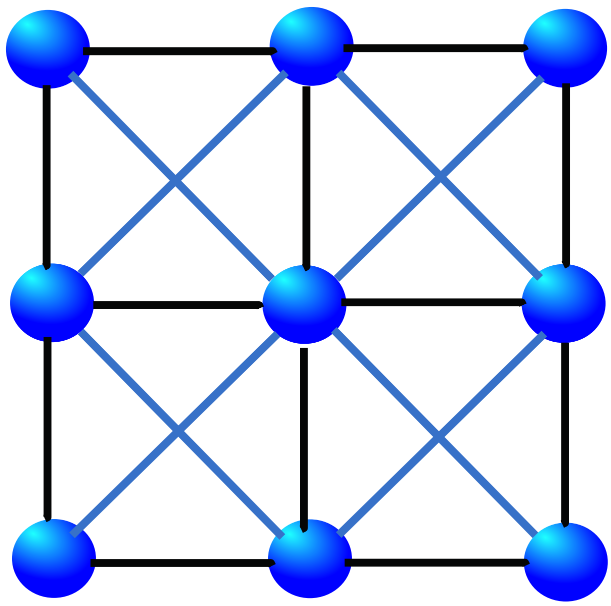
Autumn 2024 (Last updated: 2024-11-15)

Course Assignment
for

Practical Data Visualization and Virtual Reality (TNM093)
Autumn 2024

Visual Application Lab - Soft Bodies



1 Intended Learning Outcomes

The student shall, after finishing this computer exercise, be able to:

- Deepen the understanding of spring-mass-damper dynamics by creating a working simulation;
- Gain experience using web-based technologies for applying physics simulation for visualization;
- Have a basic understanding of building an interactive user interface that allows real-time adjustments and feedback in a physical simulation.

Remember: You must also work on this project ***outside*** *the scheduled time.*

2 The Setup

To build a web-based application, we recommend using JavaScript alongside the D3.js (Data-Driven Documents) library. D3.js is a powerful tool for data visualization using web standards, combining advanced visualization and interactive techniques with a data-driven approach to DOM manipulation. This allows you to fully leverage modern browser capabilities and design an optimal visual interface for your data. However, if you prefer alternative frameworks or libraries to develop your application, feel free to choose a solution that best fits your needs. You can find additional resources on D3, as well as hosting the webpage, in the previous lab instruction document.

3 Introduction to Soft-Body Simulation

Soft-body simulation is a computer simulation technique that is used to model and visualize objects that appear soft, flexible, and deformable. Soft-bodies are in constant motion and can be deformed, bent, stretched, and compressed. The ability to simulate soft-bodies takes place in numerous applications, including game development, engineering, and animation. Examples of such bodies include textiles, vegetation, and hair in motion. A soft-body simulation can also be used as an engineering tool that can model how materials and structures react to various stresses and forces.

Fabric is a specific type of deformable material that has unique physical properties. Unlike other deformable solids, the fabric is a thin, flexible material that is typically made up of fibers or threads woven together. When the fabric is subjected to external forces, it can undergo stretching, bending, and folding, meanwhile, it retains its overall shape.

4 Spring-Mass-Damper Model

Soft bodies can be described by the so-called spring-mass-damper model; the model consists of masses connected to springs [2].

In general, a soft body is represented by a grid of particles with mass connected by springs; the grid can be one-, two- or three-dimensional depending on which system is studied. At rest, all masses are equidistant from each other. If an external force is applied on a single particle or a particle is moved from its position at rest then spring forces from the neighboring particles are exerted upon the particle in order to restore the particle to its initial state of rest. These forces propagate through connections between particles to create movement in the entire system. Figure 1 shows a two-dimension grid with 9 masses connected by two types of springs:

- **structural springs** (black lines) connecting particles to each orthogonally adjacent neighbor; they are used to model the stretching and compression behavior of the system.
- **shear springs** (blue lines) connecting each particle to its diagonally adjacent neighbors; they are used to model the shearing behavior of the system.

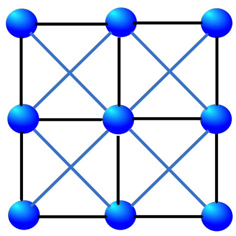


Figure 1: Grid of particles with masses connected by springs; black lines represent structural springs, blue lines represent shear springs.

Bend springs can also be implemented; they connect each particle to its neighbors two positions moved in each direction. Bend springs are used to model the bending behavior of the body.

5 Mathematical model

Each particle is subject to forces from springs, as well as other external forces acting on the system. Acceleration of the mass is determined by the sum of all forces and is described by Newton's second law of motion (1)

$$a(t) = \frac{F_t(t)}{m} \tag{1}$$

where m denotes the mass of the particle, $a(t)$ is the acceleration of the particle at time t, and $F_t(t)$ is the sum of all forces acting on the particle, both internal and external forces.

Internal forces are those forces coming from the properties of the system itself; they are produced by springs connecting particles and by the damping of the springs [1].

5.1 Spring Forces

Forces produced by springs are given by Hooke's law in (2). Hooke's law states that the force $F_s(t)$ produced by a spring with stiffness coefficient k is equal to the coefficient k times the displacement or change in length of the spring; the direction of the extension is opposite to the direction of the force.

$$F_s(t) = -k(L(t) - \ell_0) \tag{2}$$

where k denotes the stiffness coefficient of the spring, $L(t)$ denotes the displacement (or length) of the spring, and ℓ_0 is the length of the spring at rest. In particular given two particles p and q, the force produced on particle p by the spring connecting particles p and q is given by (3) where time dependence is omitted for clarity

$$\vec{F}_{s_{pq}} = -k(|\vec{r}_p - \vec{r}_q| - \ell_0) \cdot \frac{\vec{r}_p - \vec{r}_q}{|\vec{r}_p - \vec{r}_q|} \tag{3}$$

and the force produced on particle q by the spring connecting particles p and q is given by (4)

$$\vec{F}_{s_{qp}} = -k(|\vec{r}_q - \vec{r}_p| - \ell_0) \cdot \frac{\vec{r}_q - \vec{r}_p}{|\vec{r}_q - \vec{r}_p|} \tag{4}$$

where $\vec{r}_p$ and $\vec{r}_q$ denote the position vectors for particle p and particle q, respectively.

We see that forces (3) and (4) have same magnitude but opposite directions.

5.2 Damping Forces

Dampers are used in the simulation of soft bodies together with springs to dampen oscillations. They can be seen as an intrinsic property of the spring itself and can be represented as a damper element. A damper connected between two masses works against velocities to slow down the relative velocity of the two masses. The force produced by a damper connecting particles p and q is given by (5)

$$F_b(t) = b(v_p(t) - v_q(t)) \tag{5}$$

where b denotes the damping coefficient of the spring between particles p and q.

In particular, the force produced on particle p by the damper connecting particles p and q is given by (6) where time dependence is omitted for clarity

$$\vec{F}_{d_{pq}} = -b(\vec{v}_p - \vec{v}_q) \tag{6}$$

and the force produced on particle q is given by (7)

$$\vec{F}_{d_{qp}} = -b(\vec{v}_q - \vec{v}_p) \tag{7}$$

6 Numerical implementation

The physical model is implemented numerically using a step size h. At a generic time instant n acceleration of one particle is given by (8)

$$a_n = \frac{F_{t_n}}{m} \tag{8}$$

where a_n, F_n represent acceleration and total force at time instant t_n. velocity and position can be computed using a numerical approximation.

6.1 Euler Method

Euler method [1] gives the simplest numerical approximation. By applying Euler numerical approximation to the model of the system (8), velocity and position can be computed in (9) and (10)

$$v_{n+1} = v_n + ha_n \tag{9}$$

$$r_{n+1} = r_n + hv_n \tag{10}$$

The numerical approximation can become numerically unstable depending on the choice of the parameters; a critical parameter is the number of particles: when it increases (if simulating fabric) then the Euler method does not work.

6.2 Verlet Method

Verlet method [3] gives a more accurate numerical approximation and has a larger numerical stability region than the Euler method. Approximation of position and velocity are given in (11) and (12).

$$r_{n+1} = 2r_n - r_{n-1} + a_n h^2 \tag{11}$$

$$v_{n+1} = \frac{1}{2h}(r_{n+1} - r_{n-1}) \tag{12}$$

7 Tasks

We start with a simple model and we make it more and more complex. When the model becomes more complex, numerical stability becomes worse and more advanced numerical methods should be used. At the same time, it can also be necessary to reduce the step size. Numerical values for the implementation are given in Appendix A. Assume that masses are placed in a grid with a distance of 1 between each other. At the beginning of the simulation move one mass from its position at rest.

Task 1 *A system consisting of two masses connected with a spring and damper, shown in Fig. 2, is implemented. The Euler method is used for the numerical approximation.*

Figure 2: Two masses connected with a structural spring given in Task 1.

Task 2 *The system is now increased to four masses connected with only structural springs as shown in Fig. 3. For each spring a damper is implemented to dampen the oscillations. Euler method can still be used for the numerical approximation.*

Task 3 *Shear springs are now implemented to the system in Task 2 as shown in Fig. 4. For each spring a damper is implemented to dampen the oscillations. Euler method can still be used for the numerical approximation.*

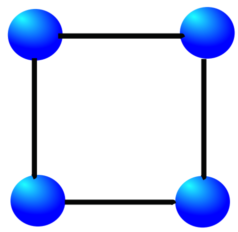


Figure 3: Four masses connected with structural springs given in Task 2.

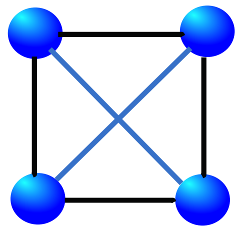


Figure 4: Four masses connected with structural and shear springs given in Task 3.

Task 4 *Now the system contains nine masses connected with both structural and shear springs as shown in Fig. 5. For each spring a damper is implemented to dampen the oscillations. The implementation becomes more numerically unstable and the Verlet method is used to have a stable simulation.*

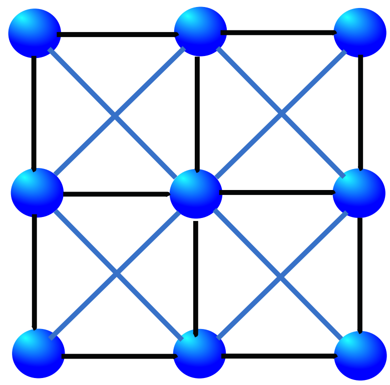


Figure 5: Nine masses connected with structural and shear springs given in Task 4.

Task 5 *Increase the number of masses to simulate the behavior of the fabric.*
Apply forces to some masses or move some masses from their position at rest.

8 Implementation

Your web-based application should **at least** fulfill 4 out of 6 following implementation requirements. Note that, when implementing the mathematical model, you should **not** use any existing library or copy-paste blocks of codes from any repository; however, you are allowed to use any existing library to implement visualizations- and interaction-related techniques.

- Be able to drag nodes to observe movement.
- Modify horizontal and vertical spring stiffness.
- Modify shear, i.e., diagonal, spring stiffness.
- Adjust global damping.
- Adjust the restoring force.

- Increase node mass.

Finally, your web-based application should support both the Euler and Verlet methods to start the simulation. The user should be able to select the desired method and run the selected simulation.

9 Reflection Document

After presenting to a lab assistant, gather your thoughts and notes from the lab to write a report. The report should be well-organized and provide clear explanations of the system behavior, code implementations, and observation, as well as a brief reflection on the project implementation, and ethical considerations for potential real-world applications of similar techniques.

References

[1] L. Ljung and T. Glad. *Modellbygge och simulering.* Studentlitteratur, 2004.

[2] D. Bourg. *Physics for game developers.* O'Reilly, 2002.

[3] L. Verlet. *Computer "Experiments" on Classical Fluids. I. Thermodynamical Properties of Lennard-Jones Molecules.* Physical Review, Vol. 19, Number 1, 1967.

A Numerical values

Numerical values for the implementation of the tasks described in Section 7.

Step size	$h = 0.01$ s
Mass	$m = 0.2$ kg
Structural spring	
length at rest	$\ell_0 = 1$ m
stiffness coefficient	$k = 20$ kg s^{-2}
damping coefficient	$b = 0.1$ kg s^{-1}
Shear spring	
length at rest	$\ell_0 = \sqrt{2}$ m
stiffness coefficient	$k = 7$ kg s^{-2}
damping coefficient	$b = 0.05$ kg s^{-1}