{\frac{3}{4}}\Big|_0^{\frac{25}{9}} = \left(\frac{1}{2}\cdot\frac{4}{5}\cdot x\cdot\sqrt[4]{x} + \frac{1}{2}\cdot\frac{4}{3}\cdot x^{\frac{3}{4}}\right)\Big|_0^{\frac{25}{9}} =$

$= \left(\frac{4x\sqrt[4]{x}}{10} + \frac{4x^{\frac{3}{4}}}{6}\right)\Big|_0^{\frac{25}{9}} = \left(\frac{2}{5}\cdot\frac{25}{9}\cdot\sqrt{\frac{5}{3}} + \frac{2}{3}\cdot\frac{5}{3}\cdot\sqrt{\frac{5}{3}}\right) - 0 =$

$= \frac{20}{9}\sqrt{\frac{5}{3}}$ Ответ: $\frac{20}{9}\sqrt{\frac{5}{3}}$ л.е.

Найти длины дуг кривых

3) $y=\frac{x^2}{2}$ от $x=0$ до $x=1$.

$$\int_0^1 \sqrt{1+\left(\left(\frac{1}{2}x^2\right)'\right)^2}\,dx=\int_0^1\sqrt{1+x^2}\,dx=\int_0^1\frac{\sqrt{tg^2t+1}}{\cos^2 t}\,dt=\int_0^1\frac{1}{\cos^3 t}\,dt=$$

$$\boxed{\begin{matrix} x=tgt & u=\sin t \\ dx=\frac{1}{\cos^2 x}dt & dt=\frac{du}{\cos u}\end{matrix}}$$

$$\int_0^1\frac{1}{(1-u^2)^2}\,du=-\frac{u}{2u^2-2}-\frac{1}{4}\cdot\ln(u-1)+$$

$$+\frac{1}{4}\ln(u+1)\Big|_0^1=-\frac{1}{4}\ln(\sin t-1)+$$

$$+\frac{1}{4}\ln(\sin t+1)-\frac{\sin t}{2\sin^2 t-2}\Big|_0^1=-\frac{x}{\sqrt{x^2+1}\cdot\left(\frac{2x^2}{x^2+1}-2\right)}-$$

$$-\frac{1}{4}\ln\left(\frac{x}{\sqrt{x^2+1}}-1\right)+\frac{1}{4}\ln\left(\frac{x}{\sqrt{x^2+1}}+1\right)\Big|_0^1=\frac{\ln\left(\frac{\sqrt2}{2}+1\right)-\ln\left(\frac{\sqrt2}{2}-1\right)+\ln(-1)+2\sqrt2}{4}$$

Ответ: $l=\frac{\ln\left(\frac{\sqrt2}{2}+1\right)-\ln\left(\frac{\sqrt2}{2}-1\right)+\ln(-1)+2\sqrt2}{4}$