How to run program:

```
$ python main.py
>> Filename?
spambase.data
>> num_splits? (int)
100
>> train_percent? (in the form of "x,x,x")
5,10,15,20,25,30
```

The dataset must be in the form:

x_1	x_2	...	x_n	y
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Where x's can be floats and integers, and y's are only binary values (0s or 1s).

Question 1:

Optimal function mapping that:

$$
\begin{aligned}
\min_{f(\cdot)} \mathbb{E}_{x,y}[J(f(x),y)] &= \min_{f(\cdot)} \int_x \int_y J(f(x),y) \cdot P(x,y)dxdy \\
&= \min_{f(\cdot)} \int_x \int_y J(f(x),y) \cdot P(y|x)P(x)dxdy \\
&= \min_{f(\cdot)} \int_x \left\{ \int_y J(f(x),y) \cdot P(y|x)dy \right\} P(x)dx \\
&\text{Find a the optimal function } f(x) \text{ for any given x} \\
&z = f(x) \\
\min_{f(\cdot)} \mathbb{E}_{x,y}[J(f(x),y)] &\cong \min_{z} \int_y J(z,y) \cdot P(y|x)dy \\
&\cong \min_{z} \int_y (z-y)^2 \cdot P(y|x)dy \\
\text{Take the gradient} \quad & \frac{\delta}{\delta z} \int_y (z-y)^2 \cdot P(y|x)dy = 0 \\
& \int_y 2(z-y) \cdot P(y|x)dy = 0 \\
& \int_y zP(y|x)dy = \int_y yP(y|x)dy \\
& z = \mathbb{E}[y|x]
\end{aligned}
$$

$$\therefore f(x) = z = \mathbb{E}[y|x] \text{ is the optimal function}$$

Question 2:

a) Summary of methods and results

$$
\begin{aligned}
N &= \text{ Number of datums} \\
d &= \text{ Number of features} \\
\alpha &= \text{ Learning rate} \\
p &= \text{ Percentage of current split}
\end{aligned}
$$

Logistic Regression

LMS update using batch GD was used for logisitic regression:

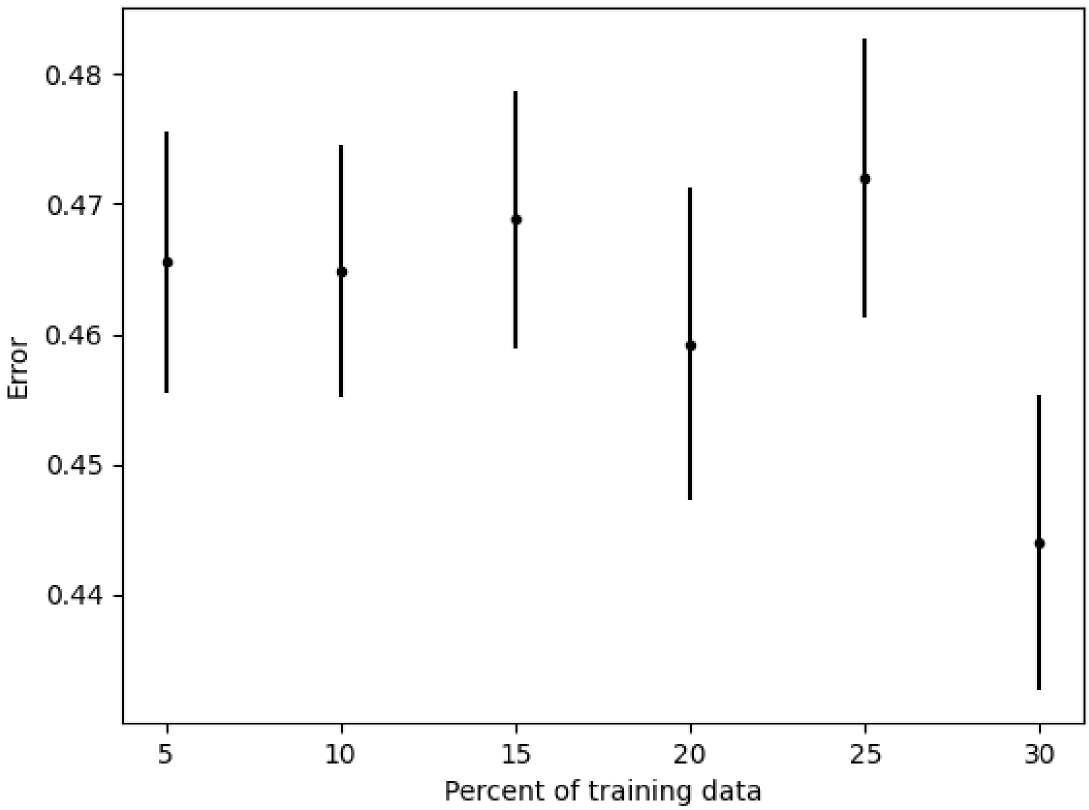
$$
\underline{\theta}^{(k+1)} = \underline{\theta}^{(k)} - \frac{\alpha}{Np} \sum_{n=1}^{Np} \nabla J_n(\underline{\theta}) \qquad \underline{\theta} \in \mathbb{R}^d
$$

$$
\nabla J_n(\underline{\theta}) = \begin{bmatrix} \frac{\delta}{\delta \underline{\theta}_0} J(\underline{\theta}_0) \\ \frac{\delta}{\delta \underline{\theta}_1} J(\underline{\theta}_1) \\ \vdots \\ \frac{\delta}{\delta \underline{\theta}_d} J(\underline{\theta}_d) \end{bmatrix}
$$

$$
\frac{\delta}{\delta \underline{\theta}_i} J(\underline{\theta}_i) = (\sigma(\underline{\theta}_i^T \underline{X}_n) - y_n) \underline{X}_n
$$

The results from the testing data with parameters

```
num_splits = 100
train_percent = [5, 10, 15, 20, 25, 30]
epochs/iterations = 350
```



Conclusion: the dataset cannot be represented better by the linear logistic model. Kernel method may produce better results.

Naive Bayers

$$P(y|\underline{x}) \propto P(\underline{x}|y)P(y)$$

Under the assumption of Naive Bayes where all features in $\underline{x}$ are i.i.d.:

$$P(\underline{x}|y) = \prod_{i=1}^{d} \frac{1}{\sqrt{(2\pi)\sigma_{i,y}^2}} \exp\left(-\frac{(x_i - \mu_{i,y})^2}{2\sigma_{i,y}^2}\right) \qquad P(y) = \frac{\sum_{n=1}^{Np} 1\{y_n = 1\}}{Np}$$

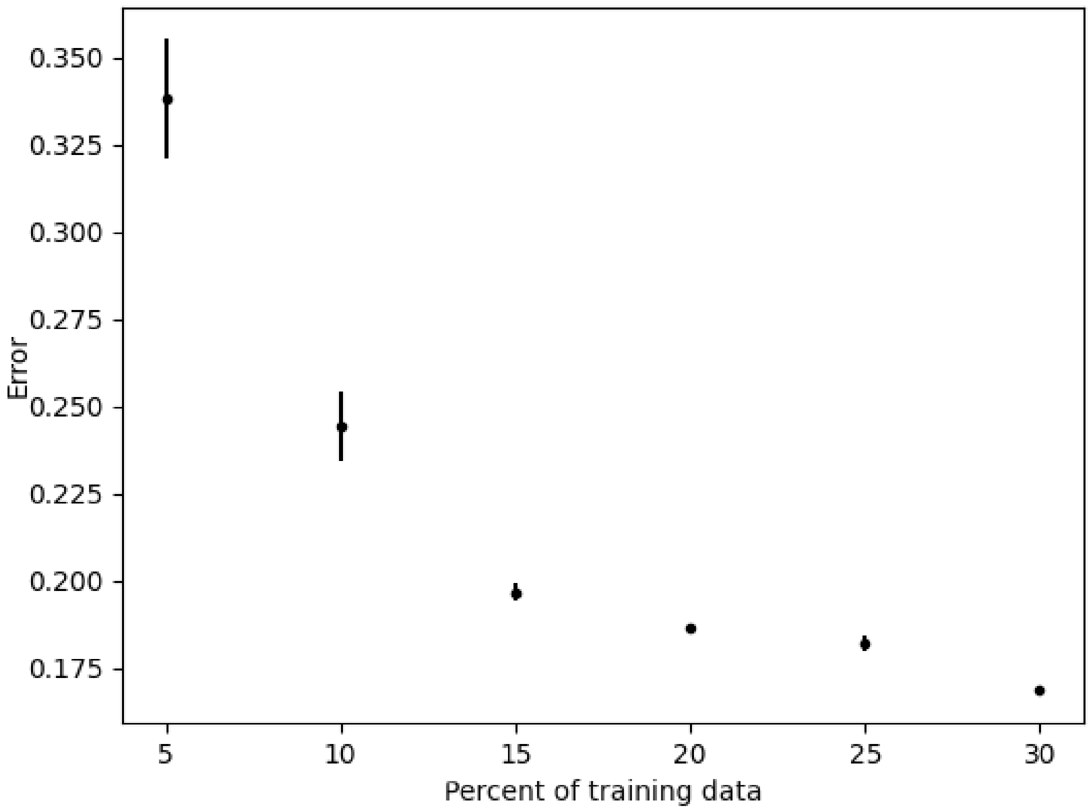
when $\sigma_y = 0$, it is set to 0.00001 to avoid divide-by-zero errors.

To calculate the mean and variance for each feature:

$$\underline{\mu_y} = \begin{bmatrix} \mu_{1,y} \\ \mu_{2,y} \\ \vdots \\ \mu_{d,y} \end{bmatrix} = \frac{\sum_{n=1}^{Np} 1\{y_n = y\}\underline{X}_n}{\sum_{n=1}^{Np} 1\{y_n = y\}}$$

$$\underline{\sigma_y} = \begin{bmatrix} \sigma_{1,y} \\ \sigma_{2,y} \\ \vdots \\ \sigma_{d,y} \end{bmatrix} = \frac{\sum_{n=1}^{Np} 1\{y_n = y\}(\underline{X}_n - \underline{\mu}_y)^2}{\sum_{n=1}^{Np} 1\{y_n = y\}}$$

The results from the testing data:



Code:

The code can be found on Github: https://github.com/DeBestTrap/Intro-to-Machine-Learning/tree/main/HW3